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PHILOSOPHICAL TRANSACTIONS

OF THE

ROYAL SOCIETY

OF

LONDON.

FOR THE YEAR MDCCCLXXXII.

VOL. 173.

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OCT 17 1883
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MDCCCLXXXIII.

ADJUDICATION of the MEDALS of the ROYAL SOCIETY for the year 1882, by the PRESIDENT and COUNCIL.

The COPLEY MEDAL to Professor ARTHUR CAYLEY, F.R.S., for his numerous profound and comprehensive researches in Pure Mathematics.

A ROYAL MEDAL to Professor WILLIAM HENRY FLOWER, F.R.S., for his valuable contributions to the morphology and classification of the Mammalia, and to Anthropology.

A ROYAL MEDAL to Lord RAYLEIGH, F.R.S., for his various Papers in Mathematical and Experimental Physics.

The RUMFORD MEDAL to Captain WILLIAM DE WIVELESLIE ABNEY, F.R.S., for his Photographic Researches and his discovery of the method of photographing the less refrangible part of the spectrum, especially the infra-red region, also for his Researches on the absorption of various compound bodies in this part of the spectrum.

The DAVY MEDAL to D. MENDELEJEFF and LOTHAR MEYER, for their discovery of the periodic relations of the atomic weights.

The Bakerian Lecture "On the Chemical Theory of Gunpowder," was delivered by Dr. DEBUS, F.R.S.

The Paper "On the Rhythm of the Heart of the Frog, and on the nature of the Action of the Vagus Nerve," by Mr. W. H. GASKELL, was appointed as the Croonian Lecture.

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OF

LONDON.

FOR THE YEAR MDCCCLXXXII.

VOL. 173.—PART I.

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ADVERTISEMENT.

The Committee appointed by the *Royal Society* to direct the publication of the *Philosophical Transactions* take this opportunity to acquaint the public that it fully appears, as well from the Council-books and Journals of the Society as from repeated declarations which have been made in several former *Transactions*, that the printing of them was always, from time to time, the single act of the respective Secretaries till the Forty-seventh Volume; the Society, as a Body, never interesting themselves any further in their publication than by occasionally recommending the revival of them to some of their Secretaries, when, from the particular circumstances of their affairs, the *Transactions* had happened for any length of time to be intermitted. And this seems principally to have been done with a view to satisfy the public that their usual meetings were then continued, for the improvement of knowledge and benefit of mankind: the great ends of their first institution by the Royal Charters, and which they have ever since steadily pursued.

But the Society being of late years greatly enlarged, and their communications more numerous, it was thought advisable that a Committee of their members should be appointed to reconsider the papers read before them, and select out of them such as they should judge most proper for publication in the future *Transactions;* which was accordingly done upon the 26th of March, 1752. And the grounds of their choice are, and will continue to be, the importance and singularity of the subjects, or the advantageous manner of treating them; without pretending to answer for the certainty of the facts, or propriety of the reasonings contained in the several papers so published, which must still rest on the credit or judgment of their respective authors.

It is likewise necessary on this occasion to remark, that it is an established rule of the Society, to which they will always adhere, never to give their opinion, as a Body, upon a subject, either of Nature or Art, that comes before them. And therefore the thanks, which are frequently proposed from the Chair, to be given to the authors of such papers as are read at their accustomed meetings, or to the persons through whose hands they received them, are to be considered in no other light than as a matter of civility, in return for the respect shown to the Society by those communications. The like also is to be said with regard to the several projects, inventions, and curiosities of various kinds, which are often exhibited to the Society; the authors whereof, or those who exhibit them, frequently take the liberty to report, and even to certify in the public newspapers, that they have met with the highest applause and approbation. And therefore it is hoped that no regard will hereafter be paid to such reports and public notices; which in some instances have been too lightly credited, to the dishonour of the Society.

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CORRECTION TO MR. G. H. DARWIN'S PAPER IN THE PHILOSOPHICAL TRANSACTIONS FOR 1880, P. 713.

For " autumnal equinox" *read throughout* " vernal equinox."

N.B.—This correction does not affect the results of the analysis.

PHILOSOPHICAL TRANSACTIONS.

I. *On the Electromotive Properties of the Leaf of Dionæa in the Excited and Unexcited States.*

By J. Burdon-Sanderson, *M.D., F.R.S., Professor of Physiology in University College, London.*

Received October 27,—Read December 15, 1881.

In 1873 I communicated to the Society the results of experiments showing that in the leaf of Dionæa, as in the excitable organs of animals, the change of form which is the visible effect of mechanical or electrical excitation, is preceded by an electrical change of a nature similar to that which occurs, under corresponding conditions, in animal muscle. In a second paper, which was read on the 14th December, 1876, I submitted to the Society, in conjunction with Mr. Page, an account of a number of experiments, made, for the most part in the Laboratory of the Royal Gardens, Kew, with the aid of Lippmann's electrometer, for the purpose of elucidating the relation between the electrical phenomena and the physiological process which they accompany. In that paper we confined ourselves to the mere relation of our observations, recognising their incompleteness, but hoping that the work of future seasons might yield better fruits. Although each year since 1876 some additions have been made to our knowledge, it has not seemed desirable to publish anything until the present occasion. If the accumulated products of so many years appear insignificant, I must ask that it may be remembered that the investigation is one of great difficulty, and that the time during each summer which is available for experiment is very short.

As almost all the experiments on which the conclusions of the present paper are founded have been of such a nature as to require the co-operation of two persons, it is obvious that the work could not have been accomplished without the aid of a practised observer. This aid I have received from Mr. PAGE, to whose technical skill and ingenuity in overcoming difficulties, whatever success may have been attained in the experimental part of the enquiry, is largely due.

I have to express my great indebtedness to the Director of Kew Gardens, Sir JOSEPH HOOKER, and to the Assistant Director, Professor THISELTON DYER, for the invariable kindness with which they have placed the resources of the Royal Gardens at my disposal.

Since 1873 several researches have been published in Germany relating to the excitatory process in Dionæa, or more generally to plant excitability. In 1876 an elaborate work appeared in Berlin by Professor MUNK, of which the aim was to acquire "a more exact knowledge of the leaf of Dionæa in its electrical relations;" and in addition two important experimental papers by Dr. KUNKEL relating chiefly to *Mimosa* have been lately published in vol. xxv. of PFLÜGER'S Archiv, under the title "Electrical Researches Relating to Animal and Vegetable Structures." For the purpose of deriving from them valuable aid, no less than for that of criticising the observations and conclusions of their authors, it will be necessary to examine these papers carefully. I therefore propose to devote an introductory section to this object.

PART I.

EXAMINATION OF THE RECENT RESEARCHES OF PROFESSOR MUNK AND OF DR. KUNKEL ON PLANT ELECTRICITY.

1. *Summary of Professor* MUNK'S *Paper.**—The introduction (§ 1) begins by referring to the discovery, made by myself in 1873, that the leaf of Dionæa is endowed with electromotive activity resembling that of muscle, and that when the leaf contracts a negative variation occurs. He states that his attention was first directed to Dionæa in November, 1873, and that, in consequence, he made experiments in 1874 and 1875, using about 100 leaves, none of which were detached from the plants to which they belonged.

§ 2 relates to the distribution of electrical tension on the surface of the leaf. After confirming the observations recorded in my first communication to the Royal Society, that in general when the Dionæa leaf is led off by the opposite ends of the outer surface of the midrib, the end furthest from the leaf stalk (distal) is found to be positive to the other end†, he states (1) that the most positive point is not the distal end of the midrib, but the point corresponding to the junction of its middle with its distal third, (p. 37) to which point, though both ends are negative, the petiole end is twice as much so as the distal end. (2) That in any line drawn across the midrib on the outer (under) surface of the leaf from side to side, a point can be found on either side which is negative to all other points, the negative point being situated nearer to the border of the leaf than to the midrib (p. 38), and finally that the distribution of tension is the same on the under surface as the upper (p. 42) and that opposite points are always equipotential.* In § 3 the difference of potential between the most negative and the most positive points of the outer (under) surface of the leaf is found to be 0·04 to 0·05 DANIELL. In § 4 the question of the seat of the electromotive forces on which this and the other observed differences are dependent is discussed.

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* Die elektrischen und Bewegungs-Erscheinungen am Blatte der Dionæa Muscipula, von Dr. HERMANN MUNK, a. o. Professor an der Universität zu Berlin. Leipzig, 1876. Aus REICHERT'S und DU BOIS-REYMOND'S Archiv für Anatomie, &c., 1876, besonders abgedruckt.

† "Herrn SANDERSON'S Angabe ist leicht zu bestätigen: man findet in der That regelmässig zwischen jenen Blattenden, nach unserer Bezeichnungsweise, einen aufsteigenden Strom."

From preliminary considerations he concludes that inasmuch as it is out of the question "das Blatt zu zerstückeln" so as to investigate the electromotive properties of its parts separately, the only way is to start from the well grounded assumption that wherever the same organisation exists there will be similar arrangement of electromotive forces, and accordingly that in the individual cell the electromotive force will show a distribution which has a definite relation to its long axis. Hence, inasmuch as the form of the cell admits of but few possibilities, we may, by ascertaining which of these possibilities when taken in relation with the known arrangement of the cells in the leaf, affords the best explanation of what is actually observed in the distribution of electrical tension on the surface of the leaf, select that which accords best with fact. In order to get at this result Professor MUNK employs a method of investigation which I shall not endeavour to describe in detail, but content myself with stating that it is founded on the assumption that it is possible to represent the living cell, as regards its electromotive properties, by a zinc cylinder having a zone of copper and surrounded by a uniform layer of a moist conductor;† and that a schema or model made up of a number of such cylinders arranged like the cells of any living part of a plant would exhibit electromotive properties similar to those of the organ it represented. Stated as shortly as possible, the actual arrangement of the parenchyma cells of the leaf of Dionæa is as follows: in the lobes the cells run all in one direction, namely, parallel to the veining and at right angles to the midrib. In the midrib the cells of the ridge (under surface) run longitudinally, but those of the trough (upper surface) are continuous with those of the cells of the lobe. For reasons which need not be here entered on (see p. 86) Professor MUNK considers that the arrangement which determines the electromotive properties of the leaf, is that of the cells of the lobe-parenchyma of which the long axes are parallel to the surface of the lobe and at right angles to the midrib. Accordingly, the leaf ought, on the assumption that the cells have the properties assigned to them (*i.e.*, that the middle of each cell is negative to its ends), to be represented by a bilateral schema consisting of two symmetrical parts, resembling in form the two lobes of the leaf, and each made up of metallic cylinders arranged like the cells in the leaf, as above described. A full account of the properties actually demonstrable in such an arrangement is given in the paper (pp. 54 and 55). It corresponds in the most important respects, as regards the distribution of electrical tension, with Professor MUNK'S account of the electrical properties of the resting leaf.

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* "An der oberen Blattfläche die gleiche Vertheilung der Spannungen herrscht, wie an der unteren Blattfläche" (p. 42). "Dass die absolute Grösse der Spannungen an der oberen und an der unteren Blattfläche die gleiche ist" (p.).

† "Die einzelne Parenchymzelle entspricht hinsichts ihrer Kräfte dem einzelnen Metallcylinder" (p. 96). DU BOIS-REYMOND's schema of an electromotive element of muscle is a copper cylinder with a zinc zone surrounded by a uniform layer of a moist conductor. For the plant electromotive element, according to MUNK, the sign must be reversed. The cylinder must be zinc and the zone copper.

§ 5 relates to the "Mechanism of the Excitatory Motion." On this subject it will be sufficient to state that the account given by Professor MUNK of the way in which the leaf closes when touched, agrees with the explanation originally given by BRÜCKE of the mechanism of the motion of the sensitive plant; namely, that the immediate cause of the change of form is the sudden passing of certain cells (in the case of Dionæa those of the upper (inner) layer of the lamina and midrib) from the state of distension into that of flaccidity; but, in addition to this, there is, he says, an active lengthening and extension of the cells of the lower or external layer.* From the context it appears that all that is here meant is that the water discharged in the sudden relaxation of the cells of the inner (upper) stratum, finds its way at once to the outer (lower). At all events, the only evidence given in support of active extension is the fact, proved by measurement, that two marks on the external surface of the expanded leaf, in a line parallel to the veining and at a measured distance from each other, are found to be further apart after the leaf has closed (p. 117).

In § 6 (p. 123) the author approaches the question which more immediately concerns our present purpose, viz.: the electrical phenomena which result from excitation. I will endeavour to give an account, first of the experiments, and secondly of the theoretical explanations—premising that, in this as in the section on the electromotive properties of the unexcited leaf, observations and theories are so mixed that it requires much patience to separate them.

The mode of experiment was always the same. The leaf was led off, either by different points on the under surface of the midrib, as in my original experiments in 1873, or by the midrib and by a spot on the external surface opposite to it. The leaf was not fixed or restrained, so that only a very few excitations could be observed before closure.

All of the experimental results are stated in the following table, in the first column of which I have entered the leading off contacts, distinguishing them by the letters *A* and *B*, in the second column whatever information is given as to the difference of potential between them before excitation. The third column gives the character of the excitatory effect as observed by the galvanometer; and the fourth, the reference to the page where the experiment is recorded.

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* "Die Bewegung kommt dadurch zu Stande dass das reizbare Parenchym erschlafft und kürzer wird, das nicht reizbare sich activ verlängert" (p. 119).

TABULAR Summary of the Results of Professor MUNK's Experiments.

	Contacts.		Electrical condition before excitation.	Excitatory effect.	Reference.	Remarks.
	A.	*B.*				
I.	Distal end of midrib.	Proximal end	Aufsteigender Strom, *i.e.* *A* 30 scale negative to *B*.	"Negativer Vorschlag," *i.e.* diminution of previous deflection, followed by "positive Schwankung," *i.e.*, increase of previous deflection.	P. 128 . . .	Electromotive force of the "positive Schwankung" said to be about equal to that of "aufsteigender Strom."
II.	Distal half .	Proximal end	Not stated; in general, the difference of potential said to be twice as great as in the previous case. (P. 38, line 7.)	"Neg. Vorschlag" so strong as to reverse the previous deflection, and to exceed the Schwankung several times in extent.	P. 131 . . .	Relation of Vorschlag and Schwankung (Doppelschwankung) to previous deflection inconstant.
III.	Distal half .	Proximal half	Contacts isoelectrical .	First effect "aufsteigend," *i.e.* in direction of the Schwankung, followed by slow swing in opposite direction, and return to zero.	Pp. 131, 132.	
IV.	Under surface of midrib at junction of middle and distal third.	Outer concave surface of lobe opposite *A*, and at most negative point.	*B* negative to *A*. *Difference* amounts to 0·05 DANIELL.	Vorschlag of 2 to 3 scale, in which *B* becomes less negative, followed by very large deflection in the opposite direction.	P. 145 . . .	In this experiment the leaf is led off by the two points of which the potentials differ most.

Professor MUNK's explanation of the *excitatory variation* is, of course, in conformity with his doctrine of the electromotive activity of the individual cell, which has been already explained ; namely, that each parenchyma cell has the endowment of a cylindrical electromotive element of which the poles are positive to the middle,* and the excitatory change produced in each cell consists in increase or diminution of the electromotive force of each cell. If the variation consisted of a single phase, it might be accounted for by supposing that all the cells of the parenchyma of the lobes are affected by excitation in the same way. As, however, it presents the characters of a "*Doppelschwankung*"—*i.e.*, consists of two phases succeeding each other, of opposite signs, this must be accounted for either by supposing (1) that all the electromotive elements in the structure are similarly affected, but not at the same time, the excitatory change being propagated from the seat of excitation in such a way that it first affects the electromotive elements at one contact, then those at the other ; (2) that all the electromotive elements are similarly affected at the same time, changes of opposite signs following each other in each element simultaneously ; (3) that there exist in the structure two sets of electromotive elements, which are affected in opposite directions by excitation, the disturbance reaching its maximum in the one set later than in the other set.

Of these three alternatives only the third appears to Professor MUNK admissible. The first he rejects on the ground that if it were true the character of the excitatory variation would be affected by the seat of excitation. This he finds not to be the case. Thus in Exp. II. (see Table) the effect should be different according as the leaf is excited by touching the proximal or distal hairs. According to Professor MUNK there is no difference (p. 136) either in this or other similar cases. He therefore concludes that the "*Doppelschwankung*" is not due to propagation of the excitatory effect from the seat of excitation. "Danach kann weder die complicirte Schwankung in irgend einer Beziehung zum Orte der Reizung stehen, noch kann überhaupt von diesem Orte der Erfolg der Reizung, innerhalb der Genauigkeitsgrenzen unserer Untersuchung, irgendwie abhängig sein" (p. 138). The second alternative is rejected on the ground that in IV. (see Table) the Vorschlag, if it occurs at all is very inconsiderable, and is very frequently absent, consisting of one phase only, which could not be the case if each cell underwent first a diminution then an increase of its electromotive force, and all acted simultaneously. The third alternative, thus adopted by exclusion, viz. : that the two sets of electromotive elements are oppositely affected by excitation, serves, Professor MUNK thinks, to explain the characters of the excitatory effect in all the cases observed by him.

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* "Die ohngefähr cylindrischen Zellen des Blattflügel-Parenchyms und der beiden Mittelrippen-Parenchyme sind mit Kräften ausgestattet der Art, dass die positive Elektricität von der Mitte der Zelle nach jedem der beiden Pole hingetrieben wird, die Pole positiv sind gegen die Mitte" (p. 97).

Professor Munk's paper does not contain any statement of his conclusions. For further reference I have arranged them as follows, in the order in which they have been referred to in the preceding sketch:—

1. Each individual parenchyma cell of the leaf corresponds, in respect of its electromotive properties, with a zinc cylinder zoned with copper and surrounded by a uniform layer of moist conductor. In other words, the nearly cylindrical cells are endowed with electromotive forces such, that positive electricity is driven from the middle of each cell towards each of its poles, *i.e.*, the middle is negative to the poles (p. 97).

2. The electromotive properties of the whole leaf (pp. 47–51) in the unexcited state are represented by those of a schema (see diagram, fig. 20, *a*), consisting of such cylinders arranged in the same order as its cells (p. 87).

3. On excitation, the cells of the upper layer of the leaf undergo a diminution of electromotive force, those of the under layer an increase, *i.e.*, the middles of the cells become less negative than before in the upper layer, more negative in the lower layer (p. 152).

4. The diphasic character of the variation is not due to the fact that the individual cells undergo opposite changes in succession, but to the interference of the opposite electromotive actions of the upper and under cells, of which the time relations are different (see diagrams 29 and 30 and p. 142).

5. The effect of an excitation is in no way dependent on its seat (p. 138).

6. The electromotive activity of the cells has no relation to their water-content (p. 158).*

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* The author's criticisms of my methods and conclusions will be best considered in connexion with the several subjects to which they refer. His paper contains some misconceptions which it will be convenient to correct in this place. One of the most important, as having led other writers into error, is founded on a casual expression of mine contained in a lecture at the Royal Institution in 1874, to the effect that the excitatory motion of the leaf is identical with muscular motion. No such expression is contained either in my communication to the Royal Society, or in the translation of it which appeared shortly afterwards in the 'Centralblatt.' ("Ueber electrische Vorgänge im Blatte der *Dionœa muscipula.*" 'Centralblatt f. d. Med. Wissensch.,' 1873, No. 53.) If Professor Munk had considered the context, he would have seen that in my lecture I meant nothing more than that both motions are expressions of the same elementary endowment of protoplasm—that of changing its form on excitation. I held then, and hold now, that, as regards what happens in the living protoplasm, the mechanical effects of excitation in the contractile parts of plants and animals are closely related. As to the mechanism by which the changes of form of the excitable organs of plants are brought about, nothing was said in my paper. I took for granted the explanation usually given, founded on Brücke's experiments. I had not become acquainted with the then recently-published researches of Pfeffer on the excitability of plants ('Physiologische Untersuchungen,' von Dr. W. Pfeffer. Leipzig: 1873).

As regards the original experiments by which I demonstrated the fact that the leaf of Dionæa responds to excitation by an electrical disturbance analogous to the excitatory variation of muscle, Professor Munk's

The researches of Kunkel,* like those of Munk, have been directed to two questions, viz. : to that of the differences of electric tension which present themselves in the uninjured leaf, and to the electromotive phenomena which follow excitation in the excitable organs of plants. As regards the first question, he has found that in leaves generally, when they are led off by non-polarizable electrodes in the usual way, differences of tension present themselves, these being always of such a nature that if one electrode is in contact with a vein, the other with the green surface between the veins, the former is positive to the latter (p. 359). But he considers this due, not to any real difference of tension between the surfaces, but exclusively to inhibition changes which begin at the moment that contact is made. As evidence that the relations of the two surfaces to water are different, he adduces the following observations.

That diffusion actually takes place at the uninjured surfaces of leaves which are in contact with water, he proves by the observation that potassium salts may be detected

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observations are, happily, in accordance with mine, as may be seen by comparing his account of the phenomena with mine:—

Sanderson, 1873.	Munk, 1876.
Wenn das Blatt so auf die Elektroden aufgelegt wird dass der normale Strom des Blattes durch eine Ablenkung der Nadel nach links angezeigt wird und die sensitiven Haare der oberen Fläche berührt werden, so schwingt die Nadel nach rechts. . . . In jedem Falle kommt die Nadel nach der negativen Schwankung in einer Stellung zur Ruhe, die weiter nach links gelegen ist als zuvor, und nimmt dann allmählig ihre frühere Stellung wieder ein. (Centralblatt, 1873, p. 835.)	Reizen wir unser Blatt bei Ableitung von den beiden Enden der Mittelrippe, so beobachten wir Folgendes: (Es sei z. B. eine Ablenkung von 30 Sc. vorhanden.) Zuerst nach der Reizung nimmt die Ablenkung sehr rasch auf 25–30 Sc. ab, und darauf nimmt sie sogleich etwas langsamer, doch immer sehr rasch auf 40–60 Sc. zu. Nun verweilt der Spiegel nur momentan auf dem Maximum der Ablenkung und kehrt dann langsam, in etwa einer Minute, zu seiner Anfangsstellung zurück (pp. 127, 128).

Notwithstanding this close correspondence, Professor Munk speaks of my description as grossly defective, "not merely in details, but in what is essential and important" (p. 124); and proceeds, that for aught I knew to the contrary, my "*negative Schwankung*" might be nothing more than a result of shifting of contacts. The serious defect, however, was not so much incompetence as an observer as inability to understand what I saw; for, referring to the very same observation further on, he says that "Herr S. den Erfolg der Reizung richtig beobachtet hat und nur seine Beobachtung durchaus missverstanden hat, indem er . . . eine negative Schwankung da zu sehen vermeinte, wo in der Wirklichkeit eine positive Schwankung mit negativem Vorschlage sich zeigte." On another of my observations (see p. 152), which he had not time to repeat, but which he thinks may be relied on though it was made "natürlich ohne von seiner Bedeutung eine Ahnung zu haben," he finds an additional prop for his own theories. In conclusion, Professor Munk congratulates himself on having substituted "an die Stelle der Spurweisen und vielfach unzutreffenden ersten Wahrnehmungen von Herrn Sanderson eine genauere Kenntniss des Dionæa-Blattes in elektrischer Beziehung."

* Kunkel. "Electrische Untersuchungen an pflanzlichen und thierischen Gebilden," Pflüger's Archiv, Bd. xxv.

in water in which such leaves have been immersed even after a few minutes' immersion. He further finds that if a freshly cut off leaf is immersed in water, the whole of the vein surface is wetted at once. The green surface of the lamina exhibits a beautiful silvery lustre indicating that air adheres to it, and that water comes very slowly and incompletely into contact with it. When the leaf is removed from the water, the veins remain permanently and completely wet, but the water runs off the rest of the surface, leaving it dry. Consequently he concludes that when moist electrodes are brought into contact with surfaces which thus differ, the diffusion processes which result from the contact must go on at different rates.

By other experiments, physical rather than physiological, he has shown that in all bodies capable of imbibition with water, electrical changes are associated with the process of imbibition. His first experiments on the subject were made with the ordinary porous cells of batteries. Similar results were obtained by the investigation of non-living vegetable structures, as *e.g.*, laminaria stems. By comparing these facts with those already stated as to the electrical phenomena observed in the investigation of leaves, he derived the conclusion that whatever differences present themselves between the different parts of leaves are due to "*Unterschied in der Benetzbarkeit*" of the surfaces led off. What the direction of this effect is, KUNKEL had determined by experiment. If a drop of water is placed on the green surface of a leaf between the veins and allowed to remain there some time and the leaf is now led off by another similar surface and by the drop, it is found that the surface first wetted is always positive to the other (p. 368). If after one electrode has been brought into contact with an ordinary surface, the other is applied to a vein, the direction of the difference is opposed to the normal. The vein is at first negative but the difference quickly diminishes and is soon reversed (p. 360). In other words, when a leaf or other similar structure is led off by two surfaces of which one is more "wettable" than the other, the former is positive to the latter, provided that both contacts have been made simultaneously.

From these observations, of which the value and interest must be recognised as showing what sort of results may arise from imbibition processes taking place at surfaces of contact, the author proceeds to draw inferences as to the nature of excitatory electromotive changes in the organs of plants, confining his attention to *Mimosa.* The excitatory effect in the swelling at the base of the leaf stalk of *Mimosa,* is described by KUNKEL, on the basis of observations made with the capillary electrometer, the organ being led off by thread electrodes applied to its opposite ends as follows. It is said to consist of a "*Vorschlag,*" in which the base becomes negative, followed by a larger excursion in the opposite direction. KUNKEL thinks that this observation is in harmony with those of MUNK. Three fragmentary experiments are given in confirmation of this, on which the theory is founded, that inasmuch as in *Mimosa* the active motions of the leaf are caused by a sudden alteration in the distribution of water in the different parts of the swelling at the base of the leaf stalk, that these "*Wasser-*

verschiebungen" are the causes of the electrical changes observed, the proof being, that in general "*Wasserverschiebungen in imbibirten Körpern*" are attended with electrical phenomena. He confesses himself unable to connect the particular phenomena observed with "*einzelne Phasen prevalirender Wasserverschiebungen.*" He thinks, however, that the first "*Vorschlag*" is not due to *Wasserverschiebung*, but to the fact that the diffusion processes which are called into existence by the application of the electrodes are disturbed by alterations in the protoplasm. It is to be noticed that these explanations, which are contained in the concluding paragraph of the paper, are not in accordance with the more general statement on p. 371. In one place it is stated that "all electrical phenomena are conditioned by *Wasserverschiebung;*" in the other, that the *Vorschlag* in *Mimosa* "cannot be referred to *Wasserverschiebung*, but to the alteration of protoplasm which precedes it." The difference is a fundamental one.

From the preceding pages it will be seen that the two observers who have investigated the electromotive phenomena which are associated with the excitatory process, viz.: Professor MUNK and Professor KUNKEL, have arrived at opposite conclusions as to their nature. According to MUNK, the electromotive activity of the plant cell stands "in no direct relation to the water it contains" (p. 158). It is KUNKEL'S main conclusion that they are determined by diffusion processes. MUNK gives as the result of his observations a complicated map of the distribution of electrical tension on the surface of the leaf of Dionæa (p. 40, figs. 14 and 15). KUNKEL starts from the proposition (which he does not stop to demonstrate) that electrical differences do not exist at all in uninjured and untouched leaves (p. 372). According to MUNK the electromotive properties of the leaf are referable to electromotors which resemble the electromotive molecules of DU BOIS-REYMOND, with the exception that their signs are reversed (p. 51). This hypothesis KUNKEL only mentions for the purpose of rejecting it (p. 371). There is, in short, but one respect in which they agree. Their descriptions of the phenomena are sufficiently explicit to make it pretty certain that they have both witnessed the same process. As regards the physiological nature of the phenomena the contradiction is complete.

PART II.

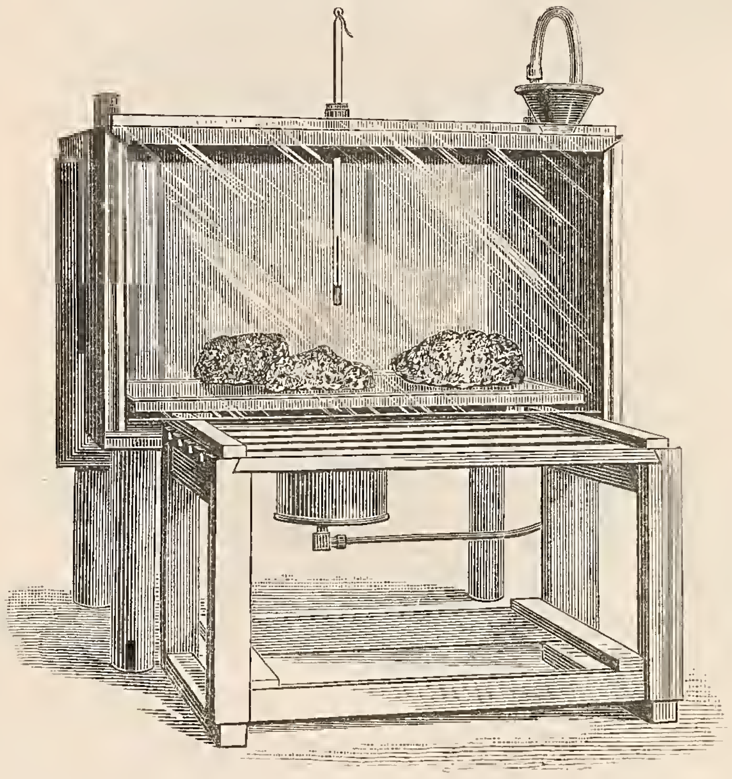
DESCRIPTION OF THE METHODS AND INSTRUMENTS EMPLOYED.

The methods used in the study of the electromotive properties of an excitable living structure, whether of plant or animal, may be grouped under four heads according as they are adapted (A) for maintaining, during the period of observation, the external conditions of temperature, moisture, &c., which are most conducive to vigorous life; (B) for the investigation of the differences of tension (electrical potential) which exist *at one and the same time* between different parts of the surface of the organ; (C) for investigating the *successive* states (tension differences) which present themselves after excitation, with reference to their duration and their time-relations with other phenomena or processes which precede or accompany them; or (D) for exciting, or otherwise temporarily modifying the physiological conditions of the organ.

The appliances used for these several purposes in the case of the leaf of Dionæa are as follows:—

A. *Means of maintaining favourable conditions of temperature, &c., during the period of observation.*—By preliminary experiments it was ascertained that the conditions most favourable to the activity of the leaf are, that the temperature of the leaf should be from 32° C. to 35° C. and that the air should be nearly saturated. For the maintenance of these conditions the plant is placed in a chamber of the following construction (fig. 1).

Fig. 1.



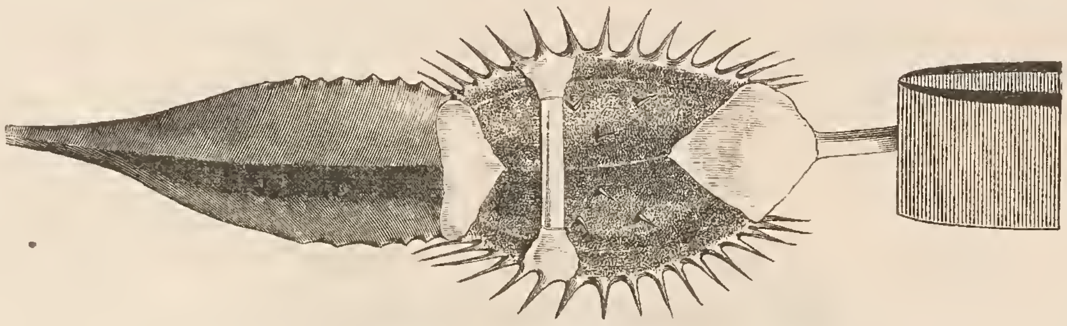
The Warm Chamber.

The sponges which serve to keep the air saturated rest on the glass plate which covers the sliding floor. On this glass plate the plant, electrode holders, &c., rest. To withdraw them, the floor is made to slide out along the rails. The upper part of the tube of the regulator is seen on the right side. (From a photograph.)

The chamber measures 47 centims. wide and 40 centims. from back to front, and is 26 centims. high. It is made of japanned tin plate. Its top, bottom, and sides are double, and separated from each other by a closed cavity of $1\frac{1}{4}$ centims. in width, which is filled with water. Its front and back are closed by panes of glass which slide up and down in grooves. The water contained in the bottom, sides, and top is heated by an ordinary gas burner, 3 inches below the bottom, along which, and up one side a chimney, $2\frac{1}{2}$ centims. by $7\frac{1}{4}$, leads. The temperature of the water is governed by an automatic regulator of the form described by Mr. PAGE in the Journal of the Chemical Society.* The chamber is provided with a sliding floor of wood, covered with glass, which serves to support the apparatus. The wires leading to the electrodes, &c., enter by openings left on either side of the floor for the purpose.

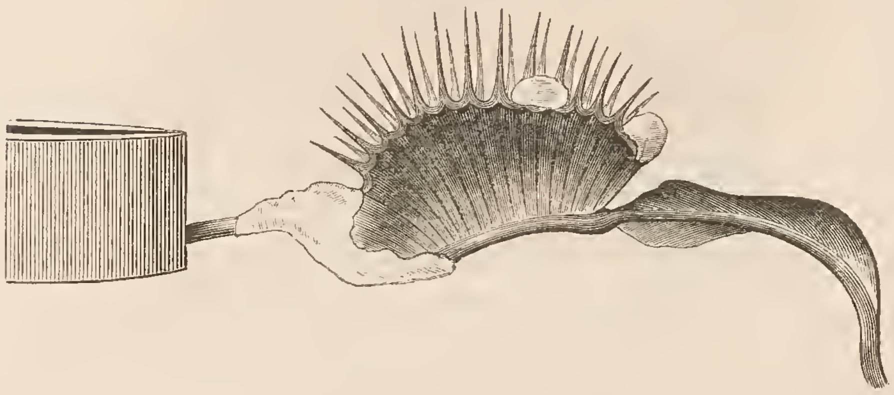
Mode of preparing the leaf for observation.—Whether the leaf is on the plant or not, it is necessary to fix it in such a way as to render it motionless. In our earlier experiments glass forceps were used, ingeniously contrived and constructed by Mr. PAGE, of which the blades were of such form that while one of them applied itself to the upper surface of the leaf when in the fully expanded state, the lower supported the midrib, for the reception of which it was suitably grooved. A figure of this instrument was given in our former paper.† We have now adopted a much better mode of immobilization which will be understood from the figure. Before otherwise interfering with the leaf, plaster of Paris, which has just been mixed with water, is introduced between the lobes at either end of the midrib, so as to bridge the space between them. A splinter of dry wood is laid across from edge to edge of the lobes, the ends of which are cemented by plaster to the marginal hairs. The leaf so prepared is unable to close (figs. 2, 3).

Fig. 2.



The Leaf, seen from above.
The two end plugs and the cross-bar, which stretches from margin to margin, serve to prevent it from closing.

Fig. 3.



The same Leaf, seen from the side. (Both figures are copied from photographs.)

B. *The galvanometer* used is a THOMSON of 5244 ohms resistance. The degree of sensibility best adapted for the ordinary purposes of the investigation is such that with a thousandth of a DANIELL and a resistance of 10,000 ohms, added to that of the galvanometer, we have a deflection of 350 scale; but for some rheotome observations, a much higher sensibility is required. With a view to more accurate reading of small deflections, a concave mirror, 10 centims. wide, is placed in front of and below the scale, by means of which that part of the graduation nearest to zero can be seen enlarged.

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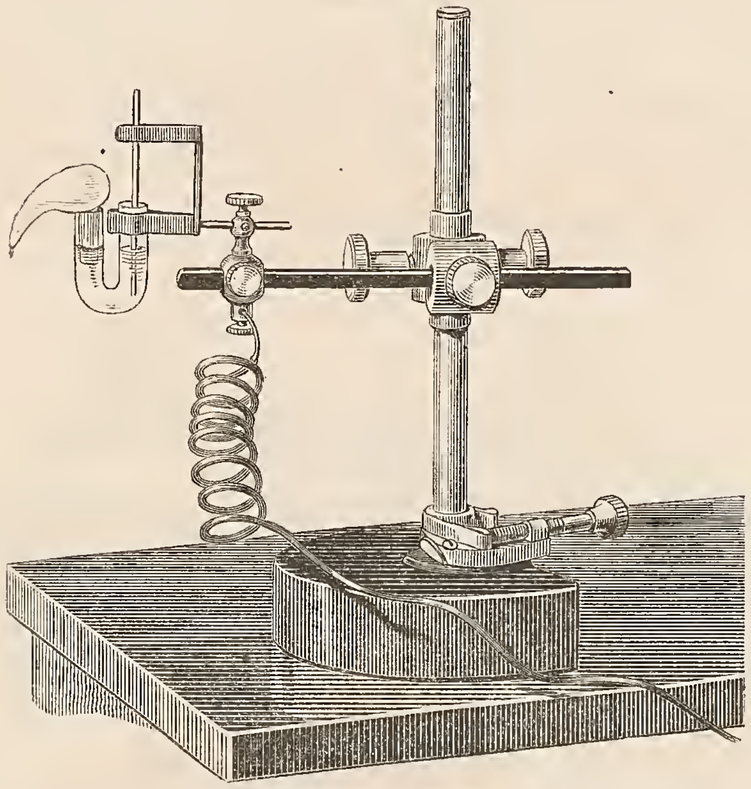
* Vol. i., 1876, p. 24.

† Proceedings, 1873, p. 413.

The electrodes.—The most important part of each electrode is a U-shaped tube containing saturated zinc sulphate, which is supported by a holder of the form shown in the figure (fig. 4). Into the end of the U contained in the clamp, sinks a zinc rod, which is held by the spring clip, and is thus brought into metallic connexion with the coiled wire. This rod is hammered smooth at the end which is intended to be immersed, polished, and amalgamated by treating it sparingly with very weak solution of mercury in nitrohydrochloric acid. Into the opposite limb of the U slips a narrow straight tube 1 centim. in length and about 5 millims. in internal width, which is charged with paste of kaolin and solution of zinc sulphate. The tube has a rim of sealing wax at the top, which prevents it from sinking too far, and is surrounded by a conical beak, shaped with the fingers out of modelling clay (kaolin made into a paste with 0·75 per cent. solution of chloride of sodium). In order to bring the modelling clay to a pencil-like point, a length of several strands of ligature thread steeped in the solution is incorporated with it. These electrodes have a resistance of about 6000 ohms and keep in order for any required time if the atmosphere is saturated or nearly so. The construction of the supports for the electrodes can be understood from the figure. It is sufficient to notice that the collar slides on the horizontal rod of ebonite. This rod can be raised or lowered, or made to rotate round the

Fig. 4.

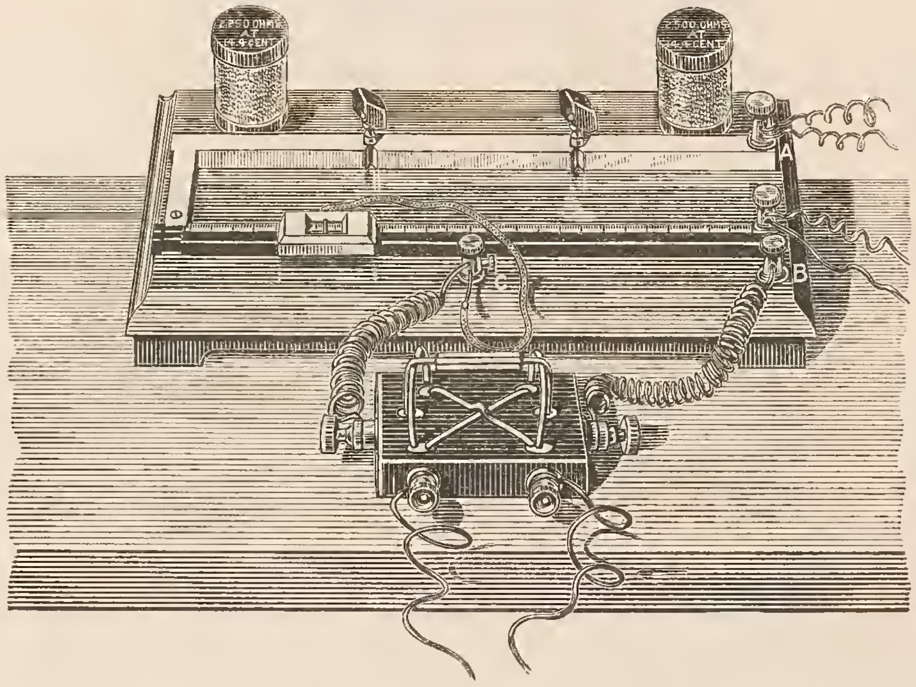


A Non-polarizable Electrode. (From a photograph.)

vertical axis, by working screws, of which the milled heads are conveniently placed for the purpose.

The compensator.—As is well known to every reader of physiological books, Professor du Bois-Reymond has applied the term to an instrument which consists essentially of a long thin wire connecting the poles of a standard cell. The wire being of uniform thickness, the difference of potential between any two points in its length is to the whole difference between the ends, as the distance between these points is to the total length. In a compensator so constructed the total difference has to be determined by a previous operation. The compensator we employ differs in no respect in principle from the potentiometer of Mr. Latimer Clarke* in which the total difference is equal to the whole electromotive force of the standard cell, and not, as in du Bois' compensator, to a certain portion of it. The form we have given to the instrument is very different from that of Mr. Clarke's instrument, and far more convenient for our purpose (fig. 5). The compensator wire, or rheochord, has a total resistance of 5 ohms. On a board, 27 centims. long, is fixed a rail of vulcanite 1 centim. broad and as much high, and 25 centims. long. Along the upper margin of this rail, which is divided into millimeters, runs a wire of platinum, of which the resistance is equal to a $\frac{1}{4}$ ohm. Each end terminates in a metal block. A second rail or bar of brass runs along the board parallel to the first. It is broken at two points for connexion with the end of the coils 1 and 2, of which the resistances are severally $2\frac{1}{4}$ and $2\frac{1}{2}$ ohms, so that when the plugs are out the total resistance between the ends of the bar is 4·75 ohms. The bar is united at one end with one of the terminals of the graduated wire. At the other it has a binding screw with two holes. The opposite terminal of the graduated wire has two binding screws, one with two, the

Fig. 5.



Compensator and Reverser. (From a photograph.)

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* "On a Voltaic Standard of Electromotive Force," Proceedings, 1872, p. 444.

other with one hole. On the vulcanite rail runs a brass sliding block or rider, the form of which is seen in fig. 5. It rests on the surface of the graduated wire by its cross wire, and is connected with the binding screw by a flexible wire. From the construction it is obvious that the difference of potential between the binding screws B and C, is to the total difference between A and B, as the length of wire, in millims. between B and the point at which the cross wire rests on the graduated wire is to 5000 millims. In other words, that each millim. of the graduation corresponds to $\frac{1}{5000}$th of the total difference. In use, A and B are connected by one pair of wires with the standard cell through a galvanometer, by the other pair with an "auxiliary battery" through a rheostat as described in Mr. LATIMER CLARKE's paper.

Measurement of electrical resistance.—It is of great importance to have the means of becoming promptly aware of the changes of resistance which occur in the course of a long observation. For this purpose we adopt the plan of from time to time measuring the deflection due to 10 millims. of wire = $\frac{1}{500}$ DANIELL. In investigating plant structures that method is pratically of great value, for the resistances we have to encounter vary, roughly speaking, between 50,000 and 100,000 ohms. As, therefore, we know approximately that our circuit, when the electrodes are in contact, is about 12,000 ohms, viz.: 6000 for the galvanometer and about as much for the electrodes, such changes as often occur are obvious enough. In the mode of leading off used in the fundamental experiment (see Part IV., p. 25) we obtain, if the contact is successfully made, with an over compensation of $\frac{1}{500}$ DANIELL a deflection of about a tenth of the deflection due to the same electromotive force when the electrodes are in contact with each other. If it is less than that, we regard the leading off as unsatisfactory. For actual measurement, we have at hand on an adjoining table a WHEATSTONE's bridge arrangement, which can be brought into connexion with our circuit without loss of time when required, by means of a switch which is substituted for the key which otherwise receives the wires from the leading off electrodes.*

Fig. 6.

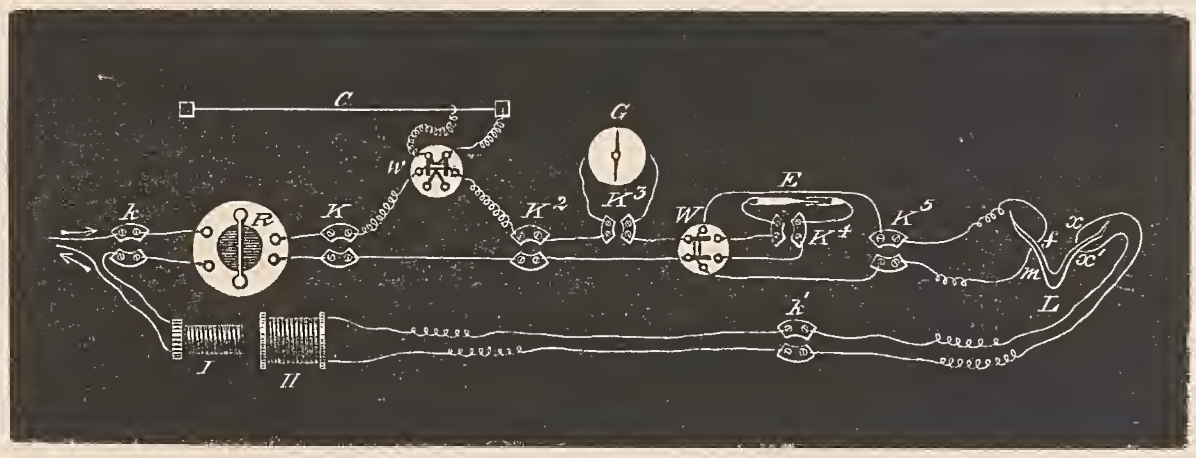

Diagram of Connexions of Compensator.

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* This corresponds to K^5 in the diagram. Here L represents the leaf, *f* and *m* the leading off contacts, *x* and *x'* the exciting contacts, I and II the primary and secondary coils, R the rheotome, C the com-

C. For the preliminary investigation of the time relations of the electromotive changes which follow excitation the most valuable instrument is the capillary electrometer. Two important improvements have been recently made in the construction of this instrument by my friend Professor Lovén,* of Stockholm. The first of these consists in constructing the tube which contains the dilute sulphuric acid, of extremely thin glass so that the capillary tube which is in contact with its internal surface can be observed with a magnifying power of at least 300 or 400 diameters (No. 10 Hartnack). The other consists in cementing the sulphuric acid tube to the mercury tube, so that no evaporation takes place. By the first, the instrument is rendered much more sensitive, by the second, variations in the strength of the sulphuric acid which arise from evaporation are avoided. When used for the purpose of observing a rapid succession of changes in the electrical relations of two led off surfaces, its superiority to the galvanometer is very striking. The movements of the mercurial column not only correspond closely in time to the actual changes which they represent, but express with very great accuracy the differences of potential which actually exist in each successive phase of a variation. In the beginning of an investigation this is invaluable, for the summary information of the duration, character, and progress of the electrical disturbance produced by excitation, which the electrometer affords, renders it possible to avoid loss of time in the subsequent application of more accurate modes of measurement. For the rougher kinds of time measurement, the use of the electrometer may be combined with that of the electro-magnetic signal, in the manner explained in our former paper. By this means the occurrence of an event can be determined within a tenth of a second by a single observation, and, of course, much more accurately by repetition. As a means of measuring electromotive force, the electrometer may be used with great advantage when the electrical states to be investigated are transitory, and at the same time so irregular that the rheotome could not be applied. For this purpose it is much better to measure the electromotive value of each excursion with the aid of the compensator than to deduce it, as has been done by Fleischl,† from the "compensation pressure." With this exception the measurements of electromotive force can be made much more accurately with the galvanometer. If it were possible to photograph the motions of the mercurial column of the electrometer it would be an addition to its usefulness.

Use of the Pendulum Myograph as a Rheotome.—The pendulum myograph (fig. 7) is essentially a seconds pendulum, the weight of which carries a glass plate so fixed

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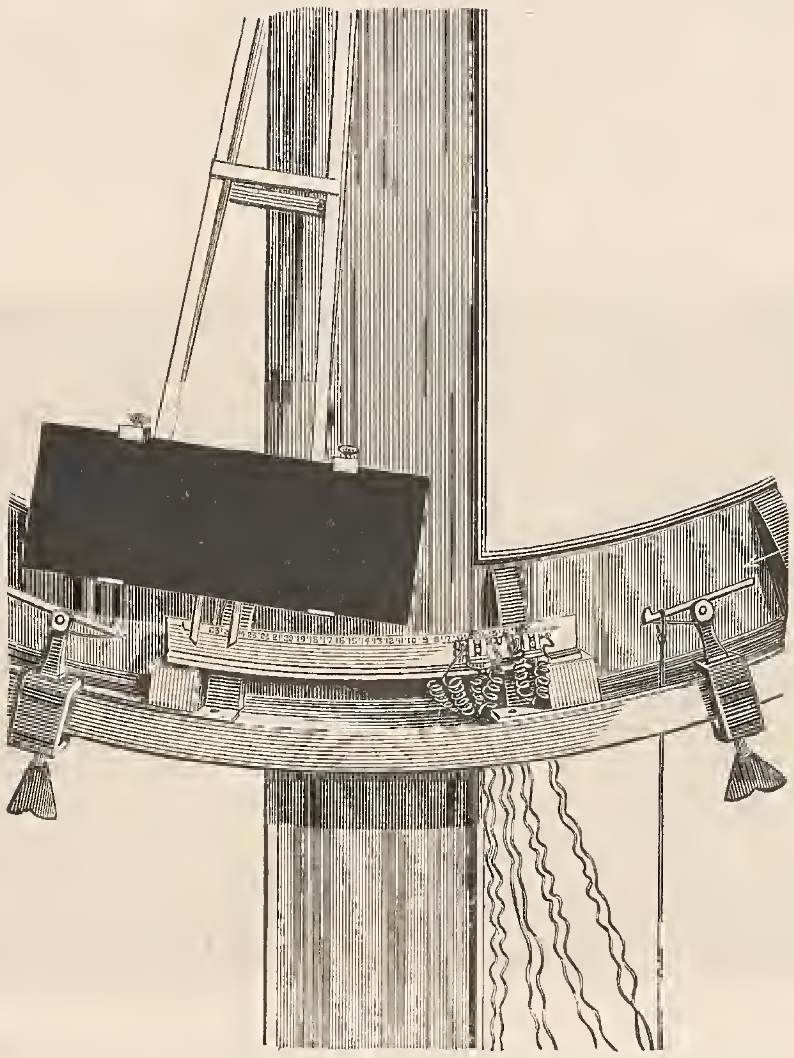
pensator, G the galvanometer, E the capillary electrometer. The wires marked with arrows lead to a battery. K, K^1, &c., are keys. W is a switch by which the wires from the electrodes may be connected either with the galvanometer (as shown in the diagram), or with the electrometer (as when the bridge is turned to the right). *w* is the reverser connected with the compensator shown in fig. 5.

* Lovén, "Om kapillarelektrometern och kviksilfertelefonen," Nordiskt medic. Arkiv. vol. xi., No. 14.

† v. Fleischl, "Ueber die Construction und Verwendung des Capillar-Electrometers, für electrophysiologische Zwecke." Archiv. f. Anat. u. Physiol., 1879, p. 269.

that the surface oscillates in its own plane. If this surface is blackened, and a tuning fork is allowed to inscribe its vibrations on it during its swing, it is easy to determine the time occupied by the pendulum in accomplishing any given distance taken at any part of its course from the tracing. I have provided my pendulum with a fixed circular scale, on which each division is at such a distance from the next that the trigger in passing from one to the other occupies one hundredth of a second. Against this scale slide four keys, as seen in the drawing. Each key is so constructed that the trigger in passing, opens it. By means of these four keys two circuits can be independently closed and opened one after the other, and the time of closing and of opening can be determined with great exactitude, according to the following plan (fig. 8).

Fig. 7.



The Pendulum-Rheotome (represented as if in motion).

The motion of the pendulum is from right to left. The detent (on the right) is provided with a wire, by means of which the pendulum can be "let off" by the observer at the galvanometer, which must necessarily be at a distance. (From a photograph.)

Provided that the events of which the time relations are to be investigated occur within a third of a second, this is an excellent form of rheotome. In opening a key of the kind employed no time is lost; consequently, the time-interval of closure of the circuit corresponds exactly to the distance between I. and II., or between III.

and IV., as the case may be. When longer periods have to be investigated the rheotome previously described by me, in relation to my experiments on the time relations of the excitatory process in the heart, answers perfectly.

D. *Methods of Excitation.*—For electrical excitation (whether by the voltaic or induction current) it is necessary to use non-polarizable electrodes. As the space between the upper surfaces of the lobes is insufficient to allow of the introduction of two electrodes of the form used for leading off, it is convenient to use a simpler form which, although not so perfect, answers the purpose completely. It is merely a zinc rod of which the amalgamated end is sheathed in wash-leather steeped in solution of zinc sulphate. The sheathed end is enclosed in a larger sheath of the same material, steeped in solution of salt, which ends in a point convenient for contact. The zinc rods are held by supports which allow of their points being brought into any required position with accuracy.

Fig. 8.

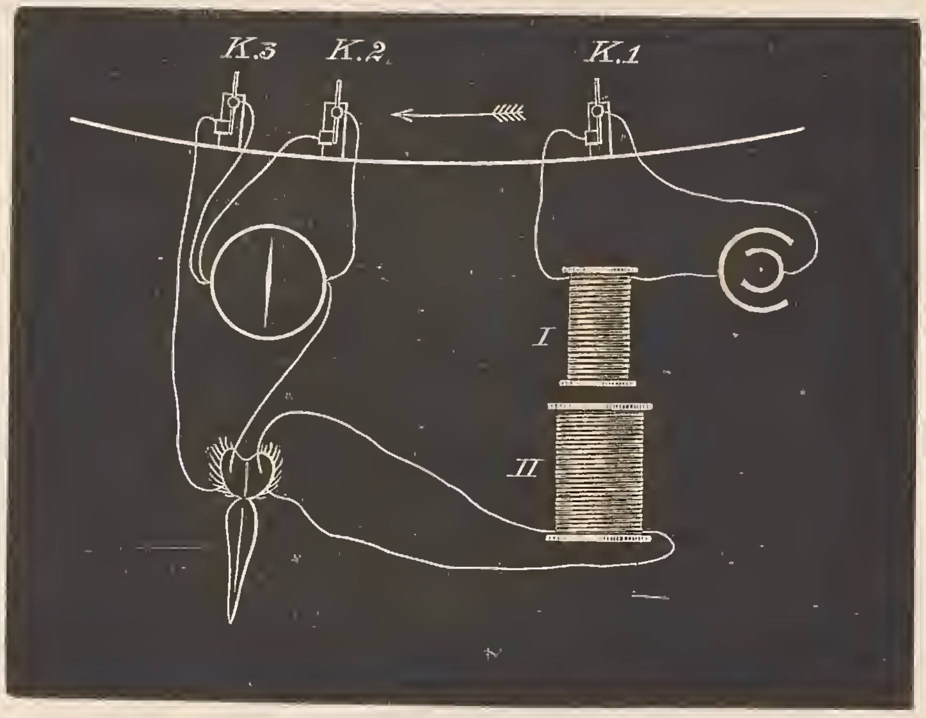


Diagram of the connexions of the Pendulum Rheotome as used for determining time of commencement of first phase, rate of propagation, &c.

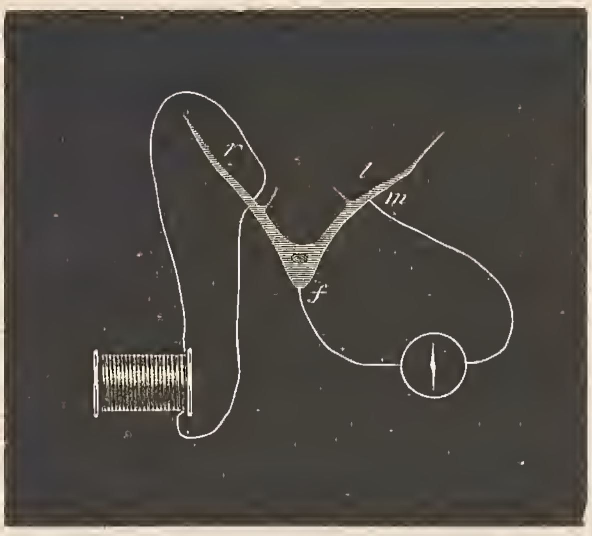
PART III.

ELECTROMOTIVE PROPERTIES OF THE UNINJURED LEAF.

The preliminary examination which was made in 1876 of the electromotive properties of the uninjured leaf appeared to show, first, that the external surface of each lobe is always positive to the internal surface, and, secondly, that in general the part of the midrib which is nearest to the sensitive hairs is positive to other parts of the external surface of the leaf. The more complete knowledge which has been acquired by work done since, justifies the statement that these two propositions express the truth, With reference to the second and less important of them, I will refer to the following observations made in the Laboratory of the Royal Gardens at Kew in the summer of 1878.

Negativity of lobe to midrib.—A leaf having been cut off and placed in the warm chamber, of which the temperature varied from 29° C. to 33° C. with the leaf-stalk in water, leading off contacts were made on the external surface of one lobe (*m*) and on the midrib *f* (see fig. 9). The difference* of potential was at first −0·006 D. The leaf was then excited by touching the right central hair at intervals of 5″ for 70′. Each excitation produced a normal diphasic variation, indicating that *m* became first negative, then positive to *f*. After each excitation the difference was compensated. After the first two it fell to −0·0026 D. and thereafter to −0·0006 D. In other words, the after-effect of excitation (see Part V.) was opposed in direction to the original difference. A second leaf was prepared in the same way and left in the chamber, of which the air was nearly saturated, all night, the temperature being 29° C. At 11.40 next morning the difference was −0·004 D., having been −0·009 D. the evening before. After excitation of the hairs of the opposite lobe it fell to −0·0026 D., and subsequently to −0·001 D. A third leaf, similarly led off and placed in a chamber as before, showed a difference of −0·0214 D., the lobes and marginal hairs being fully expanded. After the first mechanical excitation of the hairs of the opposite lobe, the marginal hairs became incurved without any further change of form of the leaf; the difference fell to −0·019 D. After a second excitation it was −0·015 D. and then sank, after repeated touches of the hairs, until, 75 minutes after the first observation, the leaf escaped from the holder without further disturbance of the leading off contacts, the last reading being −0·012 D. It remained

Fig. 9.



Leaf led off from under surface of right lobe and under surface of midrib, excited from left lobe.

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* To save unnecessary repetition in the following pages the word difference, printed in leaded type, will be used for "difference of potential." It need scarcely be added that the letter D. stands for Daniell.

in the chamber until next morning, when a fine glass rod was introduced between the marginal hairs for the purpose of excitation; the difference was now —0·007 D. The leaf was subjected to a series of excitations, each of which was followed by a normal diphasic variation. A fourth leaf was led off in the same way, but it was not, like the preceding ones, restrained by the glass holder. It was placed in the warm chamber at 1.30 P.M. and observed at intervals until 4.30 P.M. The difference varied from —0·032 D. to —0·029 D. On returning next morning it had diminished to —0·018 D. It was at once excited by gently touching the distal sensitive hair of the led off lobe, the leaf responding by a normal variation but not closing. At once the difference sunk to —0·054 D. After a second excitation of the same kind the leaf closed smartly, the subsequent difference being —0·045 D. Another leaf (not of the same series) was led off by three contacts, one (f) at the midrib, the other two at the middle of the external surface of each lobe (m and m'), so that these surfaces could be severally compared with f. Both m and m' were negative to f, the difference being m —0·018 D., and m' —0·024 D. In another similar observation the corresponding differences were m —0·038 D., and m' —0·033 D. After a few excitations these diminished to —0·011 D. and —0·004 D.

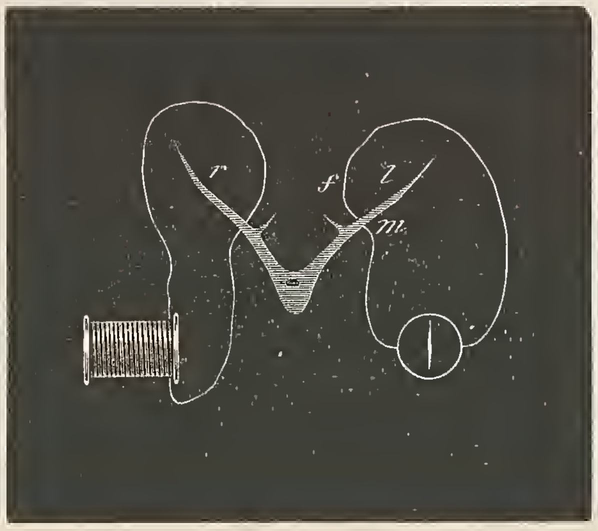
The above observations, with the exception of the two last, were made for the purpose of ascertaining the truth of Professor MUNK's statement, that the external surface of the lobe is negative to the under surface of the midrib. They tend to show that the difference in question is one of frequent (perhaps constant) occurrence in unexcited leaves, and that it diminishes in consequence of excitation.

We now proceed to the study of the second, and much more important, of the two statements made in our previous paper relating to the electromotive properties of the resting leaf. The fact that the external surface of the lobe is positive to the internal surface opposite to it, we propose to designate by the term "cross difference," which sufficiently expresses its own meaning. It will be remembered that the existence of any such difference is denied by Professor MUNK, who holds that the opposite surfaces of the lobe are isoelectrical.

In a preliminary series of observations made in the Laboratory of the Royal Gardens at the same time with those already referred to no exceptions were met with. No importance, however, could be attached to the bare fact of the negativity of the internal surface unless it could be shown to be physiological in its nature—that is, not a result of the merely chemical or physical differences of the two opposite surfaces of contact. The method which is employed for determining whether a phenomenon is physiological or not, must always be the same. It consists in observing whether its development is so associated with the functions of the living part or organism in which it has its seat as to indicate that it is dependent upon it or forms part of it. It was therefore the first step towards determining the physiological meaning of the cross difference, to ascertain whether it is modified by changes in the vital activity of the leaf, and particularly whether it is the same in unexcited and in excited leaves, and whether the change produced by excitation, if any, is in relation with the excitatory process itself.

The first observation on this subject was made in September, 1879, in the course of experiments made for other purposes. A leaf was led off as shown in the diagram (fig. 9)—that is, by opposite contacts, of which the internal corresponded to the space between the distal and proximal sensitive hairs, the surfaces of contact being as large as possible. The leaf was placed in the chamber at 38° C., and was kept there for an hour, at the end of which the two surfaces were nearly equipotential, there having been previously a difference of −0·007 D. During a period of 40 minutes the leaf was subjected to a series of 16 excitations, in the course of which the difference rose from +0·001 D. to +0·0114 D. After a second series of 15 excitations it rose to +0·032 D. In another leaf, observed the following day by the same method, the difference increased during 14 excitations from +0·0084 D. to +0·046 D. In a third the initial difference was +0·0036 D. During the following two hours the leaf was excited 28 times at nearly equal intervals. At the end of the first half hour the difference was +0·0082 D.; at the end of the second, +0·118 D.; of the third, +0·0178 D.; of the fourth, +0·024 D. In the first of these three observations the leaf was excited mechanically—that is, by touching the sensitive hairs of the opposite lobe. In the second and third, single opening induction currents were led through the opposite lobe by pointed electrodes which penetrated its external surface.

Fig. 10.



Leading off electrodes on right lobe. Exciting electrodes on left.

It was not until the next year that the opportunity offered of further investigating this effect. A leaf was led off for the purpose, as in the previous experiment, by opposite contacts, the electrode being applied to the internal surface between the distal and proximal sensitive hairs. To ensure a more satisfactory initial determination of the cross difference in the unexcited leaf, it was introduced into the chamber (of which the temperature was 20° C.) the day before, in order that any accidental difference due to initial changes at the surface of contact might be got rid of. The observations were as follows:—Before excitation the cross difference was +0·024 D.; immediately after excitation, by exciting mechanically the sensitive hairs of the opposite lobe, it was +0·031 D., but soon diminished to +0·026 D. A second excitation had a similar effect—it rose to +0·029 D. After an hour's rest in the chamber the experiment was repeated. Before excitation the cross difference was +0·025 D.; on compensating as quickly as possible it was found to have risen (about half a minute after excitation) to +0·035 D.; in five minutes it had fallen to +0·028 D. A second excitation similarly observed raised the cross difference to +0·031 D., after which it fell to +0·028 D. Four similar observations gave corresponding results, but the effect produced (increase of the cross difference), which in the first experiment of the series amounted to one-hundredth of a Daniell, did not in the later ones exceed one-five hundredth. The indication that these phenomena afford that the electrical difference which exists in the unexcited leaf of Dionæa between the opposite surfaces is in physiological relation with the excitatory properties of the leaf will receive confirmation when, in the next part of this paper, we have before us the immediate effects of excitation.

In the short communication I made to the Royal Society in 1873, in which I announced the discovery of the excitatory electrical change in the leaf of Dionæa, I stated the existence of a "leaf current" directed from the proximal to the distal end of the midrib in the uninjured leaf. I had observed that, in general, the attached end of the midrib was negative to the further end, and expressed this fact in the language commonly used in physiological writings. I had also found that the under surface of the leaf stalk in the neighbourhood of the joint by which the leaf is attached to it, was negative to the rest of the leaf stalk, and designated this negativity by the term "stalk current." I had further observed that if the current from a battery (a small Daniell's cell) was directed through the leaf stalk at the same time that the two ends of the midrib were led off to the galvanometer, the difference previously existing between the ends of the midrib was increased if the current led through the leaf stalk were in the same direction with the leaf current, diminished if it were in the opposite direction. I had finally observed that in leaves severed from the plant, the "leaf current" was curiously dependent on the length of the petiole,* so that the shorter the petiole the greater was the negativity of the proximal end as compared with the distal. Of these four observations relating to the electromotive properties of the unexcited leaf, all may be readily demonstrated in most leaves. The more complete knowledge of the subject which I now possess leads me to regard the first two as of little importance. The third possesses a greater interest, for it affords evidence that in the plant a voltaic current conducted through a living part not only influences the condition of the part through which it flows, but extends that influence to extra-polar parts, affecting them in the same way as in nerve, so that whatever may be the nature of the action in the two cases, the phenomena scarcely admit of being distinguished from each other, as may be seen from the following experiments made in the Kew Laboratory in July, 1878.

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* The experiment was related as follows:—"In a leaf with a petiole an inch long I observed a deflection of 40 scale. I then cut off half, then half the remainder, and so on. After these successive amputations, the deflections were respectively 50, 65, 90, 120." (Proceedings, vol. 21, p. 495.)

A leaf was led off at the points *f* and *m* and *f′ m′* in the diagram (fig. 11). The negativity of the attached to the free end of the leaf required 0·01 D. to compensate it, which was expressed by a deflection of 27 scale of the galvanometer. Between the two led off surfaces of the petiole there was a difference of 0·006 D., the leaf end being negative to the other. These measurements having been made, the contacts *f* and *m* were led off to the galvanometer, while *f′* and *m′* were connected with the compensator. The compensator was then reversed so as to conduct a current into the petiole opposite to that which would have been required to compensate it—in the same direction, therefore, as the natural difference—and the slide was pushed along until the needle of the galvanometer was brought to zero. The slide was then at a position indicating +0·024 D.

Fig. 11.

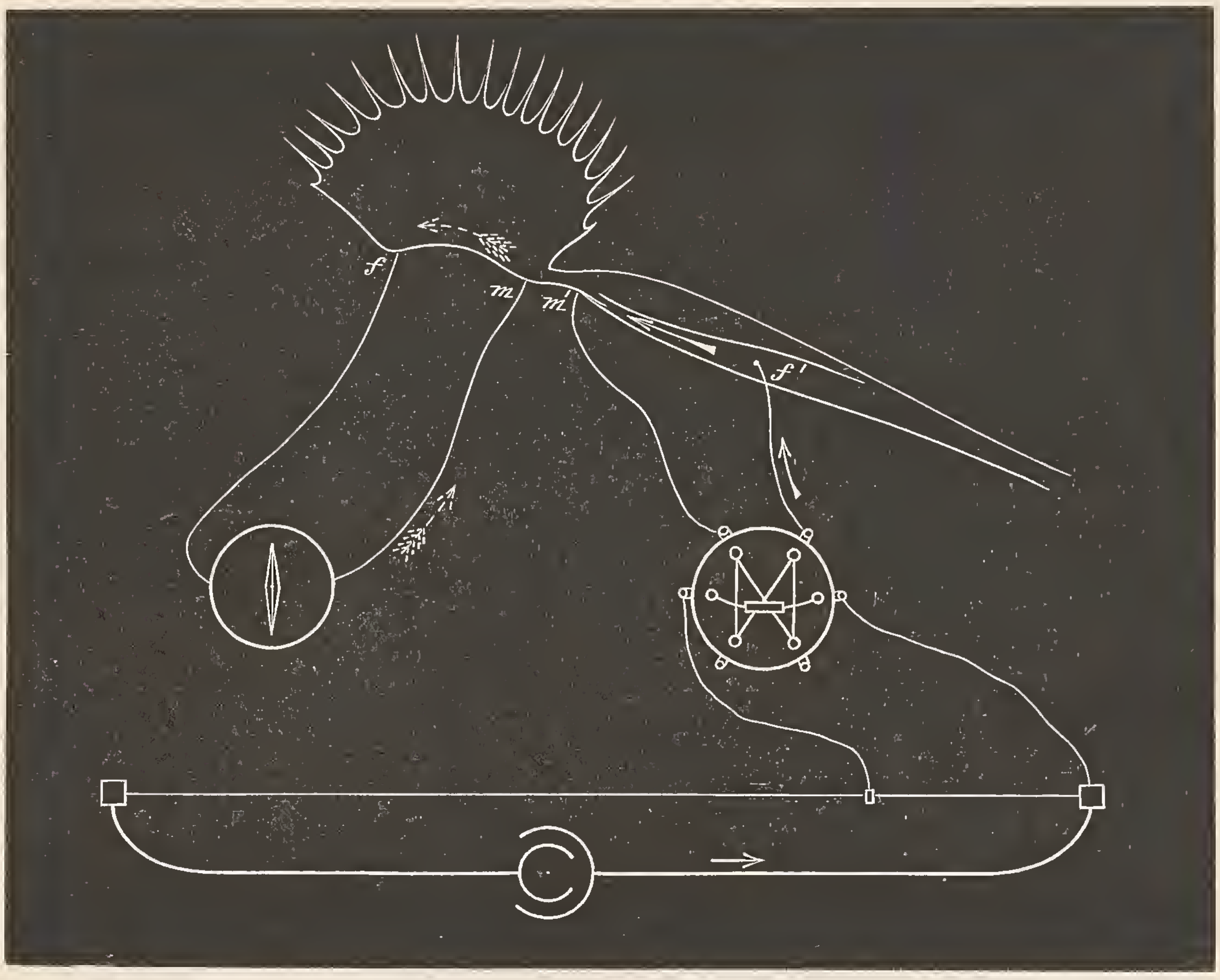

Diagram of experiment as to extra polar-influence of voltaic currents. The strong arrows indicate the direction of the derived current; the weak arrows that of the current determined by it in the leaf. N.B.—The reverser should be turned the other way.

I have since endeavoured to determine the relation between the remarkable extra polar effect (which so strikingly resembles the electrotonic variation of nerve) and the strength of the current which produces it, more accurately. For this purpose currents of various strengths, derived from a rheochord, were conducted through the leaf stalk, the leaf being led off as in the experiment last described. The effects observed were as follows:—With 25 centims. of rheochord wire,* corresponding to a difference of potential of about a quarter of a DANIELL, the difference between the two leaf electrodes was diminished when the derived current was directed from the leaf, from —0·007 to —0·0058, and increased to about —0·008 when it was reversed. With twice the length of wire (about half a DANIELL) it was diminished from —0·006 to —0·004 when the derived current was directed from the leaf. Finally, with three times the length of wire the difference, which had now fallen to —0·005, was abolished; so that when the galvanometer was uncompensated, the needle was brought to zero by the derived current through the petiole.

It was ascertained by careful experiments that the electromotive change in the leaf ceased with the current which induced it. When the experiment was so arranged that the galvanometer circuit having been open during the passage of the "polarizing" current was closed at the moment that it ceased, the needle remained motionless.

In the above experiment it is seen that the induced difference of potential between the leading off contacts was very small indeed as compared with that of the contacts by which the derived current entered and left the petiole. But it is to be borne in mind that this relation does not express the true relation between the electromotive forces, for this reason: We have already assumed that all electrical changes dependent on physiological action, have their seat in the parenchyma, and this assumption is as applicable to the leaf-stalk as to the leaf itself. Consequently the "polarizing" effect of a voltaic current led from one point to another of the surface of the leaf-stalk depends, not on the electrical difference thereby determined between the surfaces of contact, but on the differences between the end cells of the interpolar stratum of parenchyma. Now it is known experimentally, first, that the resistance encountered by a current led as above described through a leaf-stalk is enormous, and secondly, that this resistance has its seat at the surface, whereas the longitudinal resistance of the middle stratum is relatively small. The result is that the physiological or "polarizing" effect of the current on the parenchyma which is the immediate effect of the electrical difference determined by the current between the most and the least positive (or negative, as the case may be) cells of the parenchyma, is a very inconsiderable fraction indeed of what it would be if the polarizing electrodes could be applied directly to the parenchyma itself. These considerations lead me to believe that although in the experiment related on the previous page the currents used were relatively considerable, there is no reason to suppose that the electrical changes in the parenchyma exceeded those (to be described in the next part of this paper) which, in the excitable parenchyma of the leaf are associated with the excitatory state, and consequently I am led to regard it as possible that these changes may not be inadequate to produce secondary changes of the same kind in the petiole—in other words, that if the petiole were led off during the excitatory variation, it would show a variation in the same direction. Want of material has rendered it impossible to settle this important question experimentally.

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* With reference to this mode of using the rheochord, see p. 42 (Excitatory influence of the voltaic current).

The statement contained in my preliminary note in 1873, that the amputation of the petiole always increases and often doubles or trebles the electromotive force of the "leaf current," in other words, increases the negativity or diminishes the positivity of the end of the leaf which is next to it, has been confirmed by further and more exact observations. In illustration of it the following experiment may be given. A leaf was led off in the way described in my note by the two ends of the under surface of the midrib, with the fixed electrode at the distal end, it being still on the plant. The difference between the two surfaces was found to be −0·011 D. On excitation by touching a hair a normal variation was observed, consisting of a positive deflection followed by a prolonged negative after effect, that is, the same phenomena as were described in 1873. On cutting off the leaf with as long a petiole as possible, no effect could be observed on the difference. Half the petiole having been removed without effect, the difference being now −0·0112, half the remainder was removed, the result of which was that the difference increased (the observation being made as soon as compensation could be effected, that is, within a minute after the amputation) to −0·0198. In the course of a few minutes the difference sank to −0·01, that is, a little below its original amount. Amputating half of the remainder of the petiole still attached raised it at once to −0·023, but it soon sank to −0·017. Clipping off the stump of the petiole which still remained, produced only a slight increase to −0·02. Throughout these experiments the excitatory variation exhibited the same character. We have thus before us two sets of facts relating to the electrical influence of the petiole on the leaf, viz.: (1) that a current through the petiole, if directed towards the leaf, renders the attached and proximal end negative to the distal, and (2) that removal of the petiole produces the same effect. I am unable at present to explain either of these phenomena. The nature of the electrical relation between the two organs which underlies both must remain undetermined until it is possible to make further experiments.

PART IV.

The Electrical Effects of Excitation.

1. *The fundamental experiment.*—Under this term I propose to designate the observation of the electrical effects which present themselves when the leaf is led off by opposite surfaces of one lobe, while the other is excited by any of the methods

described in Part II. It is to be understood that the electrode *f*[*] is always applied to the internal surface of the leaf between the three hairs (in the trigone) and the electrode *m* to the outside surface exactly opposite *f*. What happens under this condition is always the same. After a delay—of which the duration varies with the temperature—the external surface becomes first negative, then positive, to the internal. For the purpose of giving a general view of the time-relation of the first phase of the variation, I submit the following table, which gives the results of three successive rheotome observations[†] :—

	Time after excitation at which galvanometer circuit opened.							
	0·2"	0·4".	0·6".	0·8".	1"·0.	1"·2.	1"·4.	1"·6.
Obs. I.—Deflection . .	0	3	8	22	28	25	29	16
(100 sc.=0·063 D.)	**0**	**4**	**10**	**28**	**35**	**31**	**38**	**20**
Obs. II.—Deflection . .	1	19	30	45	44	29	8	2
(100 sc.=0·045 D.)	**1**	**17**	**27**	**40**	**39**	**26**	**7**	**2**
Obs. III.—Deflection .	0	5	36	55	42	17	21	7
(100 sc.=0·0353 D.)	**0**	**3**	**25**	**39**	**30**	**13**	**15**	**5**
Mean result . . .	**0·3**	**8**	**20**	**36**	**35**	**23**	**20**	**9**

Each of the leaves employed in the experiments of which the results are given was fixed in the glass holder described in Part III., and led off, as above explained, by opposite contacts. The three observations were made consecutively; the opposite lobe to the one led off was excited by the rheotome (see fig. 6), the electrodes used being steel needles, of which the points (2 millims. apart) were inserted into the external surface of the lobe. It was consequently necessary to move them three or four times during the period of observation, care being taken that each new insertion should be at the same distance from the middle line of the leaf as its predecessor. Every excitation was repeated twice, and, if the deflections obtained differed in extent, a third time. As it was of much more importance to complete the observation while the leaf remained in a normal condition than to obtain great accuracy in the readings, they were not repeated more frequently than appeared necessary. In each experiment the leaf was introduced into the warm chamber at 32° C. some time before the observation began. The observations were made at intervals of two minutes and a half, and the number of excitations necessary for each was from 20 to 25. Consequently, each lasted nearly an hour.

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[*] In using the letters *f* and *m* to designate the leading off electrodes, it is assumed, as in former papers, for convenience of description and for no other purpose, that any difference or current is to be called negative when it is of such a nature that *m* becomes negative to *f*, and *vice versâ*. The words "negative" and "positive" therefore have only a relative meaning.

[†] The numbers in ordinary figures represent the actual deflection observed; those in black type, what they would have been had the resistance been equal in all the experiments and such that a difference of 0·05 D. would have been expressed by 100 scale of the galvanometer. The sensibility of the galvanometer was purposely reduced for this preliminary experiment, it being desirable to lengthen the periods of closure to 0·2".

The observations given in the table serve to show that the variation lasts during the whole of the first and the greater part of the second second, and that it culminates at from six to eight-tenths of a second after excitation. For these purposes it was thought best to employ long periods of closure (using the galvanometer in a very insensible state) and to make the end of one period coincide with the beginning of the next in such a way as to cover the whole period to be investigated. But such a method was quite inadequate to give information as to the question of greatest importance in relation to the excitatory variation—that of the time at which the effect commences. For this purpose it was necessary to shorten the periods of closure, and consequently to increase the sensibility of the galvanometer. We therefore substituted for the revolving rheotome the "pendulum" rheotome, of which the description is given in Part III. The instrument is so arranged for the experiment that on swinging from right to left it opens three keys in succession—K 1, K 2, K 3 (see fig. 8). Of these, the first occupies a fixed position, so that it is opened about 0·1″ after the pendulum is liberated. This position corresponds to zero of the pendulum scale. The other two keys, K 2 and K 3, can be slid along to any required distance from K 1. The binding screws of K 1 are connected with the primary circuit of the induction coil; those of K 2 with the shunt of the galvanometer, so that when this key is closed the galvanometer is "short circuited;" those of K 3 are interpolated in the galvanometer circuit between the leading-off electrodes and the shunt. Consequently, when the three keys (being previously closed) are opened in succession by the pendulum, the primary circuit is first opened by K 1, the galvanometer circuit is unbridged—that is, closed—by K 2, and again broken by K 3. As the scale of the pendulum is divided into hundredths of a second and extends over a period of a quarter of a second, this arrangement affords the means of investigating with great exactitude the events of the beginning of the variation.

The galvanometer is brought to such a degree of sensibility that with a resistance of 10,000 ohms, in addition to that of the galvanometer in circuit, 0·0001 D. gives a deflection of 97 scale. To facilitate accurate reading a slightly concave mirror is placed in front of the scale, by means of which an enlarged image of the part of the scale most used can be seen by the observer without fatigue. The leaf is led off and excited as in the fundamental experiment. For excitation, non-polarizable electrodes of the form described at D, p. 18, are used, the contacts being opposite, and corresponding in position to those of the leading off electrodes applied to the opposite lobe. The plant of which it forms part is placed in the chamber, the air of which is saturated, at 26° C.

It having been first ascertained that when K 2 was at 0·2″ of the scale of the pendulum, and K 3 at 0·04″, there was no variation, successive observations were taken with K 3 at positions further and further removed from K 2 (which remained stationary at 0·02″) with the following results :—With K 3 at 0·06″ there was a trace of movement of the needle; with K 3 at 0·08″ it had increased to −4 scale; at 0·1″ to 12; at 0·15″ to −39 sc. To complete the observation K 2 was moved to 0·1″ and K 3 to 0·2″, so that the period of closure now corresponded to the second tenth of a second after excitation; the galvanometer reading was −161 sc.; finally, between 0·2″ and 0·3″ we had −248 sc.

In four other experiments the results (stated shortly) were as follows :—

Experiment I. Temperature 32–35° C. 0·02″–0·4″, no deflection; 0·02″–0·05″, a trace; 0·02″–0·06″, −5 sc.; 0·02″–0·1″, −35 sc., −31 sc.

Experiment II. Temperature 34° C. 0·02″–0·04″, no deflection; 0·02″–0·05″, −3 sc.; 0·02″–0·06″, −6 sc.; 0·02″–0·15″, −70 sc.

Experiment III. Temperature 32° C. 0·02″–0·06″, no deflection; 0·02″–0·08″, no deflection; 0·02″–0·10″, −13 sc., 22 sc., 23 sc.; 0·02″–0·9″, −10 sc., 7 sc., after which previous negative results were verified by repetition.

Experiment IV. Temperature 36° C. 0·02″–0·04″, trace; 0·02″–0·05″, −4 sc., 6 sc.; 0·02″–0·6″, −8 sc., 10 sc.; 0·02″–0·8″, −12 sc.; 0·02″–0·1″, −16 sc.

The experiments were made consecutively, that is, on successive days, and, with the exception of Experiment IV., under precisely similar conditions. In Experiment IV. the temperature was inadvertently allowed to rise to 36°,[*] a fact which sufficiently accounts for the somewhat earlier beginning of the variation.

The experimental results which have been given justify the statement (1) that the first phase of the variation is entirely negative; (2) that it begins (at the temperature of 32° C.) about one-twentieth of a second after the excitation; (3) that its electromotive force rises rapidly at first and subsequently very gradually; (4) that it culminates about the end of the first half-second after excitation and subsides gradually towards the end of the third half-second.

In the paper which I communicated to the Royal Society with Mr. Page in 1876, I stated that the variation of the leaf did not begin until two-tenths of a second after excitation, and gave the grounds on which the statement was based. The experiments which led us to conclude that what we then called the "electrical delay" (the interval of apparent electrical inactivity) were correctly made. We repeated them under the most favourable circumstances in the Kew Laboratory in 1878,[†] with perfectly similar results, but nevertheless our interpretation of them was erroneous.

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[*] It is probably also worth noting that the day on which this experiment was made was the hottest of last summer, Friday, July 16.

[†] The observations made at Kew, in 1878, gave 0·12″ as the shortest period of delay when the

In 1876 we used an arrangement by which (1) the act of excitation (if mechanical, the depressing of a lever which brought a camel-hair pencil into contact with one of the sensitive hairs; if electrical, the breaking of the primary circuit of the induction apparatus) recorded itself on a blackened cylinder revolving at a rapid rate by clockwork; and (2) the record of the response was inscribed on the same cylinder.

In the first record, the inscription of which was effected with the aid of DEPREZ' electro-magnetic "signal," or chronograph, the loss of time was inappreciable. The response was the motion of the mercurial column of the capillary electrometer. This was observed by one of us (Mr. PAGE), who recorded the event on the cylinder by closing the current through the "signal" which the other had broken in the act of excitation. The interval between break and make of the signal current comprised: (1) the physiological delay to be measured; (2) the instrumental delay (that is the time lost, if any, in moving the mercurial column); and (3) the "personal time" or "reaction time" of the observer. The last we had no difficulty in eliminating, for Mr. PAGE's personal time was subject to very slight variation. The second we had not taken into account at all, assuming it to be non-existent. The consideration that excitatory difference in Dionæa probably begins gradually, and that if so the motion of the column is slow, and may therefore not be perceived until a longer period than the normal time after it has actually begun, led us later to conclude that our method was inadequate for the purpose, and that the only way by which the question could be settled was by the use of the rheotome and galvanometer.

The after effect.—The change which has been above described as occupying the greater part of the first 2″ after excitation is followed by a period during which the surfaces are nearly, or absolutely, in the same condition electrically as they were before excitation, but sooner or later a secondary effect begins, which I propose to call the *after effect*. During the after effect the internal surface becomes negative to the external.

In order to bring the after effect into view, the rate of the rheotome must be reduced and the periods of closure much lengthened. In our first observation the rate of revolution was 1 in 78″, and the closing time 1·4″. The leaf was excited as before at intervals of three minutes, without being placed in the chamber, the temperature of the room being 18° C., the mode of excitation being the same as in the previous series. After the first excitation the galvanometer circuit was closed between 1·3″ after excitation, and 2·6″. In this case the deflection was negative, for, in consequence of the relatively low temperature at which the observation was taken, the first phase of the

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measurement was made in the moist chamber at 32° C.—a result which exceeds the truth by about 0·05″. Assuming that the error is to be set down, not of course to loss of time in transmission of the effect to the mercurial column, but to the lengthening of the "reaction time" when the motion to be perceived is gradual, it renders all the observations made by signalling with the electrometer useless for the purpose of measuring the duration of the delay, but does not interfere with the use of this method for other purposes, particularly for determining which of the two similar electrical responses occurs earliest.

variation was prolonged so as probably to cover a great part of the closing time. In the second experiment, the closing time being 1·3″ later, namely, from 2·6″ to 3·9″ after excitation, there was a positive deflection which was not measured. In the third, 1·3″ later, the positive deflection was 90, in the following ones, successively, +71, +70, +50, +45, +30, +30, and so on. On placing the leaf in the chamber and repeating the observation a perfectly similar set of results was obtained. Between 0·6″ and 1·95″ after excitation there was a negative deflection of −380. Between 1·3″ and 2·6″ a much smaller negative of −70, these being the expression of the first phase. Then followed between 2·6″ and 3·9″ a positive deflection of +6, after which, at successive consecutive periods, +100, +75, +70, +42, +20, and so on.

From these and other similar observations made by the same method, it appears that in the fundamental experiment the external surface which, 4″ or 5″ after excitation, attains its greatest positivity as compared with the internal, returns very slowly (if at all) to its previous state. Various other methods were used for the purpose of observing the progress of the after effect with more completeness, of which the best is exemplified in the following experiment.

The method employed was as follows :—It having been ascertained that the electromotive force of the after effect at 2″ after excitation corresponded to a deflection of 160 galvanometer scale, of which the equivalent was 0·016 D., the galvanometer having been over compensated to that extent was closed at 1″ after excitation. In this way the needle was brought so nearly to rest that it followed the changes of difference without oscillation, and could without difficulty be read at 5″ intervals. The readings were :—

At 3″,	at 8″,	at 13″,	at 18″,	at 23″,	at 28″,
+9	−50	−130	−142	−149	−160.

That is, at 3″ after excitation the positivity of the external surface was a little more than 0·016, at 8″ less, at 13″ much less, and so on, so that the result in electromotive force might be expressed as follows: +0·017 D., +0·011 D., +0·003 D., +0·002 D., +0·001 D. Accordingly, in another leaf, after carefully compensating the existing difference and breaking the galvanometer circuit as before, we at once adjusted the compensator so that on closing there should be a negative difference of 0·01 D. We then excited, and 2″ after closed the galvanometer circuit. Again the needle followed the slow electrical change, its positions at 5″ intervals being as follows: +70, 0, −60, −120, −140, −150, −150, −160, &c. As the value of 0·01 D. with the preparation in circuit was 190 scale, the internal surface was still 30 scale (= about 0·0015 D.) negative. Repeating the same experiment with an over compensation of 0·015 D. the successive positions of the image were: +20, −40, −140, −200, −220, −240, −250, −260, &c. So that the observation indicated, as before, that in this instance the negativity of the internal surface exceeded the compensation used.

In a third leaf observed at the same time, the resistance being unusually low, it was impossible to get the after effect on the screen, although the difference was not larger than in the other two. On interpolating as before a length of wire corresponding to 0·009 D., and closing the circuit 2″ after excitation, the successive readings at 5″ intervals were: +200, + off scale, +230, +60, −50, −130, −210, −260, −298, −320, −345, −350, −360, &c., the reading due to over-compensation being −370; so that here, as in the other instances, the maximum difference was between 0·01 D. and 0·02 D.

Influence of repeated excitations on the intensity and duration of the after effect.—Whenever a freshly prepared leaf is observed with the capillary electrometer during the course of a succession of excitations, it is found that in the first two or three the negative movement of the mercurial column is followed by a positive one which equals or may even exceed itself in extent. As the excitations are repeated, the positive phase becomes smaller and smaller, until after eight or ten, it may very readily escape observation.

For the purpose of obtaining more exact information on this subject, the galvanometer and rheotome were used; for it was obvious that if in a succession of experiments the electrical condition of the led off surfaces could be compared, during a period after excitation corresponding to the maximum of the after effect, that is, the period during which the internal surface of the leaf is most negative, the successive deflections recorded would give a good indication of any such diminution as the observations with the electrometer indicated.

The period of closure chosen was between 3″ and 4·5″ after excitation. The deflections were, of course, all positive, but varied in extent from 175 to 21. In 21 successive excitations the mean of the first seven deflections was 157, of the second seven, 86, and of the third, 55·5. In this experiment the leaf was excited at intervals of not less than two minutes. If the excitations are repeated as often as every half minute it is found that the after effect completely disappears. This can be easily shown by the electrometer, but can be more exactly determined by the galvanometer. The best way to do this is, first to make with the galvanometer an observation of the kind described in the last paragraph (p. 30), then to subject the leaf to a succession of excitations at short intervals, and to follow these by a second galvanometer observation for comparison. The following results are obtained:—

(1.) The fourth excitation of a leaf showed an after effect of which the course was as follows, as indicated by the negative deflections observed at 5″ intervals: +300, +262, +155, +100, +74, +66, +59, &c.

(2.) After ten effectual excitations at half minute intervals the after effect was: +160, +154, +70, +49, +40, +30, +12, &c.

(3.) After ten more effectual excitations it was: +25, +5, 0, 0, &c. The leaf was then left to itself for 15 minutes and the after effect again observed. It was: +85, +130, +62, +52, +49, +40, +36, &c.; and after ten minutes' further rest: +100, +200, +104, +70, +62, +52, +47, &c.

On repeating the excitations at half minute intervals, the after effect was again entirely abolished, reappearing again after a period of rest. It was further found, in another experiment, that if, after a succession of excitations at half minute intervals, a leaf was excited twice at longer ones, *e.g.*, two minutes and a half, the after effect had, by the time the second was made, already reappeared, or if previously diminished had much increased.

In Part III. it was shown that the influence of an excitation in increasing the previous difference (that is, the positivity of the external surface) or in other words, in rendering the external surface permanently more positive than it was before excitation, depended on the interval of time which had elapsed since any previous excitation. In a leaf which has been repeatedly excited at frequent intervals, excitation leaves behind no change in the electrical relation of the opposed surfaces, because there is no after effect. In other words, the persistent change of the potential, consequent on excitation, described in Part III., is a small remainder of the after effect, as it seldom amounts to more than 0·002 D, whereas the after effect itself often attains at its maximum an electromotive force of 0·02 D. It may be roughly stated that the latter is apt to leave behind a remainder of something like one-tenth of its amount. All that can be insisted on as resulting from observation is, that the subsidence of the after effect is extremely slow and incomplete, and that whenever it exists it leaves behind it a remainder—the excitatory increase of positivity of the external surface, already described.

The excitatory process as observed when the leaf is led off by symmetrical (identical) *surfaces of contact of opposite lobes.*—The excitatory variation which is observed in a leaf led off symmetrically, that is, by corresponding surfaces on opposite lobes, is the combined result of equal and opposite electromotive changes which have their seat in the tissues between the contacts. Consequently, if in an unsevered leaf so led off we suppose an excitation to take place at the same moment from each electrode, no variation would be observed, for the electrical changes due to the one excitation would exactly balance those due to the other. The appearance of a variation would indicate inequality of action between the two lobes. If the excitatory response of the right lobe were either stronger than that of the left, or were accomplished in a shorter time, the variation would express the difference. In the former case the right would have the advantage throughout; in the latter, the right would have the advantage at first, afterwards the left.

The case in which two lobes are led off and excited simultaneously and symmetrically has not been realised experimentally. The consideration of it, however, serves to facilitate the understanding of the case which actually presents itself in which the leaf is led off by symmetrically placed contacts on opposite sides, but excited on one side only. In this case experiment shows that there is always a galvanometric effect, but that the character of the variation observed differs in different leaves very considerably —so much so indeed that at first sight the differences appear irreconcilable. Before giving an account of them it will be useful to consider what we should expect. This depends mainly on the question whether, in the transmission of the excitatory effect from the seat of excitation to the opposite lobe, time is lost or not. In the fundamental experiment we have evidence that the excitatory effect travels. For when we excite the left lobe and the right responds, some time is without doubt occupied in transmission. If, as Professor MUNK believes, the transmission is so extremely rapid that the excitatory change may be said to begin at the same moment at all parts, the case would resolve itself into the hypothetical one from which we started, viz.: that of excitation taking its rise at the same moment from both leading off contacts; and this being so, any galvanometric effect observed must be due to one of the two causes referred to above, namely, either to unequal intensity or to unequal duration of the electrical response of the two sides of the leaf. If, in addition to these inequalities, we have to consider the influence of propagation (*i.e.*, if the time lost in propagation is sufficiently considerable to tell), the interpretation of the galvanometric effect observed in an experiment in the form represented in fig. 13 becomes extremely complicated; for the modifications severally due to loss of time and to difference of intensity would interfere with each other in a way very difficult to estimate. It will be shown in a subsequent section that the rate of propagation in the excitatory disturbance is great, as compared with the duration of the excitatory effect; in other words, that its wave-length is considerable. This being so, the question whether or not the modification due to loss of time will show itself, will depend on the abruptness with which the disturbance commences, as will be seen from the following diagrams. If, for example, we suppose that the excitatory electromotive changes are so directed that the led off surface on either side becomes negative to the midrib, and that the rise of negativity on the right side is represented in two different leaves by the curve *a*, that of the left side by the curve *b*, the galvanometric effect during the period represented will be expressed in each figure by the shaded area, the depth of which is greater in the first case than in the second; so that if (as is represented in fig. 12, C) it were further diminished by a greater abruptness in the rise of negativity on the left side, it might escape observation. And it is easy to see that if, in addition to beginning more abruptly the excitatory disturbance on the left side were more intense, the influence of the loss of time in propagation would lose itself entirely in that of inequality. The two last suppositions (fig. 12, B and C) appear to afford the key to the understanding of what is actually observed when a leaf is led off by symmetrical points and excited near one of them. In this case there is always an excitatory effect, proving that the two lobes do not respond equally. Further, we have found that so far from its being true, as asserted by Professor MUNK, that it makes no difference what spot is chosen as the seat of excitation, it never happens that variations of the same character are obtained when, in two successive experiments, or in two series of experiments, the right and left lobe are alternately excited. At first sight it seemed as if this difference were referable to propagation, but more careful consideration has led to the conclusion that inequality of response has a much larger share in its production than loss of time. Ground for this conclusion is to be sought, as regards each particular case in which a symmetrically led off leaf acts unsymmetrically, in the observation that each of its two lobes when separately and successively tested shows the same inequality. Unfortunately, the nature of the case did not present itself in its true relations until too late to do this in a sufficient number of cases, but I have abundant evidence that in general the electrical reactions of the two lobes of a leaf are unequal.

Fig. 12.

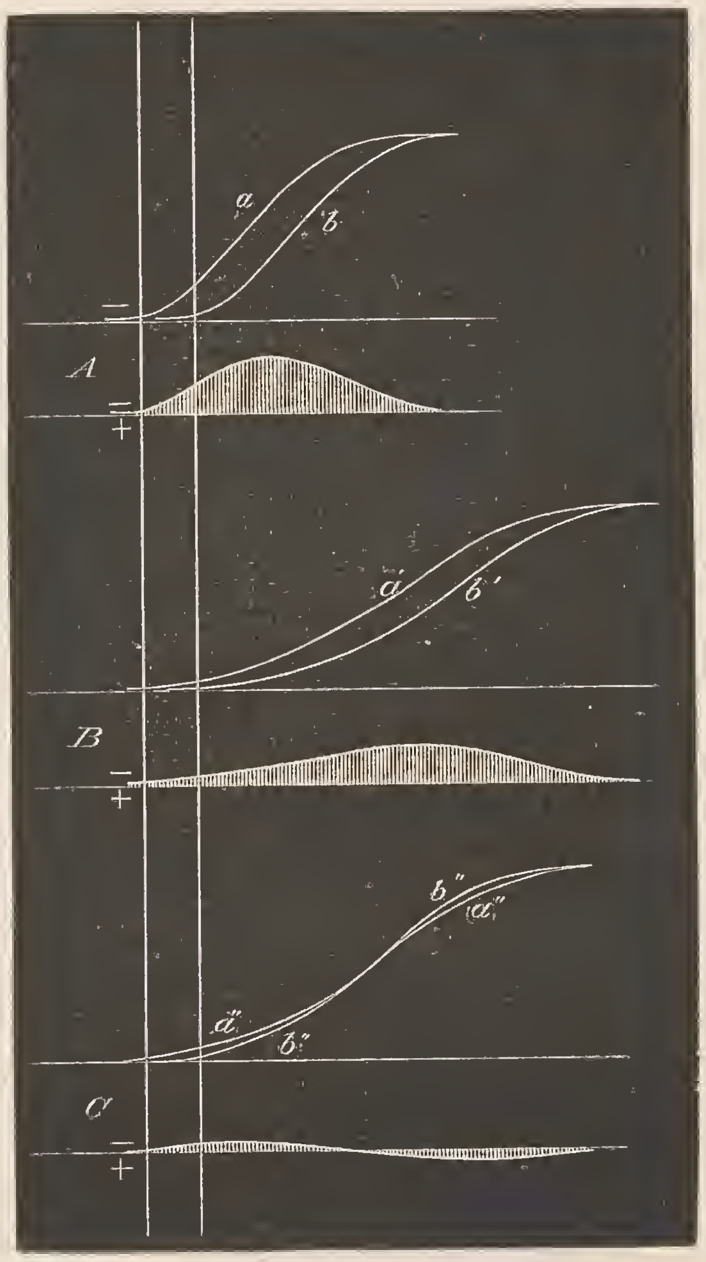

Diagram intended to illustrate the relations of the first phase of the excitatory variation when the leaf is led off by symmetrical contacts on the under surface.

In observations on symmetrically led off leaves the electrodes may be applied either to the internal or the external surface.

Symmetrical contacts on the internal surface of the leaf between the two marginal sensitive hairs.—In order to determine, if possible, the general character of the variation in this form of experiment, four observations were made in succession with the electrometer. The leaves used were led off by symmetrical contacts between marginal hairs of opposite lobes, and subjected, first on one side and then on the other, in each experiment, to two series, each consisting of ten successive excitations. The table shows the results. The letter *f* or *m* indicates that, in the case to which it relates, the fixed or movable electrode was on the excited lobe. In all the experiments the excitation was effected by touching a sensitive hair with a camel-hair pencil. The hair touched is indicated. After each observation the difference was compensated. The mean differences are given in the last column. All of the leaves were tested by observing the excitatory effect in each lobe separately when led off as in the fundamental experiment.

No. of experiment.	Which lobe excited.	Which hair touched.	Which electrode on excited lobe.	Character of variation.	Difference.
I.	R.	Proximal .	M.	Monophasic, −39·1	+0·011 D.
	L.	Ditto .	F.	Diphasic, +11·7, −21·2 . . .	+0·007 D.
II.	L.	Proximal .	F.	Monophasic, +6·2	−0·01 D.
	R.	Ditto .	M.	Diphasic, −2, +25·4	−0·02 D.
III.	R.	Proximal .	M.	Diphasic, −3·2, +18·8. . . .	−0·007 D.
	L.	Ditto .	F.	Monophasic, +22·2	−0·005 D.
IV.	L.	Distal . .	F.	Diphasic, +3·4 pause . . . . +7·4 (from zero)* . .	+0·023 D.
	R.	Ditto . .	M.	Diphasic, +9·8, −2·9	+0·023 D.

In leaves I. and II. the difference between the two lobes was so great that the leaves could scarcely be considered normal. In leaf III. the variation was normal on both sides, but the first phase was larger on the left than on the right (−8·5, +4·3 on the left, −4·4, +7·8 on the right). In leaf IV. the first phase was nearly equal on both sides (−30·0 on the left, −31·6 on the right), and was followed in both cases by a normal after effect. But these facts afford no clue to the understanding of the results recorded in the table, excepting in so far as they show that in a well-balanced leaf—*i.e.*, one which responds to excitation with some degree of equality—whichever lobe was excited directly had the advantage over the other in the compound variation. That this was so was confirmed by the observation that in leaf IV. the negative phase of the variation, as observed with opposite contacts (the fundamental experiment), was much smaller (−18·4 on the left, −26·6 on the right) when the opposite lobe was excited than when a hair adjacent to the leading off electrode was touched. The only general conclusion that I venture to draw from the experiments is a negative one, namely, first, that the differences which are observed between the excitatory effects in the two cases are as little due to mere inequality of response in the two lobes as to propagation; for such inequality would not account for the effect produced by changing the seat of excitation. We must suppose that, independently of any difference in physiological activity which may exist between the lobes, the electrical disturbance is more intense in those parts of the leaf which are nearest to the seat of excitation than in those which are more distant from it. This being admitted, and account being taken of the complicated arrangement of the electromotive structures concerned, the incongruity of the results need no longer surprise us.

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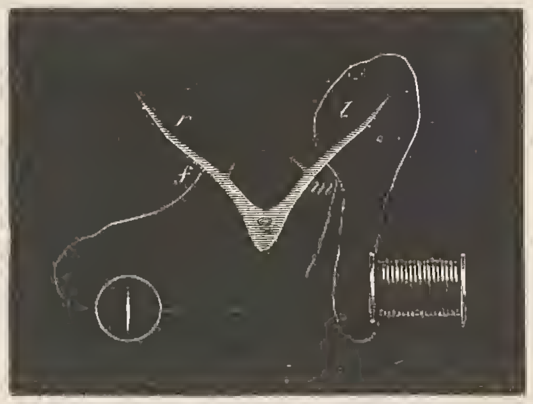
* To be read thus:—In the first phase, the meniscus moved from zero of the scale to −3·4, in the second, after a pause, from 3·4 to 7·4 in the same direction.

Symmetrical contacts on the under surface, opposite the space between the sensitive hairs (fig. 13).—The same considerations apply to this case as to that in which the leaf is led off from the upper surface; but inasmuch as the under surface presents no appreciable difference of structure, whereas in the upper we have to do with the sensitive hairs, the prospect of obtaining congruous results is not so hopeless. It will be sufficient to refer to one or two experiments made during the autumn of 1880, in which the galvanometer and rheotome were used for comparison of the excitatory effects.

Fig. 13.



Leaf led off by symmetrical contacts on under surface of lobes.

The first leaf used for the purpose was one in which symmetrical results could hardly be expected. It was a large well-grown leaf which contained the remains of a digested fly; and when, after having been excited an equal number of times on either side, it was allowed to close, the incurvature of the left lobe was much greater than that of the right. This leaf was excited electrically (the exciting contacts, x, being on the internal surface) and led off symmetrically, the electrodes touching the external surface exactly opposite the trigone on either side, the fixed electrode being on the right side. The rheotome gave the following result, the period of closure being 0·2″:—

	x On left lobe.	x On right lobe.
	M.	F.
At 0·4″	−37·5	+50
At 0·6″	− 9·0	+41
At 0·8″	+33·0	+40

The numbers given are the means of two accordant observations at each period, before each of which the difference was compensated.

In a second leaf, led off in precisely the same way, the general character of the excitatory effect was observed with the electrometer. The fixed electrode was on the left lobe. When that lobe was excited electrically (as in the last experiment) the variation was diphasic, the first phase being positive. When the right lobe was excited (x on right lobe), the variation was also diphasic, with opposite signs. The rheotome gave the following, the closing time being 0·075″:—

	x On left lobe.	x On right lobe.
	F.	M.
At 0·10″	+ 9	− 7
At 0·20″	+24	−18
At 0·30″	+27	−20
At 0·40″	+20	− 9
At 0·50″	+ 4	− 4

The numbers express in each instance the means of two observations, before each of which, as usual, the difference was compensated. The resistance exceeded 160,000 ohms, notwithstanding that the utmost pains were taken in making the leading off contacts.

In a third experiment we had at our disposal a large vigorous leaf. The fixed contact was on the right lobe. The leaf was subjected to 48 excitations during a period of 70 seconds, to all of which it responded. Its resistance was such that 100 scale of the galvanometer corresponded to 0·45 D. The excitations gave with the rheotome (the duration of closure of the galvanometric circuit being one-fifth of a second) the following results:—

	x On right lobe.	x On left lobe.
	F.	M.
At 0·20″	+28	−33·5
At 0·4″	+44	−24·3
At 0·6″	+50	− 3·0
At 1·0″	0	0

These observations show, even more strikingly than the previous ones, how completely Professor MUNK is mistaken in his statement that, provided the leading off is the same, the seat of excitation is a matter of indifference. They also serve to strengthen the conclusion that the intensity of the electrical disturbance is greater in the neighbourhood of the seat of excitation than at a distance from it. No other explanation would account for the results.

Rate of propagation of the excitatory disturbance.—The only certain method of determining the rate of transmission of the excitatory change in Dionæa consists in measuring the interval of time after excitation at which the electrical effect first becomes appreciable, by a series of observations in which a leaf, led off as in the fundamental experiment, is excited alternately in the neighbourhood of the leading off electrodes, and at a corresponding part of the opposite lobe. In such an experiment the interval of time between excitation and effect in the one case will be greater than in the other by a fraction of a second. That fraction, multiplied by the distance in millims., will give the rate per second in millims.

In our former paper we attempted to arrive at a conclusion on the subject by a method which we now know to be inapplicable. I will first refer to certain experiments which were made on this principle in the Kew Laboratory in 1878, for the purpose of confirming the statements contained in our former paper (1876). A plant was placed in the chamber at 30° C., and led off as above stated. The excitatory variation on the electrometer scale was − 12 followed by + 10. The leaf was excited by touching the sensitive hairs, alternately on opposite sides, the time of excitation and that of response being determined by the method referred to in Part II. During the period of observation the temperature rose to 37° C. The observations were made in four series, in which the means of the time-intervals between excitation and response were as follows :—

1.	Excitation of the hairs of the same side	0·08″
2.	,, ,, ,, ,, opposite side	0·28″
3.	,, ,, ,, ,, same side	0·116″
4.	,, ,, ,, ,, opposite side	0·243″

In another leaf the experiment was made first at the temperature of the room, afterwards at that of the chamber 32° C. Six observations gave :—

1.	Excitation of the hairs of the same side	temp.	18° . .	0·10″
2.	,, ,, ,, ,, opposite side	,,	18° . .	0·27″
3.	,, ,, ,, ,, same side	,,	32° . .	0·05″
4.	,, ,, ,, ,, opposite side	,,	32° . .	0·12″
5.	,, ,, ,, ,, same side	,,	18° . .	0·43″
6.	,, ,, ,, ,, opposite side	,,	18° . .	0·78″

Here the excitation was by induction shocks, the external surface being punctured by fine electrodes for the purpose. This observation shows the combined effect of temperature and local diminution of excitability. The influence of the latter is still more strikingly shown in the following series, the temperature throughout being 18° C.

Leaf fresh :—	
1. Excitation of the same side	0·15″
2. ,, ,, opposite side	0·26″
After repeated excitation :—	
3. Excitation of the same side	0·50′
4. ,, ,, opposite side	0·84″
Later :—	
5. Excitation of the same side	0·53″
6. ,, ,, opposite side	0·78″
Still later :—	
7. Excitation of the same side	1·54″
8. ,, ,, opposite side	3·50″

These Kew experiments showed that at the ordinary temperature of summer the interval of time between excitation and response is considerable, and that its length depends on the distance of the seat of excitation from the led off surfaces, and is much longer in injured leaves than in fresh ones. But for reasons which I have already stated in discussing the fundamental experiment, it is certain that time measurements made by the method of signalling are incapable of giving an answer to the question of the rate at which the excitatory effect is propagated. During the hot weather of last summer (July, 1881) we had the opportunity of making experiments with the pendulum rheotome which proved in the clearest way that, under favourable conditions, the rate of propagation in the leaf of Dionæa is extremely rapid.

From the extreme difficulty which was met with last summer in obtaining plants in vigorous condition, I was prevented from making as many observations as I deemed desirable. Of those which were made I think it best to submit the results of one only, which was perfectly satisfactory :—

Period of closure.	Same lobe excited.	Opposite lobe excited.
0·02″ to 0·04″*	0	0
0·02″ ,, 0·06″	−11	0
0·02″ ,, 0·08″	−22	0
0·02″ ,, 0·10″	−31	0
0·02″ ,, 0·125″	..	9
0·02″ ,, 0·15″	..	26
0·10″ ,, 0·20″	..	50

In this experiment the plant was placed in the chamber at 38° C. and remained at that temperature throughout. The leaf was led off as in the fundamental experiment. The excitation was by induction shocks. The exciting electrodes were applied on the led off lobe close to the distal and proximal hairs respectively, so that a straight line connecting the contacts would pass through both of them. On the opposite lobe they were applied opposite to each other in the space between the hairs. The observations with excitation on the opposite side were made first, so that the effect of "exhaustion," if any, in increasing the delay, would tell against the result. Two observations were taken at each period, which coincided in most cases exactly. The weather was extremely warm and sunny. In other leaves the time interval between the two sets of results never exceeded that observed in the one recorded, so that I think I am justified in saying that the rate of transmission is at least 200 millims. per second. But it must be carefully borne in mind that this only applies to plants at hot-house temperature, as was strikingly shown in all of the experiments. A leaf, which in the chamber at 32° C. responded to an excitation of the opposite lobe when the galvanic circuit was opened at 0·08″ after excitation, gave no trace of a variation with a closing time which lasted to 0·20″, when the experiment was repeated as soon as possible afterwards in the room at 21°. On shifting K 2 to 0·20″, so that the galvanometer circuit remained closed from one-fifth of a second after excitation onwards, the image shot off the screen.

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* Trace at 0·05.

PART V.

EXCITABILITY.

The investigation of excitability in Dionæa is a matter of great difficulty, partly on account of the rapid changes which it undergoes, partly on account of the difficulty of so applying any excitant that its exciting action can be assumed to be constant. The results of our observations as to the behaviour of the leaf when excited will be best stated in relation to the different methods of excitation.

1. *Mechanical excitation.*—It was observed by Mr. DARWIN ('Insectivorous Plants,' p. 289) that when a single human hair was fixed in a handle, and cut off so that one inch projected, and "the extremity of the hair was then brought by a slow movement laterally into contact with the tips of a filament, the leaf" excited "instantly closed," or, "after two or more touches of the same kind;" whence he concluded that they were extremely "sensitive to a momentary touch" which was best adapted to enable the trap to close on an insect. To this I have to add another observation, which is at first sight so improbable that I have become convinced of its truth with great reluctance, viz.: that the touch is much more efficacious when it is a living touch, that is, when communicated by an instrument (particularly a hair or a camel-hair pencil) held in the hand, of which the motion is muscular, than by any mechanical arrangement which I have been able to contrive. The two forms of mechanical excitation which have been used are (1) a lever, of which the axis works in a firm support of which the height can be varied at will, at the end of which a camel-hair pencil is fixed of which

the point is downwards, and so constructed that when it is depressed by the finger the signalling circuit is closed and the tip of the camel-hair pencil brought into contact at the same moment; and (2) a similar lever of which the motion is due to the action of an electro-magnet included in the signalling circuit. Whichever of these forms was employed, and whatever care was taken in setting the pencil so as to ensure its hitting the hair to be excited without fail, it was invariably found that after a few excitations no further response was obtained; whereas it is always easy to repeat an experiment which I made with Dr. FRANCIS DARWIN in 1873, in which we succeeded in exciting a leaf during an hour at two minutes' intervals, without a single failure of the electrical response. This may, I think, be due to the relative rudeness (if such a word may be applied) of the mechanical motion as compared with the other. I thought at first that it might be attributed to the sameness of the motion—*i.e.*, to the circumstance that when the blows are inflicted mechanically, however gently, they are all in the same direction and each acts on a part which has been acted on before. As, however, I have not found that altering the direction of the motion makes any perceptible difference in the result, this explanation fails. What is to be learned from the observation is this: the very slightest touch is an "adequate" stimulus; any beyond this is more than sufficient and injurious. Its exhausting influence is, however, evidently perfectly local; for the loss of sensibility of one hair fails to determine any similar loss in others which have not been excited. If in any leaf a single hair, excited repeatedly by mechanical means, has ceased to respond, you can at once reproduce the result by acting on another hair, which will respond just as readily as if the leaf were intact.

Electrical excitation.

1. *Single induction shocks.*—If dry metallic electrodes are applied to the internal surface of the leaf, induction shocks conducted through them are without effect, however near the electrodes may be—the reason being, no doubt, that the surface is so perfectly dry that but little current passes between the platinum points. This difficulty may be overcome, either by using the pointed wires so that by piercing they can be brought into contact with the cells beneath, or by covering the surface with a layer of conducting material such as kaolin, made into paste with half per cent. solution of common salt. I used the former of these methods in my first experiments in 1873, as well as in those of which the results were published in 1876. For obvious reasons it is preferable to use non-polarizable electrodes, to which the form described in Part II. is the best that can be given. I have always used the small induction apparatus of DU BOIS-REYMOND without removing the core, and one DANIELL cell in the primary circuit. In all experiments opening induction shocks have been exclusively used. As a rule, the electrodes are applied to opposite points of the upper and under surface of one lobe. With reference to the action of such currents, so applied, it has been ascertained: (1) That when in a succession of excitations the secondary coil is

gradually brought nearer to the primary, it is observed that no effect whatever follows the passage of the induction current until the secondary coil is approximated to a relatively short distance, usually about 10 centims. from the primary. (2) That the excitatory effect produced at this distance is unmodified by further approximation. (3) That the distance of the secondary coil at which the first response takes place, when in a succession of excitations it is gradually approximated, is much greater when the induction current is directed from the upper to the under surface than in the contrary case; in other words, that the leaf responds to a weaker current when it is directed downwards than when it is directed upwards; and, finally (4) That if one current of an intensity a little less than sufficient to evoke a response be followed at a short interval of time by a second current of similar intensity, the latter may be effectual. This last fact will be reverted to further on, under the head of "Summation."

Excitatory influence of the voltaic current.

For the purpose of studying the influence of a weak voltaic current, a rheochord of platinum wire was used, of which the length was 14 feet and the resistance 20 ohms; this was interpolated in the circuit of a Daniell cell. Currents could be derived from it and conducted through the leaf by connecting one electrode with the end of the rheochord, the other with any point of its length by a slider. In this case the derived current might, in consequence of the relatively high resistance of the plant, be regarded as proportional to the length of wire between the end block and the slider. By comparing this rheochord with the compensator it was found that each foot of wire corresponded to a tension difference of about 0·06 D. and hence the whole wire to about 0·84 D. With this contrivance the following observations were made. In the first series the leaf was led off to the galvanometer as in the fundamental experiment, but a switch (a Pohl's Wippe, without cross wires) was introduced into the circuit, of such construction that for one or more seconds a derived current from the rheochord could be sent through the electrodes, the galvanometer circuit being for the same period broken. As care was taken to compensate immediately before the switch was thrown over, this produced no movement of the needle, but served to show in what way the passage of the current modified the electromotive properties of the leaf. The effects observed were as follows:—(1) When a very weak current was used (1 foot of wire, corresponding to a difference of potential of 0·06 D.) a slight effect, resembling polarization, was observed—*i.e.*, a transitory deflection indicating the existence in the leaf of a current opposed to the current which had been led through it, its direction being upwards when the anode was on the upper surface, and *vice versâ.* It was ascertained that this was not due to polarization of the surfaces of contact of zinc with zinc sulphate solution, for when the experiment was repeated with the clay points in contact with each other, *i.e.*, without the interposition of the leaf, no deflection was observed. (2) When the derived current was doubled (2 feet of wire) the after effect was always positive, whatever was the direction of the current. In other words, the effect showed itself in either case as an augmentation of the previous difference. Consequently the downwards current left behind it as its after effect, a current in the same direction as itself. By increasing the current, this effect, whether its direction is upwards or downwards, could be increased in about the same proportion. When the experiment was made in a leaf of which the opposite lobe was led off to the electrometer, no effect was observed during or after the passage of the current, showing that the effect was not propagated, that is, was not an excitatory effect and did not cause one. When similar currents were led in a similar way through the leaf stalk by electrodes on opposite surfaces, a polarization took place, but no effect resembling the one described was observable. (3) When (1) and (2) were alternated, that is when the current was passed through the leaf a number of times in succession in the same direction, but alternately with different strengths of current (1 foot and 2 feet of wire), the effect was, of course, alternately positive (with the stronger current) and negative (with the weaker). In both cases the effect was evanescent, and the previous difference was unaffected. (4) When the length of wire was increased to about 5 feet (a current five times as great as in (1)), different lengths, however, being required in different leaves, the effect was that of an excitation, that is, there was a negative deflection which was cut short by a larger one in the opposite direction. When this was compared with that of a mechanical excitation of one of the hairs of the opposite lobe the two variations, although resembling each other, were not identical. For if in a succession of observations a momentary (tenth of a second) closure of a downward current of sufficient strength to excite the leaf was alternated with mechanical excitation of a hair of the opposite lobe so as to compare the effects, it was found that in the variation which followed the electrical excitation, the first phase was smaller, the second phase larger than in those produced mechanically.

	Mechanical.		Electrical.
	Previous difference.	Variation.	Variation.
1st series	+0·0003	− 36 +192	− 3 + off scale (*i.e.*, over 600)
2nd series	+0·0028	− 67 +132	−10 + ditto
3rd series	+0·0035	−102 +110	−11 +500

Each series consisted of six mechanical and as many electrical excitations. The mean results of each series are given in the columns headed "Variation."

(5) The excitatory effect produced in (4) was followed by an "after effect" which subsided very gradually in the ordinary way, leaving behind it a lasting increase of the previous cross difference, and when a leaf was subjected to successive excitations, the

variation gradually changed its character just as it does when mechanical excitations follow each other at short intervals. In a leaf of which the cross difference was, at the beginning of the experiment, abnormal, and which was excited 58 times in succession in half an hour, the variation being observed alternately with the electrometer and galvanometer, the cross difference changed sign from −0·009 D. to +0·005 D. The mean difference and variation were in three successive periods as follows :—

	Cross difference.	Galvanometer.		Electrometer.	
1st period	−0·0023 D.	−40·8	+137	−29·8	+1·8
2nd period	0	−51·0	0	−23·2	0
3rd period	+0·0036 D.	−61·4	0	−21·0	0

In another similar experiment the cross difference changed, in a series of 32 excitations, from −0·011 D. to +0·01 D., the second phase (positive deflection) diminished from +210 scale to +90 scale, while the first phase (negative) increased from −15 to −220. (6) When (3) was repeated with the difference that the lengths of wire were respectively 3 feet and 5 feet, instead of 1 foot and 2 feet in two series *a* and *b*, so that the currents conducted through the leaf were, in series *a*, only sufficiently strong to produce the effect (2), viz. : a single positive deflection, whereas in series *b* they were sufficient to give rise to an excitation, it was observed that in series *a* the electrical relation between the upper and under surface remained unaltered, but that in series *b* the diphasic variation was followed by an after effect which subsided with the usual increase of the cross difference already described and exemplified.

The preceding facts may be summed up as follows :—The closing of a voltaic current directed from the upper to the under surface of the leaf of Dionæa produces different effects according to its strength. Very weak currents give rise to feeble polarization. Currents of moderate strength produce a transitory increase of the cross difference, that is, make the under surface more positive to the upper surface than it was before. Strong currents produce this transitory increase of the cross difference, but, in addition, produce an excitation which is followed by a lasting increase of the cross difference. In the successive deflections observed, these two effects are summed ; so that the negative phase is diminished and the positive phase increased.

The explanation of these phenomena depends on the meaning to be assigned to the remarkable modification of the electrical state of the leaf produced by moderate currents. On this point we can only express ourselves negatively. It is not analogous to an excitatory effect, for it is not capable of propagation : it is not identical with the after effect for it is transitory : it is not a polarization effect, for its direction is always the same as that of the cross difference whatever the direction of the current which produces it. This fact also makes it impossible to identify it with Kunkel's currents. The utmost that can be stated with reference to it is, that it indicates the occurrence of a local change in the path of the current, which is probably of the same nature as that which, when stronger currents are used, excites a propagable explosion.

By the experiments already related it has been shown that when a leaf is traversed by a voltaic current in a direction vertical to its surface for a short period (one-tenth of a second to a second) from the upper to the under surface, excitation follows as soon as the difference between the two electrodes applied to the two surfaces of a leaf at opposite points, amounts to something like half the electromotive force of a DANIELL cell. But they afford no information either as to the time or place at which the excitation occurs. It remains to be determined whether it occurs at the closure or opening of the circuit, and whether at the anode or cathode. The first of these questions was easily decided in experiments in which the leaf was led off as in the fundamental experiment and excited by currents for periods of sufficient length to render it possible to distinguish easily between the effects of opening and closing. The method consisted in making a series of observations in which the current was alternately directed upwards and downwards, and lasted from five to seven seconds, the opposite lobe being led off to the electrometer. A succession of experiments were made on different leaves in which various strengths were used, from one DANIELL, without rheochord, to two GROVES, always with the same result, namely, that the leaf responded at make, and at make only, whatever was the strength of the current or its direction. When strong currents were used, as will be noticed further on, the excitation at the closure of the circuit was succeeded by another from three to six seconds later, and might be easily mistaken for a break effect, if the moment at which it occurred happened to follow that at which the current was opened.

To the question whether excitation proceeds from the anode or cathode, we have failed entirely in our efforts to obtain an answer. The modes of experiment used were as follows:—The leaf having been led off to the electrometer as in the fundamental experiment, non-polarizable electrodes for excitation were applied on the surface of the opposite lobe and on the petiole. In some cases the lobe electrode was on the under surface, in others on the upper. A reverser was introduced into the exciting circuit, so that the current of two GROVES could be directed either towards the leaf or towards the leaf stalk. In either case the excitatory effect showed itself at make, and make only. But when the electrode was on the upper surface it was anodic, and when it was on the lower surface, cathodic. At first sight it seemed as if this indicated a difference of action between the poles, but on consideration it appeared to be another form of the result already recorded, viz.: that in the lamina the downwards current is more effectual than the upwards. A current *from* the leaf stalk which passes out at the under surface acts in the same way as a current *to* the leaf stalk which enters at the upper surface, because in the excitable part the direction of both is descending. The next attempt to settle the question was by a rheochord experiment, in which the leaf was excited by electrodes applied symmetrically to corresponding surfaces on the right and left lobes respectively, the one lobe (right or left as the case might be) being led off to the galvanometer as in the fundamental experiment. The time of closure of the exciting circuit (two LECLANCHÉ'S) was one-twentieth of a second. The galvanometer circuit was also closed for the same period, an interval of 0·03″ intervening between the opening of the one circuit and the closure of the other. In the first leaf experimented on, the results were remarkable. In a series of twelve observations in which the current was directed across the leaf, alternately from and to the led off lobe, the needle remained motionless when, as in the first case, the anode was next the leading off surface, but swung to the negative side from 12 to 14 scale when the cathode was in the same position. As repetition confirmed the result it was clear that, as regards the particular leaf under observation, the proximity of the cathode was favourable to the early occurrence of the excitatory effect. Doubt was, however, suggested by the consideration that here, when the cathode was on the left lobe, the direction of the current near the leading off electrodes was downwards, that is, favourable to the excitatory effect, whereas in the opposite lobe it was unfavourable, and this surmise we confirmed by the observation that in another leaf, in which an exactly similar experiment was made, with the exciting electrodes applied to the internal surface, the response was strongest when the anode was on the led off lobe. All that was learned from these attempts was, that by the method employed, which seemed to be the only one likely to yield an answer to the question, no answer could be obtained. The want of material made it impossible to proceed further, but even had it been forthcoming I should have been at a loss how to use it.

Influence of the Duration of the Current on the Effect.

1. *Short periods.*—In order to determine the shortest period of closure of a voltaic current directed transversely from the upper to the under surface of a leaf, experiments were made in which the time of closure was gradually shortened by means of the pendulum rheotome. It was found that there was no diminution of effect until the duration of the closure was less than one-hundredth of a second. Beyond this, the response became uncertain when two DANIELL'S with a rheochord of 20 ohms were used, and disappeared when the closing time was reduced to 0·007. Want of material made it impossible to investigate the important question of the influence of strength of current on the minimum. It was, however, ascertained that it was necessary to lengthen it when weaker currents were used.

2. *Currents of long duration.*—When a voltaic current of five seconds' duration and of moderate strength is led through the leaf from the upper surface between the sensitive hairs to the under surface, there is a response at make, and no other. When stronger currents are used (from one DANIELL to two GROVES) the first variation is followed by a second. If the current is continued for a long period (30 seconds) a succession of excitatory variations takes place at irregular intervals, as shown in the following

examples. If the leaf is expanded and not restrained, it may close at the first excitatory variation, *i.e.*, one to one-and-a-half second after the closure of the current. But if the experiment is made at a low temperature (18° to 20° C.) this event does not happen until after the third, fourth, or fifth variation. To observe these facts the current of a GROVE cell is led through one lobe, while the other is led off (in the usual way, to the electrometer, and the time after closure of the circuit noted by the watch, as in the following experiments :—

I. Current of one GROVE from above downwards for half a minute; excitatory variation at 3″, 9″, 13″, 19″, 23″, 30″.

Same current directed upwards—variation at closure only.

II. Current of one GROVE from above downwards for one minute; variation at 1″, 15″, 25″, 34″, 38″, 45″.

Same current directed upwards—variation at closure only.

III. Same leaf. Current directed through contacts previously used for leading off; leaf led off by lobe previously excited. Variation at 2″, 8″, 22″, 32″.

Same current in opposite direction—variation at closure only.

Summation of stimuli.—To prove summation it must be shown that two equal excitations, each of which alone is inadequate to evoke a response, do so by summation, that is, when they follow each other at a short interval. If this is so, there is evidence that the first, although it is apparently without effect, yet produces a change in the excitable structure which renders it more excitable than it was before. The best method consists in making two alternating series of excitations, in one of which single opening induction shocks, just inadequate to produce excitatory effect, are used, while in the other, two induction currents of equal strength follow each other at a variable interval.

1st Series. Distance of coil 7·6 centims. Temperature of chamber 32° C. Interval 0·02.″ Ten excitations, of which Nos. 2, 4, 6, 8 were single, 1, 3, 5, 7, 9 were repeated. All of the latter were effectual; all of the former ineffectual.

2nd Series. Distance of coil 7·4 centims. Temperature 32° C. Interval 0·1.″ Ten excitations; six single, five repeated ($\frac{1}{10}$″). All of the latter were effectual; all but one of the former ineffectual.

3rd Series. Distance of coil 7·3 centims. Temperature 32° C. Interval 0·2.″ Sixteen excitations, of which half single, the rest repeated ($\frac{1}{5}$″). All of the latter were effectual; all except two of the former ineffectual.

4th Series. Another leaf. Distance of coil 8 centims. Temperature 32° C. Four excitations; two single, two repeated at intervals of $\frac{1}{8}$″. Both of the latter effectual; the other ineffectual.

Four excitations; two single, two repeated at intervals of $\frac{2}{8}$″. The latter effectual; the former ineffectual. In this series it could easily be seen that the excitatory effect followed the second excitation.

5th Series. Eighteen double excitations at intervals of half a second; of these eight were fruitless. Of the ten responses, half occurred at the first excitation, the remainder at the second.

These experiments show that at all intervals between 0·02″ and 0·4″ summation occurs, and that at 0·5″ the result becomes uncertain.

Another method may be used which consists in subjecting a leaf to repeated excitations by successive series of inadequate induction shocks at a fixed interval—*e.g.*, at a twentieth of a second, gradually increasing the number in each series until a response is obtained. In an experiment of this kind a leaf responded in a room at 20° C. when the number of opening shocks amounted to 23. In the chamber at 32° C. it responded after 7. It is to be noted that, in an experiment of this kind, the distance of the secondary coil from the primary must very slightly exceed that at which single shocks are responded to without fail. This method is useful as a means of testing excitability under different conditions.

PART VI.

RELATION BETWEEN THE EXCITATORY PROCESS AND THE MECHANICAL EFFECT.

The moment at which the mechanical effect begins can be easily measured graphically by the following method. A cork is prepared, with a hole across its axis for the reception of the round horizontal arm of a support resembling that shown in fig. 4 (electrodes). On this the cork can be rotated. The cork is cut so as to present a surface somewhat smaller than that of a lobe of the leaf. To this a leaf is cemented with plaster of Paris by the external surface of one lobe, leaving the midrib free. To the border of the opposite lobe a very light straw lever is fixed by the same means, in the position it would assume if it were a prolongation of the middle marginal hair. This having been done, the lever is made horizontal by rotating the cork which, with the leaf, is brought into such a position that its motions are inscribed on a blackened cylinder, of which the horizontal movement should be 1 centim. per second. The leaf is excited mechanically by a camel-hair pencil of which the motion breaks a circuit. The moment at which this happens is recorded on the same cylinder by a DEPREZ' time marker. It is thus learnt that the mechanical response, at temperatures from 15° to 20° C., begins at an interval of from a minute-and-a-half to two minutes after excitation, so that in the fundamental experiment it coincides with the close of the first phase of the variation. If this observation is made at a low temperature, as *e.g.*, 15° C., and the hairs are touched with great care, the first excitation produces no perceptible motion. At the fifth or sixth the lever slowly rises and continues to do so for several seconds (see Proceedings 1876, p. 415). To record this effect, the best plan is to leave the lever free and record its position at the end of every half second on a smoked glass plate. The result is shown in the figure. It is seen that in each case the rate of ascent of the lever is greatest during the first half second, after which it gradually declines, and that the total effect consequent on each excitation is very variable, being dependent on accidental differences in the way in which the sensitive hair is touched.

The mirror experiment.—The effect of series of frequently repeated mechanical excitations on leaves at relatively low temperatures may be observed with greater accuracy by another method. The leaf is attached to the rotating cork in exactly the same way as in the experiment last described, but instead of a lever a very light glass mirror, such as is used for reflecting galvanometers, is cemented to the external surface of the free lobe, near the margin. By means of this mirror the image of a horizontal slit is thrown on a vertical scale which is so graduated that the angular movement of the lobe can be measured with great delicacy. An experiment of this kind gives the following results. The leaf having been led off to the electrometer by electrodes applied to the upper surface of the attached lobe and to the midrib, was subjected to 22 excitations, each of which consisted in touching very gently one of the hairs of the attached lobe. The total angular movement (rotation) of the mirror was 167°. This was accomplished in 20 excitations, all of which were effectual in the sense that each produced a normal variation.

Number of excitation.	Effect.	Sum of effects.
1	°·0	°·0
2	·0	·0
3	0·5	0·5
4	0·4	0·9
5	0·4	1·3
6	0·8	2·1
7	1·0	3·1
8	4·1	4·5
9	3·5	8·0
10	4·0	12·0
11	5·5	17·5
12	7·5	25·0
13	13·0	38·0
14	15·0	53·0
15	42·0	95·0
16	34·0	129·0
17	10·0	139·0
18	11·0	150·0
19	13·0	163·0
20	4·0	167·0

The fact that in this and in other similar experiments it is possible to excite the leaf once, twice, or a greater number of times without any appreciable movement of the image, at first sight seems to indicate that the electrical effect is independent of the mechanical. But before accepting such an inference it must be remembered that we cannot be at all sure that the interstitial movement of liquid, which in the leaf as in all the moving organs of plants is the efficient cause of change of form, may not begin without making itself visible by any change in the curvature of the lobe, however delicate our means of observation may be. It appears to me probable that every effectual excitation—every excitation which is marked by electrical disturbance—is also followed by diminution of the water-charge of the excited protoplasm. At first, this may either expend itself entirely or produce only such small effects as, in the experiment last related, presented themselves in excitations 3 to 7.

In our previous paper the phenomena described above were referred to as evidences of summation. It will be better in future to limit the application of this word to the still more remarkable phenomenon which has been described under the heading "Summation" in Part V., p. 47; for those phenomena are strictly analogous to the effects to which the term "Summation of stimuli" has been applied by STIRLING.[*] The term "Summation" is in itself as applicable to the process now under consideration as to the other, but it differs from it in this respect—that it is rather a summation of effects than a summation of excitations. The process with which it is most comparable in the physiology of animal excitable tissues is that which is known as the *Treppe* or staircase.[†] By this term is designated the fact that when the ventricle of the heart of the frog is subjected, after a period of rest, to a series of effectual excitations, the contractions become stronger and stronger, although the excitations are all of equal intensity. In both processes the mechanical effect of the first excitation is much smaller than that of the second, the second smaller than that of the third, and so on, but in the *Treppe* the difference which is large at first rapidly diminishes, whereas in the leaf it as rapidly increases. Further, the total mechanical effect of the first excitation is so small as to be barely perceptible, whereas in the heart the first contraction (*i.e.*, the weakest) is at least half as vigorous as the strongest. But in proceeding with our comparison we find that in both, the change of form is opposed by a resistance which has its seat in the structure of the organ, and is removed step by step when it is subjected to repeated excitations. In the heart this resistance diminishes rapidly at first, more slowly afterwards: in the lamina of the leaf it does not begin to yield until after several excitations, provided that they are only just strong enough to provoke an electrical response. This difference is unimportant, and becomes almost imperceptible if somewhat stronger excitations are substituted for the minimal ones. In this case the effect of the very first excitation may be large, that of the second larger, the third still larger, and so on, but with decreasing, not increasing increments, just as in the *Treppe*. The most fundamental difference is that which concerns the relation between the resistance and the mechanism of contraction in the two cases. In the ventricle only a fraction of the mechanical effect, even of the first contraction, is due to abolition of resistance. At each repetition of the excitation this fraction diminishes, until after a dozen or so of excitations the successive mechanical effects become sensibly equal. From the first, therefore, the change of form is due to something more than mere diminution of resistance. In the leaf the removal of resistance is everything. Consequently, so soon as a sufficient number of excitations have occurred to accomplish this, it is not only incapable of resuming its original form, but of any further response to excitation until, after many hours, the resistance is restored.

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[*] STIRLING, "Ueber die Summation electrischer Hautreize." LUDWIG'S Arbeiten, 1874, p. 223.

[†] See BOWDITCH, "Ueber die Reizbarkeit der Muskelfasern des Herzens." LUDWIG'S Arbeiten, 1871, p. 156.

If we go a step further and enquire what this resistance consists of, we are led, in accordance with the conclusion that has been arrived at by every physiologist who has investigated the mechanism of the changes of form of the excitable organs of plants, to identify it with turgescence. By turgescence we understand the power which living protoplasm possesses of retaining water. In the case of cells which are excitable, the immediate effect of excitation is suddenly to diminish this power, and thereby to produce a diminution of volume of the cells which is equal to that of the water (probably holding diffusible bodies in solution) which is discharged into the intercellular spaces.

ADDENDUM.

Received November 3, 1881.

[*The following is a Summary of the most important Experimental Results embodied in Parts III., IV., and V.*]

1. When different parts of the surface of the uninjured and unexcited leaf of Dionæa are compared by the method of compensation described in Part II., electrical differences present themselves even when the points selected are symmetrical. In this case the differences observed are accidental. They are due either to surface conditions which disappear when the leaf, with its leading off electrodes in position, is allowed to remain in saturated air (differences of turgidity of the surface layers of cells) or to accidental physiological differences between the two lobes, which do not disappear.

2. Similarly, when unsymmetrical points of either surface of the leaf are led off, the differences observed are for the most part accidental. In one case, however—that of the comparison of the middle of the under surface of either lobe with the under surface of the petiole—the negativity of the former to the latter is a result which occurs so constantly that it must be assumed to have a physiological meaning.

3. If any two points opposite to each other on the upper and under surface of either lobe are compared, it is usually found that the upper surface is negative to the lower. Whether this is so or not in the unexcited leaf, it becomes so after one or more excitations; in other words, the effect of mechanical or electrical excitation, whether applied in the neighbourhood of the surfaces compared or at a distance from them, is to produce a lasting change in their electrical relation of such a nature that on repeating the comparison of the two surfaces the *under surface* is always *found to have become more positive* (*less negative*) *than it was before excitation.*

4. If the same observation is made while the leaf is subjected to mechanical or electrical excitation, it is seen that at the moment that excitation takes place the under surface becomes suddenly negative to the upper. By the rheotome we learn that the change last mentioned does not begin until $\frac{1}{20}$th of a second after its cause, that it culminates about the middle of the first second, the maximum difference amounting to as much as 0·08 DANIELL, and then gradually subsides.

5. It is further learnt by the same mode of observation that in the course of the second second or towards its close, the electrical relation of the surfaces to each other is reversed, the under surface becoming positive to the upper. The difference of potential in this direction having rapidly augmented until it amounts to about 0·02 DANIELL, very slowly subsides into the lasting effect of excitation described in 1.

6. When two symmetrical points on the under surfaces of opposite lobes are similarly compared during excitation, the excitatory effect differs in character according to the relation between the seat of excitation and the leading off contacts, the surface of the lobe which is directly excited becoming negative to the other, consequently by changing the positive seat of excitation the direction of the first effect may be reversed.*

7. When a voltaic current is led across the lobe from the upper to the under surface by electrodes applied opposite to each other near the sensitive hairs, but not touching them, at the same time that the electrical state of the opposite lobe is observed as in 4, a response occurs at the moment that the current is closed, provided that its strength is such that the available electromotive force amounts to about half that of a DANIELL cell and that the temperature is not below 30° C. No response occurs at opening the current. If a stronger current (two DANIELLS) is used and the direction is downwards, the response at closing the current is followed by several others. This effect does not happen when the current is directed upwards.

8. A voltaic current directed from the upper to the under surface, which is too weak to evoke an excitatory response, produces an increase of the positivity of the under surface, limited to the part of the lobe through which the current passes, which lasts several seconds after the current is broken. Its direction shows that it is not due to "polarization."

9. Voltaic currents of less than $\frac{1}{200}$th second duration, though of moderate strength, do not excite Dionæa. Weaker currents cease to act when their duration is less than $\frac{1}{100}$th second. But the relation between strength and duration has not been ascertained.

10. Opening induction currents such as are yielded by DU BOIS-REYMOND'S smaller inductorium when the secondary coil is at a distance of 8 to 10 centims. from the

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* It cannot be concluded either from the result stated above or from the preceding ones that in Dionæa, as in muscle and nerve, the excited part becomes *negative* to other parts. For it is as consistent with the facts observed to say that the upper surface becomes positive as the immediate result of excitation, as to say that the under surface becomes negative.

primary, may for many purposes be used more advantageously than any other mode of excitation, for by means of them a leaf may be subjected to repeated excitations for several hours without failure or appreciable exhaustion.

The minimum effectual strength of an opening induction current depends on its direction, weaker currents being responded to when they are directed downwards than in the contrary case. Induction currents which are inadequate to evoke a response are yet followed by a local change in the electrical state of the surfaces through which they are led which resembles that produced by weak voltaic currents.

11. *Summation of stimuli.*—Two inadequate opening induction currents which follow one another at any interval greater than $\frac{2}{100}''$, or less than $\frac{4}{10}$ths of a second, may evoke a response. In this case the response follows the second excitation. When a leaf is subjected to series of inadequate induction currents at short intervals (*e.g.*, $\frac{1}{20}$th of a second) the response may occur after a greater or less number of excitations, according to the temperature at which the experiment is made and the strength of the current.

12. *Summation of effects.*—In a series of mechanical excitations each of which is just adequate to produce an electrical response, those which occur earliest are followed by no visible change of form. Of the later members of the series each produces a measurable movement, the extent of which becomes greater each time that the excitation is repeated until eventually the leaf closes.]

CONCLUSION.

In the preceding paragraphs all reference has been omitted to the physiological meaning of the electromotive phenomena which have been described. The moment has now arrived at which it appears necessary to offer such an explanation as the knowledge acquired justifies, as to their relation to the vital processes with which they are associated.

According to Professor Munk the electromotive properties of the leaf of Dionæa may be accounted for on the supposition that in each cell the ends of the cell are, in the resting state, positive to the middle, and that in excitation the difference of potential between the ends (poles) of the cell and its middle (equator) undergoes a sudden diminution. This theory must be rejected at once, on the ground that it fails to explain the fundamental experiment. The electromotive forces it supposes to be in operation act in directions parallel to the surface of the leaf—at right angles, therefore, to the path of the currents which show themselves when the two opposite surfaces of the leaf are led off to the galvanometer. Under these conditions it is impossible that the latter can be the expression of the former.

On the other hand, I accept as fundamental the doctrine that whatever physiological properties the leaf possesses, it possesses by virtue of its being a system of living cells. The first question, therefore, to be determined is that of the electromotive endowments of the individual cell.

Each of the two lobes of the leaf of Dionæa is made up, for the most part, of cylindrical cells. The long axes of these cells (parenchyma cells) are all parallel to each other, and to the vascular bundles which run out from the midrib towards the margin. A transverse section of the leaf, made in such a way as to include a vascular bundle, shows that there are two or three layers of them, the most superficial of which are covered by the epidermis of the under and upper surfaces of the leaf. As they are alike in structure, it may be assumed that they have the same or similar functions.

If it were possible, without injury, to investigate the electromotive properties of one of these cells individually, by leading off its opposite surfaces, we should probably find them to be isoelectrical both at rest and when excited; for the electrical differences which exist in the cell are probably between the living inside and the non-living outside, and not between different parts of the surface. Consequently the electrical differences which show themselves between the opposite sides of the leaf blade which is made up of layers of such cells cannot be due to the summation of smaller differences between the two sides of the individual layer, for these are isoelectrical. They must, therefore, be referred to the contact of different layers of cells with each other. But if electrical differences exist at the surfaces of contact of the cells, they must be associated with physical differences of other kinds. With respect to the nature of this difference it appears to be very probable that the most important element is migration of water. For on the one hand we know that in consequence of the surface evaporation, migration of water certainly exists, while on the other we have proof in the experiments of Dr. KUNKEL, that such migration cannot occur without producing electrical differences.

As has been so conclusively proved by the experiments of PFEFFER, those motions of the organs of plants which follow excitation are attended by diminution of the turgor or water-charge of the protoplasm of the excitable cells. The displacement of water occasioned by this discharge must unquestionably be attended by electrical change. In Dionæa the mechanical effect of an excitation can be seen from one to two minutes after the moment at which the excitation takes place. It is at the same moment that the second phase, or after effect, begins. In their origin, culmination, progress, and duration the two changes are closely associated. It therefore appears extremely probable that the two are causally related—that the positivity of the external surface is an expression of the electrical differences which exist between contiguous strata in different degrees of turgor—the less charged being negative to the more.

The same theory affords us a complete and satisfactory explanation of the gradual change of potential which always results from repeated excitation, as well as for the fact so constantly observed, that the "after effect" which follows each of the first few excitations to which a leaf is subjected, is much greater in extent than those which occur after later ones, and that eventually the after effect fails entirely, reappearing after a period of repose.

I need scarcely say that the first phase of the variation—the effect which immediately follows excitation and has an opposite sign to the after effect and a much higher electromotive force—does not admit of a similar explanation; for it cannot be imagined that a change which spreads over the whole lamina in less than one-twentieth of a second can be dependent on migration of water. The excitatory disturbance which immediately follows excitation is an explosive molecular change, which by the mode of its origin, the suddenness of its incidence, and the rapidity of its propagation is distinguished from every other phenomenon except the one with which I have identified it, namely, the corresponding process in the excitable tissues of animals.

Of the nature of this preliminary disturbance (to which alone the term excitatory variation ought to be applied, it alone being the analogue of the "action current" of animal physiology) we know nothing. Just as in the case of muscle and nerve, the proof that it is attended by any chemical alteration or by the separation of any product of disintegration is wanting, so it is here. In the one case as in the other, we must regard the electrical change as a visible sign of an unknown molecular process. That it is not the primary change which occurs in protoplasm when it enters into the state of excitation we have evidence in two facts, first, that even when the exciting agent is an induction current which passes through one of the opposite surfaces by which the leaf is led off to the galvanometer, so that no time whatever is lost in transmission, there is still an interval between excitation and response of about 0·03″, during which interval molecular changes are obviously in progress; and, secondly, that excitation just insufficient to evoke a response gives effect to a second which follows it, provided that the interval between the first and the second does not exceed one-third of a second—a result which can only be explained on the supposition that the first excitation actually effectuates a molecular change, notwithstanding that the visible sign is wanting.

The direction of the excitatory effect in the fundamental experiment is such as to indicate that in excitation, excited cells become positive to unexcited, whereas in animal tissues excited parts always become negative to unexcited. The apparent discrepancy will probably find its explanation in the difference of the structural relations of the electromotive surfaces.

II. Lucifer: *a Study in Morphology.*

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Communicated by Professor HUXLEY, *Sec. R.S.*

Received April 6,—Read April 28, 1881.

[PLATES 1–11.]

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SECTION I.—INTRODUCTORY.

THE general anatomy of the adult *Lucifer* has been satisfactorily made known by the observations of SOULEYET, HUXLEY, HENSEN, DANA, SEMPER, CLAUS, DOHRN, and FAXON; and the only facts which I have to add relate to the structure of the reproductive organs.

The earliest recorded observations upon this subject are by DANA ('United States Exploring Expedition during the Years 1838, 1839, 1840, 1841, and 1842,' under the Command of CHARLES WILKES, U.S.N., vol. xiii., part 1). In plate 44, fig. 9, *b*, *h*, and *m″*, he gives a very correct representation of the male reproductive organs and spermatophore of an adult male specimen of *Lucifer* (*acestra*); but his description of these figures (p. 670) shows that he was completely at a loss for an interpretation of the parts which he has represented, and had no idea of their true function.

Later students have entirely overlooked these figures by DANA, and there has been some doubt whether *Lucifer* is an adult animal at all, rather than the young form of some other Decapod.

In 1861, SEMPER (Reisebericht des Herrn Dr. SEMPERS. Ein Schreiben an A. KÖLLIKER; Zeit. f. Wiss. Zool., xi., 1861, pp. 100–108) re-discovered and described the male organs, and also the female organs, of a large, transparent, and probably new species which he found at Zamboanga.

He gives no figures, and his short account, which is in the main correct, is as follows:—

"Die Geschlechtsöffnung ist einfach, liegt bei beiden Geschlechtern in der Mittellinie des Bauches, dicht hinter dem letzten Brustfusse. Der Hode besteht aus einer, in der Mittellinie des Thorax, dicht unter dem Magen liegenden Samendrüse, an deren hinteres Ende, dort wo der kurze Samenleiter entspringt, sich mehrere Nebendrüsen ansetzen. Der Same wird, noch unentwickelt, in einen birnförmigen grossen Spermatophor eingeschlossen. Das hinterste Ende dieser männlichen Drüse reicht bis in die Mitte des ersten Hinterleibgliedes, das vorderste bis ziemlich dicht an den Schlund.

"Das Weibchen hat zwei Eierstöcke, die vom Ende des sechsten Hinterleibgliedes an dicht unter dem Darm, sich bis in die Mitte des Thorax erstrecken, hier biegen sich die beiden Samenleiter nach unten, und schwellen dann zu zwei grossen Taschen an, die eine kleine Tasche umfassen; die Geschlechtsöffnung ist einfach; ein einziger Spermatophor steckt mit seinem spitzen Ende darin. Entwickelte Zoospermien habe ich nicht beobachtet. Weibliche Begattungsorgane fehlen. Die Entwickelungsgeschichte ist mir unbekannt geblieben."

The male organ has two external openings; they are not on the median line, and their position in the body does not correspond to that of the female orifice; but in other respects my own observations show the correctness of this description.

As SEMPER does not give any account of the general structure of these sexual individuals, CLAUS ("Ueber einige Schizopoden und niedere Malacostraken Messinas," Zeit. f. Wiss. Zool., xiii., 1863, pp. 433–437) held that the adult nature of *Lucifer* must still be a matter of uncertainty; but in 1871 DOHRN verified SEMPER'S account from alcoholic specimens ("Untersuchungen über Bau und Entwickelung der Arthropoden," von Dr. ANT. DOHRN, Zeit. f. Wiss. Zool., xxi., 1871, p. 357), and showed that the mature animals have the form which had been described by THOMPSON ('Zoological Researches and Illustrations,' 1829, p. 58) as characteristic of the genus.

In the following year SEMPER ("Zoologische Aphorismen, von C. SEMPER: I. Einige Bemerkungen über die Gattung *Lucifer*," Zeit. f. Wiss. Zool., xxii., 1872, p. 305) published a second paper, in which he gave two good figures of the male and female reproductive organs (taf. xxii., figs. 3 and 4), but added nothing to his earlier description.

During my own studies upon the development of the larva I found an abundant supply of adult specimens of both sexes, and am thus enabled to give a more complete account of the structure and relations of the reproductive organs.

Plate 9, fig. 75, is a side view of the carapace (*c*) and the first abdominal somite (*a*) of an adult male, showing the first abdominal foot (*Pl.* 1) and the basal joints of the third maxilliped (*Mp.* 3), and the first, second, and third thoracic limbs (*Pr.* 1, *Pr.* 2, *Pr.* 3). The testis (*t*) consists of a series of about eight pouches or follicles, which hang down into the body cavity under the anterior end of the intestine (*i*). The body of the animal is so thin that it is almost impossible to get a good dorsal view without crushing the specimen; but a very careful examination of the side view seems to show that there is only a single organ on the median line of the body, as SEMPER states. On each side of the intestine, along the line where the testis joins its wall, a small tubular vas deferens (*vd*) arises, and runs backwards along the side of the intestine nearly to the end of the first abdominal somite, to which it seems to be attached (at *l*) by a ligament. It then bends outwards and forwards upon itself to form a second much larger portion (*sp*), which is parallel to and outside of the first portion, and reaches nearly to the anterior edge of the first abdominal somite. The third or terminal portion (*sv*) has a large cavity, thick walls, and it runs down to an external opening which is situated on the outer edge of the sternal surface of the thoracic region, behind the basal joint of the third pereiopod, and therefore in the position which would be occupied by the basal joints of the fourth or fifth pereiopods if they were present.

There is a vas deferens, made up of these three portions, on each side of the body, and the ventral nerve chain (*tg*) passes between their external openings.

The more anterior follicles of the testis are almost perfectly transparent, but the development of the male cells in the posterior ones gives to them a faint granulation. The first division of the vas deferens (*vd*) has a small cavity, thin walls, and as it usually seems to be entirely empty it is probable that the passage of the male cells from the testis through it to the second division (*sp*) takes place quickly. The second division (*sp*) has a very large cavity, and in it the male cells become arranged in a single layer around the surface of a central core, which is formed of some dense transparent adhesive substance.

The spermatophore appears to pass into the third chamber (*sv*) before it is completely formed, as all those which were seen in the second chamber consisted only of a central core and a layer of male cells, while those which were contained in the thick-walled third chamber had an outer enveloping capsule.

I found several specimens with a fully-developed spermatophore on one side of the body and none on the other side, and was thus enabled to thoroughly satisfy myself of the presence of two vasa deferentia, and two external openings.

I was unable to discover how the spermatophore is transported to the body of the female, or what part the clasping organ (*c*) upon the first pleopod of the male performs during the act of copulation.

Upon several occasions I observed a male clinging to the basal joints of the first antennæ of a female, but as I never succeeded in getting the pair under a lens without separating them, I made no careful examination. Copulation usually takes place during the daytime, or at least this was the case in every instance which I observed. In several cases I found female specimens with a single fresh spermatophore attached to the opening (Plate 9, fig. 74, *o*) of the seminal receptacle (*sr*). This opening is situated between and a little anterior to the basal joints of the third pair of thoracic limbs (*Pr.* 3 of fig. 74). As the spermatophore gradually discharged its contents, it was easy to see that both the central core and the investing layer of spermatozoa escaped from the outer sheath and passed into the seminal receptacle. In all the breeding females which I observed the spermatozoa filled the posterior, and the transparent core of the spermatophore the anterior half of the spermatic receptacle, as shown in fig. 74. The ovary is very long (fig. 74, *ov*), and it lies under the intestine, reaching from the fifth abdominal somite to the posterior edge of the carapace, where it bends upon itself at right angles and runs down to its external opening, which is upon or close to the median line of the ventral surface, a little in front of the third pair of pereiopods. The wall of the ovary is so very thin and delicate that I was not able to detect it at all except when it was filled with ripe ova. These are very much elongated, granular, and slightly opaque; and there does not seem to be any shell around them. They are very elastic, and undergo great changes of shape as they pass through the small oviduct.

Oviposition occurs between 9 and 10 o'clock in the evening, and occupies only a few minutes.

After the eggs are laid they are spherical, transparent, and each one has a rather thick shell. They are attached, in a loose bunch of twenty or more, to the last pair of thoracic limbs, and in order to save space I have shown them in fig. 74, although the specimen from which the figure was drawn had not laid any of its eggs.

As I obtained very few ripe females, I was not able to sacrifice one of them to study the reproductive organs under pressure, and I am therefore unable to decide whether any parts of this system are double; but I feel confident that there is only one spermatic receptacle, and the opening of the oviduct seems to be upon the median line.

We found a few adult specimens out at sea, but, while I was able to learn little about their habits, I think that they are not strictly pelagic, but that their proper home is the salt marshes close to the ocean.

They were met with in the greatest abundance about half-a-mile inside Old Topsail Inlet, near a large marsh, during the first hour of the ebb tide, on calm evenings when the tide turned between 7 and 8 P.M.; and I infer that they leave the marshes at this time to breed in the ocean. All the mature females which we found, with one exception, were captured under these peculiar conditions; and we never failed to find them at this spot when the tide turned about sunset and the water was calm.

Owing to this singular limitation there are only a very few favourable evenings for procuring the eggs in a single season; and until the animals can be made to thrive and multiply in confinement, it must always remain an extremely difficult matter to procure the eggs in abundance.

Up to the present time our knowledge of the early stages of *Lucifer* has been extremely meagre.

In his report on the Crustacea of the United States Exploring Expedition, DANA described (p. 634) an organism under the name of *Erichthina demissa*, and figured it in plate 42, fig. 3. CLAUS ('Crustacean System,' p. 13) gives a figure of the same organism at a latter stage of development, and calls attention to the numerous features of resemblance to the *Protozoëa* stage of development of *Penæus.*

Only a few months before his death, the lamented WILLEMÖES-SUHM collected a number of specimens of *Erichthina* in the South Pacific, and, associated with them, a sufficient number of later stages to assure him that *Erichthina* is the larva of *Lucifer*. His account (" Preliminary Remarks on the Development of some Pelagic Decapods," by R. von WILLEMÖES-SUHM, Ph.D., Proc. Royal Soc., Dec. 9, 1875, p. 132–4) is very brief, and as it contains all that is known about the metamorphosis of this extremely interesting form, I quote it in full:—

" Very similar to that of *Sergestes* is the development of *Leucifer*. Here the earliest *Zoëa* of a species from the Western Pacific has got at first no eyes, then sessile ones came out, and the animal then presents the form which DANA has called *Erichthina demissa*; and which CLAUS suspected to be not a Stomatopod but a Schizopod larva. After the second moulting this *Erichthina* gets stalked eyes, and very long setæ on all its appendages, becoming a rather long, very delicate *Zoëa*. It now enters the *Amphion* stage, but never gets more than four pairs of pereiopods, and loses another pair of these when it moults for the youngest *Leucifer* stage, in which two pairs of pereiopods are absent.

"The next question after having found this out, was, of course, whether *Amphion*, *Sergestes*, and *Leucifer* leave the egg as a *Zoëa*, or whether there is a preceding *Nauplius* stage. My own impression is that in the two first-named genera this is not the case, as the youngest *Zoëas* which I caught had all the same size, and as none of them was without the large lateral stalked eyes. As for *Leucifer*, the question appears to me to be doubtful; for it is, from what I have seen, quite possible that my youngest *Zoëa*, which has only got a central eye, may be preceded by a *Nauplius*. Of course, the simplest thing would be to get the eggs; but there is the difficulty, for *Amphion* is caught very rarely, and has never been obtained at any other time but between 8 and 12 P.M., when it is extremely difficult by lamplight to find out the youngest stages. *Sergestes* larvæ are commoner, appearing also in the daytime, and *Leucifer* is sometimes caught in abundance. I hope, therefore, that I shall succeed in completing my researches about this question, especially as far as the two latter genera are concerned.

" H.M.S. 'Challenger,' Honolulu, Sandwich Islands.
"July 30, 1875."

As the sad death of this lamented naturalist, only a short time after, put an end to this as well as to his other researches in all departments of zoology, I take pleasure in stating that I have fortunately been able to complete his observations upon *Lucifer*, and to furnish a very perfect account of its entire metamorphosis, as well as a few important facts with reference to its development in the egg.

At the end of April, 1880, I found a single specimen of *Lucifer* with two eggs attached to one of its appendages, and I was led by the great importance and interest of the subject to make every effort to trace its life-history. For four months I met with no success whatever, but about the 1st of September I found a few advanced larvæ, and traced them to the adult, and I then succeeded in finding earlier stages and tracing them as far as the stages which I had previously found, but it was not until the last week of my season at the sea-shore that I succeeded in hatching the *Nauplius* from the egg, and the last gap in my series was bridged by a moult which occurred only a few hours before my departure.

As the result of my four months' efforts I can now state that I have seen the eggs of *Lucifer* pass out of the oviduct. I have seen the *Nauplius* embryo escape from the same egg which I had seen laid, and I have traced every moult from the *Nauplius* to the adult in isolated specimens. There is therefore no Crustacean with the metamorphosis of which we are more thoroughly acquainted than we now are with that of this extremely interesting genus.

Not only is it true, as WILLEMÖES-SUHM has pointed out, than DANA'S *Erichthina demissa* is a larval stage of *Lucifer*, but that DANA'S *Sceletina armata* is a later stage in the same series, while some of the forms which he includes in his genus *Furcilia* are also, in all probability, *Lucifer* larvæ.

The occurrence of a free *Nauplius* stage of development in the life-history of one of the higher Crustacea is a matter of such profound significance in the scientific discussion of the phenomena of embryology in general, that it can hardly be accepted without question so long as there is any possibility of error. Two of the observers who have testified to its occurrence have based their conclusions upon evidence which would be perfectly satisfactory in any ordinary case, but as they did not actually trace all the stages of development their statements do not stand the severe analysis which the importance of the case demands, and certain naturalists have therefore refused to give them unqualified acceptance.

The third observation was made so many years ago, and the larva is so briefly described, that it would not be safe to assume, in the absence of all corroborative evidence, that it is a *Nauplius* at all.

In December, 1838, DANA found in the harbour of Rio de Janeiro great numbers of specimens of a Schizopod, which he described ('United States Exploring Expedition during the Years 1838-1842,' under the command of CHARLES WILKES, U.S.N., vol. xiii., part i., p. 654) as *Macromysis gracilis*. In the brood-pouches of some of his specimens he found an abundance of eggs and developing embryos, several of which are shown in his plate 45, fig. 5. He made no careful study of their structure; his notice of them in the text is only a few words; and his figures are very small, and show the embryos in dorsal view, as seen under a very low magnifying power, but they are so much like Fritz Müller's figures, that we must acknowledge that the credit of the first discovery of a Malacostracan *Nauplius* belongs to Dana, and that up to the present time this is the only case in which a *Nauplius* has been traced to an egg which could be definitely identified as that of a specific adult Malacostracan, although his account is so imperfect that in itself it is certainly not sufficient to prove the existence of the *Nauplius* stage at all.

In 1861 Fritz Müller found, at Desterro, in Brazil, a single specimen of a *Nauplius* ("Die Verwandlung der Garneelen." Erster Beitrag von Fritz Müller, in Desterro, Arch. f. Naturgeschichte, 1863, p. 9), which he traced, through other specimens which were also collected in the ocean, to a form which he believed to change into the youngest *Zoëa* of a species of *Penæus*. The series of stages is so satisfactory that there is no reason for doubting the accuracy of his conclusion, but the chances for error, in the attempt to trace Crustacean development from isolated specimens, are so very great that the statement has not received unqualified acceptance.

The only other recorded observation of a Malacostracan *Nauplius* is not among the Decapods, but in the more embryonic Schizopods. These observations, which were made by Metschnickoff, would tend to corroborate those by Müller, but they are unfortunately open to the same criticism. He did not actually rear the larvæ and trace them to a specific adult, and although there would in ordinary cases be no doubt of the correctness of his conclusion, a careful analysis of his papers will show that there certainly is a possibility of error.

In the spring of 1868 he collected from the surface of the ocean at Messina a few early stages in the development of a Crustacean, which he believed to be *Euphausia mulleri* (Claus), and showed ("Ueber ein Larvenstadium von *Euphausia*" von El. Metschnickoff in Petersburg, Zeit. f. Wiss. Zool., xxix., 1869, p. 479, taf. xxxvi.) that it passes through a well-marked *Nauplius* stage, of which he gives three figures.

The following year, at Villafranca, he collected a good supply of young larvæ and floating eggs in advanced stages of development, and was thus enabled to supplement his first paper by a second ("Ueber die Naupliuszustände von *Euphausia*," von Elias Metschnickoff, Zeit. f. Wiss. Zool., xxi., 1870, p. 380, taf. xxxiv.) in which he gives a minute account of the *Nauplius* from the time it leaves the egg until it changes into a form somewhat similar to the youngest stage of *Euphausia*, which had been previously described by Claus ("Ueber einige Schizopoden und niedere Malacostraken Messinas," von Prof. Dr. C. Claus, Zeit. f. Wiss. Zool., xiii., 1863, p. 422). Claus had supposed this to be the stage in which the larva leaves the egg, and he says (p. 450), "Diese Larve bin ich geneigt für die jüngste aller freieren Entwickelungsformen der *Euphausia* anzusehen." He subsequently learned, however ("Untersuchungen zur Erforschung der Genealogischen Grundlage des Crustacean-Systems," p. 9), that he had been in error, since he afterwards found, in an Atlantic and also in a Mediterranean species, an earlier *Protozoëa* stage, which changed into the *Zoëa* described in his first paper. It therefore follows that METSCHNICKOFF studied something else, or that he was in error in believing that he had traced his *Nauplius* directly to what CLAUS has shown to be a somewhat late stage in the development of *Euphausia*. METSCHNICKOFF'S only reason for believing that his *Nauplius* is a young *Euphausia* is its resemblance to CLAUS'S larva, and as there is certainly an error here, we are not justified in giving unqualified acceptance to his statement that it is an *Euphausia* larva. It seems very probable, indeed, that this is the case, but in the absence of the direct evidence which could only be afforded by actually tracing it back to an *Euphausia* egg, or forwards to the adult *Euphausia*, I do not think that the existence of a Malacostracan *Nauplius* can be said to be established by these observations, for they do not stand the severe test which is demanded by their unusual importance, and I think the facts justify the statement that, up to the present time, there has been no unquestionable evidence of the occurrence of such a stage of development in the higher Crustacea.

The present series of observations is complete at both ends, and I have not relied upon surface-collecting to fill a single gap, but have traced every stage in isolated captive specimens, and the possibility of error seems to be entirely out of the question.

The close resemblance between the *Nauplius* of *Lucifer*, and MÜLLER'S and METSCHNICKOFF'S larvæ, renders it almost certain that they also are Malacostracan larvæ, but before this corroborative evidence was furnished, it was certainly quite possible, although hardly reasonable, to doubt whether this was true of either of them.

II.—THE SEGMENTATION OF THE EGG, AND FORMATION OF THE FOOD-YOLK AND PRIMITIVE DIGESTIVE CAVITY.

Unusual difficulties attend the study of the early stages in the embryology of *Lucifer*, and the observations which I have been able to make are incomplete, and leave many gaps to be filled and many interesting points to be decided by future investigations ; but the facts which I have made out are so novel, so different from all that was previously known of the early stages of Arthropod development, and they throw so much light upon the relation of the peculiar and greatly modified form of segmentation characteristic of the group to the less modified form of segmentation presented by the more normal eggs of other animals, that it seems best to give my results in their present incomplete state.

I am the more willing to do this, because the peculiar difficulties of the subject leave little hope for the attainment of more complete results in the future.

The eggs are so loosely attached to the appendages of the female that they are broken off by the slightest roughness of handling, and it is very difficult to obtain them by collecting the egg-bearing females. Even when great numbers of mature specimens are captured in the breeding season, with the greatest care and delicacy, very few of them, much less than 1 per cent., are found to have eggs attached to their limbs when the collection is examined.

If the mature animals could be induced to thrive and multiply in confinement, there would be no difficulty in obtaining a sufficient supply of eggs, but until this can be done it must be extremely difficult to procure them in sufficient numbers for exhaustive study.

During the early stages the eggs are so delicate that they are soon destroyed by the confinement and compression to which it is necessary to subject them while they are under examination, and it is therefore impossible to watch very many stages in a single egg.

When we add to this that the eggs are laid about 9 o'clock in the evening, and must be studied between this time and daylight, after several hours of laborious collecting, by eyes that have been already severely taxed with looking over the collections and picking out the transparent and almost invisible adults by an artificial light, and examining each one of them with a lens to find those which carry eggs, the difficulty of the subject will be appreciated.

The eggs are spherical, transparent, and they contain extremely little food-material. This is uniformly distributed over the whole egg in minute globules, which have nearly the same colour and refractive index as the surrounding protoplasm.

The egg undergoes total regular segmentation, and a true segmentation cavity occupies the place filled by the large central yolk-mass in the eggs of other Arthropods.

It first divides into two equal portions (Plate 1, fig. 1); then, by a cleavage at right angles to the first, into four (fig. 5); then into eight (fig. 8); then into sixteen (fig. 10); and so on.

At the stage shown in fig. 10 the inner ends of the sixteen spherules are seen to be separated from each other by a central space, the segmentation cavity, which persists, and is shown at later stages in figs. 11, 13, 15, 16, 17, 19, and 20, at *b*.

In fig. 10 the egg will be seen to be spherical, and all the segments have their broad ends at the surface; but in the next stage one pole of the egg becomes a little flattened, and in an optical section the spherule (*c*), which occupies the centre of the flattened area, is seen to have its broad end nearest the centre of the egg.

Most of the food-material has meanwhile disappeared from the other spherules, which are now quite transparent, while the spherule (*c*) still contains as much as ever, but apparently no more than there was contained at an earlier stage in an equal area of any part of the egg. In an optical section of the same egg, in a plane at right angles to that of fig. 11, the spherule (*c*) shows a trace of a fissure, which a little later divides it into two (see fig. 12, *c*).

Plate 2, fig. 13, is an optical section, like the one given in fig. 11, of a somewhat older egg; and fig. 14 an optical section of the same egg at right angles to fig. 13. The outline is a little more flattened on one side than it is in fig. 11, and the spherule (*c*) is completely divided by a radial fissure into two, and these project into the segmentation cavity (*b*) a little more than they did before.

In fig. 15 the flattening has become a deep pit (*d*), and the spherules (*c*) have been pushed quite into the segmentation cavity, and the adjacent cells have begun to move in the same direction. This change is more marked in fig. 16; and in fig. 17 the egg consists of a double wall of cells, the ectoderm and the endoderm, surrounding a primitive digestive cavity (*d*), and separated from each other by the segmentation cavity (*b*), in which the two cells (*c*) are situated. Each of these also shows traces of a division into two.

These changes are more marked in fig. 19; and in fig. 20 the opening of the primitive digestive cavity is much reduced in size, and the cavity itself does not lie exactly in the axis of the egg, but at one side of it.

A more minute examination of the segmentation brings out a number of interesting points; one of them is the rhythmical character of the process, which is not a continuous uniform change, but a series of stages of activity, separated from each other by periods of rest.

The egg shown in Plate 1, fig. 1, was laid about 10 o'clock P.M., and about 10.35 it was in the condition which is represented in the figure. As I had not been watching it I did not observe the first division, and when first seen it was in the resting condition, and the two spherules were not sharply defined, but pressed together.

During the next fifteen minutes no external change was visible, and the drawing was made at 10.50 P.M. It then entered upon the second period of segmenting activity, and in five minutes the two spherules were well defined, as shown in fig. 2; and in five minutes more (fig. 3) one of them showed traces of division into two. In ten minutes more (fig. 4) this division was completed, and traces of a similar change had made their appearance in the other spherule, which was also perfectly divided into two at the end of five minutes more (fig. 5). This stage ended the second period of activity, which was twenty-five minutes long.

During the whole of this time the egg showed gradual and uniform change, which was sufficiently rapid to be distinctly visible. Although four so-called stages are figured, there was no division into stages, but a continuous change without interruption.

The four spherules now began to flatten down, and in five minutes the egg was in the condition which is shown in fig. 6, and it then remained without any external change for more than ten minutes. The second period of rest, measured from the time when the four spherules began to shrink together to the time when they began to swell out and enter upon the third period of active segmentation, was therefore more than fifteen minutes long.

At 11.40 the four spherules were once more sharply defined (fig. 7), and changes went on uniformly until, at 12.15 A.M., each was perfectly divided into two, as shown in fig. 8, which marks the end of the third period of activity, thirty-five minutes long.

I was not able to watch this egg pass into the next resting stage, as it had been so long under observation (1 hour 45 minutes) that its development was arrested at this point; but another egg in this stage of development was seen to pass into the resting condition, as shown in fig. 9, and it then remained quiet for about fifteen minutes, showing no external indications of change during this time.

At the end of the third period of rest the spherules again became prominent, so that the outline of the egg was exactly like that of fig. 8, and the egg entered upon the fourth period of activity, soon dividing into sixteen spherules (fig. 10), arranged around a segmentation cavity.

In about twenty-five minutes from the beginning of this period of activity the spherules began to flatten down once more, and the egg passed into the fourth resting stage, but it was not observed beyond this point.

The alternation of activity with rest was observed at much later stages, but after the gastrula invagination makes its appearance the cells of the endodermic portion of the egg do not undergo active change at the same time with those of the ectoderm, and the egg has one set of periods of activity for each layer. As development goes on the periods of rest grow longer and the periods of activity shorter, and the spherules do not flatten down while at rest.

The egg which is shown in optical longitudinal section in fig. 16 was in the field of the microscope for nearly twenty minutes, while I was examining another specimen. An occasional look at it showed that it was not changing, but at the end of this time I noticed that the outer ends of the ectoderm cells directly opposite the orifice of invagination were notched, as is shown in the figure. Activity spread in all directions from this point, and in less than five minutes all the cells were notched, and those nearest the centre of the area of activity were perfectly divided into halves. In about five minutes more all the ectodermal cells had divided, and this layer had the appearance shown in fig. 17—which, however, was drawn from another specimen.

This last egg remained in the condition shown in the drawing for fifteen minutes from the time it was first observed, and a movement of the appendage to which it was fastened caused it to roll over and present its formative pole for examination before the beginning of the next period of activity, which is shown in surface view in fig. 18. The manner of division was simply a repetition of that which has just been described.

The cells nearest the centre of the formative area became notched, and then divided into halves; and the activity gradually spread over the egg in all directions, until, in a few minutes, all the cells which were visible were at some stage of division.

The rapidity and uniformity with which this change spread over the egg rendered it an extremely interesting and impressive sight, and I know of no other case in which segmentation is so perfectly regular at such an advanced stage of development.

The activity did not affect the endoderm cells in either of these cases, but at a later stage (fig. 20) they were seen to be in an active segmenting condition at a time when the ectoderm cells were at rest. I was not able to keep this egg alive long enough to watch the completion of the process, for it had been under the microscope for some time before the stage shown in the figure was reached; but the division of the endoderm cells appears to go on much more slowly than that of the ectoderm cells.

This phenomenon, the alternation of periods of rest with the periods of active segmentation, does not seem to have received from embryologists the attention which it deserves. A number of observers have pointed out that in many animals, among the Mollusca especially, the distinctness of the spherules becomes more or less completely obscured after each division, and that this state persists until just before the next division, when the spherules swell out and again become conspicuous. The change of form does not seem to be at all general, and in most accounts of segmentation nothing of the kind is recorded.

I believe that it is a secondary phenomenon, and that the essential thing is the alternation of rest with activity; and I am confident that careful time records of segmentation will show that this occurs in nearly every case, sometimes with and sometimes without the accompanying change of form.

I have observed it in *Physa, Limnæus,* and *Planorbis,* where segmentation is total and nearly regular; in the Oyster, where the egg has a rudimentary food-yolk and segmentation is irregular; in a bony fish with a large food-yolk and a discoidal segmentation; and in *Lucifer.* Other investigators working under my guidance have observed it in *Amblystoma* and in oligochætous and polychætous Annelids. These are all the cases in which I have been able to test the matter since my attention has been attracted to the subject; and as the alternation was found to occur in every case, although the animals are so widely separated and present such diverse modes of segmentation, I feel justified in assuming that the phenomenon is general, and will be found in all eggs which can be properly examined by watching and timing them while segmentation is going on.

The cause of rhythmical physiological change is an extremely interesting question; and as the segmenting egg exhibits the phenomenon in the greatest possible simplicity, it would seem to be a peculiarly favourable subject for investigation.

The phenomena which have been described seem to show that segmentation is not due to the action of any purely molecular force, like polarity, but is essentially a vital activity, and in a paper on the embryology of the fresh-water *Pulmonates* ('Studies from the Biological Laboratory of the Johns Hopkins University,' vol. i., part ii.) I have ventured the following explanation.

During the period of segmentation the protoplasm of the whole egg (of *Physa*) gradually becomes more and more transparent, on account of the gradual disappearance of the granular food-material which it contains, and the rhythmical character of the process of segmentation would seem to admit of a simple explanation on the supposition that the physical properties of the protoplasm offer a resistance which must be overcome before the force which is set free by the assimilation and reduction of the food-material can exert itself to bring about the active changes of segmentation. During a period of rest the process of digestion and assimilation accumulates a store of energy which, at length, becomes sufficient to overcome this resistance, and to initiate a period of activity which lasts until the whole of this reserve of force has been expended in the rearrangement of the protoplasm. The physical properties of the protoplasm now reassert themselves, and tend to reduce the whole egg as nearly as possible to a spherical form once more, and the egg then remains inactive until the supply of energy again becomes great enough to overcome the resistance.

If this is the true explanation we should expect to find the alternation of rest and activity much more general than the change of form, for the degree of consistency of the protoplasm or the amount or character of the food-material, or the way in which it is distributed through the egg, may prevent the second set of changes from showing themselves. This is precisely what we do find, and in the bony fishes, where the large food-yolk would prevent any marked change of form, we find the first set of changes well marked, but with no trace of the second set.

Leaving this subject for the present, I wish to say a few words about another interesting phase of the early stages of *Lucifer*. We cannot fail to be impressed by the very remarkable departure from ordinary Arthropod segmentation, nor can we overlook the fact that in all the points of difference from the eggs of allied forms, the eggs of *Lucifer* show a most suggestive resemblance to the ordinary unspecialized ova of other Metazoa.

In an ordinary Arthropodan egg we have, as the outcome of the process of segmentation, a central mass of food-yolk, which may or may not be divided into segmentation products, and which completely fills the segmentation cavity; and an outer investing layer of blastoderm cells; that is, the egg undergoes a centrolycethal segmentation.*

In most Crustacea the early stages of segmentation are regular, and apparently total, but the lines of cleavage do not pass entirely through the egg, and the spherules are united to each other by a central mass of food-yolk. When segmentation is somewhat advanced the products of segmentation become more or less pyramidal, with the bases of the pyramids at the surface, and their apices fused together at the centre of the egg. The outer ends of the pyramids then become transparent and separate off as a blastoderm, while the inner portions usually fuse together, more or less perfectly, to form a central food-yolk, which fills the space which in ordinary eggs constitutes the segmentation cavity. A small portion of the blastoderm then becomes invaginated to form the primitive digestion cavity, and the remainder becomes the ectoderm.

The centrolycethal type of segmentation presents great variations in the different groups of Arthropods, but in nearly all cases its peculiarities are so well marked

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* The whole subject of segmentation has been so ably and exhaustively reviewed by BALFOUR in his recent work on 'Comparative Embryology,' that it does not seem necessary to burden this paper with a long list of references to the literature of Arthropod segmentation, or to enter into an exposition of the present state of our knowledge of the subject. All the essential facts and opinions may be found on pages 79–99, 317–379, and 425–433 of vol. i. of the 'Comparative Embryology.'

that it is difficult to trace any resemblance to the various forms of segmentation which occur in other groups of animals. In *Lucifer* the case is reversed, and we have a type of segmentation which is obviously similar to that of the Echinoderms, Annelids, Molluscs, Tunicates, Vertebrates, &c., but is less obviously related to that of the eggs of closely allied forms. The resemblance to what may be called "normal" segmentation is so plain that it need not be dwelt upon, but the relation between the egg of *Lucifer* and an ordinary centrolycethal egg is by no means clear.

It seems probable, however, that since the food-material which has not been assimilated becomes centralised, after segmentation is somewhat advanced, in the single spherule *c*, of fig. 11, this spherule must correspond to one of the yolk-pyramids of an ordinary Crustacean egg. This then divides, by radial fission, into two portions (fig. 13, *c*), and it seems probable that the food-material then becomes restricted to their central ends, while the outer protoplasmic ends separate off as a pair of blastoderm cells (fig. 15), thus leaving the two masses of food-yolk (*c*) inside the segmentation cavity. While I was investigating the subject I regarded the spherule *c*, of fig. 11, as a primary mesoblast, which became pushed into the segmentation cavity, and then divided up to form the mesoderm; and I expressed this view without comment in a preliminary abstract of the subject ("Embryology and Metamorphosis of the Sergestidæ," Zoologischer Anzeiger, iii., p. 563). In most cases where the origin of the mesoderm has been most carefully studied, it originates by the separation of the inner ends of the cells which are to give rise to the endoderm, either before or during or after the invagination takes place; the mode of origin of these spherules in *Lucifer* and their position in the egg agree with what we should expect if they belong to the mesoderm, but the great quantity of food-material which they contain would hardly be looked for in this case, and favours the view that they are yolk-pyramids rather than mesoblasts.

As I examined no eggs between fig. 20 and fig. 21, the later history is uncertain, but a reference to figs. 21, 22, 23, and 24, which are about twenty hours later than fig. 20, shows that the region of the digestive tract of the *Nauplius* is marked by the presence of a number of large polygonal masses of what appears to be food-yolk, and it seems probable that these are the derivatives of the spherules *c*, of fig. 20. I was not able to actually witness the change from fig. 11 to fig. 15, and cannot state with absolute certainty that the spherules *c* divide into a central and a peripheral portion. Fig. 15 seems to indicate that this is the case, but in the absence of direct observation of the change, it is possible that the two cells which in fig. 15 lie below the cells *c*, are the ones which were at its sides in fig. 11.

If each of the cells *c* gives rise to a blastoderm cell, we should expect to find two more cells in fig. 15 than in fig. 18, but the number is the same. This is hardly a safe guide, however, for while the drawings are careful copies from Nature, they are not from the same egg, and the cells are so wedged together that vertical sections in different planes would not intersect the same number in all cases, and there may have been two more in fig. 15 than in fig. 13.

I think, then, that the facts indicate that *c* of fig. 11 is a yolk-pyramid, rather than a primary mesoblast, and that after it divides into two, as in fig. 13, each part gives rise to a central portion *c*, and a peripheral endoderm cell.

If we accept this view and regard the cell *c* as a yolk-pyramid, two views as to the relationship between the egg of *Lucifer* and an ordinary Crustacean egg at once suggest themselves.

We may hold that *Lucifer* presents the primitive or ancestral form of segmentation, of which centrolycethal segmentation is a secondary modification. In this case we may suppose that as the supply of food-material gradually increased, new food-bearing cells or yolk-pyramids were added until all the cells were included, and the segmentation cavity was entirely filled and obliterated by them.

According to the other view, we may hold that the segmentation of the *Lucifer* egg is a secondary modification, which has been brought about by the gradual reduction of the amount of food-material, and its restriction, at last, to a single one of the cells of the segmenting egg.

There does not seem to be much difficulty in deciding which of these views is most satisfactory and probable. *Lucifer* is undoubtedly a very primitive Malacostracan, but it can hardly be regarded as a primitive Crustacean ; and the occurrence of perfectly centrolycethic segmentation in the Copopods, Phyllopods, Amphipods, and Isopods, as well as in the Decapods—forms below as well as forms above *Lucifer*—forbids us to believe that the egg of *Lucifer* is ancestral, or the unmodified descendant of an ancestral type of egg ; and we must therefore believe that the egg of *Lucifer* has been simplified by the loss of the greater part of its food-yolk.

A change of this kind is not without a parallel, and I have shown ("The Acquisition and Loss of a Food-Yolk in Molluscan Eggs," 'Studies from the Biological Laboratory of the Johns Hopkins University,' vol. i., part iv.) that the resemblance between the segmenting egg of the Oyster and a Molluscan egg with a food-yolk can only be explained by the supposition that the Lamellibranchs have inherited a rudimentary food-yolk which was functional at some past time, and that the assumption gives an explanation of all the peculiarities of oyster segmentation.

If we accept this view, and regard the egg of *Lucifer* as simplified by secondary change, it is extremely instructive to note that the loss of a food-yolk has brought it back to a type of segmentation which is directly comparable with that of ordinary Metazoan eggs, and we must therefore believe that a segmentation cavity is potentially present in all centrolycethic eggs, or else that the segmentation cavity of the egg of *Lucifer* is not homologous with that of ordinary eggs.

III.—General Account of the Metamorphosis of Lucifer.

The most instructive method of studying the metamorphosis of *Lucifer* is to trace each part of the body through the series of changes which it undergoes from its first appearance until it assumes the adult form; but as this method of comparing the successive stages in the development of each organ necessarily involves references to other organs, it seems best to give first a general account of the whole structure of the larva at each stage of development, and afterwards to go over the same ground more rapidly in a different way, and to trace the history of each appendage.

The egg Nauplius.

About thirty hours after oviposition the eye spot and appendages of the *Nauplius* became visible inside the egg-shell, as shown in a ventral view in Plate 2, fig. 21, and in a dorsal view in fig. 22. If the egg-shell is torn at this stage the embryo escapes, and swims about quite vigorously for a short time, but soon dies. The various parts of the body are much better shown in the swimming embryo than while it is contained in the egg, and I therefore give, for comparison with figs. 21 and 22, a dorsal view (fig. 23) and a ventral view (fig. 24) of an embryo which has thus been set free.

Fig. 23 shows an embryo of exactly the same age as those in figs. 21 and 22, while fig. 24 was drawn from an embryo a few hours older. The difference in the outline of the body is not due to this difference in age, however, but to a slight change in the point of view. In all four figures the letter *e* marks the anterior end of the body, and fig. 22 is a view directly opposite to fig. 21. Fig. 23 is in the same position as fig. 22, but the embryo shown in fig. 24 was in such a position that more of the anterior surface and less of the posterior surface was visible than in the other figures.

On the median line of the ventral surface the labrum (figs. 21 and 24, *L*) is very conspicuous at the anterior end of the body, and behind it there is a double row of four pairs of bud-like eminences, arranged in a longitudinal series. The first pair (figs. 21 and 24, *g*) are much larger than the others, and the depression which separates them on the median line is less marked than it is in the three pairs which lie behind. It is rather difficult to decide with certainty what this pair of buds becomes, but in the larva which Metschnickoff studied the changes were more gradual than they are in *Lucifer*, and he was therefore able to trace their history more satisfactorily, and to show that they become the metastoma. Their position with reference to other parts indicates that they have the same history here, and that the other three pairs of buds are the first and second maxillæ and the first pair of maxillipeds (*Mx.* 1, *Mx.* 2, and *Mp.* 1).

Three pairs of much larger appendages are folded down on to the sides of the body, within the egg; and when the embryo is set free they are seen to be the first antennæ

(*A*), the second antennæ (*An*), and the mandibles (*M*). They are not divided into joints or rings, although the second antennæ and the mandibles are biramous, and consist of a basal portion or protopodite, an expodite, and an endopodite. All three pairs have hairs projecting from their tips, and these lengthen considerably within a few minutes after the embryo is freed from the egg. The first antennæ are nearly as long as the second, and both pairs, as well as the mandibles, are organs of locomotion, to row the animal through the water. The motions of the larva are very erratic and violent, and consist of a series of quick leaps produced by vigorous backward strokes of the appendages.

The outline of the body will be understood by a reference to the figures. When the second maxillæ are in the centre of the field of view, as in fig. 21, the outline is pear-shaped, with the broad end of the pear at the posterior end of the body; but when the metastoma is in the centre this is reversed, and the broad end is in front. This difference is due to the fact that the dorsal region is much wider than the labrum and series of buds, which together form a ridge along the ventral surface.

In a dorsal view the simple eye (*Oc*) is seen as a black spot on the middle line, near the anterior end of the body. It did not show any traces of a division into halves at any stage of development which was observed.

The ocellus lies upon a large rounded granular body, which is imperfectly divided into halves by a notch upon its posterior margin. This body consists of the fused cerebral ganglia.

The dorsal portion of the posterior region of the body is swollen and rounded, as shown in figs. 21 and 23; and near its lateral margins there are a pair of small, but very conspicuous, dark pigment-spots (*i*), which might easily be mistaken at this stage for ocelli, since they have almost exactly the same size and colour. These two pigment-spots are very conspicuous during all the early stages of the metamorphosis, and their position during the later stages (figs. 25, 26, 27, 34, 35, and 47, *p*) shows that the portion of the *Nauplius* body which bears them becomes the thoracic, not the abdominal, region of the adult.

In the interior of the enlarged posterior portion of the body there is a huge mass of polygonal highly-refractive bodies, which appear to constitute a food-yolk, and which surround the digestive tract of the embryo. I have already given my reasons for believing that those bodies are derived from the spherule which becomes pushed into the segmentation cavity during the early stages of development. If this is their origin they must increase in size between the stage shown in fig. 20 and that shown in fig. 21. This is not at all an unusual occurrence, and in the fresh-water Pulmonates the yolk-spherules which surround the digestive tract continue to grow until a very advanced stage of development. I found so few eggs at this stage that I was afraid to sacrifice any of them by attempting to study their internal structure under pressure, and I am not able to give an account of the digestive tract or of the other internal organs.

When the embryo is set free from the egg it is seen to be inclosed by a delicate cuticle, which is shown, around the antennæ, in figs. 23 and 24. It is soon stripped off by the vigorous movements of the larva, and in fig. 24 it has been torn from all the appendages except the first antennæ (*A*).

In a dorsal view a number of muscular fibres are seen to extend outwards and forwards from the median line of the body to the basal joints of the antennæ.

The posterior end of the body is not notched, the anus is absent, and there is no trace of the telson or of the carapace.

The first free Nauplius *stage.*

About thirty-six hours after oviposition the larva escapes from the egg as a *Nauplius*, $\frac{8}{1000}$ inch long, which is shown in side view in Plate 3, fig. 25. There is now no difficulty in keeping it alive and rearing it, and it swims very actively by vigorous strokes of its two pairs of antennæ. Its movements are very characteristic, and much like those of a Copepod or Cirrhiped *Nauplius*.

The most important differences between it and the egg *Nauplius* are the segmentation of the locomotor appendages, the lengthening of their hairs, the increased size and dendritic form of the pigment-spots (*p*), and the appearance of the telson (*Tl*), as a projecting fold furnished with two pairs of short spines or hairs, in the ventral surface of the posterior end of the body.

As regards the more minute structure of the appendages, the first antennæ (fig. 25, *A*) are five-jointed, and the hairs, which are more than half as long as the limb, are borne on the terminal joint.

The second antenna consists of a two-jointed basal portion or protopodite which carries two rami, one of which (fig. 25 *ex*), is obscurely divided into three nearly equal joints, while the other (fig. 25, *en*), is divided into eight very distinctly marked joints. Both at this stage and later the appendage possesses considerable power of rotation, and sometimes the branch *ex*, and sometimes the branch *en*, is on the outer surface. It is therefore very difficult to decide from an examination of this appendage alone which branch is the exopodite and which the endopodite ; but, as I shall show further on, a comparison with other appendages at a later stage indicates that the eight-jointed ramus is the endopodite, although the limb is frequently, and perhaps generally, carried in a position which brings this branch on to the outside. At this stage the locomotor hairs of both branches are confined to the tips of the terminal joints. The first and second joints of the endopodite are quite short, while the other six are longer and nearly equal in length.

The mandible consists of a short unjointed basal segment, which carries a one-jointed endopodite, and an obscurely three-jointed exopodite. Each branch carries three hairs, which are somewhat longer than the limb, and the entire length of the appendage, including the hairs, is about equal to the length of the first or second antenna, without its hairs. There are no cutting blades or hooks upon the basal joints of either pair of antennæ or the mandibles.

The labrum (*L*) is somewhat larger and more prominent than it was at the stage before, and the anus is still absent.

The second free Nauplius *stage or meta*-Nauplius.

In about twelve or fourteen hours the *Nauplius* sheds its skin and assumes the form shown in Plate 3, fig. 26. From the prominence of the region of the hind body, and the presence of a carapace, CLAUS has distinguished this stage of development, in allied forms, by the name of meta-*Nauplius.*

I did not actually witness the change, and am not sure of the exact length of the first free *Nauplius* stage, but it is not more than eighteen, and probably no more than twelve hours long. A *Nauplius* which had hatched from the egg some time during the latter part of Monday night was placed, alone, in a watch-glass of sea-water, and changed into the one from which fig. 26 was drawn before 9 P.M. on Tuesday evening.

The differences between this and the preceding stage are sufficiently great to attract the attention at first sight. The length, as measured from the ocellus to the posterior end of the body, has increased from $\frac{8}{1000}$ inch to $\frac{9}{1000}$ inch. The labrum (*L*) is longer and more prominent. The first antennæ (*A*) are unjointed, and the joints of the second antennæ (*An*) and mandibles (*M*) are almost absent.

The hairs at the tips of the endopodites of the second antennæ and mandibles (*en*) are irregularly plumose, and a long slender slightly curved hair is carried by each of the larger joints of the endopodite of the second antennæ.

On the inner posterior edge of the basal joint of the mandible, a short stout curved hook or blade has made its appearance. The four pairs of buds on the ventral surface, posterior to the labrum, are in the same condition as before, but the telson (*T*) is quite prominent, notched or forked, and furnished with two pairs of short stout spines, the inner pair being much longer than the outer. A well-marked fold (*c*) of the surface of the body now marks the posterior and the lateral edges of the carapace, but this line is not continued on to the anterior end of the body, and the posterior edge is not yet raised or separated from the hind body as it is, according to METSCHINCKOFF, in the last *Nauplius* stage of *Euphausia.*

The pigment-spots (*p*) are drawn out in such a way as to surround a large rectangular area, at the posterior end of the carapace, and in the region where the heart is placed at the next stage.

The digestive tract is now visible in a side view. The œsophagus (*œ*) runs upwards and forwards from the mouth, situated under the overhanging tip of the labrum, and then bends backwards and upwards to open into the floor of the stomach (*s*); the side walls and top of the stomach could be made out without difficulty, but I was not able to decide whether its ventral wall is complete or not. It is divided by a fold or flap in its dorsal wall into a small rounded anterior chamber, into which the œsophagus opens, and a longer posterior chamber, with its dorsal wall very thick, which gives rise at its posterior end to the intestine (*i*). The greater part of the anterior chamber lies in front of the œsophageal opening. On each side of the stomach there is a group of polygonal yolk-cells (*l*), which are by no means as conspicuous as they were at an earlier stage. The intestine is small, with thin walls, and it follows the dorsal curvature of the body to the anus, which was visible in a ventral view just in front of the spines of the telson, at the point marked (*a*) in fig. 26. The cerebral ganglia (*ga*), and the ocellus (*oc*), are still visible, and underneath the stomach there is an elongated granular body (*n*), obscurely divided into segments, which is, without doubt, the rudimentary ventral nervous system.

As it was necessary to keep this larva alive I did not dare to use much pressure whilst examining it, and was therefore unable to make a very thorough study of its internal structure.

The first Protozoëa *stage.*

On Tuesday evening, September 28th, at 9.30 P.M., the *Nauplius* which has just been described was placed alone in a watch-glass of sea-water, and at 9 A.M. on Wednesday, the 29th, it had changed into the larva which is shown in dorsal view in Plate 3, fig. 27. The number of segments and appendages of this larva and its general form and proportions are like those of the *Euphausia*, *Penæus*, and *Sergestes* larvæ at the stage of development which CLAUS has proposed to call a *Protozoëa* ('Crustacean System,' p. 2). The precise time when the change took place could not be learned, but there is reason to believe that it was not much later than the middle of the night. On September 14th I obtained, by dipping with a surface-net, a *Protozoëa*, which I studied and drew. It was of exactly the same size ($\frac{20}{1000}$ of an inch measured from the tip of the rostrum to the bases of the spines of the telson) as the one which moulted from the *Nauplius*, and it agreed with this in every respect except that the free segments of the hind body, shown in fig. 27, were wanting. It hardly seems probable that there are two stages of exactly the same size between 9.30 P.M. and 9 A.M., and it is much more probable that the body segments do not become distinct until some time after the moult, and as the larva had them at 9 A.M., I infer that it was nearer the end than the beginning of the first *Protozoëa* stage, and that the change had taken place some hours before I examined it.

CLAUS is inclined to believe that the difference between FRITZ MÜLLER'S last figure of the *Nauplius* of *Penæus* and his first figure of the *Protozoëa* is so great that there must be a gap in the series of observations. The isolated *Nauplius* of *Lucifer* passes through quite as great a change in twelve hours, and its length increases from $\frac{9}{1000}$ to $\frac{20}{1000}$, or more than 100 per cent., and there does not seem to be any necessity for supposing that FRITZ MÜLLER has missed a stage in order to account for the change in his larva.

In the case of *Lucifer* the actual increase in size is not very great, but the carapace becomes folded out over the body, and the thick posterior portion of the body of the *Nauplius* becomes pulled out into the long free movable hind body of the *Protozoëa*, so that the length is more than doubled, while the vertical thickness of the body is correspondingly reduced. The shape of the larva when seen from one side will be understood by a reference to Plate 4, fig. 35, for although this figure was drawn from an older larva, it correctly represents a side view of fig. 27 in all essentials.

The most marked differences between the meta-*Nauplius* of *Lucifer* and the *Protozoëa* are due to the development of the carapace and the hind body. The carapace (fig. 27) is horse-shoe shaped, with smooth lateral and posterior edges, and it forms about one-half of the total length of the body. On the median line of the anterior edge it is drawn out into a long rostrum (*R*), at the base of which are the cerebral ganglia (*ga*) and the ocellus (*Oc*). On the median line of the posterior edge of the dorsal surface there is a shorter dorsal spine (*ds*), and at the outer angles of the posterior edge a pair of lateral spines (*ls*), which are a little longer than the dorsal one. The side view (fig. 35) shows that the sides of the carapace have folded down on to the sides of the body, and all the appendages, except the antennæ, are almost completely covered by it. The appendages are so nearly alike in this and the next stage that it will be most convenient to describe them together.

The stomach (*s*) is now divided into a pair of anterior or cephalic, and a pair of posterior or hepatic lobes, and between the cephalic lobes a number of muscular fibres run upwards and forwards from the œsophagus to be attached (at *m*) to the carapace. The intestine is small and straight (*i*), but it is not of uniform character, and is divided into a series of small enlargements separated from each other by constricted portions.

The last of these enlargements is much more constant than the others, and its walls are attached to the integument of the abdomen by a number of small muscles.

It exhibits regular pulsations, which seem to draw water into and out of the anus (*a*), which is on the ventral surface of the telson.

The heart (*h*) is compact, short, situated near the posterior edge of the carapace, and it gives rise to a single median and two lateral anterior arteries.

The hind body is about as long as the carapace, and it is divided into four somites and a long unsegmented region (*abd*). The study of the appendages shows that the four somites are those which carry the third pair of maxillipeds (*Mp.* 3), and the first, second, and third thoracic somites (*T* 1, *T* 2, and *T* 3). There are no traces of appendages on any of them. The end of the unsegmented region of the hind body forms a well-marked flattened telson (*T*), which is slightly notched on the median line, and carries four pairs of stout spines, and one pair of very small ones. The small ones are nearest the median line; the third pair are the longest and largest, and the fifth pair spring from the edges of the telson, some distance from the end.

The second Protozoëa *stage.*

As my season's work at the sea-shore ended the day the *Nauplius* shown in fig. 26 turned into the *Protozoëa* shown in fig. 27, I was not able to trace the development of that specimen; but on September 14th I had captured and drawn a larva in the same stage, and this moulted, while isolated in a watch-glass, into the second *Protozoëa* which is shown from above in Plate 3, fig. 34, and from the right side in Plate 4, fig. 35.

This larva measures $\frac{27}{1000}$ inch from the tip of the rostrum to the fork of the telson. The appendages are like those of the first *Zoëa* in number and structure, but there is a well-marked difference in the shape of the body. The carapace is somewhat elongated, its anterior edge is less perfectly rounded than before, and a pigment-spot (fig. 34, *E*) represents the future compound eye.

The pouches of the stomach (*s*) are much more conspicuous than before, and the œsophagus (fig. 34, *œ*) is visible in a dorsal view, between its anterior or cephalic lobes. The four somites of the hind body (*Mp.* 3, *T* 1, *T* 2, and *T* 3) have become short, but there is, as yet, no trace of their appendages. The unsegmented portion of the abdomen (*abd*) has increased in length, as have also the spines of the telson (*T*). The two pairs of antennæ have substantially the same form that they had during the *Nauplius* stage, and they are still the chief locomotor organs. The larva swims by jerks like a *Nauplius* or a Copepod.

The appendages at this as well as at the preceding stage are as follows (see Plate 4, fig. 35): the long uniramous first antennæ (*A*); the biramous second antennæ (*An.*); the cutting mandibles (*M*); the biramous first and second maxillæ (*Mx.* 1, *Mx.* 2); and two pairs of biramous maxillipeds (*Mp.* 1, *Mp.* 2).

The first antennæ consist at both stages (figs. 27, 34, and 35, *A*) of a long cylindrical basal joint which carries a few short hairs, and a short pointed terminal joint or flagellum, which ends in two long rather thick sensory hairs.

The second antennæ (figs. 27, 34, and 35, *An*; and fig. 36) are the chief locomotor organs, and are made up of a short stout two-jointed basal portion, a longer unjointed exopodite (*ex*), with four long terminal swimming hairs, and a longer endopodite (*en*), which is made up of two short proximal rings, and a series of six longer joints, each of which carries one, and the terminal one four, long slender swimming hairs.

Underneath the rostrum (fig. 35, *R*) there is a little elevation upon which the ocellus (*Oc*) is situated.

The labrum (fig. 35, *L*) has been carried on to the ventral surface of the body, and its anterior angle has become produced into a short stout, sharp spine, which is extremely small during the first *Protozoëa* stage.

As has been stated, the compound eye is represented at the second stage by a pigment-spot (fig. 35, *E*).

The mandibles (*M*), (figs. 27, 34, 35), have become reduced to cutting blades, which are visible in a dorsal view, and all traces of the *Nauplius* limb have disappeared.

During the first *Protozoëa* stage (Plate 3, figs. 28 and 29) it has only one denticle, which is large and pointed, and situated at the posterior angle of the cutting edge; but at the second *Protozoëa* stage (Plate 4, fig. 37) a number of smaller denticles have appeared in front of the long one. The mandibles are never quite symmetrical, but the outline of the left always differs a little from that of the right.

The external surface of the first maxilla of the first *Protozoëa* is shown in fig. 30, and the posterior surface of that of the second *Protozoëa* in Plate 4, fig. 38. It consists, at both stages, of a basal portion made up of two joints with cutting hairs (fig. 38, 1 and 2); a two-jointed endopodite (*en*), with three long slender hairs; and an exopodite or scaphognathite (figs. 30 and 38 *sc*), with three long slender hairs. In the first stage (fig. 30) the hairs of the scaphognathite are simple, but in the second stage (fig. 38) they are plumose.

The posterior surface of the second maxilla of the first *Protozoëa* is shown in Plate 3, fig. 31, and that of the second *Protozoëa* in Plate 4, fig. 39. It consists of a many-jointed basal portion (*b*), a two-jointed endopodite (*en*), and a scaphognathite or exopodite (*sc*). The whole inner edge of the appendage carries short stout hairs; the tip of the endopodite a few somewhat longer hairs; and the scaphognathite three slender plumose hairs, which are much longer in the second than in the first stage.

The first maxilliped (figs. 32 and 40) is very similar to the second antenna, and consists of a two-jointed basal portion, a four-jointed endopodite, and an unjointed exopodite. The inner edge is set with short stout hairs, which are simple in the first, but irregularly plumose in the second *Protozoëa* stage. The terminal joint of the endopodite carries four long slender simple hairs, and the tip of the exopodite four long straight slender hairs, which are plain in the first but regularly plumose in the second stage.

The second maxilliped of the first *Protozoëa* is shown in fig. 33, and that of the second *Protozoëa* in fig. 41. It is essentially like the first maxilliped in structure, but much smaller, and apparently of little functional importance.

In the second stage there is a small convoluted shell gland (fig. 35, *sg*), which appears to open at the base of the first maxilla; but the constant and violent movements of the limbs render it difficult to decide with confidence exactly what its relation to them is, and it is possible that its opening is upon the base of the second instead of first maxilla.

In the second *Protozoëa* stage the two pigment-spots (*p*) on the carapace become extremely dendritic, and a pair of anal pigment-spots (Plate 3, fig. 34, *pp*) make their appearance on the telson on each side of the anus.

At this stage the area, when the œsophageal muscles are attached to the carapace, is somewhat peculiarly marked by six little circles arranged in a pentagon, as shown, highly magnified, in fig. 35*a*.

The last Protozoëa *stage* (Erichthina).

The change from the last stage to the next one in the series was actually observed in several specimens, and more than fifty larvæ passed through it in the laboratory.

After the moult the larva, which is shown from the ventral surface in Plate 4, fig. 42, and in outline in fig. 42*a*, has the characteristics of DANA's genus *Erichthina*.

Its length, from the tip of the rostrum to the end of the telson, has increased to about $\frac{27}{1000}$ inch, and most of the increase is in the hind body. The carapace also is somewhat elongated (it was a little flattened by pressure in the specimen which was drawn), and the outline of the anterior edge is no longer regularly curved. At the base of the rostrum there is a slight eminence where the integument is pushed out a little by the optic ganglion, and at the outer angle there is a much larger eminence which is the rudimentary cornea of the compound eye. The eye itself is now represented by a large conspicuous pigment-spot (fig. 42*a*, *E*).

The appendages have undergone extremely little change, and they are, as before, as follows: the first antennæ (*A*), the second antennæ (*An*), the mandibles (*M*), the two pairs of maxillæ (*Mx.* 1 and *Mx.* 2), and two pairs of maxillipeds (*Mp.* 1 and *Mp.* 2). The second antennæ are still the chief organs of locomotion.

The hind body is much longer than it was at the stage before, and it is now somewhat longer than the carapace. It now consists of nine free segments and an unsegmented portion (*A* 5, 6). The first of the free segments (fig. 42, *Mp.* 3) is much narrower than any of the others, and its outer edges are marked by enlargements which appear to be the rudimentary appendages, the third pair of maxillipeds. None of the segments which follow it show a trace of the appendages, and the thoracic and abdominal ganglia are not yet visible.

The four segments which follow next after the one with the bud-like processes have rounded posterior edges, while the posterior edges of the next four are pointed. The later history seems to show clearly that those with rounded edges are the first, second, third, and fourth thoracic somites, and that the following ones are the first, second, third, and fourth abdominal somites. It will be seen, then, by a comparison of this with the earlier and later stages, that the somites of the body are all developed in regular order, from in front backwards, but that the first abdominal somite follows immediately after the fourth thoracic, while the fifth thoracic is never developed. At this stage the long unsegmented region (*A* 5, 6), represents the fifth and sixth abdominal segments and the telson. The two anal pigment-spots are larger than they were during the stage before, and from this time to maturity their colour is a dirty reddish-brown instead of black.

The "Zoëa" *stage* (Elaphocaris *stage of* Sergestes.)

After the next moult, which was observed in a great number of specimens, the larva passes into a stage which is directly comparable, so far as the appendages are concerned, with the *Elaphocaris* stage of *Sergestes,* although the most conspicuous features of the *Elaphocaris* larva, the long compound spines, are not present in *Lucifer.* It is now about $\frac{50}{1000}$ inch long, and it is shown in a dorsal view in Plate 5, fig. 44, and, more highly magnified, from below in Plate 4, fig. 43. In a side view (fig. 45) it still agrees pretty closely with fig. 35; its body is carried in the same attitude, and the antennæ are still the chief organs of locomotion. The fully-developed appendages are, as before, the first and second antennæ, the mandibles, two pairs of maxillæ, and the first and second pairs of maxillipeds, but the third pair of maxillipeds, four pairs of thoracic appendages, and the swimmerets or appendages of the sixth abdominal somite are now present as rudimentary buds.

The compound eye (figs. 43 and 45, *E*) is now well advanced in development, although there is as yet no trace of a stalk, and the cornea is simply a modified portion of the integument of the carapace.

The carapace is longer, narrower, and more rectangular in a dorsal view than it was at the last stage, and it makes only about one-third of the total length of the body of the larva. Its pigment-spots are very large, dendritic, and conspicuous, but their colour has changed from black to dark reddish-brown.

The anterior lobes of the stomach (fig. 44, *s*) have lengthened and approached each other on the median line, and they now reach forwards nearly to the optic ganglia.

The appendages which were present during the *Protozoëa* stage have essentially the same structure now, and the differences are very slight. The number of cutting hairs on the basal joints of the first maxilla (fig. 46) has increased; the hairs on its endopodite are plumose, and one of those carried by the scaphognathite is much longer than the other two. This is the case also with the second maxilla (fig. 47), and the hairs along its inner edge have become almost as long and slender as those at its tip. The first maxilliped (fig. 48) is almost exactly like that of the *Protozoëa;* but the second (fig. 49) is much more developed, and the hairs on its exopodite are plumose.

The hind body is now divided into its full number of segments; that of the third pair of maxillipeds (*Mp.* 3); the first, second, third, and fourth thoracic somites (*T* 1, *T* 2, *T* 3, and *T* 4); and the six abdominal somites, but the telson (*T*) is not yet completely distinct from the last abdominal somite. The thoracic somites are shortened and crowded together, and each of them carries a pair of bilobed buds, the rudimentary thoracic appendages. These buds are crowded together in a double row on the median line of the ventral surface of the body, and outside them is a pair of much larger buds (figs. 43 and 45, *Mp.* 3), bilobed also, but pointing backwards; the rudimentary third pair of maxillipeds.

The future history of the larva seems to show conclusively that the inner set of buds are, as indicated in fig. 43, the first four pairs of thoracic limbs or pereiopods. The side view (Plate 5, fig. 45) shows that there is no other pair in front of or behind them, and the fifth thoracic somite is entirely wanting, nor are its appendages present at any stage in the development of *Lucifer*.

The abdomen is much longer than it was at the last stage, and all its segments (fig. 43, *A* 1, *A* 6) are present, although the last one (*A* 6) and the telson (*T*) are not yet entirely separated.

The ventral surface of the sixth abdominal somite is armed with a pair of long stout spines over the base of the swimmeret, or sixth abdominal appendage, which is shown in fig. 43 as a long, bilobed pouch or bud, which reaches nearly to the tip of the telson. The third, fourth, and fifth abdominal somites carry, close to the anterior edge of the ventral surface, irregular groups of reddish-brown pigment-spots, which do not seem to be present in all specimens. The thoracic spots (fig. 44) and the anal spots (fig. 45) are usually a little more red than before, but they are nearly black in some specimens. The abdominal ganglia, which could not be distinctly made out in the last *Protozoëa*, are now very conspicuous, as shown in the ventral view (fig. 43). They lie near the posterior edges of the somites, and their halves are united in the median line, although the commissures between the ganglia are quite widely separated.

The spines on the telson have lengthened, but their number, arrangement, and relative size is the same as before. Their proximal ends from the base about half-way to the tip are marked by fine serrations, which appear to be short hairs, which have not been perfectly extended.

Schizopod or Sceletina *stage* (Acanthosoma *of* Sergestes).

Up to this time the mode of locomotion has been by means of short, jerking *Nauplius* leaps, and the two pairs of antennæ have been, as they were when the larva left the egg, the chief organs of locomotion. The structure of these appendages has remained extremely constant through all the moults, but they now change their character entirely, and lose their locomotor function.

The change which is undergone by the larva at the end of the *Zoëa* series is very much greater than it has been at any preceding moult, except that between the *Nauplius* and the first *Protozoëa*, and in some respects it is even greater than it was at that time. After the moult it is a Schizopod (Plate 6, fig. 50), about $\frac{70}{1000}$ inch long, with seven pairs of long jointed biramous swimming feet, fringed with long slender hairs. The swimmerets are also present as functional appendages, with long fringing hairs.

This stage differs from those which have gone before in this, that it persists with slight change for several moults, while there has been considerable change at each of the preceding moults. It is shown from below in fig. 50, as it appears immediately after the moult which follows the stage shown in fig. 43.

The figure was drawn from a *Zoëa* which was captured at the surface of the ocean, carefully examined and compared with fig. 43, and found to agree with it exactly. It was then placed alone in a small beaker of sea-water. The next day it was found to be moulting, and the drawing (fig. 50) was made from it immediately after the completion of the moult. Other specimens, like fig. 50, were kept until they changed their skins, and assumed a form a little larger than fig. 50, but similar to it in all respects except that the abdominal appendages were now present as small buds. Some of these were kept until they changed into larvæ like the one which is shown, less highly magnified, from the side, in fig. 54. The abdominal appendages were now quite long, but still rudimentary, and the general form of the larva from above or below, as well as the form, number, and arrangement of the thoracic appendages and mouth parts, was like fig. 50.

When seen from above or below (fig. 50) the carapace has nearly the same shape that it had during the *Zoëa* stages, but it now makes less than one-third of the total length of the body, and a side view (fig. 54) shows that it is now only a little deeper than the body, so that the basal joints of the thoracic limbs and maxillipeds are exposed below its inferior border. The posterior dorsal spine and the two postero-lateral spines have disappeared, and a pair of long antero-lateral spines (fig. 54, *s*), nearly half as long as the rostrum, have made their appearance underneath the eyes. The rostrum (fig. 50, *R*) has the same shape and about the same relative length as before, and the ocellus (*Oc*) is still present at its base.

The compound eye (*E*) is mounted upon a movable stalk, which is quite short during the first Schizopod stage, but it soon lengthens, as shown in fig. 55, which is a dorsal view of the anterior end of the carapace of the larvæ shown in fig. 54.

The first antenna has undergone more change at this than at all the previous moults together. It is now about as long as the carapace, and each of the two long cylindrical joints (fig. 50), which make up its basal portion, carries on its inner edge three long slender two-jointed delicately plumose hairs. The base of the proximal joint is swollen and carries a small hook-like process on its inner edge. The two long sensory hairs have disappeared from the tip, which is unsegmented, pointed, and ends in a bunch of short hairs. This appendage changes slightly with each moult, and in the third Schizopod stage (fig. 54) the distal half of the proximal joint (fig. 56) has separated from the proximal joint, so that the shaft is made up of three instead of two portions. The hook is still present on the swollen base of the first joint, and behind it the otocyst (*e*) has made its appearance. The terminal joint or flagellum has now lengthened, and it carries three long sensory hairs which spring from about the middle of its outer surface.

The changes which the second pair of antennæ undergo at this moult are even greater than those which take place in the first pair. Their locomotor function is lost; the long swimming hairs have disappeared; and in the first Schizopod stage (fig. 50) the appendage is quite rudimentary, unjointed, less than one-half as long as the first antenna, and divided into an exopodite and an endopodite which are nearly equal in length, although even at this stage the endopodite is a little the longest. Each ramus ends with a pair of very short hairs.

The appendage now changes with each moult, and in the third Schizopod stage (fig. 54) the exopodite has become a scale (fig. 57, *ex*) while the endopodite (*en*) has elongated, and now forms a seven-jointed flagellum, about as long as the first antennæ or the carapace. The basal joint (fig. 57, *b*) is thick and swollen, the two proximal joints of the flagellum (2 and 3) are short; the next (4) long, and the other four about equal in length, and about half as long as the joint (4).

Through all the Schizopod stages the structure of the labrum (*L*) is about as it was in the *Protozoëa* and *Zoëa*, and its interior angle is still produced into a short stout sharp spine.

The mandibles are cutting jaws with no trace of a palpus, and at the first Schizopod stage (fig. 51) the denticles are numerous and of nearly uniform size. In the last Schizopod stage (fig. 58) a second set of denticles has appeared on the outer surface of the blade a short distance from the cutting edge.

The first maxilla (fig. 52) is very much like that of the *Protozoëa* and *Zoëa*, but the cutting hairs upon the two basal joints (1 and 2) are more numerous, and a small slender plumose hair has appeared near the edge of each joint. The scaphognathite is small and has only two hairs, which are less regularly plumose than before.

The scaphognathite of the second maxilla (fig. 53, *sc*) is now rudimentary and has no hairs. The hairs on the inner edge of the appendage are shorter than they were during the *Zoëa* stage, and all of them are plumose and about equal in length.

The first maxilliped (fig. 50, *Mp.* 1) has not changed very much, although its joints are nearly absent. The exopodite is about as long as the endopodite, and all the hairs on the appendage are short and plumose.

The second and third maxillipeds and the four pairs of thoracic appendages are well developed, as a series of long biramous or Schizopod feet, which are essentially alike in form and structure, and, with the telson and swimmerets, now form the locomotor apparatus of the larva, which no longer swims by jerks but darts through the water with great rapidity, and is able to offer considerable resistance to the suction of a dipping tube. Each swimming foot consists of a two-jointed basal portion or protopodite, a long four-jointed endopodite, and a much shorter exopodite. The exopodite is flat, pointed, and its outer or distal half is marked by a series of six pairs of notches, or annulations, close together. The terminal joint carries a pair of long slender unplumose hairs, and a pair of similar hairs springs from each annulation, so that there are fourteen hairs in all on each exopodite, arranged so as to form a large fan-shaped paddle at the tip of the limb. The terminal joint of the endopodite is much shorter than the others, and it carries six long plumose hairs. The first appendage in this series, the second maxilliped (fig. 59, *Mp.* 2), is somewhat rudimentary: the endopodite is scarcely longer than the exopodite, and its hairs are short. The next or third maxilliped (*Mp.* 3) is more like those which follow, but its hairs are shorter. The first, second, and third pereiopods are about equal in length, and they have the typical structure which has just been described; but the endopodite of the fourth (*Pr.* 4), like that of the second maxilliped, is shorter than the exopodite, although its hairs are very long.

At the last Schizopod stage (fig. 54) the series of limbs, shown from above in fig. 59, is about as it is in the first stage, but the hairs on the endopodites of all the appendages, except the last, are short. A comparison of one of these appendages with the second antenna of the *Nauplius* or *Protozoëa* or *Zoëa* shows great similarity, and I am therefore disposed to believe that the long jointed ramus of the antenna is homologous with the long ramus of the thoracic limb, and consequently the endopodite.

The abdomen is very much longer in proportion to the carapace than it was at the "*Zoëa*" stage, and a comparison of figs. 50 and 54 with fig. 43 will show that it has become flattened from side to side, while its vertical thickness has greatly increased. All six somites are distinct, but at the first Schizopod stage there are no traces of any abdominal feet except the swimmerets, which are large and perfect. In the second Schizopod stage the first five pairs of pleopods are represented by short buds, and in the last Schizopod stage (fig. 54) they have nearly or quite their full size, but are still rudimentary.

The posterior edge of the ventral surface of each abdominal somite carries a couple of spines (fig. 50) pointing backwards. They are small on all the somites except the last, and they appear to correspond to those which, from their great size, have given the name *Acanthosoma* to the larva of *Sergestes* at the same stage of development. The sixth abdominal somite also has a small median dorsal spine.

The telson (*T*) is movable, greatly elongated, three times as long as wide, and its spines have become very small, although in number, arrangement, and relative size they agree with those of the *Zoëa* and *Protozoëa.*

The sixth pleopod or swimmeret consists of a short thick basal joint, a long flat exopodite which is serrated along its inner edge and free extremity, but smooth along its outer edge; and a flat endopodite serrated on both sides. Each serration carries a long slightly curved plumose hair, and the outer edge of the exopodite has a small tooth at its outer end. From the base to the tooth the outer border is nearly straight and parallel to the inner border, but the end of the appendage is prolonged into a rounded tip which reaches beyond the tooth. In the first Schizopod stage there are eight hairs on the inner border and four on the end of the exopodite, or twelve in all; and there are eight hairs on the endopodites, but the number of serrations and hairs increases rapidly with each moult, on each division of the limb, and they are much more numerous in the last Schizopod stage, as shown in fig. 54.

A large reddish-brown pigment-spot (fig. 54, *p*) has now appeared on each side of the fourth abdominal segment, and the anal spots are large, with a dull red tinge. The spots on the carapace disappear at the end of the *Zoëa* series.

The Mastigopus *stage.*

After the next moult the larva (Plate 7, fig. 60) assumes a form which is essentially like that of the adult, but with numerous slight differences, the most important of which are the shortness of the flagellum of the first antenna and the absence of the neck or elongation of the carapace. In these respects, as well as in the number, character, and relative size of the appendages, it now agrees very closely with the young *Sergestes* or *Mastigopus.*

The size of the thorax is reduced, while the abdomen has grown larger and longer. The exopodites of the maxillipeds and first three pairs of pereiopods have disappeared, together with every trace of the fourth pereiopod. The abdominal appendages are perfect; the first is made up of an elongated basal joint, which carries a single terminal branch of about the same length as the basal joint, but pointed and fringed with long slender swimming hairs. The four appendages which follow are each furnished with two terminal branches instead of one, but are similar in other respects. The larva now sheds its skin several times, and grows with each moult; but the process of change into the adult is, with the exception of the elongation to form the neck, simply a process of growth, as the appendages and somites all have essentially their adult character.

A larva about one-fifth of an inch long, two moults after the last Schizopod stage, is shown from the side, magnified about fifty diameters, in Plate 7, fig. 60. The first antenna (*A*) is a little more than twice as long as the eye-stalk, and consists of a stout three-jointed basal portion, which forms about two-thirds of the total length of the appendage, and a short, thin, two-jointed flagellum. The scale (*ex*) of the second antenna is only a little longer than the eye, while the flagellum (*en*) is more than half as long as the body of the animal, measured from rostrum to telson, and is made up of thirteen small joints and two thicker basal joints.

The carapace has elongated considerably, and the neck (*n*) makes nearly half its length. The anterior end of the carapace has a dorsal rostrum (*R*), two much shorter lateral spines (*ls*), and a very small spine on each side close to the anterior edge and about half way between the rostrum and the lateral spine. The cephalic lobes of the stomach extend into the neck, and reach nearly to the basis of the eye-stalks. The coiled antennal gland (*g*) has made its appearance. The carapace proper (*c*) has a pair of anterior spines, but none on its posterior margin. The labrum (*L*) has a much greater relative size than it had during the Schizopod stages, but its spine disappears at the end of the last Schizopod stage. The mouth parts and thoracic limbs have their adult character, and will be noticed at length in the description of the adult. A reddish-brown pigment-spot has now appeared between the bases of the eye-stalks;

another at the base of the telson; and the dorsal surfaces of the fifth, fourth, and sometimes the third abdominal somites are irregularly marked, near their posterior edges, by patches of the same colour. The anal pigment-spots are of a dirty red colour.

The Lucifer *stage.*

The specimen from which fig. 61 was drawn was a little more than half an inch long, or about half as large as an adult specimen. It differs in several particulars, besides size, from an adult male, but in all respects except size and the presence of reproductive organs it is exactly like a mature female. Its appendages are like those which are shown in figs. 63 to 70, although these were drawn from an adult female specimen.

The adult structure of our American species has been described by FAXON ('Studies from the Biological Laboratory of the Johns Hopkins University,' vol. i., part iii.); but as he had only a single male specimen, which had been preserved in alcohol, his account was necessarily somewhat incomplete.

The first antenna (Plate 7, fig. 61, and Plate 8, fig. 66, *A*) is about as long as the carapace and neck, and it is divided into two nearly equal portions, the base (fig. 66, 1) and the flagellum (fig. 66, 2). The base is divided into three joints, the first about as long or a little longer than the eye, the second much shorter, and the third still shorter. The large ear occupies the centre of the proximal end of the first joint. On the outer end of the first joint and on the second there is a row of six short, equal, plumose hairs, three on each joint. The flagellum is made up of ten joints; the first and second are thicker than the others, and the first carries two and the second three sensory hairs. The terminal joint of the flagellum is much longer than the other, and carries a few very short hairs at its tip.

The second antenna (figs. 61 and 66, *An*) is, in the fully-grown specimen, almost twice as long as the first, and nearly or quite as long as the body. It consists of a very short basal joint (fig. 66, 3), which carries the scale (*ex*) and the flagellum (*en*). The scale is somewhat longer than the eye, flat and narrow, and its inner edge carries nine and its tip three long, slender, plumose hairs, which are about half as long as the scale itself. The flagellum tapers gradually from the base to the tip, and is made up of twenty-four joints, each of which carries a pair of very short hairs. The joints at the tip of the flagellum are a little longer than those at the base. The living animal usually carries these appendages extended before it, and diverging a little at their tips. It occasionally throws them back along the sides of the body, but only for an instant at a time.

The eye-stalk tapers gradually from the base to the tip, and there is no abrupt distinction between the stalk and the eye proper, as there appears to be in other species. The length of the eye, with its stalk, is a little less than that of the true carapace.

The neck makes a little more than three-fifths of the total length of the carapace, and its vertical diameter is more than half that of the thorax. It has a median dorsal rostrum (fig. 61, *R*), which is much smaller relatively in the adult than in the young, and two antero-lateral spines (*ls*). About half-way between the rostrum and the lateral spine the anterior edge of the neck has an extremely minute spine on each side, as in the younger stage last described. The cerebral ganglia (*cg*) occupy the ventral half of the anterior end of the neck, and the long commissures can be seen at *co.* running back to join the ventral nervous system. The cephalic lobes of the stomach (*s*) and the antennary gland (*g*) occupy the dorsal portion of the neck.

The true carapace (*c*) does not reach down on to the sides of the body as far as the basal joints of the thoracic limbs and mouth parts, and both these and their ganglia (figs. 75 and 76, *tg*) are visible below its free edge. Its edges are smooth, but there is a small spine at its anterior end.

The labrum (fig. 61, *L*) is massive and prominent, but there is no trace of a spine.

The inner surface of the mandible (fig. 62) is marked by a number of parallel ridges, one for each denticle; and there is a second, and a faint trace of a third, series of denticles on the outer surface (fig. 63). There is no trace of a mandibular palpus.

The scaphognathite of the first maxilla (fig. 64) has disappeared, the endopodite is rudimentary, while the second basal joint is very much larger than the first, and carries about fifteen stout short hairs arranged in three rows. The first joint has four much larger unequal hairs, which are serrated. The outer edge of the first and both edges of the second joint carry a single delicate plumose hair each. Fig. 65 shows the inner surface of the second basal joint.

The second maxilla (Plate 8, fig. 67) is more like that of the larva. There is a three-jointed inner portion with short stiff hairs, and an extremely large scaphognathite (fig. 67, *sc*), which is long and narrow, and united to the body of the appendage by a very narrow stalk. The outer end carries three rather stiff, short, plumose hairs, and five similar but somewhat longer hairs arise from the inner surface between the outer end and the area of attachment. The inner end carries four plumose hairs, three of which are almost as long as the scaphognathite itself, while the fourth appeared to be broken off in the four specimens which I dissected out.

The first maxilliped is a short, stout, two-jointed appendage (Plate 8, fig. 68), convex on its outer but flat on its inner surface, and fringed with short, stout, plumose hairs.

The second maxilliped (Plate 7, fig. 61, *Mp.* 2, and Plate 8, fig. 70) is a long jointed limb, bent into a knee, and formed of six joints. It is fringed by long plumose hairs, which, on all the joints except the first and second, are arranged in a single row. The first and shortest joint has no hairs; the next, or second, has one row of five and one row of three; the next, or third, has six hairs; the next, or fourth, and the fifth have ten each; and the terminal joint has six.

The next or third maxilliped (fig. 61, *Mp.* 3) is a long, slender, six-jointed limb, with a double row of short hairs.

The first pereiopod (fig. 61, *Pr.* 1) is four-jointed, and shorter than the last maxilliped.

The second and third pereiopods (*Pr.* 2 and *Pr.* 3) are nearly equal, and twice as long as the first; they are four-jointed, have a double row of small hairs along the anterior edge, and the last ends in a small curved hairy claw.

They exhibit no trace of gills or of endopodites, and there is no stump to indicate the position of the fourth pereiopod, which disappeared at the end of the Schizopod period.

The first abdominal appendage of immature specimens or of mature females (Plate 9, fig. 74, *Pl.* 1) is made up of a thick basal portion, which is unjointed in young specimens but two-jointed in mature ones, and a pointed annulated terminal portion which is fringed with swimming hairs. In the nearly grown but immature male (fig. 76) there is a little bud or projection (*a*) near the base of the anterior surface of the long basal joint. In the sexually mature male (fig. 75) this bud has become the clasping organ which has been described by MILNE-EDWARDS, DANA, SEMPER, DOHRN, CLAUS, FAXON, and others; and another smaller process or tooth has appeared upon the distal one of the two joints into which the base of the limb has now divided.

The second, third, fourth, and fifth pleopods consist, in the young of both sexes, and in the mature females, of a long unjointed basal portion and two hairy terminal branches. In the adult male the second pleopod has a third and smaller terminal branch, as CLAUS has pointed out (Zeit. f. Wiss. Zool., xiii., 434).

The first, second, third, fourth, and fifth abdominal somites end below in short spines, and they are all about equal in length, except the fifth which is nearly twice as long as any of the others. It has a median dorsal spine on its posterior edge, and the very young specimens also have a pair of postero-lateral spines, as shown in Plate 7, fig. 60. In older specimens this pair of spines disappears, as shown in Plate 9, fig. 72, and in the adult female the somite undergoes no further change. When the male reaches sexual maturity, however, the lower edge of the somite becomes produced, as described by DANA, on each side into the hooks shown in fig. 73. In our species the smaller one of these hooks is near the middle of the somite, and the larger one about half way between it and the posterior edge.

As shown in figures 72 and 73, the telson of an adult specimen is only about half as long as the swimmerets. The tip of the telson of an adult female is shown from above in fig. 71.

In the male the telson becomes somewhat bent (fig. 73, *T*) as maturity is reached, and a rounded anal papilla becomes developed in its lower surface, while the telson of the adult female remains like that of immature specimens of both sexes.

The exopodite of mature specimens usually has about twenty hairs, and the endopodite sixteen. The exopodite is longer and wider than the endopodite, and it is alike in both sexes until maturity is reached, when it becomes somewhat modified in the male, This sexual difference has been pointed out by DOHRN (Zeit. Zool., xxi., 1871, p. 358), but it seems to have escaped the notice of all other observers.

In the young and in the mature female (fig. 72) the rounded tip projects beyond the tooth (*a*), but as the male approaches maturity the outer edge lengthens, thus pushing the tooth out, as shown in fig. 73, until the end of the appendage becomes square instead of rounded. It is extremely interesting to notice that in *Lucifer*, as in so many other animals, the adult female is infantile in all the secondary points of difference from the male.

General view of the metamorphosis of Lucifer.

A review of the facts which have been described in this section indicates that some of the changes are much more significant than others, since the number of moults is much greater than the number of distinct larval type.

The meta-*Nauplius* is obviously a *Nauplius* with the rudiments of structures which are to appear after the moult, and it must therefore be regarded as a *Nauplius* prepared for the change into a *Protozoëa*, rather than a distinct stage of development.

There is no such break between the first *Protozoëa* and the last *Zoëa* as there is between the first *Protozoëa* and the *Nauplius*. The rudimentary pereiopods and swimmerets of the so-called *Zoëa* are nothing but a preparation for the next stage of development, and the supposed necessity for finding a stage which can be directly compared with the *Zoëa* of the higher Decapods does not justify us in making two larval types out of the unbroken series of *Protozoëa* and *Zoëa* forms.

It is obvious that the three Schizopod stages are modifications of a single larval type, and the presence of rudimentary pleopods in the second and third stages must be regarded as a preparation for the next stage of development.

There is no abrupt break between the so-called *Mastigopus* and the young *Lucifer* when it is a little older and the neck has appeared.

On the other hand, there is a real break between the *Nauplius* and the *Protozoëa*, and the change from one to the other is accompanied by profound structural changes. This is the case also with the transition from the *Zoëa* to the Schizopod stage ; and with that from the Schizopod stage to the young *Lucifer* stage. The same thing is true to a lesser degree of the change from the immature *Lucifer* to the adult male.

The metamorphosis may then be divided into the following well-marked stages, each of which except the last, and in all probability the last also, persists through more than one moult :—

1. A *Nauplius* stage.
2. A *Protozoëa* stage.
3. A Schizopod stage.
4. An immature *Lucifer* stage, which persists in the female.
5. An adult male stage.

If we neglect the features which, at the end of each stage, make their appearance as preparation for the next, we may describe each stage as follows :—

The *Nauplius* has three pairs of locomotor appendages, the first antennæ, the second antennæ, and the mandibles; and there is a large labrum without a spine, and the carapace and telson are absent. There is an ocellus, but no compound eyes.

The *Protozoëa* has two pairs of antennæ, which are like those of the *Nauplius*. The mandible is reduced to a cutting blade. There are two pairs of biramous maxillæ, with scaphognathites, and two pairs of biramous maxillipeds. There is a long hind body, ending in a flat telson. The labrum has a spine. The carapace is large, and has a rostrum, a median dorsal and two lateral posterior spines; and its free edges reach down beyond the basal joints of the appendages. There is an ocellus, but no stalked eyes.

The Schizopod stage is characterized by the great change in the two pairs of antennæ, which are no longer like those of the *Nauplius*, but have the characteristics of those of the adult. All the mouth parts and four pairs of thoracic limbs are present, and all posterior to the first pair of maxillipeds are biramous and locomotor. The abdomen has six somites and a movable telson. The swimmerets are present, but the other abdominal appendages are not.

The ocellus persists, but the stalked eyes are also present. The carapace has a rostrum and two antero-lateral spines, but those at the posterior edge have disappeared. The edges of the carapace do not reach over the basal joints of the thoracic limbs, and the body is flattened vertically. The labrum still has a spine.

The young *Lucifer* and the adult female have a long flagellum on the first antenna, a flagellum and scale on the second; the ear and antennary gland are present; the neck is elongated. The fourth pereiopod has disappeared, and the others, as well as the maxillipeds, have lost their exopodites. The first pleopod has one terminal branch, the next four two branches each; the sixth abdominal somite has a smooth lower edge. The telson is straight and the outer end of the exopodite of the swimmeret is rounded.

The adult male has a clasping organ on the first pereiopod, three rami on the second, two teeth on the lower edge of the sixth abdominal somite, a square end to the exopodite of the swimmeret, and a bent telson.

It is true that these five stages merge into each other somewhat, and that they are complicated by the presence of the rudiments of organs which are be functional at the next stage; but after all these secondary modifications are allowed for, it will be seen that each stage is sharply and definitely marked, and separated by a pronounced gap from the stages before and after.

The significance of these five stages can be best inquired into after the corresponding stages of other *Sergestidæ* have been examined, and I will return to the subject further on, in a section on the general relationships of the group.

IV. History of the Appendages of Lucifer.

For convenience of reference I will now describe the changes which each appendage undergoes at each stage of development, going over the same ground once more, but in a different way.

The first antenna.

In the egg *Nauplius* (figs. 21, 23, and 24, *A*) this appendage is unjointed, more than half as long as the body, and it carries a terminal tuft of hairs.

In the first free *Nauplius* (fig. 25, *A*) it consists of five nearly equal joints; it is nearly as long as the body of the second antenna, and its tip carries two long simple hairs and two much smaller hairs.

In the last *Nauplius* stage, or meta-*Nauplius* (fig. 26) the joints have disappeared; it is only about two-thirds as long as the body, and it carries only the two long hairs at the tip.

In the first *Protozoëa* stage (fig. 27, *A*) it is made up of a long cylindrical basal joint with a few short hairs, and a much shorter terminal joint, which is pointed, and carries the two long hairs as before.

The structure of the appendage does not change until the end of the *Zoëa* series, and it is shown at *A* in figs. 34, 42, 43, and 44.

At the first Schizopod stage (fig. 50, *A*) the basal portion is made up of one very long cylindrical joint, with a hook near its swollen base, and a much shorter distal joint. Three long, two-jointed, plumose hairs spring from the inner edge of the second joint, and three more from the inner edge of the distal third of the basal joint. The terminal portion has lost the two long hairs which it had at earlier stages.

In the last Schizopod stage (fig. 54, *A*, and fig. 56) the distal third of the basal joint has separated off as a distinct joint (fig. 56, 2) upon which the three hairs are situated. The ear has made its appearance, behind the hook, on the swollen base of the first joint. The terminal joint (4) carries three sensory hairs, which arise upon its outer surface about half way between its tip and base.

In the *Mastigopus* stage (fig. 60) the terminal joint has lengthened to form a two-jointed flagellum, and the appendage is more than twice as long as the eye.

In the young specimens which have attained to the adult form (fig. 61, *A*) the appendage is about as long as the carapace and neck, and in the adult (fig. 66, *A*) the flagellum (2) is about as long as the basal portion (1). It consists of ten joints, the terminal one longest, and the first and second thick. The first carries two and the second three sensory hairs.

The basal portion is thick, cylindrical, three-jointed, with six plumose hairs, and the ear nearly fills the enlarged base.

The second antenna.

In the egg *Nauplius* (fig. 24, *An*) this is unjointed, more than half as long as the body, divided into two nearly equal rami, with hairs at their tips.

In the first free *Nauplius* stage (fig. 25, *An*) a two-jointed basal portion carries a three-jointed exopodite and an eight-jointed endopodite. The appendage is nearly as long as the body, the two rami are about equal in length, and each has three long simple hairs at its tip. In the last *Nauplius* stage (fig. 26) the joints are obscure; the endopodite is longer than the exopodite; it has long hairs along its side, and those at the tip are plumose. In the *Protozoëa* stages (figs. 27, 34, 35, 42, 43, and 45, *An*, and fig. 36) it consists of a two-jointed basal portion (fig. 36), which carries an unjointed exopodite (*ex*) with long, slender, non-plumose terminal hairs, and an eight-jointed endopodite (*en*) with eight long hairs arranged along its side and tip. The first and second joints are very short, while the other six are longer and nearly equal.

In the first Schizopod stage (fig. 50, *An*) the appendage is rudimentary, its joints are absent, and the exopodite is almost but not quite as long as the endopodite. The appendage is only half as long as the first antenna. In the last Schizopod stage (fig. 54; *An*, and fig. 57) the exopodite has become a scale, which is only half as long as the seven-jointed flagellum which has become developed from the endopodite; the basal joint is simple, very large, and the appendage is as long as the first antenna.

The flagellum now grows rapidly, and in the adult (fig. 61, *An*, and fig. 66, *An*) it has twenty-four joints, and is more than half as long as the body. The antennal gland opens into its base, and the scale is longer than the eye, and carries twelve long plumose hairs.

The mandible.

In the egg *Nauplius* (fig. 24, *M*) this is biramous, unjointed, and tipped with hairs. In the first free *Nauplius* (fig. 25, *M*) it is short, and made up of a stout basal joint; a two-jointed exopodite with three long slender hairs, two of which are carried by the terminal and one by the proximal joint; and a shorter endopodite with three long simple hairs. In the last *Nauplius* stage (fig. 26, *M*) the joints of the exopodite have disappeared, the three hairs on the endopodite have lengthened and become plumose, and the inner edge of the basal joint carries a hook or blade. From the beginning of the *Protozoëa* series to maturity the mandible is a cutting blade, with no trace of a palpus, and the number of its denticles gradually increases with age.

The metastoma.

The manner in which the metastoma originates in the *Nauplius* as a pair of buds similar to those which become the maxillæ, as well as the fact that it persists in closely-allied forms as a pair of limb-like structures, seems to show, as Claus has pointed out ('Untersuchungen,' &c., p. 15), that the Decapod metastoma is morphologically a pair of appendages; that it has been formed by the simplification and union of structures homologous with the limbs; and that this pair of appendages was originally furnished with a body-somite and a pair of ganglia. CLAUS's reason for the homology is the resemblance between the Decapod *Protozoëa* and the larva of Phyllopods and Copepods, and the manner in which these parts are developed in the Nauplii of *Lucifer* and *Euphausia* seems to be an additional reason for accepting his view.

The first maxilla.

This appendage is rudimentary during the *Nauplius* stages, but, as shown in fig. 21, *Mx.* 1, it is represented by a pair of buds several hours before birth.

In the *Protozoëa* and *Zoëa* series it has the form shown in fig. 46, which was drawn from the appendage of a larva in the last *Zoëa* stage. Its characteristics are developed gradually, and it is somewhat simpler during the earlier *Protozoëa* stages than it is in fig. 46. Fig. 30 shows it as it appears in the first *Protozoëa* when seen from the outside. It consists of a basal portion (fig. 46) made up of two joints (1 and 2), which carries a short obscurely-jointed endopodite (*en*) and a scaphognathite (*sc*). In my description of this and the other mouth parts of *Lucifer* I have accepted CLAUS's homology ('Untersuchungen,' &c., p. 16), and regard the two basal joints as the equivalent of the basal portion of the antenna, or of one of the thoracic limbs; the jointed palpus as the homologue of the inner ramus of the antenna, or the limb proper of one of the thoracic appendages; and the scaphognathite as the homologue of the exopodite of one of the thoracic appendages, or of the antennæ. In all these appendages the exopodite is shorter than the endopodite, unjointed, and set with long hairs, the plumose character of which is well marked. The scaphognathite of the maxilla agrees with the exopodite of the second antenna and of the other appendages in this respect, while the palpus of the maxilla agrees with the endopodite of the second antenna, and with that of the mandible of the *Nauplius* and with the thoracic limbs of the adult *Lucifer*, in consisting of several joints with one or more, usually simple, hairs at each joint.

The inner edges of the basal joints of the maxilla carry cutting hairs, and the second joint is largest. The endopodite carries five long slender hairs which are simple in the earlier and plumose in the later stages. The scaphognathite carries three hairs which are equal and simple in the earlier *Protozoëa*, but plumose in the *Zoëa*, where one is very much longer than the other two.

The structure of this appendage undergoes extremely little change from the time it appears in the *Protozoëa* to maturity. In the Schizopod larva (fig. 52) the second basal joint (2) has become much larger than the first (1), and its cutting hairs are more numerous than before; a small slender plumose hair has made its appearance on the edge of each joint. The endopodite (*en*) is obscurely three-jointed, and the scaphognathite (*sc*) has only two long plumose hairs.

In the adult (fig. 64) the scaphognathite is absent; the endopodite is rudimentary and the second joint of the base (2) is very much wider than the first (1), and has fifteen cutting hairs arranged in three rows, while the first joint has only four very much longer serrated cutting hairs. The basal joint has only one plumose hair as before, but the second joint has one on each side of the blade.

The second maxilla.

The second maxilla is present as a bud (fig. 24, *Mx.* 2) in the egg, and it becomes functional in the first *Protozoëa*, and persists without very much change to maturity.

In the first *Protozoëa* (fig. 31) it has a long, many-jointed basal portion (*h*), with slender simple hairs on its inner edge; a two-jointed endopodite (*en*) with three simple hairs on its tip, and two on the second joint; and a small scaphognathite with plumose hairs.

In the last *Zoëa* (fig. 47) the hairs on the inner edge are plumose, and one of the three hairs on the small scaphognathite is much longer than the others.

In the Schizopod stage (fig. 53) the limb is thick and long, the scaphognathite is rudimentary, and the endopodite is small, and has no terminal hairs.

In the adult (fig. 67) the endopodite and all but three of the joints of the basal portion are absent. The first of these (3) is the largest and has a broad edge, with a number of cutting hairs, while the others (2 and 1) are narrow and have three hairs each. All these hairs are simple. The scaphognathite is elongated, and is now about as long as the body of the appendage, to which it is joined by a narrow neck. The inner end has four plumose hairs, three of which are about as long as the appendage, while the fourth was short and apparently broken in all the specimens which I examined. The outer half of the scaphognathite has three short straight plumose hairs on its outer end, and five somewhat longer ones on its inner side.

The first maxilliped.

The first maxilliped is represented by a bud in the egg *Nauplius* (fig. 21 *Mp.* 1) and it becomes functional in the first *Protozoëa*, and then consists (fig. 32) of an unjointed exopodite (*ex*) with four long terminal hairs; a four-jointed endopodite (*en*), with three long terminal simple hairs, and a shorter hair springing from the inner edge of each joint; and an obscurely two-jointed basal portion with short simple hairs on its inner edge.

In the *Zoëa* stage (fig. 48) the hairs on the inner edge and on the exopodite are plumose, and the endopodite is long and six-jointed.

In the Schizopod stage (fig. 50, *Mp.* 1) the joints are obscure; the exopodite is nearly as long as the endopodite; all the hairs are plumose, and about equal in length, and there is a double row along the inner edge of the appendage.

In the adult the appendage (fig. 68) is extremely simple, short, stout, two-jointed, flattened on its inner and rounded on its outer surface, with a fringe of short, stout, equal, plumose hairs around the edge of the flattened surface.

The second maxilliped.

It is difficult to decide with certainty whether this appendage is represented by a bud in the *Nauplius* or not. If the first pair of buds become the metastoma, as seems probable from their position with reference to the mandibles and from the analogy of the *Euphausia nauplius*, the second pair of maxillipeds are not represented, but if the first pair of buds are the rudimentary first maxillæ the last pair are the second maxillipeds. At any rate the appendages are present in the first *Protozoëa* (fig. 33), and they are essentially like the first pair, but much smaller.

In the last *Zoëa* stage (fig. 49) they are larger, although still smaller than the first, and their inner edges carry only three short hairs which are not plumose.

In the Schizopod stage (fig. 59, *Mp.* 2) a long basal joint carries a four-jointed endopodite and an unjointed exopodite of nearly equal length. The outer half of the exopodite is fringed by fourteen long, simple hairs, and the terminal joint of the endopodite has a few short plumose hairs.

In the next stage the exopodite is absent, and the long six-jointed limb (fig. 70) is bent into the shape which is so characteristic of the adult *Sergestidæ*.

The basal joint (1) is quite short and stout. The next joint (2) is longer and has five plumose hairs, almost as long as the joint, on one side and three on the other.

The next joint (3) is the longest, and carries six plumose hairs. The next (4) is about as long as the second, and the bend in the limb occurs in this joint and between it and the third. It carries ten plumose hairs about as long as those in the other joints, and arranged in a single close rank. The fifth and sixth joints are shorter than any of the others except the first; they are about equal in length, and the fifth carries ten, the sixth six long plumose hairs.

The third maxilliped.

This appendage makes its appearance as a bilobed rudiment (figs. 43 and 45, *Mp.* 3), at the end of the *Zoëa* series, and it becomes developed into a Schizopod foot, at the next or first Schizopod stage (fig. 59, *Mp.* 3). A stout basal portion which appears to be two-jointed, carries an unjointed exopodite, and a four-jointed endopodite. The latter branch is the longest, and its tip carries four rather short plumose hairs. The outer half of the exopodite carries fourteen long simple hairs.

At the end of the Schizopod period the limb loses its exopodite entirely, lengthens and becomes a slender six-jointed leg, fringed by a double row of short hairs, as shown in fig. 61, *Mp.* 3.

The history of this appendage in *Lucifer* shows that that there is no reason, except the arbitrary system borrowed from the higher Decapods, for classing this appendage with the mouth parts, instead of with the thoracic limbs.

It appears much later than the first and second pairs of maxillipeds, or at the same time with the thoracic limbs. It agrees with these latter in all its subsequent changes and in its adult structure, and must be regarded as forming one of the thoracic series. I have employed the recognised name, third maxilliped, to prevent confusion, but the appendage is in no sense a mouth part. In fact, the only reason for holding that the missing appendage in *Lucifer* is the fifth pereiopod, instead of the last maxilliped, is the tacit assumption that the appendages must follow a definite serial order from in front backwards. We do not know that this assumption is justifiable in all cases, and it is therefore perfectly possible that the appendage which is usually called the third maxilliped of *Lucifer* may really be the first pereioped. I think the probability is in favour of the accepted homology, but the use of the term "third maxilliped" in the present paper for the appendage in question must not be regarded as evidence that the homology is accepted without question.

The pereiopods.

At the end of the *Zoëa* series four pairs of pereiopods, the first, second, third, and fourth, are represented by buds (figs. 43 and 45), while the fifth is entirely absent, as Dana pointed out in the 'Report on the Crustacea collected by the United States Exploring Expedition,' p. 634. Willemöes-Suhm (Proc. Roy. Soc., vol. 24, p. 134), calls attention to the same fact: the total absence of this somite at all stages of development. In the Schizopod stage each of these appendages is biramous (fig. 59), and similar to the last maxilliped, although the first three pairs (fig. 59, *Pr*. 1, *Pr*. 2, and *Pr*. 3) are longer.

At the end of the Schizopod series of stages the entire fourth pair and the exopodites of the other three pairs disappear, and the endopodites lengthen to form the long slender limbs of the adult (fig. 61, *Pr*. 1, *Pr*. 2, and *Pr*. 3). They are four-jointed, with a double row of short hairs along the anterior edge, and the first is only half as long as the second and third, which are nearly equal, and almost as long as the carapace and neck. The third ends in a short, curved hairy claw, too small to be shown in the figure.

The first abdominal appendage.

This is present as a rudimentary bud at the end of the Schizopod series, but does not become functional until the *Lucifer* form is reached. In the young it consists of a long unjointed base, and a single pointed tip, fringed with swimming hairs (fig. 61, *Pl*. 1). In older specimens the basal portion divides into two joints, and in the young male or the young or mature female the appendage has the form shown in fig. 74. As the male approaches maturity a small process, shown in fig. 76, appears on its anterior face, and becomes modified in the mature male into the clasping organ (fig. 75, *c*), while a second process (*d*) appears a little nearer the tip of the limb.

The second abdominal appendage.

This appears at the same time with the first, and developes two terminal branches. In the mature male a third shorter one is added.

The third, fourth, and fifth abdominal appendages.

These all develop at the same time with the first and second; they have two terminal branches and are alike in both sexes.

The sixth abdominal appendage.

This is present as a rudiment in the last "*Zoëa*," and it becomes fully developed in the first Schizopod larva.

It consists of a basal joint which carries a long, wide, and flat exopodite, and a narrower shorter endopodite.

In the young and in the mature female the outer end of the exopodite is rounded, but it is nearly square in the mature male.

The labrum.

The labrum is large and conspicuous in the *Nauplius*, but it has no spine. The spine is present from the first *Protozoëa* stage to the last Schizopod stage, but it is absent in the adult.

The compound eyes.

These make their appearance as rudiments in the last *Protozoëa*, but they are not perfectly developed or stalked until the last Schizopod stage. The homology of the stalked eyes of the Malacostracan has been a matter of some uncertainty. They are usually enumerated in the list of appendages, and the typical Crustacean is supposed to have a corresponding somite. CLAUS has pointed out ("Zur. Kenntniss der Malacostrakenlarva," Würzb. Zeitschr. ii., 1861. p. 33) that no especial taxonomic importance can be attached to their presence or absence; and their mode of origin in *Lucifer* certainly gives no support to the view that they have been produced, like the mandibles, by the gradual specialisation of a pair of ordinary appendages. They do not resemble ordinary appendages at any stage, but are formed directly, and the fact that the period of their development is spread over several moults renders their history quite different from that of the appendages. As I shall show further on, serially homologous organs do not necessarily owe their resemblances to inheritance from the unspecialised organs of a remote ancestor, and I think that the presence of a distinct occular segment in *Squilla* compels us to recognise an homology between the stalked eye and an ordinary appendage, although it is no doubt true that all the groups in which stalked eyes occur cannot be traced back to a common stalked-eyed ancestor, and also true that the stalked eyes themselves cannot be traced back to ordinary appendages.

The ocellus.

This is present from the first *Nauplius* stage to the end of the Schizopod series.

Explanation of Table I.

This table is designed to show at a single view the condition of each appendage at each stage of development.

For convenience I have included the compound eyes, the ocellus, and the labrum, but do not wish to imply that these structures are or are not homologous with ordinary appendages, and I have omitted the metastoma, although I have no doubt that this should be included in a list of the appendages.

In the table the word "same" indicates that the condition of the appendage is the same as it was at an earlier stage, and does not refer to other appendages in the same vertical line.

TABLE I.—Comparative table to show the history of the appendages of *Lucifer*.

	Nauplius.	First *Protozoëa* or *Erichthina* Stage.	Last *Zoëa*.	First Schizopod or *Sceletina* Stage.	Last Schizopod.	Young of both sexes and adult female.	Adult male.
Compound eye	Absent	Same	Not stalked	Stalked	Same	Same	Same
Ocellus	Present	Same	Same	Same	Same	Absent	Absent
First antenna	Has nauplius form	Same	Same	Adult form, but rudimentary	Adult form	Same	Same
Second antenna	Has nauplius form	Same	Same	Rudimentary	Adult form	Same	Same
Labrum	No spine	Spine	Same	Same	Same	No spine	Same
Mandible	Biramous limb	Adult form	Same	Same	Same	Same	Same
First maxilla	Bud	Biramous	Same	Same	Same	Scaphognathite absent, endopodite rudimentary	Same
Second maxilla	Bud	Biramous	Same	Endopodite and scaphognathite rudimentary	Same	Endopodite absent, scaphognathite large	Same
First maxilliped	Bud	Biramous	Same	Same	Same	Rami absent	Same
Second maxilliped	Bud	Biramous	Same	Same	Same	Exopodite absent	Same
Third maxilliped	Absent	Same	Bud	Biramous	Same	Exopodite absent	Same
First pereiopod	Absent	Same	Bud	Biramous	Same	Exopodite absent	Same
Second pereiopod	Absent	Same	Bud	Biramous	Same	Exopodite absent	Same
Third pereiopod	Absent	Same	Bud	Biramous	Same	Exopodite absent	Same
Fourth pereiopod	Absent	Same	Bud	Biramous	Same	Absent	Same
Fifth pereiopod	Absent	Same	Same	Same	Same	Same	Same
First pleopod	Absent	Same	Same	Same	Bud	One ramus	Clasper
Second pleopod	Absent	Same	Same	Same	Bud	Two rami	Three rami
Third pleopod	Absent	Same	Same	Same	Bud	Two rami	Same
Fourth pleopod	Absent	Same	Same	Same	Bud	Two rami	Same
Fifth pleopod	Absent	Same	Same	Same	Bud	Two rami	Same
Sixth pleopod	Absent	Same	Bud	Biramous	Same	Exopodite round at tip	Exopodite square at tip

V. The Metamorphosis of Acetes.

While I was studying the development of *Lucifer*, I found during the month of September a few specimens of the very similar larva which is shown from above in Plate 9, fig. 79, and from the side in fig. 78.

Several specimens were placed by themselves in tumblers of sea-water, where they passed through the stages shown in Plate 11, figs. 84, 85, and 90. Only one of my specimens reached this last stage, and as this one moulted on the last day of the season I was not able to trace it any further, and as I collected no adult specimens of the same kind, its precise systematic position must at present remain in some uncertainty. The close similarity which I shall point out between its larval stages and those of *Lucifer* and *Sergestes* renders it very probable that it is a Sergestid, and the analogy of these forms also indicates that the larva shown in fig. 90 has in all probability nearly or quite attained to the mature form. This larva differs from the other two forms in the possession of small claws at the tips of the last three pairs of pereiopods, and as this is characteristic of Milne-Edwards' genus *Acetes*, and only three genera of Sergestidæ—*Lucifer*, *Acetes*, and *Sergestes*—have been described, I think we may conclude that we have to do with the development of an American species of this genus. At any rate, whatever the systematic position of the adult may be, the fact that the *Protozoëa* is in most respects intermediate between the simple *Protozoëa* of *Lucifer* and the extremely modified *Protozoëa* of *Sergestes*, gives this form so much interest that it seemed best, for the sake of comparison, to embody all that I was able to learn about its metamorphosis in the present paper.

At the earliest stage which was observed, the larva (figs. 77, 78, and 79) is a "*Zoëa*" $\frac{67}{1000}$ inch long, and a comparison of fig. 79 with fig. 44, or of fig. 77 with fig. 43, will show that it is essentially like the last *Zoëa* of *Lucifer*, although the minor differences are both numerous and conspicuous.

The number of somites and appendages is alike in both forms, and the appendages are alike in most respects, although each one of them shows distinctive characteristics of its own.

The carapace (fig. 79) makes about one-half the length of the body, and it is much more flattened than it is in *Lucifer*. It has a rostrum (*R*) and a median dorsal spine, but the postero-lateral spines (*sp.*) point outwards and backwards, instead of directly backwards, and there are a pair of anterior spines as long as the rostrum, projecting over the eyes. The two large pigment-spots which give such a characteristic appearance to the carapace of *Lucifer* are entirely absent, and the thoracic segments and appendages are covered by its posterior edge.

The eyes are mounted upon distinct stalks, while they are sessile in *Lucifer* at the same stage.

The abdominal somites are short and wide, and coloured by bright-red pigment-spots, and their lower edges are produced into strong projecting spines.

The telson is deeply cleft, and its halves diverge from each other like a swallow's tail feathers, so that the posterior ends of the rudimentary swimmerets are visible between them in a dorsal view, as shown in figs. 77, 78, and 79. The spines on the telson are similar in arrangement to those of *Lucifer*, but much longer.

A comparison of the Lucifer *Zoëa* (fig. 44), the Acetes *Zoëa* (fig. 79), and the Sergestes *Zoëa* (CLAUS'S 'Crustacean-System,' taf. vi., fig. 1) at the same stage of development, brings out the extremely interesting fact that the *Acetes* larva stands between the very simple *Zoëa* of *Lucifer* and the remarkably complicated *Elaphocaris* larva of *Sergestes* in nearly every feature in which the two differ. In *Lucifer* the eyes are sessile; in *Acetes* they have short stalks; and in *Sergestes* the stalks are very long.

In *Lucifer* the spines over the eyes are absent; in *Acetes* they are present and simple; and in *Sergestes* they are very long and compound.

In *Lucifer* the postero-lateral spines are parallel to the long axis of the body; in *Acetes* they are oblique, so that they project a little beyond the outline of the body; and in *Sergestes* they are at right angles to the long axis, and compound.

The carapace, including the rostrum, makes about one-third of the total length of the body of the Lucifer *Zoëa;* about one-half of that of the Acetes *Zoëa;* and more than two-thirds of the total length of the Sergestes *Zoëa.* The abdominal somites of the Acetes *Zoëa* are shorter and wider than those of the Lucifer *Zoëa*, and this change is carried still further in the Sergestes *Zoëa.* In the Lucifer *Zoëa* the sixth abdominal somite is the only one which has ventral spines, and these point backwards. All the abdominal somites of *Acetes* have spines, and they point backwards and a little outwards, while in *Sergestes* they all point directly outwards.

The telson is slightly notched in *Lucifer;* deeply forked in *Acetes;* and in *Sergestes* the prongs of the fork diverge so much as to form a right angle.

These facts are extremely interesting, as they seem to show that the *Elaphocaris* is a larva essentially like that of *Lucifer*, which has passed through a remarkable process of secondary modification, resulting in the acceleration of the development of the eyes, and the production of a forked telson, and a very spiny body. The larva of *Acetes* has been modified in the same direction but to a much less degree. It may be asked why we are to assume that the Lucifer *Zoëa* is the primitive form, and the *Elaphocaris* larva the secondary modification rather than the reverse; but a little thought will show that the distinctive features of the *Elaphocaris* stand in direct relation to the environment, as weapons of defence, sense organs, or locomotor apparatus, while the distinctive marks of the Lucifer *Zoëa* are features of general or typical resemblance to the corresponding larva of *Euphausia* and *Penæus.*

I did not succeed in finding the *Protozoëa* from which the *Zoëa* shown in fig. 79 is derived, but I think it extremely probable that future research will show that an unknown larva which has been figured by DOHRN and CLAUS is the *Protozoëa* of *Acetes*, or else of a new closely-related genus of the Sergestidæ.

In his "Untersuchungen über Bau und Entwickelung der Anthropoden" (Zeit. f.

Wiss. Zool., xxi., 1871), DOHRN describes the "Larve eines unbekannten Krebses" from the Indian Ocean (p. 377), which is shown in his plates 29 and 30, figs. 62 to 67. In his 'Crustacean-System' (taf. iv., figs. 2 to 7) CLAUS gives much more satisfactory figures of what appears to be the same larva, and speaks of it as a "Phyllopodenähnlichen Protozoëa unbekannter Herkunft." Its close resemblance to the *Protozoëa* of *Lucifer* renders it extremely probable that it is the *Protozoëa* of a Sergestid, and as the *Protozoëa* of *Lucifer* and that of *Sergestes* are known, this must be the larva of *Acetes*, or of some closely-related unknown form.

The carapace is nearly smooth, rounded, and there is no trace of a rostrum, and it makes more than three-quarters of the total length of the body.

The compound eyes are present and well developed, but they are sessile, and there is no indication of the stalk. The first antenna is seven-jointed, and the two terminal joints are thin and long.

The second antenna is nearly twice as long as the first, and very thick. Its short stout basal portion consists of two joints, and carries a short two-jointed exopodite, with three long terminal non-plumose swimming hairs, and a very large twelve-jointed endopodite with a long swimming hair at each joint.

CLAUS's figures show that the appendages at the back of the antennæ are very much like those of *Lucifer*, and the same ones are present; that is, the mandibles, first and second maxillæ, and first and second maxillipeds.

The hind body is segmented, and ends in a broad, flat, deeply-cleft telson, with six pairs of irregularly plumose hairs, the third pair very much longer and thicker than the others.

A comparison of CLAUS's figure with fig. 27 of this paper will show that most of the differences between this unknown larva and the first *Protozoëa* of *Lucifer* are of the same kind as the differences between the Acetes *Zoëa* (fig. 79) and the corresponding stage of *Lucifer* (fig. 44).

At a time when the eyes of *Lucifer* are rudimentary and sessile they are perfect and stalked in *Acetes*, and at a time when they are entirely absent in *Lucifer* DOHRN's larva has them sessile and rudimentary but distinct.

The *Zoëa* of *Acetes*, like this larva, has its telson deeply forked; its hairs are plumose, and the third is much longer than the others. These resemblances, and the great length of the carapace, render it very probable that this unknown larva is the *Protozoëa* of *Acetes*.

I will now continue my description of the appendages of the *Zoëa*.

The first antenna (fig. 77, *A*) is uniramous, and it consists of a long, cylindrical, two-jointed shaft, and a single short flagellum, which shows obscure traces of a division into three joints. The basal joint of the shaft is a little more than half as long as the second joint, and it carries a single short sharp hair on the inner side of its distal end. The second joint has two much longer hairs on its distal end, and one about half way between its ends. The flagellum makes about one-fifth of the total length of the appendage, and it carries four terminal hairs, two of them about as long as those on the terminal joint of the shaft, and two nearly three times as long.

The second antenna is the chief locomotor organ, and (as shown in fig. 77, *An*) it consists of a thick two-jointed basal portion, which carries a two-jointed exopodite (*ex*) and a ten-jointed endopodite (*en*).

The proximal joint of the exopodite is about twice as long as the terminal joint, and it carries two long hairs on its outer end, and two more near the base. The terminal joint has, at its tip, one short hair, and four which are about as long as the limb. The endopodite consists of four short rings, and a series of six joints like those of the corresponding organ of the *Nauplius*, *Protozoëa*, and *Zoëa* of *Lucifer*. The terminal joint carries four, and each of the five other joints one long swimming hair, and none of these hairs are plumose.

On the basal portion of the appendage there is a large bright-red pigment-spot, which forks and runs along the exopodite and endopodite, about half way to their tips.

The labrum (fig. 77, *L*) is smaller than that of *Lucifer*, with a spine and a large red pigment-spot.

The mandible (fig. 77, *M*, and fig. 80) has small irregular denticles along its cutting edge, and these reach to the tip of the long tooth which occupies the posterior angle of the blade. The mandibles of two specimens were dissected out, and in each case there was a little hairy pad (*m*) upon the posterior surface. It could also be seen in the entire animal (as shown in fig. 77). It is possible that this pad is the mandibular palpus, but it seems much more probable that it is half of the lower lip or metastoma, for no palpus is present on the mandible of *Lucifer*.

The first maxilla (fig. 77, *Mx.* 1, and fig. 81) is quite different from that of *Lucifer* (fig. 46) at the same stage, but the difference is in minor points, and there is essential agreement in general structure. The two basal joints or blades are long and slender, and their hairs are also longer and thinner than they are in *Lucifer*. The endopodite (*en*) is placed nearly at right angles to the base, and is distinctly three-jointed. It carries five hairs as it does in *Lucifer*, and they are similarly placed, but longer. The three hairs on the scaphognathite are about equal in length, and the plumules on their sides are short and irregular.

The second maxilla (fig. 77, *Mx.* 2, and fig. 82) is much like that of the Lucifer *Zoëa* (fig. 47), but the three hairs at the tip are more than twice as long as those on the inner edge of the appendage, and they are irregularly plumose, while they are simple in *Lucifer*.

The first maxilliped (fig. 77, *Mp.* 1, and fig. 83) differs from that of *Lucifer* (fig. 48) in the same way, and the exopodite carries seven instead of four hairs, and these are as long as the appendage, and two-jointed.

The second maxilliped (fig. 77, *Mp.* 2) is about as long as the first, but it does not seem to be of much functional importance. It is usually carried stretched back along the hind body, as shown in the figure, and its hairs are short. As in *Lucifer* at the same stage, the exopodite is as long as the endopodite.

The third pair of maxillipeds, and the first, second, third, and fourth pairs of thoracic limbs are represented by buds, as in *Lucifer* at the same stage. The bud for the third maxilliped (fig. 77, *Mp.* 3) is bilobed, longer than the others, and it points backwards outside the other buds. The buds for the first three pairs of periopods are bilobed, in contact on the median line, and about equal in size. Those for the fourth pair are much smaller, and are hidden in a ventral view by the buds for the third pair, but they can be seen in side view (as shown at *T* 4 in fig. 78). There is no trace of the fifth pair of pereipods either at this or at any later stage. CLAUS figures buds for the fifth pair in the *Zoëa* of *Sergestes,* and also in the next or *Acanthosoma* stage of *Sergestes;* but the study of the *Zoëa* of *Acetes* shows even more satisfactorily than is the case in *Lucifer* that these appendages are entirely absent, and it seems safe to believe that this is the case in *Sergestes* also until the larva of the latter has been carefully examined with reference to this particular point.

The abdominal appendages, with the exception of the fifth pair, are entirely absent; but each abdominal somite has a pair of long ventral spines. The swimmerets are represented by long bilobed buds, which project beyond the fork or notch in the telson. The abdominal ganglia are very much more conspicuous than they are in *Lucifer.*

The distribution of pigment is somewhat different from what we find in the Lucifer *Zoëa,* and nearly all the pigment-spots are bright-red. There is a large spot of red and one of reddish-yellow on the eye stalk, a red spot on the labrum, a large red and very dendritic spot on the second antenna, red spots on the dorsal surface of the posterior edge of the third, fourth, and fifth abdominal somites on the median line; red spots on the ventral surface of the first, second, and third at the bases of the spines; a red and a brown spot at the base of the spine on the fifth; a brown spot at the base of the spine on the sixth, and a red spot on the base of the swimmeret. The anal spots are large and bright-red.

On September 20th I found several specimens of the stage which has just been described. Fig. 79 was made from one of them, which was then placed in a glass of water by itself, and the next day it was found to be moulting. In the evening the moult was found to be finished, and the larva was swimming actively. The drawing given in fig. 84 was made from it without injuring it, and later stages were also drawn from the same specimen.

The larva, $\frac{70}{1000}$ inch long, has undergone very great change, and although it is an *Acanthosoma,* it presents many important differences from both *Lucifer* and *Sergestes.*

The abdomen has lengthened so that the carapace makes less than one-third the total length of the body, and the dorsal and postero-lateral spines have disappeared.

The abdominal spines stand out from the body, and the swimmerets have become the chief locomotor organs. The spine has disappeared from the labrum; the two pairs of antennæ have changed from the larval to the adult form; the endopodite of the fourth pereiopod, and the first three pairs of pleopods are represented by long buds.

The first antenna (fig. 84, *A*) consists of a three-jointed shaft about as long as the carapace, and two terminal flagella. The basal joint of the shaft makes half the total length of the appendage, and the other two are about equal to each other. On the inner edge of the distal two-thirds of the shaft there are eight long, similar equidistant, plumose hairs, and there are two short spines on the outer edge. The inner flagellum is short, and carries one long slender terminal hair. The outer one is more than twice as long, and carries two thick sensory hairs.

The exopodite of the second antenna (fig. 84, *ex*) has become a scale, only one-third as long as the endopodite, which is now a ten-jointed flagellum about as long as the carapace.

The second and third maxillipeds (fig. 84, *Mp.* 2 and *Mp.* 3) and the first, second and third pereipods (fig. 84, *T* 1, *T* 2, and *T* 3), are Schizoped-like, but they are of very slight functional importance, and their endopodites are folded forwards on the ventral surface, like the maxillipeds of *Squilla*, so that it is impossible to study the mouth parts without dissection. The endopodite of the fourth pereiopod has entirely disappeared, and the limb is represented only by its exopodite. The five exopodites are about alike, and they all end in long slender swimming hairs: those of the four pereiopods (*T* 1 *ex*, *T* 2 *ex*, *T* 3 *ex*, and *T* 4 *ex*) are bent outwards and upwards towards the dorsal surface, as in the maxillipeds of a Crab *Zoëa*, but those of the second and third maxillipeds (*Mp.* 2 *ex* and *Mp.* 3 *ex*) are more nearly parallel to the endopodites. The endopodite of the second maxilliped (*Mp.* 2) is free and movable, but those of the third maxillipeds (*Mp.* 3) and of the first, second, and third pereiopods are covered by a delicate cuticle, and are almost immovable.

I did not actually witness the next moult, but four days later the larva, $\frac{80}{1000}$ inch long, was in the stage shown in fig. 85. The exopodites of the thoracic limbs have become reduced to rudiments, the limbs themselves have stretched out and are now functional, as are the three pairs of abdominal feet.

The first antenna (fig. 86) has not changed much, but its base is swollen and the otocyst has appeared.

The second antenna is now half as long as the whole body, its flagellum is ten-jointed, and red pigment has appeared at its base and tip (fig. 87). The outer end of the scale carries nine long plumose hairs arranged on the tip and inner edge.

The second maxilliped (fig. 88, *Mp.* 2) is bent into a knee, and is fringed by sixteen plumose hairs. Its exopodite is rudimentary, but longer than in any of the appendages which follow.

The third maxilliped (fig. 88, *Mp.* 3) is long, slender, six-jointed, with a rudimentary endopodite.

The other three limbs (fig. 88, *T* 1, *T* 2, *T* 3) are six-jointed and they end in enlarged chelæ. The first is the shortest; the second is about as long as the third maxilliped, and the third is still longer. The fourth is now represented only by a small rudiment and a ganglion; and the fifth is entirely absent, as it has been at all stages.

The three pairs of pleopods are alike in structure, and each consists of two joints about equal in length (fig. 89). The outer half of the terminal joint is toothed and carries six pairs of long slender non-plumose hairs, so arranged as to form a paddle.

The rostrum (fig. 85) is long and curved, and it has a single secondary spine in its upper surface. A pair of very small spines have also appeared at its base.

This specimen had nearly completed its moult into the stage shown in fig. 90 on the last day of my stay at the seashore—five days after fig. 85 was drawn.

Fig. 90 was drawn from another specimen, $\frac{15}{100}$ inch long, which was captured at the surface on September 25th.

The eye-stalks are long and very movable, the flagellum of the second antenna is considerably longer than the body, the five pairs of thoracic limbs have developed gills, and the fourth and fifth pleopods have appeared; but in other respects the structure is nearly as it was in the preceding stage. The endopodites of the maxillipeds are pointed, but those of the three pereiopods end in rudimentary chelæ. The endopodite of the third pereiopod is much longer and thicker than the others, and its tip reaches nearly to the rostrum.

The buds for the first three pairs of pleopods are long, obscurely jointed, and they meet each other on the median line. There are as yet no traces of the fourth and fifth pairs. The spines on the abdominal somites are long and sharp. Those on the first three somites point outwards and forwards, those on the fourth point almost directly outwards, and those on the fifth outwards and backwards. The sixth abdominal somite has lengthened, and is now about as long as those of the others. The telson is short and shield-shaped, with two pairs of long and one pair of very short spines, and the swimmerets are perfectly formed and fringed with long plumose swimming hairs.

The exopodite is long, narrow, with a smooth outer edge which ends in a tooth, and a rounded point. It carries fifteen hairs: ten on the inner edge, two on the tip, and three between the tip and the tooth. The endopodite is nearly as long and wide as the exopodite, and it carries nineteen hairs: two at the end, eight on the outer, and nine on the inner edge.

The ocellus is still present, and the pigment-spots have nearly the same arrangement as before, but some of them are now yellow or green instead of red.

The eye-stalk is about as it was before, and the ocellus is still present and double. As regards the more minute structure of the appendages the first antennæ are now about as long as the carapace, and most of the increased length is in the flagellum, which now consists of seven joints. The secondary flagellum is still quite short. The shaft of the antenna is three-jointed, as before, but the basal joint is much lengthened, and now makes more than half the total length. The auditory organ at its base is now very conspicuous, and the inner edge of the shaft carries eleven hairs instead of six; five of these are on the basal joint, three on the second joint, and three on the third.

The scale of the second antenna has lengthened and is now more than half as long as the first antenna, including the flagellum. Its inner end carries eight, and its tip three hairs, and the outer edge of the tip is toothed. The swollen base of the flagellum of the second antenna carries a large red pigment-spot, and the flagellum, which is considerably longer than the body of the animal, is also marked by bright-red pigment throughout the greater part of its length.

The second maxilliped has become completely bent upon itself, and it bears a close resemblance to that of the adult *Lucifer*, although it carries a gill, as do the second maxillipeds and the three pairs of thoracic limbs. All traces of the exopodites have disappeared from all these appendages, but their structure and comparative length are about as before.

The first, second, and third pairs of abdominal feet have increased in length, and the first is now almost as long as the last thoracic limb. The second is a little shorter; the third is still shorter, and has acquired a second terminal branch, which is as yet rudimentary.

The fourth and fifth pleopods, which have now made their appearance, are much smaller than the others, and each has one large and one small terminal branch.

The swimmerets and telson are very similar to those of the immature *Lucifer*, although the telson is shorter and wider. The exopodite has fifteen hairs on its inner edge, two on its rounded tip, and four between the tip and the tooth. The endopodite has nineteen hairs. The surface of the carapace is finely punctated, and the rostrum has no secondary spine. The spine has disappeared from the first abdominal somite, and the one on the third somite is longer than any of the others. The dorsal surface of the third somite is bent, so that the abdomen is no longer perfectly straight. Large conspicuous red pigment-spots have appeared on the lower edges of the second, third, fourth, and fifth abdominal somites.

As the series of drawings which I have given was made from such a small number of specimens, I am unable to contribute much information as to the changes of the mouth parts, and must leave this, as well as the exact determination of the adult form and systematic position of the species, to future research.

In his 'Facts for Darwin' FR. MÜLLER has figured a larva (fig. 33) which is extremely like, if not identical with the one shown in fig. 90, and he regards it as the young of a Prawn, closely related to *Penæus*. CLAUS has suggested ('Crustacean-System,' p. 35) that it is much more likely to prove to be a young Sergestid than a Prawn, and the facts regarding its metamorphosis which I have given above, certainly seem to point in the same direction. An earlier stage of development is given in FR. MÜLLER'S fig. 32, and a comparison with my fig. 84 will show that the same larva at an earlier stage might, when crushed by a cover glass, present very much the same appearance as this larva. If they are the same FR. MÜLLER is certainly mistaken in his statement that fig. 33 follows directly after fig. 32, without the intervention of a Schizopod stage, for the metamorphosis is really quite complicated, and a true Schizopod stage exists, although it is of extremely short duration.

VI.—Relation between the Larvæ of Lucifer, Acetes, Sergestes, Penæus, and Euphausia, and the Significance of the Decapod Zoëa and the Crustacean Nauplius.

The general significance of the peculiar type of Decapod metamorphosis, of which *Lucifer* is now the most thoroughly known illustration, has been discussed with the greatest ability and knowledge of the facts by Claus in his 'Untersuchungen zur Erforschung der Genealogischen Grundlage des Crustacean-Systems.' My own acquaintance with the phenomena of Crustacean morphology in general is very far from being sufficiently extended and minute to qualify me for a critical discussion of this work; but while the facts in the life-history of *Lucifer* seem to tend to a similar conclusion, and even to place it upon a much firmer basis than before, they also indicate that Claus's views cannot receive unqualified acceptance in their present shape.

I shall not venture at present upon the broader aspects of the question, but I wish to draw attention to the resemblances and differences between the various larval stages of *Lucifer* and those of a few closely-related forms. The materials which are at present available for a comparison of this kind are extremely scanty, for there is no other closely-related form in which all stages, from the egg to the adult, have been actually traced in a single species by rearing captive specimens.

Comparison of Lucifer *and* Acetes.

The genus which shows the closest similarity to *Lucifer* is *Acetes,* but in this case we are ignorant of both the early and the later stages. During the last "*Zoëa*" stage the resemblance between the two forms is well marked, and is shown in such features as the similarity in the shape of the carapace and hind body; in the length and structure of the two pairs of antennæ; in the mode of locomotion; by rowing with the antennæ; in the presence of an ocellus; the presence of a spine on the labrum; the close similarity of the mouth parts and maxillipeds; the rudimentary structure of the thoracic limbs and swimmerets; the total absence of the fifth thoracic somite; and the absence of the first five pairs of pereiopods. Notwithstanding these resemblances the differences are quite conspicuous. The eye is sessile in *Lucifer,* stalked in *Acetes.* The shaft of the first antenna is one-jointed in *Lucifer,* two-jointed in *Acetes.* The endopodite of the second antenna has two basal rings in *Lucifer,* four in *Acetes.* The two lobes of the metastoma are conspicuous in *Acetes,* and could not be made out at all in *Lucifer.* The abdominal somites are rounded in *Lucifer,* and spiny in *Acetes;* and the telson is deeply forked in the latter, slightly notched in the former. In a word, the resemblances between the two are general rather than detailed, and the differences are specific differences of the same character as those between closely related adult animals. A comparison of column 2 of Table V. with column 1 will show these resemblances and differences in tabular form.

If we assume the correctness of the extremely probable assumption that Dohrn's and Claus's unknown larva is the earliest *Protozoëa* of *Acetes*, the resemblances between it and the corresponding larva of *Lucifer* (compare fig. 27 with Claus, fig. 2, taf. iv.) are much greater than they are at a later stage. The chief differences are the presence in *Acetes* of rudimentary compound eyes; the great length of the carapace; the absence of a rostrum and spines; the great number of joints in the first and second antenna, and the difference in the length of these two appendages; the deep notch in the telson. The close similarity between the two larvæ at this stage will be seen by comparing column 1 of Table IV. with column 2.

After the moult which ends the *Zoëa* series the differences between the *Acetes* larva (fig. 89) and the *Lucifer* larva (fig. 53) become much greater, although they do not obscure the fundamental similarity between the two forms. In each of them the carapace makes less than one-third the total length of the body, and it has a rostrum and two antero-lateral, but no postero-lateral or dorsal spines. The first antenna has lost its swimming hairs, and has developed one flagellum in each form and two in *Acetes*. In both forms a series of long plumose hairs has appeared on the inner edge of the shaft of the appendage. In both forms the second antenna has lost its locomotor function and assumed the adult form, but it is rudimentary in *Lucifer* and well developed in *Acetes*.

The ocellus is present and the eye stalked and movable in both.

The fifth thoracic somite and its appendages are entirely wanting in both forms. The fourth is biramous in *Lucifer*, and similar to the ones before it, but in *Acetes* the limb proper has disappeared and the appendage is represented only by an exopodite. The second and third pairs of maxillipeds, and the first, second, and third pairs of pereiopods are essentially alike in structure in both forms, but in *Acetes* the endopodites are rudimentary, covered by a cuticle, and functionless. The swimmerets are present and very similar in the two forms, but the other abdominal appendages are absent in *Lucifer*, while the first, second, and third pairs are developed, but rudimentary in *Acetes*. The abdominal somites have acquired ventral spines in both forms, but these are very small in *Lucifer* and long and prominent in *Acetes*. The telson is long and narrow in *Lucifer* and short and wide in *Acetes*. The relation between the two forms at this stage of development will be seen by a comparison of columns 1 and 2 of Table VI.

The later history of the two genera can hardly be divided into parallel stages. *Lucifer* keeps all its Schizopod limbs for at least two more moults, and as shown in fig. 54, acquires the rudiments of all the abdominal feet at one time, and before the fourth pair of thoracic limbs and the exopodites of the others and of the maxillipeds disappear, while *Acetes* (fig. 85) loses its exopodites at once, and the maxillipeds, thoracic limbs, and antennæ become like those of an adult Sergestid some time before the appearance of the five pairs of pleopods; and these do not appear together, but in two sets.

It is interesting to note that although the changes which the two forms undergo at successive moults do not admit of exact comparison with each other; the outcome, after a few moults, is almost exactly the same, as will be seen by a comparison of fig. 60 with fig. 90.

The number and character of the somites and appendages is now the same, and while the two forms differ greatly in outline and proportion, the young *Acetes* is essentially like the young *Lucifer*, except in the length of the flagellum of the second antenna, the presence of chelæ on the thoracic limbs, the presence of gills, and the absence of a "neck." The outcome of the process of development is alike, but the paths followed diverge from each other to converge again at this stage.

Comparison of Lucifer *and* Sergestes.

The metamorphosis of *Sergestes* is more like that of *Lucifer* than is the case with any other known Crustacean except *Acetes*, but our knowledge of the development of *Sergestes* is incomplete, and we have no assurance that the various stages which have been described belong to the same species.

In 1870, Dohrn described a remarkable larva ("Untersuchungen über Bau und Entwickelung der Decapoden, No. 10, Beiträge zur Kenntniss der Malacostraken und ihrer Larven, Part 4, Beschreibung einer neuen Decapoden-Larve," Zeit. f. Wiss. Zool., xx., p. 607) which he collected at the surface at Messina, and which he was unable to refer to any adult form. He proposed for this larva the provisional name *Elaphocaris*. *Elaphocaris* is a *Zoëa* which so far as its appendages are concerned does not differ much from the last *Zoëa* of *Lucifer*, but its abdomen is very spiny, and the spines on the carapace are drawn out so that each one of them is nearly half as long as the body, and they are fringed with rows of long secondary spines which are hooked at their tips, and so arranged as to give to the body a very grotesque appearance, and the larva does not, at first sight, show any similarity to the simple *Erichthina* larva of *Lucifer*.

Claus had several years before described ("Ueber einige Schizopoden und niedere Malacostraken Messinas," Zeit. f. Wiss. Zool., xiii., 1863) a larval Crustacean with swimmerets, biramous thoracic limbs, and a very spiny body, which he calls an *Acanthosoma*. This same larva, or a very closely related form, had been figured and described nearly twenty-five years before by Dana ('Crustacea,' p. 664, plate 44, fig. 5) as *Sceletina armata*.

In the same paper Claus gives a figure of a young Crustacean, which had previously been described by Leuckart under the name of *Mastigopus*, and shows that it is in all probability a young *Sergestes*.

In his 'Untersuchungen zur Erforschung,' &c., he describes an *Elaphocaris* at a much younger stage than Dohrn's figure, and shows that this larva, Dohrn's *Elaphocaris*, his own *Acanthosoma*, and Leuckart's *Mastigopus* are successive stages in the development of *Sergestes*.

From independent researches in the South Pacific, WILLEMÖES-SUHM also ascertained (Proc. Royal Soc., Dec. 9, 1875, p. 133) that *Elaphocaris* is the larva of *Sergestes*, and he traced its development through the *Acanthosoma* stage, which, from its resemblance to *Amphion*, he calls the *Amphion* stage.

These various observations, and especially those by CLAUS, give us a pretty complete acquaintance with the metamorphosis of *Sergestes* from the first *Protozoëa* stage to maturity.

The first *Protozoëa* (CLAUS, 'Untersuchungen,' taf. v., fig. 1) has, like the *Protozoëa* of *Lucifer*, locomotor antennæ, a spine on the labrum, a partially segmented hind body, and a very spiny telson. The mandibles, first and second maxillæ, and first and second maxillipeds are like those of the corresponding *Lucifer* larva. In addition to the spiny carapace it presents the following conspicuous differences from the *Lucifer* larva. The eyes are stalked, movable, and compound. The first antenna has seven joints. The endopodite of the second antenna has no small rings at its base. There is a third pair of maxillipeds. Five thoracic somites are represented in the figure. The telson is very deeply cleft. The relation between the larva and the first *Protozoëa* of *Lucifer* will be seen by a comparison of columns 1 and 3 of Table IV.

The next stage which CLAUS describes (taf. vi., fig. 1) is no doubt separated from the first by one or more intermediate stages. The rostrum has developed a pair of long secondary compound spines at its base, which do not correspond to anything in the corresponding larvæ of *Acetes* and *Lucifer*.

The thoracic limbs are represented by *five* pairs of rudimentary bilobed buds. There are five free abdominal somites without appendages, and the sixth and telson are represented by an unsegmented region, which carries a pair of long bilobed pouches, the rudimentary swimmerets.

The relation between *Elaphocaris* and the corresponding larvæ of *Acetes* and *Lucifer* may be understood by a comparison of column 3 of Table V. with columns 1 and 2.

In the next or *Acanthosoma* stage ('Untersuchungen,' taf. v., fig. 6) the two pairs of antennæ assume the adult form, and the thoracic limbs and swimmerets become developed as they do in *Lucifer*, and the carapace loses its posterior spines, although there are three in place of one pair of anterior spines. The telson is distinct from the last abdominal somite, and all the abdominal somites have projecting spines.

The eye-stalks are much longer than they are in the other two forms. The first antenna has a secondary flagellum, as in *Acetes*, and the scale and flagellum of the second antenna are well developed.

The exopodites of the maxillipeds and pereiopods are very long, many-jointed, except in the first maxilliped, and they are longer than the endopodites in all the pereiopods.

The fifth pair of pereiopods are present and like the others. The swimmerets are very long and slender, and the telson very short and forked.

This stage, like the corresponding stage of *Lucifer*, and unlike that of *Acetes*, persists for more than one moult, and the five pleopods make their appearance together, as rudimentary buds, before the exopodites of the pereiopods and maxillipeds disappear.

The third column of Table VI. shows the resemblances to *Lucifer* and *Acetes* at this stage.

In the immature or *Mastigopus* stage (see CLAUS's 'Ueber einige Schizopoden und niedere Malacostraken, Messinas') the three forms are almost exactly alike, except as far as the generic distinctions are concerned, and the young *Sergestes* scarcely differs from the young *Lucifer* except in the absence of a neck, the length of the flagellum of the second antenna, and the presence of rudiments of the fourth and fifth pairs of pereiopods.

Comparing the whole course of development of the three forms, as far as it is known, we notice that while the larval stages of *Sergestes* are much more different than those of *Acetes* from the corresponding stages of *Lucifer*, the character of the change at each moult is much more like what we have in *Lucifer* than what we have in *Acetes.*

We cannot fail to notice, in the second place, that the attempt to express the facts of the metamorphosis of these forms, so far as we know them, in a tree-like diagram, would result in a tree placed upside down, with the branches which represent the three Protozoëas much more divergent than those which represent the three young Sergestids. A similarity of type runs through the whole metamorphosis, but it is no more marked at the early stages than it is in the late stages, while the secondary differences are much more conspicuous during the *Zoëa* and *Acanthosoma* stages than they are as we approach the adult form.

While this is true it is also true that if we imagine a metamorphosis which shall agree with these three in all their common features, but shall have none of the features which they do not all share, we shall have something much more like the metamorphosis of *Lucifer* than that of *Acetes* or *Sergestes,* and we must therefore regard the life-histories of these three forms as somewhat divergent modifications of a form of development which is at present more closely adhered to by *Lucifer* than by the other two, and in this metamorphosis we must recognise a *Protozoëa* stage when the two pairs of antennæ are locomotor, the ocellus present, the labrum furnished with a spine, the carapace armed with posterior dorsal and lateral spines and a rostrum; the two pairs of maxillæ, and two pairs of maxillipeds present, and the thoracic and abdominal segments without appendages. This stage persists, with slight modification, through several moults in all of them, and is followed by an *Acanthosoma* stage, in which the carapace has a rostrum and antero-lateral spines, and a smooth posterior edge; the eyes are stalked; the two pairs of antennæ have their adult character; there are at least four pairs of pereiopods with swimming exopodites; the swimmerets are large and have their adult form, and the other abdominal appendages are absent. The duration of this stage and the mode of transition to the next varies in the three forms, but it is followed in all by what may be called a *Mastigopus* stage, characterised by the general features of the family.

In all three forms the somites, and with the exception of the swimmerets the appendages also, develop in serial order from in front backwards.

The interesting question whether we are to attribute to this typical form of development a fifth thoracic somite and appendages must, I think, be left in doubt. A comparison of the Sergestid larvæ seems to indicate its absence, but wider comparison with *Penæus* and the Schizopods seems to lead to the opposite view.

Comparison of Penæus *with the Sergestidæ.*

In order to render the comparative tables as complete as possible, I have added columns showing the corresponding stages of *Penæus* and *Euphausia.*

FRITZ MÜLLER has described a number of stages in the development of a species of *Penæus* ("Verwandlung der Garneelen," Arch. f. Naturgeschichte, 1863, pp. 8–23, taf. ii.). The series commences with a *Nauplius* which may belong to the same species, although we have no certainty of this. In a second paper ("Ueber die Naupliusbrut der Garneelen," Zeit. f. Wiss. Zool., xxx., 163–166) he gives, in reply to doubts which had been expressed to him by SPENCE BATE, ALEX. AGASSIZ, PAUL MAYER, and others, the following reasons for believing in the specific identity of all the forms in his series:—1st, the peculiar mode of locomotion; 2nd, the resemblance in colour; 3rd, the great length of both pairs of antennæ; 4th, the character of the mandible; 5th, the presence of four pairs of buds in the *Nauplius*, and four corresponding pairs of limbs in the *Zoëa;* 6th, the similarity in the structure of the heart, digestive tract, and liver in the *Nauplius* and the youngest *Zoëa;* 7th, the presence of frontal organs in both stages. As all the points except the colour would apply to any Crustacean which passes through a *Protozoëa* stage, there is certainly nothing more than a presumption that the whole of his series represents a single species; but as there is no doubt that the *Nauplius* belongs to *Penæus* or to some closely-related form, I have included it in the table.

FR. MÜLLER'S account of the later stages is supplemented by a few additional observations of other species by CLAUS ("Untersuchungen," &c., pp. 11 and 41, taf. ii. and iii.), and I have compiled the columns in the tables from both sources.

The first *Nauplius* stage (Table II., column 3) appears to be more simple than that of *Lucifer*, as MÜLLER failed to observe any buds to represent appendages posterior to the mandibles.

The *Nauplius* stage is followed by a meta-*Nauplius* stage (Table III., column 2), which is distinguished from that of *Lucifer* by the great size of the blade of the mandible, by the presence of frontal organs, and by the shortness of the carapace.

The next stage is a *Protozoëa* (Table IV., column 5), with a rounded carapace without spines or rostrum, four basal rings and six terminal joints in the endopodite of the second antenna, a spine on the labrum, two pairs of maxillæ, two pairs of maxillipeds, and a long hind body which, according to CLAUS, is divided into six thoracic and five abdominal somites, and terminates in a deeply-forked telson with seven pairs of spines.

This stage persists with slight change for several moults, and at the last the buds for the thoracic limbs and swimmerets appear. According to CLAUS, the rudiments of all the abdominal appendages can be seen at an earlier stage.

The passage from the last of the *Protozoëa* series to the first Schizopod stage is attended by a complete change in the structure of the antennæ, and these now assume the adult form. The carapace also acquires two antero-lateral spines and two more at the base of the rostrum. At this time it is much like *Lucifer*, as shown in column 4 of Table VI., but the endopodites of the third pair of maxillipeds and of the pereiopods are rudimentary, and shorter than the very long-jointed exopodites.

The significance of the various stages in the metamorphosis of the higher Crustacea is one of the most interesting questions in the whole field of morphological science, and it has given rise to at least its due share of speculation, but it will not be out of place to examine the relation between the facts which have been described and the various theoretical views which have been expressed upon the subject. In the case of the Sergestidæ it is obvious, in the first place, that the adult *Lucifer* and *Acetes* also, if *Acetes* be an adult, are little more than mature representations of the *Mastigopus* stage, complicated in the case of *Lucifer* by the formation of a neck, and in the case of *Acetes* by the presence of gills, and chelæ on the pereiopods. There can also be little doubt that the Schizopod stage of development in the Sergestidæ and *Penæus* bears a similar relation to the adult Schizopods, especially to *Amphion*, the adult character of which seems to be established by WILLEMÖES-SUHM'S observations (Proc. Roy. Soc., Dec. 9, 1875).

The significance of the *Zoëa* stage in the higher Decapods is one of the most vexed points in Crustacean morphology. We have shown that in the Sergestidæ and in *Penæus* the so-called *Zoëa* stage is nothing but a preparation in the *Protozoëa* for the next or Schizopod stage; that it involves no changes of structure except those which are related to the form which it is to assume after the next moult, and that the *Zoëa*, as a distinct stage, is absent. The life-history of these forms would therefore lead us to suspect that the Brachyuran *Zoëa* is a secondary modification of the more primitive *Protozoëa*, and we may perhaps see in the larval skin which many Crab-*Zoëas* shed soon after or even before they leave the egg, and which usually has a conspicuously forked and very spiny telson—a remnant of the unmodified *Protozoëa* stage.

DOHRN ('Geschichte des Krebsstammes, Jenaische Zeitschr.,' 1871) and FRITZ MÜLLER ('Für Darwin') have held that the typical *Zoëa*, with segmented abdomen and suppressed thorax, is the ontogenetic recapitulation of an ancestral form which has formerly existed as an adult, and DOHRN even goes so far as to recognise the still more remote ancestor of this *Zoëa* type in an embryo ("Untersuchungen über Bau und Entwickelung der Arthropoden; eine neue *Nauplius*-form: Archizœa gigas," Zeit. f. Wiss. Zool., xx., 597), which WILLEMÖES-SUHM has recently shown ("On the

Development of *Lepas fascicularis* and the '*Archizœa*' of Cirripedia," by R. von Willemöes-Suhm, Ph.D., Proc. Roy. Soc., Dec. 9, 1875, pp. 129–130) to be the *Nauplius* of a Barnacle, in all probability *Lepas australis.*

Claus, on the other hand, believes that the *Zoëa* has no such ancestral significance ("Untersuchungen," &c., p. 31). That it has been formed by secondary modification of the *Protozoëa*, and that the views of Müller and others, that the *Zoëa* presents a picture of the remote ancestor of the Malacostraca, is fundamentally erroneous; and not only this, but that the *Protozoëa* itself is the result of the extreme secondary modification of an ancestral form which Claus proposes to call an Urophyllopod, and which he believes to have had the following characteristics ("Untersuchungen," p. 23): A greatly developed shield-like carapace, produced by a fold of the integument in the region of the maxillæ, and probably armed with median and unpaired spines; two maxillary segments and appendages, eight somites of the mid-body with appendages, and six abdominal somites with swimmerets and telson; a many-chambered heart; compound eyes, probably stalked; a first antenna with sensory hairs; locomotor second antennæ, in which the exopodite was probably a scale; the mandible probably lacked a palpus; the metastoma was represented by a pair of paragnathi; the maxillæ had their basal joints modified for mastication, their endopodites reduced to a jointed palp, and the exopodite modified to form a scoop or scaphognathite for regulating the flow of the respiratory current under the carapace.

The following eight pairs of appendages were more like Schizopod feet, and each of them carried a basal gill-plate; the six pairs of abdominal appendages had large basal joints with two branches and gill-plates.

Claus believes that we may recognise in *Nebalia*, which has stalked eyes, a scale on the first antenna; only one long flagellum on the second antenna; a mandibular palp; a highly specialised, long jointed endopodite on the first maxilla; two long limb-like rami on the second maxilla; eight pairs of phyllopod-like thoracic limbs with jointed endopodite, flat, spiny exopodite and gill; six pairs of pleopods, the last two rudimentary; and a seventh somite between the sixth abdominal somite and the deeply-forked telson ("Ueber den Bau und die Systematische Stellung von Nebalia," Zeit. f. Wiss. Zool., xxii. p. 323–330), a very slight modification of this ancestral Urophyllopod.

He gives on pages 69–71 of his "Untersuchungen," &c., a long, minute, and extremely ingenious explanation of the way in which this Urophyllopod stage of development became converted by secondary modification into the Malacostracan *Protozoëa*, and afterwards, by still greater modification in the same direction, into the typical *Zoëa* of the higher Decapods.

The facts which have been detailed and tabulated with reference to the metamorphosis of the Sergestidæ and *Penæus* seem to substantiate at least a portion of this view, and to show that the typical *Zoëa* is a secondary modification of the *Protozoëa*;

but a comparison of these forms with the metamorphosis of *Euphausia*, upon which CLAUS lays especial emphasis, seems to demand a directly opposite interpretation.

If the *Zoëa* has been produced by a secondary modification of the *Protozoëa* we should expect to find the characteristics of the *Protozoëa* better preserved in the Schizopods than in the lower Decapods, and if we find in the Schizopods certain features of the typical *Zoëa*, which are absent in the *Protozoëa* of the lower Macroura, we can hardly accept without question the interpretation which sees, in secondary modification of the latter, the origin of the *Zoëa*. In *Euphausia* the somites appear in regular succession, from in front backwards, but the somites of the abdomen acquire appendages before the pereiopods appear, and there is a stage when the abdomen is fully developed and the thorax almost absent; a stage which, therefore, resembles the Brachyuran *Zoëa* more perfectly than any stage in the development of *Lucifer*, *Acetes*, *Sergestes*, or *Penæus*.

We have no complete history of any one species of *Euphausia*, but the observations of METSCHNICKOFF (Zeit. f. Wiss. Zool., xix., pp. 479–481, and xxi., pp. 390–401), and CLAUS (Zeit. f. Wiss. Zool., xiii., pp. 442–454, and "Untersuchungen," &c., pp. 9 and 33) give us a tolerably complete account of the metamorphosis of the genus.

METSCHNICKOFF's larva is extremely like that of *Lucifer*, although there are many differences. It is interesting to note that it leaves the egg in a much more rudimentary form, passes through a greater number of moults, and attains to much greater structural complexity than *Lucifer* during the *Nauplius* stage. We can select three stages which agree pretty closely with the egg *Nauplius*, the first free *Nauplius* and the last, or meta-*Nauplius*, of *Lucifer*, but between, after, and before these stages there are others which are not found in *Lucifer*.

The youngest *Nauplius* (Zeit. Zool. xxi., fig. 2) is so much less advanced than the egg *Nauplius* of *Lucifer* six hours before hatching, that it does not seem probable that it normally leaves the egg in this condition.

It has an oval body, without ocellus, mouth, or labrum, and there is no trace of more than three pairs of appendages or of the carapace. At the next stage the swimming hairs of the first three pairs of appendages are fully developed, and the anus, notch, and two spines of the telson are present. In these respects it is more advanced, but in the rudimentary condition of the labrum and metastoma less advanced, than the first free *Nauplius* of *Lucifer*. The buds for the first and second maxillæ and the first pair of maxillipeds are present, but continuous across the median line of the body. According to METSCHNICKOFF, the larva shown in fig. 3 of his first paper is in the next stage of development; but I can scarcely believe that it belongs to the same species, for the ocellus is absent, and the hairs on the three pairs of locomotor appendages are much more rudimentary than they are in fig. 3 of the second pupa.

The next stage (fig. 4) of the second paper agrees with the first free *Nauplius* of

Lucifer, so far as the form and number of the appendages is concerned; but the last pair of buds are biramous, and the carapace and telson are well developed. The next stage (fig. 5) of the second paper is more advanced in nearly every respect than the second free *Nauplius* or meta-*Nauplius* of *Lucifer*. The mandible is rudimentary, but still bilobed, with no trace of a blade. The outline of the carapace is free from the body, and its anterior and posterior edges are spiny. It has frontal organs, and the basal joint of the second antenna carries five recurved hooks.

According to the author, figs. 2 and 3 of the first paper show the next stage; but the structure of the hairs on the antennæ, the fact that they are plumose, and the very deep notch in the telson, seem to indicate that this is another species. However this may be, the structural complexity at this and the next (first paper, fig. 6) stage is much greater than we find it in *Lucifer* at the end of the *Nauplius* series.

It will be observed that, while METSCHNICKOFF'S larva and the *Nauplius* of *Lucifer* are essentially alike, there is at no time an actual agreement, since certain structures, as the carapace, become developed earlier, and others, as the labrum, later than they do in *Lucifer;* and certain structures, as the frontal organs and the hairs on the base of the antennæ, are entirely absent in *Lucifer*.

In column 4 of Table II. I have compared fig. 4 of METSCHNICKOFF'S second paper with the first free *Nauplius* of *Lucifer*, and in column 3 of Table III. his fig. 5 with the last *Nauplius* stage of *Lucifer*.

The various *Protozoëa* stages are shown by CLAUS in plate 1 of the "Untersuchungen," &c. The early *Protozoëa* (Table IV., column 5) is much like that of *Lucifer*, but the carapace is serrated, there is only one pair of maxillipeds, and, according to CLAUS there is a fifth thoracic somite. In the last *Zoëa* stage (Table V., column 5) all the abdominal somites and the rudimentary swimmerets are present, but there is no trace of the second and third pairs of maxillipeds or of the pereiopods.

Up to this point the course of development has followed essentially the same line as in the *Sergestidæ*, but, as we should expect, the *Protozoëa* series is not followed by a larval Schizopod stage, but by a series of moults during which the adult characteristics are gradually acquired. In the loss of the posterior spine of the carapace, the acquisition of antero-lateral spines, and the change in the antennæ from the *Nauplius* form to the adult form, the moult is like that of *Penæus* and the Sergestidæ; but the second and third maxillipeds and the pereiopods appear one at a time in succession from in front backwards, and the abdominal feet appear before the pereiopods. There is no *Zoëa* stage it is true, but the course of development differs from that of *Penæus* and the Sergestidæ in the very feature in which the larvæ of these forms differ from a typical *Zoëa*—the irregular manner in which the pereiopods appear.

I am therefore unable to give CLAUS'S interpretation of the significance of these larvæ unqualified acceptance at present, and feel that our groundwork in this department of knowledge can be made sure only by new observations. Every naturalist who can trace the whole life-history of a single species of any of the genera of lower

Malacostraca by actual moults, will not only help us to a sound and thorough appreciation of the significance of Crustacean embryology, but will also contribute to a better knowledge of the relation between ontogeny and phylogeny in the whole province of biology.

The phylogenetic significance of the *Nauplius* stage of development seems to me to rest upon a much firmer basis, and there are many reasons for believing that this is really an ancestral form. Its occurrence in so many widely-separated groups of Crustacea shows its great antiquity, and if it does not represent the adult form of the ancestral Crustacea, but a later larval form which has been produced by secondary modification of the original course of development, this secondary modification must have taken place very early in the history of the group, at a time when the adult forms were very primitive and unspecialised. A sufficient difference between the habits and surroundings of a young animal and those of the adult to favour secondary modification of the young is much less probable in an early unspecialised form, with simple habits, than it is in later and higher forms; and the older a larval form can be shown to be, the more probable does it become that it at one time existed as an adult.

The great age of the *Nauplius* stage and its definite structure therefore indicate that it is ancestral, and nothing except the supposed necessity for believing that the primitive Crustacean had a great number of somites and appendages seems to oppose this view.

I shall try to show further on that the serial homology shown by the parts of the body of one of the higher Crustacea cannot be fully accounted for by assuming, with BALFOUR ('Comparative Embryology,' p. 418), that the primitive Crustacean had, in addition to its three pairs of appendages similar to those of existing Nauplii, a long segmented body with simple biramous appendages; and I shall also try to show that this homology can be accounted for without any such supposition, so that the peculiarities which BALFOUR points out—1st, that the mandibles have the form of biramous swimming feet; 2nd, that the second pair of antennæ are biramous swimming feet; 3rd, that the body shows no traces of segmentation; 4th, that the heart is absent; 5th, that the ocellus is the sole organ of vision—must be allowed their full weight, and must not be opposed by any *à priori* assumption of the theoretical need for a greater number of somites and appendages.

TABLE II.—Comparative Table of the Parts of the Nauplii of *Lucifer*, *Penæus*, *Euphausia*, *Apus*, and *Palæmonetes*.

	First free *Nauplius* of *Lucifer*, $\frac{8}{1000}$th inch long (Plate 2, fig. 25).	First free *Nauplius* of *Penæus* (MÜLLER, "Verw. der Garneelen," fig. 1).	Free *Nauplius* of *Euphausia* (METSCHNICKOFF, Zeit. Zool., xxi., taf. xxxiv., fig. 4).	Free *Nauplius* of *Apus* (BALFOUR, fig. 208 from CLAUS).	Egg *Nauplius* of *Palæmonetes* (FAXON).
Compound eyes	Absent	Same	Same	Same	Rudimentary
Ocellus	Present	Same	Same	Same	Not observed until later
Frontal organ	Absent	Same	Same	Absent	Absent
Carapace	Absent	Same	Present	Indicated by fold	Absent
First antennæ	Uniramous, unjointed, locomotor	Same	Uniramous, obscurely jointed, as long as second, locomotor	Same, but shorter than second	Rudimentary, uniramous
Second antennæ	Biramous, unjointed, locomotor, no blade	Same	Biramous, jointed, locomotor, no blade	Biramous, unjointed, locomotor, has basal hook	Rudimentary, uniramous
Labrum	Large, without spine.		Moderately large, no spine	Very large, no spine	Small at this stage, but large a little later
Mandible	Biramous, unjointed, no blade	Same	Same	Same	Rudimentary, uniramous
Metastoma	Rudimentary	Absent	Rudimentary	Five rudimentary body segments	A long unsegmented body without telson
First maxilla	Rudimentary	Absent	Rudimentary		
Second maxilla	Rudimentary	Absent	Rudimentary, with spine		
First maxilliped	Rudimentary	Absent	Rudimentary, bilobed		
Second maxilliped	Absent	Absent	Absent		
Third maxilliped	Absent	Absent	Absent		
First pereiopod	Absent	Absent	Absent		
Second pereiopod	Absent	Absent	Absent		
Third pereiopod	Absent	Absent	Absent		
Fourth pereiopod	Absent	Absent	Absent		
Fifth pereiopod	Absent	Absent	Absent		
First abdominal	Absent	Absent	Absent		
Second abdominal	Absent	Absent	Absent		
Third abdominal	Absent	Absent	Absent		
Fourth abdominal	Absent	Absent	Absent		
Fifth abdominal	Absent	Absent	Absent		
Sixth abdominal	Absent	Absent	Absent		
Telson	Absent	Present, with spines	Well marked, deeply notched, has spines		
Anus	Absent		Absent		

TABLE III.—Comparative table of the parts of the last *Nauplius* or meta-*Nauplius* of *Lucifer*, *Penæus*, *Euphausia*, and *Apus*.

	Second free *Nauplius* of *Lucifer*, $\frac{9}{1000}$th inch long (Plate 2, fig. 26).	Meta-*Nauplius* of *Penæus* (MÜLLER, Zeit. Zool., xix., fig. 2).	Meta-*Nauplius* of *Euphausia* (METSCHNICKOFF, Zeit. Zool., xx., taf. xxxiv., fig. 5).	Second free *Nauplius* of *Apus* (BALFOUR, after CLAUS).
Compound eyes	Absent	Same	Same	Same
Ocellus	Present	Same	Same	Same
Frontal organ	Not observed	Same	Present	Same
Carapace	Indicated by fold	Well marked	Same, margins toothed	Elongated backwards
First antenna	Uniramous, unjointed, locomotor, long, sensory hairs	Same	Same	Same, but shorter than second
Second antenna	Biramous, unjointed, locomotor, no blade	Biramous, locomotor	Biramous, locomotor, jointed, five blades	Biramous, locomotor, obscurely jointed, has blade
Labrum	Large, no spine	Moderately large	Same	Very large
Metastoma	Rudimentary, double ?	?	Rudimentary	Not mentioned
First maxilla	Rudimentary	Same	Same	Same
Second maxilla	Rudimentary	Same	Same	A number of rudimentary body segments
First maxilliped	Rudimentary	Same	Same	
Second maxilliped	Absent ?	Rudimentary ?	Absent	
Third maxilliped	Absent	Absent	Absent	
Thoracic sornites	Absent	Absent	Absent	
Abdominal sornites	Absent	Absent	Absent	
Telson	Well marked, not forked, has spine	Deeply forked, with spines	Notched, has spines	Forked, has spines
Anus	Present	Present	Present	Present

TABLE IV.—Comparative table of the parts of the first *Protozoëa* in *Lucifer*, *Acetes* (?), *Sergestes*, *Penæus*, and *Euphausia*.

	1. First *Protozoëa* of *Lucifer*, $\frac{20}{1000}$th inch long (Plate 2, fig. 27).	2. CLAUS's larva ('Untersuchungen,' &c., plate iv., fig. 2).	3. *Protozoëa* of *Sergestes* (CLAUS, 'Untersuchungen,' &c., plate v., fig. 1).	4. *Protozoëa* of *Penæus* (MÜLLER, 'Verwandlung,' &c., plate ii., fig. 4).	5. *Protozoëa* of *Euphausia* (CLAUS, 'Untersuchungen,' &c., plate i., fig. 2).
Compound eye	Absent	Rudimentary	Stalked	Rudimentary	Absent
Ocellus	Present	Same	Same	Same	Same
Frontal organ				Present	Same
Shell gland	Present after first moult				Present
Carapace	$\frac{1}{2}$ length of body, rostrum, posterior, dorsal, and lateral spines	$\frac{3}{4}$ length of body, postero-lateral spines	More than $\frac{1}{2}$ length of body, rostrum, postero-lateral and dorsal spines	$\frac{1}{2}$ length of body, no rostrum or spines, edges smooth	More than $\frac{1}{2}$ length of body, no rostrum or spines, edges serrated
First antenna	As long as second, jointed, uniramous, locomotor	Not quite as long as second, many joints	Seven joints	Same as *Lucifer*	Same
Second antenna	Biramous, jointed, locomotor, no blade	Same	Same	Same	Same
Labrum	Large, small spine	Small, no spine described	Has spine	Same	Same
Metastoma		Bilobed	Bilobed		Bilobed
Mandible	Blade, no palp	Same	Same	Same	Blade and rudiment of limb
First maxilla	Adult form, scaphognathite	Like *Lucifer*	Same	Same	Same
Second maxilla	Has scaphognathite	Same	Same	Same	Same
First maxilliped	Large, biramous	Same	Same	Same	Same
Second maxilliped	Rudimentary, biramous	Same	Large, biramous	Rudimentary, biramous.	Somite, no appendage
Third maxilliped	Somite, no appendage	A segmented region	Rudimentary	Segment, no appendage	Same
First pereiopod	Somite, no appendage	A segmented region	Somite, no appendage	Same	Same
Second pereiopod	Somite, no appendage	A segmented region	Somite, no appendage	Same	Same
Third pereiopod	Somite, no appendage	A segmented region	Somite, no appendage	Same	Same
Fourth pereiopod	Somite, no appendage	A segmented region	Somite, no appendage	Same	Same
Fifth pereiopod	No somite	A segmented region	Somite, no appendage	Same	Same
Abdominal somites	Represented by long unsegmented region about $\frac{1}{3}$ the length of the body	Same, less than $\frac{1}{4}$ length of body	Same, less than $\frac{1}{3}$ length of body	Same, $\frac{1}{3}$ length of body, six abdominal somites appear, according to CLAUS, before next moult	Same, less than $\frac{1}{3}$ length of body
Telson	Notched, four pairs large spines, one pair small ones	Deeply forked, six pairs of spines	Still more forked, with six pairs of spines	Deeply forked, seven pairs of spines	Posterior edge straight, seven pairs of spines

TABLE V.—Comparative table of the parts of the last *Protozoëa* of *Lucifer*, *Acetes*, *Sergestes*, *Penæus*, and *Euphausia*.

	Last *Protozoëa* of *Lucifer*, $\frac{50}{1000}$th inch long (Plate 5, fig. 43).	Last *Protozoëa* of *Acetes*, $\frac{67}{1000}$th inch long (fig. 77).	*Elaphocaris* of *Sergestes* (CLAUS, 'Untersuchungen,' taf. vi., fig. 1).	Last *Protozoëa* of *Penæus* (MÜLLER, 'Verwandlung,' &c., fig. 7).	Last *Protozoëa* of *Euphausia* (CLAUS, 'Untersuchungen,' taf. i., fig. 4).
Compound eye	Rudimentary	Stalked, short	Stalked, long	Stalked, short	Rudimentary
Ocellus	Present	Same	Same	Same	Same
Carapace	Dorsal spine, two postero-lateral spines, and rostrum, all simple	Same	Same, all compound	No posterior spines	No rostrum, dorsal spine only
First antenna	*Nauplius*-like, two jointed	Same, three jointed	Same, seven jointed	Like *Lucifer*, but with sense hairs	Like *Lucifer*
Second antenna	*Nauplius*-like	Same	Same, but without basal rings, no endopodite	Like *Lucifer*	Same
Labrum	Has spine	Same	No spine	Has spine	No spine
Mandible	Adult form, no palp	Same	Same	Same	Same
First maxilla	Two basal joints, jointed endopodite, scaphognathite	Same	Same	Same	Same, without scaphognathite
Second maxilla	Has scaphognathite	Same	Same	Same	Same
First maxilliped	Biramous, large	Same	Same	Same	Rami very short and simple
Second maxilliped	Small, biramous	Same	Large	Large	Somite, without appendages
Third maxilliped	Bilobed bud	Same	Small, but functional	Bilobed bud	Somite, without appendages
First pereiopod	Somite, rudimentary appendages	Same	Same	Same	Somite, without appendages
Second pereiopod	Somite, rudimentary appendages	Same	Same	Same	Somite, without appendages
Third pereiopod	Somite, rudimentary appendages	Same	Same	Same	Somite, without appendages
Fourth pereiopod	Somite, rudimentary appendages	Same	Same	Same	Somite, without appendages
Fifth pereiopod	Somite and appendages absent	Same	Somite, rudimentary appendages	Same	Somite, without appendages
First abdominal somite	Somite, no appendages	Same	Same	Same	Same
Second abdominal somite	Somite, no appendages	Same	Same	Same	Same
Third abdominal somite	Somite, no appendages	Same	Same	Same	Same
Fourth abdominal somite	Somite, no appendages	Same	Same	Same	Same
Fifth abdominal somite	Somite, no appendages	Same	Same	Same	Same
Sixth abdominal somite	Somite united to telson, appendages present as bilobed buds	Same	Same	Same	Same
Telson	Notched	Forked	Deeply forked	Same	Straight

TABLE VI.—Table of the parts of the Schizopod larva of *Lucifer*, *Acetes*, *Sergestes*, and *Penæus* at the first stage.

	Schizopod stage of *Lucifer* (Plate 6, fig. 50).	*Acanthosoma* stage of *Acetes* (fig. 84).	*Acanthosoma* of *Sergestes* (CLAUS, 'Untersuchungen,' taf. v., fig. 6).	Schizopod stage of *Penæus* (MÜLLER, 'Verwandlung,' &c., taf. 11, fig. 7).
Ocellus	Present	Same		
Compound eye	Stalk short	Same	Stalk, very long	Stalk short
Carapace	$\frac{1}{3}$ length of body, rostrum, two antero-lateral spines, no posterior spines	Same	$\frac{1}{3}$ length of body, rostrum, six pairs of antero-lateral spines, no posterior spines	$\frac{1}{3}$ length of body, rostrum, two pairs of antero-lateral spines, no posterior spines
First antenna	Plumose fringing hairs, long shaft, one short rudimentary flagellum	Same, with a second rudimentary flagellum	Same	Same
Second antenna	Rudimentary at first, but it acquires scale and flagellum at next moult	Scale and flagellum	Same, flagellum long	Scale large, flagellum rudimentary
Labrum	Has spine			No spine
Mandible	No palp		Same	Same
First maxilla	Like that of *Protozoëa*		Same	Same
Second maxilla	Scaphognathite rudimentary		Same	Same
First maxilliped	Rami equal		Endopodite longest	Same
Second maxilliped	Short exopodite, long endopodite, both free and movable	Exopodite alone free and movable	Equal, both free	Biramous
Third maxilliped	Short exopodite, long endopodite, both free and movable	Exopodite alone free and movable	Endopodite longest	Biramous
First pereiopod	Short exopodite, long endopodite, both free and movable	Exopodite alone free and movable and chelate	Exopodite very long, no chela	Endopodite rudimentary, exopodite very long, jointed
Second pereiopod	Short exopodite, long endopodite, both free and movable	Exopodite alone free and movable and chelate	Exopodite very long, no chela	Endopodite rudimentary, exopodite very long, jointed
Third pereiopod	Short exopodite, long endopodite, both free and movable	Exopodite alone free and movable and chelate	Exopodite very long, no chela	Endopodite rudimentary, exopodite very long, jointed
Fourth pereiopod	Short exopodite, long endopodite, both free and movable	Endopodite absent	Exopodite very long, no chela	Endopodite rudimentary, exopodite very long, jointed
Fifth pereiopod	Somite absent	Same	Exopodite very long, no chela	Endopodite rudimentary, exopodite very long, jointed
First abdominal	Somite present, appendages absent	Appendages rudimentary	Like *Lucifer*	Same
Second abdominal	Somite present, appendages absent	Appendages rudimentary	Like *Lucifer*	Same
Third abdominal	Somite present, appendages absent	Appendages rudimentary	Like *Lucifer*	Same
Fourth abdominal	Somite present, appendages absent	Same	Same	Same
Fifth abdominal	Somite present, appendages absent	Same	Same	Same
Sixth abdominal	Like adult	Same	Same	Same
Telson	Long, narrow, notched	Short, wide, notched	Short, deeply cleft	Very long, forked

VII. Serial Homology and Bilateral Symmetry in the Crustacea.

The Phyllopods and the highest Brachyura are connected with each other by a tolerably complete series of intermediate forms, and as we pass this series in review we cannot fail to notice that, as has been so frequently pointed out by morphologists, each successively higher form is a little in advance of the one next below it in the degree to which the functions and structure of the somites and appendages are subordinated to the individuality of the organism as a whole.

In the lower forms the body is made up of a series of nearly similar somites, and the appendages, with the exception of those at the anterior end of the body, are essentially alike in structure and their functions are indentical throughout the series. The greater part of the body of such a Crustacean as *Artemia* consists of a series of similar somites, and in *Apus* we find more than sixty pairs of limbs which agree with each other so perfectly in function as well as in structure that any one of them might be substituted for any other without involving any essential change in the structure of the animal as a whole.

At the other end of the series we have Crabs with the primitive distinctness of the somites so obscured by the centralised individuality of the whole organism that it cannot be traced at all without careful study and comparison of various stages in the life of a number of forms.

Comparing the various appendages of a Crab with each other we find that their functions are not at all alike. The mandibles are nothing but masticating organs, and the power which they once had, and which they still retain in the *Nauplius* of *Lucifer* to aid in locomotion, has entirely disappeared.

Other appendages have become organs for procuring food, or weapons of offence or defence; others have become walking legs; others long oars or paddles; others again have lost all limb-like functions, and are changed into accessory reproductive organs; whilst others again have entirely disappeared.

In accordance with this specialisation of each appendage to a particular function, a corresponding structural change has been brought about, and it is only after careful study of the younger stages that we perceive the mandibles, maxillæ, foot-jaws, walking and swimming legs, and copulatory organs of an adult Crab to be as strictly homologous with each other as are the unspecialised appendages of *Apus.*

The integration of the somites into a centralised whole has been accompanied by a differentiation of each appendage from the others, and a specialisation to a restricted function.

An adult Crab resembles and differs from one of the higher Macroura in about the same way that it resembles and differs from its own *Megalops* larva, and the transition from the larval form to the adult form is accompanied, like the transition from an adult low Crustacean to a high one, by increased dependence of the various parts on each other, by the increased prominence of the general individuality over the individualities of the somites or metameres, and by the increased structural and functional specialisation and differentiation of each appendage as compared with the others.

This series of changes is so well exemplified by the study of adult and larval Crustacea; it is so remarkable and interesting; so very conspicuous and unquestionable, that it has long attracted the attention and called forth the speculation of morphologists. It is natural to suppose that the process of change which is open to our observation through study and comparison of living Crustacea, is a continuation of a similar process which went on in the remote past. There seems then at first sight to be reason for believing that, if we could go far enough back, we should find the individuality of the whole organism gradually disappearing and giving place to the separate individualities of the component somites; that we should find the specialisation of the appendages gradually disappearing, until we should at last find, as the remote ancestor of the Crustacea, a series or community of independent organisms, each one essentially like the others, and able to provide for its own wants and to lead an independent existence when accidentally or naturally detached.

This view has been advocated at length by HÆCKEL ('Generelle Morphologie,' 1866) and by SPENCER ('Principles of Biology,' vol. xi., 1867), and used by both these writers as an explanation of the origin of all segmented or compound animals and plants. It has been accepted, with more or less qualification, by many other writers, although HUXLEY ('Oceanic Hydrozoa') and METSCHNICKOFF (Zeit. f. Wiss. Zool., xxiv.) have pointed out that, even in the Siphonophoræ, where the individualities of the units in the compound are extremely well marked, the view that the organism has been evolved by the gradual integration and specialisation of originally independent Zooids is attended with serious difficulties.

So far as we can see there is no reason why the Crustacea might not have originated in this way, by the gradual integration and differentiation of a community of independent metameres, but the evidence which is attainable seems to directly oppose the belief that this has actually happened. We are able to trace the higher Decapods back, very satisfactorily, to a Phyllopod-like ancestor with a long series of undifferentiated somites and appendages, but even here the somites are simply parts of the body, and they furnish no more evidence than those of a Crab to show that they ever were the independent organisms of a community.

When we attempt to go still further back we find that the facts of embryology, if they show any thing whatever about the phyllogeny of the Crustacea, lead us back to a *Nauplius* with three inter-dependent somites and three pairs of specialised appendages, rather than to a form with a great number of unspecialised somites and similar appendages.

Turning now to a somewhat different aspect of the subject, we notice that, if we confine ourselves to structure, and leave out of sight the question of origin, there is the closest similarity between serial homology and the homology between the corresponding organs of allied animals which OWEN has proposed to distinguish by the term "special homology."

The structural relation between one appendage of *Lucifer*, say the first pereiopod, and another, such as the swimmeret, is identical with the relation between the pereiopod of *Lucifer* and that of *Squilla*, or a Crab or Lobster. In both cases we have a fundamental similarity of plan, which is independent of external conditions; and joined to this essential similarity, we have a more superficial diversity of structure which is plainly due to difference in the functions of the appendages, and their relations to the external world. The resemblance between the two kinds of homology does not stop here. Tracing the ontogeny of the appendages we find that there is much less difference between the larval pereiopods of *Lucifer* and those of the Lobster than there is between the appendages of the adults, and we find exactly the same thing when we compare the pereiopod and swimmeret of the same individual at earlier and earlier stages of development.

There is precisely the same resemblance between symmetry and special homology. The right and left claws of the Common Crab (*Callinectes*) are not exactly alike, since the cutting edge of one claw is sharp and set with pointed teeth, while the edge of the other is thick, with thick blunt crushing tubercles. The two appendages are alike in plan or homologous, but each is fitted for a specialised function by a slight structural peculiarity. In this case, as in the others, the differences are less marked, and the common plan more closely followed, in the larva than in the adult.

Serial homology and bilateral symmetry are thus seen to be like special homology in all purely structural features. In each case the homology is a resemblance which is independent of external conditions, but which may be obscured by secondary modifications whenever external conditions render it necessary.

In each case, too, the secondary modifications become less marked, and the underlying plan more evident as we pass back from the adult to earlier and earlier stages of development. We must therefore include all three kinds of homology in a single class or category, and the employment of one term to denote the phenomena of special homology, of another for serial homologies, and a third for bilateral homologies, and others for other sorts of general homology must not be allowed to obscure the fact that they are all different forms of the same thing, essential similarity joined to superficial diversity. The terminology which has been employed by BRONN, HÆCKEL, LANKESTER, and others for the different kinds of homology is valuable, and the only reason why I have not made use of it is that the more familiar terms, "serial homology" and "bi-lateral symmetry" answer every purpose equally well in treating of the Arthropods. HÆCKEL'S subdivisions are natural, but they are simply subdivisions of a great class of similar phenomena, which must still be included under the general term "homology."

Special homology may be defined in two ways, morphologically and phylogenetically. From the morphological point of view an homology is a similarity in essential plan of structure, which may be obscured by differences due to diversity of function. From the phylogenetic point of view it is a resemblance which is due to community of origin or heredity from a common ancestor, while the differences between homologous organs are due to the divergence of allied forms, and to the selection and perpetuation, through natural selection, of variations which are in accordance with changed conditions of life.

Now are the phenomena of serial and lateral homology like those of special homology in this second or phylogenetic sense, as well as in a morphological sense?

On the assumption that the remote ancestor of the Crustacea was a community of independent organisms, all of which had inherited their organisation from the same parent, we might answer that serial homology is like special homology when viewed from a phylogenetic stand-point, and if we assume that this series was at first double, and that the progress of centralisation suppressed one side of each metamere as the community became gradually fused into a bilateral organism, we may make the same statement regarding symmetry.

A process of evolution of this sort is not impossible, and in some cases there seems to be evidence that it has actually occurred. Pyrosoma is clearly a community of independent Ascidians, which has been brought by natural selection into a form which has a certain degree of individuality of its own, independent of that of the component units; although in this case the peculiar form of the community has called for little differentiation, and the polymorphism is therefore very slight.

The salpa-chain is a bilateral community, and in Doliolum we have a similar community which exhibits considerable polymorphism. If this process were carried a little further we might ultimately have a bilaterally symmetrical organism in which corresponding parts in the series or on opposite sides should be strictly homologous by descent; but we are not therefore justified in assuming that all instances of serial and lateral homology have originated in this way, and even if we were a more careful analysis will show that the assumption does not remove all the difficulties.

If we grant, for the sake of argument, that the Crustacea are not the descendants of a *Nauplius*, but of a remote ancestor which consisted of a community of independent metameres, we shall still be forced to recognise a bond of relationship between the limbs of a Decapod, which is very much more recent than that which they owe to common descent from the parent of the group of Zooids which formed the ancestral community.

A reference to the figures will show that the first, second, and third thoracic limbs of the adult *Lucifer* agree with each other, or are homologous, in certain features which are not present in a Schizopod. The exopodite is absent and the endopodite is long and slender in all of them, and it carries short hairs along its entire length, while, in the Schizopod-larva, the exopodite is present, and the long hairs are restricted to the tip of the stout endopodite. We must therefore recognise a bond of union or homology between these three appendages which has determined that they shall be like each other in the adult *Lucifer*, and the assumption that this similarity is due to heredity from the parent of the imaginary metameres which joined together to form the primitive Crustacean is out of the question, for we know that no further back than the Schizopods these appendages had quite a different structure.

The study of serial or lateral homology in other groups of animals forces us to the same conclusion, and compels us to recognise a persistent bond of union between them which cannot be due to what we usually understand by heredity.

On the assumption that the Vertebrates are the descendants of a community of metameres, the genetic relationship between a man's arm and a bird's wing must be almost infinitely closer than that between a man's arm and his leg, and this again much more recent than that between his right and his left arm. The arm and wing inherit their homology from the anterior limb of the common ancestor of man and the birds, but man's arm and leg have no common ancestor more recent than the limb of the parent of the imaginary metameres which gave origin, by their union, to the ancestor of the Vertebrates, and the common ancestor of the right and left arms must have been still more remote.

When we compare man's arm and leg we find that they have homologous features which are not only more recent than the time when man's ancestors diverged from the ancestors of the birds, but more recent than the separation of the anthropoid and simian stems. They resemble each other in the texture of the skin and in the shape of the nails, and these resemblances are strictly homological, that is, they are not due to external conditions, but in spite of them; and we meet with countless similar resemblances all through the animal kingdom. They are not accounted for by the "metamere" theory, even if this is fully accepted, for in many cases they are not old, but are of recent acquisition.

In the case of the Crustacea the assumption that the remote ancestor of the group had a many-jointed body does not account for them; and as the supposed necessity for an explanation of serial homology is the only reason for believing that this remote ancestor had a great number of body-segments, it is clearly illogical to reject the embryological evidence that this ancestor was a three-jointed *Nauplius*, in order to hold an hypothesis which fails to account for the facts which are supposed to render it necessary.

Explanation of the Plates.

All the figures where the magnifying power is not stated were drawn with a power of 160 diameters (Zeiss, Oc. 1, Obj. D); but the actual amplification of the drawings is not uniform. In copying the original sketches it has been convenient to reduce the size of some of them, and no inference as to relative size should be drawn from any of them except where measurements are given.

In order to render the figures as truthful and lifelike as possible, the animals were subjected to very little confinement while under examination, and as their incessant and violent movements rendered the use of a camera impossible, they are not drawn to a fixed scale.

In all the Plates the capital reference letters are used to denote the same parts, as follows:—

A. First antenna.
A. 1 to *A.* 6. The series of abdominal somites.
An. Second antenna.
C. Carapace.
E. Compound eye.
L. Labrum.
M. Mandible.
Mp. 1, *Mp.* 2, *Mp.* 3. The first, second, and third maxillipeds.
Mx. 1, *Mx.* 2. The first and second maxillæ.
Oc. Ocellus.
Pl. 1–*Pl.* 6. The six pairs of abdominal appendages.
Pr. 1–*Pr.* 4. The four pairs of thoracic limbs.
R. Rostrum.
T. 1–*T.* 4. The four thoracic somites.
T. Telson. (*Tl.* in Plate 3, fig. 26.)

PLATE 1.

The letter *a* in figs. 1 to 8, which were all drawn from the same egg, marks the same point in all.

Fig. 1. An egg during the period of rest which follows the first period of segmenting activity.

Fig. 2. The same egg at the beginning of the second period of segmenting activity, five minutes after the stage shown in fig. 1.

Fig. 3. The same egg five minutes later. One of the primary segments is beginning to divide into two.

Fig. 4. The same egg ten minutes later. One of the primary spherules is perfectly divided into two, and the division of the other is less advanced.

Fig. 5. The same egg five minutes later, and completely divided into four equal segments. This stage ends the second period of segmenting activity.

Fig. 6. The same egg, fifteen minutes later, during the second period of rest.

Fig. 7. The same egg, ten minutes later, entering upon the third period of activity.

Fig. 8. The same egg, twenty-five minutes later, divided into eight equal spherules. This stage ends the third period of activity.

Fig. 9. Another egg during the third period of rest.

Fig. 10. Another egg near the end of the fourth period of activity, and divided into sixteen equal spherules, arranged around a central segmentation cavity.

Fig. 11. Optical section along the principal axis of a somewhat older egg, showing the yolk spherule, *c*, and the segmentation cavity, *b*.

Fig. 12. Optical section of the same egg at right angles to the one shown in fig. 11.

b. The segmentation cavity.

c. The yolk spherule.

PLATE 2.

Fig. 13. An optical section of an egg somewhat older than the one shown in fig. 11.

Fig. 14. Optical section at right angles to that of fig. 13.

Fig. 15. Optical section of an egg a little older than the one shown in fig. 13.

d. Orifice of invagination.

Fig. 16. Optical section of a still older egg in the same position as figure 15.

Fig. 17. Optical section of a still older egg in the same position.

Fig. 18. Surface view of the formative pole of the egg shown in fig. 17.

Fig. 19. Optical section along the principal axis of a still older egg.

Fig. 20. A similar section of a still older egg.

Fig. 21. Ventral view of an embryo in the egg-shell 24 hours after oviposition.

e. Anterior end of body.

f. Large spherules in the region of the digestive tract.

g. Metastoma.

Fig. 22. Dorsal view of the same embryo.

(*e* and *f* as in fig. 21.)

h. Cerebral ganglia.

i. Pigment spots.

m. Muscles.

Fig. 23. Similar aspect of the same embryo artificially removed from the egg shell. (Letters as in fig. 22.)

PLATE 3.

Fig. 24. Ventral view of the same embryo, seen from a point of view a little anterior to that of fig. 21.

ex. Exopodite.

en. Endopodite.

(The other letters as in fig. 21.)

Fig. 25. Side view of the *Nauplius*, $\frac{8}{1000}$th inch long, as it leaves the egg 36 hours after oviposition.

g. Metastoma.

p. Pigment spots.

ex. Exopodite.

en. Endopodite.

Fig. 26. Side view of a *Nauplius*, $\frac{9}{1000}$th inch long, and a little older than the one shown in fig. 25. The animal is a little flattened by pressure. In a ventral view of the same larva the anus was visible at the point marked *a.* in the figure.

œ. Œsophagus.
s. Stomach.
l. Large cells around stomach.
i. Intestine.
ga. Cerebral ganglia.
n. Ventral nerve-chain.
p. Pigment spots.

Fig. 27. Dorsal view of a *Protozoëa*, $\frac{20}{1000}$th inch long, which moulted from the *Nauplius* shown in fig. 26, about 96 hours after oviposition.

ga. Cerebral ganglia.
s. Stomach.
h. Heart.
i. Intestine.
p. Pigment spots.
a. Anus.
m. Muscles of œsophagus.
ls. Postero-lateral spines of carapace.
ds. Posterior dorsal spine of carapace.
abd. Unsegmented portion of abdomen.
ex. Exopodite.
en. Endopodite.

Fig. 28. Right mandible of the same specimen seen from below.

Fig. 29. Right mandible seen from behind.

Fig. 30. Back or outer surface of first maxilla of the same specimen.

sc. Scaphognathite.

Fig. 31. Posterior surface of left-second maxilla of the same specimen.

sc. Scaphognathite.
b. Basal joint.
en. Endopodite.

Fig. 32. Left-first maxilliped of the same specimen; posterior surface.

ex. Exopodite.
en. Endopodite.
b. Basal joint.

Fig. 33. Right-second maxilliped of the same specimen.

ex. Exopodite.
en. Endopodite.
b. Basal joint.

Fig. 34. A *Protozoëa*, $\frac{27}{1000}$th inch long, and a little older than the one shown in fig. 27.
œ. Œsophagus.
pp. Anul pigment spots.
(Other letters as in fig. 27.)

PLATE 4.

Fig. 35. Side view of another larva at the stage shown in fig. 34.
ga. Cerebral ganglia.
s. Stomach.
sg. Shell gland.
h. Heart.

Fig. 35A. Surface view of the area of attachment of the œsophageal muscles of the same larva.

Fig. 36. Second antenna of the same specimen.
b. Basal joint.
ex. Exopodite.
en. Endopodite.

Fig. 37. Mandible, at same stage, seen from below.

Fig. 38. First maxilla of left side at same stage.
1 and 2. The two cutting joints of the basal portion.
ex. Exopodite.
en. Endopodite.

Fig. 39. Left-second maxilla at same stage.
b. Basal portion.
en. Endopodite.
sc. Scaphognathite.

Fig. 40. Left-first maxilliped at same stage.
ex. Exopodite.
en. Endopodite.

Fig. 41. Left-second maxilliped at same stage.
ex. Exopodite.
en. Endopodite.

Fig. 42. Ventral view of the same larva after the next moult and $\frac{35}{1000}$th inch long.

Fig. 42A. Outline of the anterior end of fig. 42, more enlarged.
b. Basal joints of appendages.
ex. Exopodites.
en. Endopodites.
sc. Scaphognathite.

PLATE 5.

Fig. 43. Ventral view of the last *Protozoëa* (DANA'S *Erichthina demissa*), $\frac{50}{1000}$th inch long.
ex. Exopodite of second antenna.
en. Endopodite of second antenna.

Fig. 44. Dorsal view of the same larva (ZEISS, A. 2).
ds. Median dorsal spine of carapace.
ls. Postero-lateral spines of carapace.

Fig. 45. Side view of the same larva at the same stage.
ds. Median dorsal spine of carapace.
tg. Thoracic ganglia.
i. Intestine.

Fig. 46. Posterior surface of left first maxilla of the same larva.
1 and 2. Cutting joints of basal portion.
en. Endopodite.
sc. Scaphognathite.

Fig. 47. Left second maxilla of the same larva.
b. Basal joint.
sc. Scaphognathite.
en. Endopodite.

Fig 48. Left first maxilliped of same larva.
b. Basal joint.
ex Exopodite.
en. Endopodite.

Fig. 49. Left second maxilliped of same larva.
(Letters as in fig. 48.)

PLATE 6.

Fig. 50. First Schizopod stage (this stage is the equivalent of DANA'S genus *Sceletina*; of CLAUS'S *Acanthosoma* stage; and of WILLEMÖES-SUHM'S *Amphion* stage). The larva passes, by a single moult, which was frequently observed in isolated specimens, from the stage shown in fig. 43 to the one which is shown in this figure.

Fig. 51. Mandible at same stage.

Fig. 52. First maxilla at same stage.

Fig. 53. Second maxilla at same stage.

Fig. 54. *Sceletina* larva, two moults later (ZEISS, A. 2).
s. Antero-lateral spine.
p. Pigment spot on fourth abdominal somite.

Fig. 54A. Outline of anterior end of carapace.
Fig. 55. Dorsal view of the anterior end of the same larva.
Fig. 56. First antenna of the same larva.
Fig. 57. Second antenna of the same larva.
Fig. 58. Mandible at same stage.
Fig. 59. Locomotor appendages of the left side, at same stage, seen from above.

PLATE 7.

Fig. 60. Side view of a young *Lucifer*, about $\frac{25}{100}$th inch long, which was produced by the moulting of a larva like that shown in fig. 54 (this stage agrees pretty exactly with the *Mastigopus* stage of *Sergestes*).
    g. Antennal gland.
    n. "Neck."
    c. Carapace.
Fig. 61. Half-grown *Lucifer*, about $\frac{1}{2}$ an inch long.
    g. Antennal gland.
    cg. Cerebral ganglia.
    co. Commissures to ventral ganglia.
    s. Cephalic pouch of stomach.
    h. Heart.
    c. Carapace.
Fig. 62. Inner surface of mandible of adult.
Fig. 63. Outer surface of mandible of adult.
Fig. 64. Outer surface of first maxilla of adult.
Fig. 65. Inner surface of same.

PLATE 8.

Fig. 66. Head of a small female about $\frac{2}{3}$rds of an inch long, seen from below.
Fig. 67. Second maxilla of same.
Fig. 68. First maxilliped of right side of adult.
Fig. 69. Inner surface of same.
Fig. 70. Second maxilliped of adult.

PLATE 9.

Fig. 71. Dorsal view of tip of telson of adult female.
Fig. 72. Side view of last abdominal somite and swimmerets of adult female.
Fig. 73. The same parts of an adult male.

Fig. 74. Side view of posterior half of carapace and first abdominal somite of a mature female, to show the reproductive organs. In order to reduce the number of figures this specimen is represented with its ovaries full of ripe eggs, while a large bunch of developing eggs are attached to the basal joints of the last pair of thoracic appendages, but these two features are never exhibited completely at the same time in a single individual.

ca.	Carapace.
h.	Heart.
i.	Intestine.
tg.	Thoracic ganglia.
ov.	Ovary.
od.	Oviduct.
sr.	Seminal receptacle.

Fig. 75. The corresponding part of the body of an adult male.

c.	Carapace.
h.	Heart.
i.	Intestine.
tg.	Thoracic ganglia.
a.	First abdominal somite.
t.	Testis.
vd.	First division of vas deferens.
sp.	Second division.
sv.	Third division.

Fig. 76. First pleopod of young male.

Fig. 78. Side view of the larva shown in Plate 10, fig. 77 (Zeiss, A. 2).

Fig. 79. Dorsal view of the same larva.

Fig. 80. Mandible of the same larva.

Fig. 81. First maxilla of the same larva.

en.	Endopodite.
sc.	Scaphognathite.

Fig. 82. Second maxilla of the same larva.

sc.	Scaphognathite.
en.	Endopodite.

PLATE 10.

Fig. 77. Ventral view of the last *Protozoëa* stage of *Acetes* $\frac{67}{1000}$th inch long.

ex.	Exopodite of second antenna.
en.	Endopodite of second antenna.

Fig. 85. The specimen shown in fig. 84, after another moult and $\frac{80}{1000}$th inch long.

Fig. 86. First antenna of the same larva.

Fig. 87. Second antenna of the same larva.

PLATE 11.

Fig. 84. Ventral view of the specimen shown in fig. 81, after the next moult and $\frac{70}{1000}$th inch long.

ex. Exopodite.
en. Endopodite.

Fig. 83. First maxilliped of the larva shown in Plate 10, fig. 77.

Fig. 88. Second and third maxillipeds and periopods of the larva shown in Plate 10, fig. 85.

Fig. 89. First pleopod of same larva.

Fig. 90. An older specimen $\frac{15}{100}$th inch long, which was captured at the surface.

III. *On the Structure and Development of the Skull in Sturgeons* (Acipenser ruthenus *and* A. sturio).*

By William Kitchen Parker, *F.R.S.*

Received April 14,—Read May 5, 1881.

[Plates 12–18.]

INTRODUCTION.

Several years ago the late Mr. William Lloyd procured for me from Hamburgh seventeen young Sturgeons (*Acipenser sturio*); these were from 7 to 8 inches in length. Valuable as these specimens were, they were far too much developed for embryological purposes; and no pains were spared by me to obtain, if possible, newly-hatched embryos and small "fry" of this type.

More lately it was suggested to me by Mr. Balfour that I should write to Professor W. Salensky, of Kasan, who had been working at the development from the egg of *Acipenser ruthenus*, the small Sturgeon or "Sterlet."† My application to him was promptly and most kindly responded to, and in a short time I received a considerable number of newly-hatched and very small young of that species, ranging from 5½ to 14½ millims. in length. Half these were for Mr. Balfour, and the rest for me. His researches are embodied in that inestimable work, the second volume of his 'Comparative Embryology;' mine are here offered to the Society in the accustomed form.

I purposely refrain at present from working out the structure and development of the trunk and limbs (I have laboured at these regions, and shall be ready to resume that part of my work when this is done); but other workers are from time to time taking up those parts, and when once the cephalic skeleton is mastered what remains will be a comparatively easy task.

I am more anxious to be prepared for my own limited work by acquiring a thoroughly clear conception of the embryology of each type; in this my best helper is Mr. F. M. Balfour. Professor Huxley is, and always has been, my most valued critic and counsellor in all that relates to broad and philosophical views of animal life generally, and of the life of the Vertebrata in particular.

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* The skull described in my last paper as that of *Discoglossus pictus* (Phil. Trans., 1881, Plate 20, figs. 7–11, p. 112) was prepared from a badly-preserved, half-grown *Rana esculenta*. I am indebted for this correction, and for a genuine *Discoglossus*, to M. Boulenger.

† This species rarely exceeds the length of 3 feet (Günther, 'Study of Fishes,' p. 361).

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First Stage.—Newly-hatched embryos of Acipenser ruthenus *from 5½ millims. to 6½ millims. in length.*

The specimens figured by me (Plate 12, figs. 1–3) are rather smaller than the one figured by Mr. BALFOUR ('Comp. Embryol.,' vol. ii., p. 88, fig. 53) which measured 7 millims. in total length, but his description may serve for mine, which were nearly as much developed as his slightly larger specimen.

After describing the development of the embryo, Mr. BALFOUR explains its peculiar outspread form in its unhatched condition (*op. cit.*, p. 86, fig. 52), and says:—"The further changes which take place are, in the main, similar to those in other Ichthyopsida, but in some ways the appearance of the embryo is, as may be gathered from fig. 52, rather strange. This is mainly due to the fact that the embryo does not become folded off from the yolk in the manner usual in the Vertebrates; and as could be shown in the sequel, the relation of the yolk to the embryo is unlike that in any other known Vertebrate. The appearance of the embryo is thus something like that of an ordinary embryo slit open along the ventral side and then flattened out. Organs which properly belong to the ventral side appear in the lateral parts of the dorsal surface. Owing to the great forward extension of the yolk the heart (fig. 52, B) appears to be placed directly in front of the head." Then, after describing the progress made in the development of the brain and organs of special sense, Mr. BALFOUR says:—"At the sides of the cephalic plate the visceral arches make their appearance, and in fig. 52, A and B, there are shown the mandibular (*Md.*), hyoid (*Ha.*), and first branchial (*Br.*) arches, with the hyo-mandibular (spiracle) and hyo-branchial clefts between them. They constitute peculiar concentric circles round the cephalic plate, their shape being due to the flattened form of the embryo already alluded to." And then, further on (p. 88) our author says:—"Before hatching, the embryo has to a small extent become folded off from the yolk both anteriorly and posteriorly; and has also become to some extent vertically compressed. As a result of these changes, the general form of its body becomes much more like that of an ordinary Teleostean embryo. The general features of the larva after hatching are illustrated by figs. 53, 54, and 55. Fig. 53 represents a larva of about 7 millims., and 54 a lateral and 55 a ventral view of a larva about 11 millims.* There are only a few points which call for special attention in the general form of the body. In the youngest larva figured" [fig. 53, 7 millims. long; and see also Plate 12, figs. 1–3 of the present paper of somewhat younger larvæ] "the ventral part of the hyomandibular cleft is already closed: the dorsal part of the cleft is destined to form the spiracle (*sp.*). The arch behind is the hyoid, on its posterior border is a membranous outgrowth, which will develop into the opercular membrane. In the older larvæ a very rudimentary gill appears to be developed on the front walls of the spiracular cleft, but I have not succeeded in satisfying myself about its presence; and rows of gill papillæ have appeared on the hyoid, and the true branchial arches (figs. 54 and 55, *g*)." [The mandibular gill, about which Mr. BALFOUR speaks doubtfully, is according to my observations a thickened mass of hypoblastic cells lining the front wall of the first or "spiracular" cleft. This mass is crescentic (Plate 12, figs. 2, 4, and 7, cl^1.), and is slightly grooved along its hinder margin; on each side of this groove the low ridges are imperfectly divided into little rounded masses, which appear to me to be, evidently, the rudiments of branchial papillæ; at the lowest computation, this soft, tuberculate mass of cells is homologous with the tracts of cells which do develop into branchial papillæ in the Elasmobranchii.] "The biserially-arranged gill papillæ of the true branchial arches are of considerable length" [see Plate 12, figs. 4–9], "and are not yet covered by the operculum, but they do not form elongated thread-like external gills like those of the Elasmobranchii. The oral cavity is placed on the ventral side of the head; it has a more or less rhomboidal form. It soon however (fig. 55) becomes narrowed to a slit with projecting lips, which eventually becomes converted into the suctorial mouth of the adult. The most remarkable feature connected with the mouth is the development of provisional teeth (fig. 55) on both jaws. These teeth were first discovered by KNOCK ('Die Beschr. d. Reise z. Wolga Behufs. d. Sterlettbefructung,' Bull. Soc. Nat., Moscow, 1871). They do not appear to be calcified, and might be supposed to be of the same nature as the horny teeth of the Lamprey. They are, however, developed like the teeth, as a deposit between a papilla of subepidermic tissue and an epidermic cap. The substance of which they are formed corresponds morphologically to the enamel of ordinary teeth. As they grow they pierce the epidermis and form hollow spine-like structures with a central axis filled with subepidermic mesoblastic cells. They disappear after the third month of larval life. In front of the mouth two pairs of papillæ grow out, which appear to be of the same nature as the papillæ in the suctorial disc of Lepidosteus (figs. 54, 55, p. 89). They are very short in the embryo, represented in fig. 53. Soon, however, they grow in length (figs. 54 and 55, *st.*)" [Plate 12, figs. 4, 5, 7, 9], "and it is" [*more than*] "probable that they become the barbels, since both sets of structures occupy a precisely similar position."

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* These do not correspond precisely to the specimens worked out by me, and of which I also have given figures illustrating the outward form. My first Stage includes larvæ from 5½ millims. to 6½ millims. in total length; Stage two, 8½ millims. to 9½ millims.; and Stage three, 13½ millims. to 14½ millims. The largest larva of this species in those sent to me was 14½ millims. ($\frac{7}{12}$ths of an inch) in total length.

"The openings of the nasal pits are at first single, but the opening of each becomes gradually divided into two by the growth of a flap on the outer side (fig. 54, *ol.*)" [Plate 12, figs. 4, 7, 9, *ol.*]. "It is probable that the two openings of each nasal sack, so established in these and in other Fishes, correspond to the external and internal nares of the higher Vertebrata. At the time of hatching there is a continuous dorso-ventral fin" [Plate 12, figs. 1, 2], "which by atrophy in some parts, and hypertrophy in other parts, gives rise to all the impaired fins of the adult, except the first dorsal and the abdominal. The caudal part of the fin is at first symmetrical, and the heterocereal tail is produced by the special growth of the ventral part of the fin combined with an atrophy of the dorsal part."*

Referring the reader to Mr. BALFOUR's work and to my own plates, I shall now simply describe the figures of the general form of the first stage, and then those of the second and third, in which also will be given a detailed account of their skeletal structures.

The fourth, fifth and sixth stages will be of the other species (*A. sturio*), and that in a thoroughly metamorphosed condition.

The smallest of my specimens (5½ millims. long, Plate 12, fig. 1) is intermediate between figs. 52 and 53 of Mr. BALFOUR (*op. cit.*, pp. 87, 88); the original of his fig. 52 (A and B), may be found in Professor SALENSKY's Russian work (plate 7, figs. 56 and 58), whilst his fig. 53 is only slightly larger than my second specimen (Plate 12, figs. 2 and 3).

In the smallest specimen, scarcely hatched at the time when it was put into the spirits, the large hind-brain (C^3.) and the lesser mid- and fore-brain (C^2., C^1.) are seen arranged in an accurate series. The auditory sac (*au.*) is below and behind the wide, front part of the hind-brain, the eye (*e.*) below and between the fore- and hind-brain, and the olfactory sac (*ol.*) on the side of and below the fore-brain, in front of the eye. The mouth (*m.*) is a considerable rhomboidal space on the ventral aspect of the head, and behind it we see three folds, two clefts, and a general hollow space behind the third fold. These folds are the rudiments of the mandible, hyoid, and first branchial (*mn.*, *hy.*, br^1.), the clefts are the hyomandibular (cl^1.) and the hyo-branchial (cl^2.), and the fossa behind leads to the tissues that are preparing to form the rest of the branchial arches with their intervening clefts. The last third of the spinal region is behind the anus (*an.*); it is the tail, and the intestinal opening is but a small distance behind the huge enlargement caused by the yolk-mass; that mass is three-fifths the length of the embryo. A vertical line drawn through the hind-brain and eye-ball would bisect a special enlargement in front of the yolk-mass. This enlargement, caused by the heart (*h.*), would be partly in front of that line, and looking at the embryo from the upper end of the line we should see the heart lying *in front of the head.*

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* I am glad to be able to give, and the reader will not be sorry to have, an account of these early larvæ in Mr. BALFOUR's own words. Of course I have been over the same ground, but my work has been made much easier by my friend's kindness in sending me his proofs as soon as they were in type.

The liver (*l.*) would be behind the lower end of the line, for most of the yolk-mass which is included in the rudimentary stomach lies *in front of the liver*, contrary to what takes place in the Lamprey, Frog, Elasmobranch, and the Amniota (Balfour, *op. cit.*, p. 90, fig. 56). According to that excellent observer "the peculiar flattening out of the embryo on the yolk is due to the mode in which the yolk becomes enveloped by the hypoblast" (p. 91).

In an embryo more developed, but slightly smaller (Plate 12, figs. 2, 3, 6½ millims. long) than that figured by Mr. Balfour (his fig. 53 was 7 millims. long), the yolk-mass is only one-third the length of the embryo—in the last it was half as long—and the tail is now also about one-third the length of the whole larva. The head is now fairly free from the yolk-mass, at least as far as to the hyoid fold (*hy.*), but the heart (*h.*) is a very short distance behind the mouth (*m.*), and the end of the intestine (*an.*) is seen to be three times as far from the yolk-mass (*y.*). The tail is at present perfectly diphycercal; the notochord running along the middle of the upturned tail, and the azygous fin is equally above and below the axis.

The mouth (fig. 3, *m.*) is a crescentic slit, with a small anterior bay in the middle of its anterior convex margin, and with a definite labial rim. In front of the oral opening and under the fore-brain (C^1.), behind a transverse ridge that passes from side to side parallel with the mouth, there are two pairs of pupiform enlargements with their narrow ends behind, and those of each pair approximated; these are the rudiments of the barbels (*bb.*). There are four visceral folds with their clefts, the two anterior larger folds being the mandibular and the hyoid; from the latter a free edge is growing, the opercular fold (*op.*); between the two bars the first (or hyomandibular) cleft (cl^1.) is closed ventrally, already, its permanent uppermost part will be the spiracle. The convex front lip of that limited cleft is thickening inside, and tends to produce a mandibular gill. The nasal sac (*ol.*) is still a shallow pouch with a thickened rim, the outer coats of the eye (*e.*) are closing over the lens, and the auditory sac (*au.*) is a smallish oval mass showing no evident involution.

In this larger embryo the basis-cranii and visceral arches are still composed of "embryonic cartilage;" these skeletal structures can be better studied in somewhat more advanced larvæ.

Second Stage.—Larva of Acipenser ruthenus, *from* $8\frac{1}{2}$ *to* $9\frac{1}{2}$ *millims. in length.*

The larvæ at this stage have a very *Selachian* appearance (Plate 12, figs. 4–8); altogether they have made a very great advance in development since the last stage: I shall describe first the external form, then a dissection of the visceral arches, and after that a series of transverse sections.

The azygous fins are beginning to be subdivided (fig. 7); now, the axis (notochord) turns downwards behind, but the tail is still diphycercal; it has become almost as long as the trunk, measuring from the anal aperture (*an.*), so that the post-anal region is very much longer, relatively, than in the last stage (fig. 2). The azygous fin is more definite along the trunk, almost reaching as a crest to the head, and the pectoral fins (*p.f.*) that then were at most mere thickenings of the tissue, are now well-developed, free, auriform paddles.

The yolk-mass (*st.*) makes only half the bulging it did: it is now quite portable; the intestine (*in.*, *an.*) is now seen as a definite ventral cavity behind the stomach and yolk, and is nearly as long as the wide part in front.

The heart (figs. 5 and 7, *h.*) is now fairly behind the operculum (*op.*), but is still seen from the outside as a double swelling.

The mesocephalic flexure is less, and the various parts of the brain (C^1., C^2., C^3.) are very visible through the diaphanous membrano-cranium.* So also the auditory organs show clearly through the skin, and the common tube of the anterior and posterior canals (*au.*, *p.s.c.*), and the single lesser tube forming the horizontal canal (*h.s.c.*), are very apparent without dissection.

The eye-ball (*e.*) is rapidly finishing and its socket is being formed, and there is a distinct suborbital ledge. The olfactory sac (*ol.*) is lateral and close to the eye, it has acquired a very definite superficial membranous capsule; this is sending downwards a triangular flap which tends to subdivide the opening.

The "barbels" (*bb.*) are now well formed, and look in the larger specimens of this stage like formidable *tusks*; they arise from a recess in front of the upper lip, which has over it a fold of the fore face, and in front of it, below the fore brain (Plate 12, fig. 5), there is between the olfactory sacs (*ol.*) a very remarkable concavity; it is nearly circular, is as large as one of the olfactory sacs, is almost as deep, and has in front of it a crescentic fold, and on each side a lateral lip-like margin. Its somewhat produced hind margin lies between the two inner barbels, and its lateral walls run up to those barbels. In the lesser larvæ of this stage (figs. 4, 5, *m.*) the mouth is still somewhat rhomboidal, but in the larger (fig. 7) its edges come very near together. The lips are now thick and large, the front is emarginate, the hind lip has an apiculate lobe, at the mid-line, looking forward; each is armed with the small, clear, pointed teeth (*t.*).

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* Through want of proper food these larvæ soon began to starve. Moreover, the spirit in which they were preserved contracted their tissues; the figures *purposely* show this contracted appearance.

There are, now, seven cartilaginous arches on each side, the two first are large, the rest smaller; they are all out of sight, more or less, the operculum hiding the two first, and the gills those behind; the clefts also are largely hidden from view by these structures.

In the smaller specimens at this stage (fig. 4) the form of the two first arches can be seen through the skin, the pterygo-quadrate bar running within the upper lip, MECKEL'S cartilage within the lower jaw, and the hyoid arch (*hy.*) inside the fore part of the opercular fold (*op.*). The opercular fold of the mandible is very limited on account of the closing in of the ventral part of the first cleft (*cl*^{1}.), but its serial homology with the large next fold (*op.*) is manifest. Inside it is thickened by a mass of cells from the first hypoblastic pouch, and this mass, somewhat obscurely, shows on its surface two rows of short mammillate projections—arrested gill papillæ.

But the next fold, with its lining, is greatly developed (Plate 12, figs. 4–7, *op.*), and its inner face is covered with rather long papillæ, like those which grow freely from the proper branchial arches (*br.*). Membranous, at present, this free fold of the hyoid arch is destined to contain three ganoid scutes, the upper of which (the opercular) will be one of the largest of the huge plates developed in this fish.

The rich growth of simple longish papillæ on the other arches hides them from view; these gills, however, are much shorter than those seen in the large embryos of Selachians.

When the skin and the gills are removed from the side of the head in one of the larger larvæ at this stage—9½ millims. in length—then the form of the newly chondrified bars is seen (Plate 13, fig. 6).

I have already remarked upon the *Selachian* appearance of the larvæ at this stage (Plate 12, fig. 7); a dissection, like the one under notice, displays much more than superficial Selachian characters. Even in the adult the Sturgeon has not made much advance towards the Teleostean culmination of the fish-type, but the dermal scutes have been largely dominated, in the head, by the cartilaginous endo-skeleton; and in the face, behind the mandible, we find many of the bars ensheathed in their own "ectostoses."

The hyostylic type of skull, which is not seen in the generalised forms, such as *Notidanus* and *Cestracion*, but is common to Skates and ordinary Sharks, is in the "Acipenseridæ" carried to its uttermost pitch of perfection, and here, in this minute larva, recently hatched, and scarcely chondrified, the Selachian type of skull is overpassed, and *at least two* additional segments are seen on each side. These additional joints in the highly subdivided and modified primary hyoid bar are expressly for the purpose of throwing the mouth out far away from the skull; the jaws are suspended on an extremely hypertrophied *upper hyoid segment.*

This hypertrophy of the "epi-hyal" segment is correlated with the atrophy of the uppermost, or suspensorial part of the pier of the mandible, which pier, even in this early larva, is *below the middle* of the hyoid arch Plate (13, fig. 6, *pg.q.*, *mn.*, *hm.*, *c.hy.*).

This is the more remarkable in a type that might have been expected to show something primitive in its visceral arches, seeing that it is only a *Chondrosteus Ganoid*, with a huge, permanently undivided notochord, and a totally unossified chondrocranium.

Even in the Salmon (Phil. Trans., 1873, Plates 1–3), the embryos of which are much larger and more tractable, I did find, *as cartilage*, the primitive facial bars; and their metamorphosis into the highly modified hyostylic type could be traced step by step; but this cannot be done in *Acipenser* nor in *Lepidosteus* (my next subject). In *Scyllium* (Zool. Trans., vol. x., plate 34) this was also possible; but in the Skates, *Raia* and *Pristiurus* (ibid., plates 35 and 39) the earliest tracts of true hyaline cartilage were, already, half metamorphosed, for all the *uppermost* segments (ibid., plate 35, fig. 4) were developed as distinct cartilages, although the remainder of each arch was continuous, and its segmentation could be traced afterwards.*

Here, in *Acipenser*, whilst the cartilage is still very soft, the segments of the highly subdivided hyoid arch are all apparent, although the branchial arches are continuous tracts (Plate 13, fig. 6); but their "pharyngo-branchial" segment—so very distinct in the embryo of the Skate—is already half severed from the main bar (Plate 13, fig. 4, *p.br.*, *e.br.*).

The pier of the mandibular arch, instead of growing up to the basis cranii, is a falcate cartilage (the pterygo-quadrate, *pg.q.*), which lies half-way down the face, and passes forwards and downwards. It is flat, obliquely-placed, and somewhat uncinate in front; behind, it is thick and rounded, and forms a condyle which lies in the concavity of its own lower segment, the mandible (*mn.*). This segment is thick behind; in front of its condyloid depression it sends upwards a coronoid swelling, whilst, behind, it ends in a rounded angular process. The rest of the bar is rounded; it lessens towards its ventral end, and is curved the opposite way to the pterygo-quadrate.

The rounded quadrate region of the upper bar is attached by ligamentous fibres to the lower end of the hyoid pier; that pier is the hyomandibular (*hm.*), the lower, rounded end of which is being segmented off as the symplectic (*sy.*). The whole bar is a massive phalangiform cartilage, gently bent backwards, altogether running downwards and a little forwards, and articulated above by an oblong condyle, to the "tegmen tympani" of the ear-capsule, under the horizontal canal (*h.s.c.*). It is one-third larger than the pterygo-quadrate, but its own lower piece, the cerato-hyal, is somewhat less than the mandibular pier.

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* In Selachians themselves, the best of all Fishes in which to study the development of these parts, it is next to impossible to maintain a consistent nomenclature of the elements of the visceral arches. In the Dog-fish (*Scyllium*, ibid., plates 37, 38) the hyomandibular is evidently the serial homologue of the epibranchials; in the Skate (*Pristiurus*, ibid., plate 35) the uppermost part of all the arches is developed as a separate nucleus of cartilage, and the metapterygoid and hyomandibular naturally classify themselves with the succeeding pharyngo-branchials. Again, the subdivided hyoid pier of *Acipenser*, which also carries the mandibular apparatus, is not divided into a normal pharyngo- and epi-hyal, but the epi-hyal, apparently, is itself subdivided, and there is no pharyngo-hyal.

The cerato-hyal element is a flattish rod, pinched in the middle, and dilated at both ends; its upper end is articulated to the postero-inferior face of the rudimentary symplectic by means of a small intercalary nucleus of cartilage—the inter-hyal (*i.hy.*). The lower end of the bar is nearly segmented off as a short semi-oval hypo-hyal (*h.hy.*). The branchial arches (*br.*) are about half the size of the hyoid—supposing it to have been in one large bar; there are *five* of them, and they decrease in size from before, backwards. The top of each is semi-distinct from the rest as a pharyngo-branchial (*p.br.*); that part is turned inwards, the rest of the arch is curved outwards, and turns inwards and forwards, below, where it joins the basi-branchial bar (Plate 13, fig. 4, *b.br.*).

Second Stage (continued).—*Transversely-vertical and vertical sections of the head of a larva of* Acipenser ruthenus, $9\frac{1}{2}$ *millims. in length.*

The transverse sections (Plate 12, figs. 1–3, and Plate 13, figs. 1–5) will further illustrate the structure of the visceral arches, and also show the state of the basis-cranii at this stage.

Section 1.—The first section (Plate 12, fig. 10) is through the fore-brain (C^1.), and the fore part of the mid-brain (C^2.), the eye-balls (*e.*), and the barbels (*bb.*); here the only cartilaginous tracts are the trabeculæ (*tr.*), which are sub-oval in section, obliquely placed, and separated by a distance equal to their own width; they are sub-concave on their upper surface, where they lie under the thalamencephalon as it narrows to the infundibulum.

Section 2.—The rounded ends of the trabeculæ are free for a very short distance in the extremely short head of this larva, which has at present no rudiment of the huge pre-cranial rostrum afterwards to be developed. In this section (Plate 12, fig. 11) the trabeculæ are wider, flatter, and are in close contact with each other; they lie here under the infundibulum (C^1.) which is covered now directly by the mid-brain (C^2.). Here the fore part of the oral cavity is cut through, so that the front row of teeth (*t.*) are seen, and on each side the outer barbels (*bb.*) are just in view; this section is behind the eye-ball. Above the mouth two oval masses of cartilage, more solid than the trabeculæ, are seen; these are the pterygo-quadrate bars (*pg.q.*) at their fore end.

Section 3.—The next (Plate 12, fig. 12) is close in front of the auditory capsules, through the widest part of the mid-brain (C^2.) where it joins the hind-brain, in the middle of the mouth, and the huge Gasserian ganglia (V.), whose cerebral roots are brought into view. Here the trabeculæ (*tr.*) are in close contact, are wider than in the last section, and they curve well round the base of the hind part of the infundibulum (*inf.*).

Here the pterygo-quadrate cartilages (*pg.q.*) are very solid and curved, and have a sharp edge lying over the "adductor mandibulæ" muscle (*ad.m.*), (see also SALENSKY, *op. cit.*, plate 18, fig. 165, *ad.m.*); these muscles lie outside the concave face of these oblique cartilages; the front teeth and lip are still seen. Above the front face of the auditory capsules (here, by mistake, lettered *e*), the auditory involutions (*aq.v.*) are seen.

Section 4.—The next slice (Plate 13, fig. 1) contains the hinder half of the auditory involution (*aq.v.*), and below it the auditory capsule (*au.*) is seen, in the region of the anterior canal; there is some cartilage in the infero-lateral part of the wall of the capsule. This is behind the mid-brain, and the hind-brain (see figs. 1–3, C^3.) is very large; large masses of ganglionic cells are seen belonging to the 5th, 7th, and 8th nerves (V., VII., VIII.). Outside, the lip of the spiracle (cl^1.) is cut through, and below the brain the large notochord (*nc.*) is shown near its apex. This lies in and between the moieties of the investing mass (*iv.*), which are nearly square—just a little hollow above, and convex below. Below these parts the oral cavity is laid open; right and left of it the thinner hind part of the pterygo-quadrate (*pg.q.*) is severed, and, below, the teeth of the lower lip (*t.*, *l.l.*).

Section 5.—Here the hinder part of the ganglionic mass belonging to the 7th and 8th nerves (Plate 13, fig. 2, VII., VIII.), and some fibres of their cerebral roots are cut through; the spiracle ($cl^{1\prime}$.) is here laid open; above it a cartilaginous tract of the auditory wall is seen. The anterior and horizontal canals, and the vestibule (*a.s.c.*, *h.s.c.*, *vb.*) are laid open. The notochord (*nc.*) is enlarging, and is being enclosed in the square moieties of the investing mass (*iv.*) which lie somewhat under it. Below the mouth the mandibles (*mk.*) are laid bare throughout their entire length, on account of their almost directly transverse position, and over their articular hollow the articular knob (*q.*) of the pterygo-quadrate cartilage is shown *in situ*. The back of the lower lip (*l.l.*) is seen, whose toothed front face was shown in the last section.

Section 6.—Here, on each side of the narrowing hind-brain (Plate 13, fig. 3, C^3.), the auditory capsules are cut through in their widest part, and both the arch and ampulla of the posterior canal (*p.s.c.*) are cut and laid open, as well as the deep saccular part of the vestibule (*vb.*). The infero-lateral shell of auditory cartilage is joining itself on to the more thin and outspread part of the investing mass (*iv.*—line too high); the notochord (*nc.*) is placed directly above the moieties of cartilage. The pharyngeal cavity is here very large, and in its walls we see the large, massive, arcuate epi-hyal element, undergoing segmentation into a long hyo-mandibular and a short symplectic piece (*hm.*, *sy.*). Here the directly transverse position of the parts gives, in one section, the whole of the hyoid series, for the symplectic is seen to be followed, lower down, by the little inter-hyal (*i.hy.*), and that by the long cerato-hyal (*c.hy.*), which is giving off from its sub-triangular end the short hypo-hyal (*h.hy.*). This is the front face of the section, and behind and within the hyoid we see the lower half of the first branchial arch and the fore part of the basi-branchial bar ($c.br^1$.); the branchial arch, like the hyoid, is becoming subdivided below, to form the hypo-branchial. The front part of the ganglionic mass of cells for the 9th and 10th nerves (IX., X.) is cut across.

Section 7.—These most successful and illustrative sections* display all that is sought for; here (Plate 13, fig. 4), on each side of the narrowing hind-brain (C^3.), the notochord has become suddenly twice as wide as in the last, and it has now gained a lower position; the investing mass (*iv.*) is horn-shaped in section, and the base of each "horn" sits on the supero-lateral face of the huge notochord. The back wall of the auditory capsule (*au.*) is cartilaginous, and at this part is some distance from the basal plate (*iv.*); in that interspace the ganglion of the vagus (X.) is shown.

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* The sections were made by my son Mr. W. N. Parker.

Here, happily, another visceral arch, the second branchial (*p.br.*, $e.br^2$., $c.br^2$., *b.br.*) is exposed in one slice; the segments are only indicated by the crowding of cells at the line where segmentation will take place. Within and behind the second branchial arch the third ($c.br^3$.) is partly seen, and the large, simple gills (*g.*) are shown fringing the folds of the section, below; they are clavate, because of the loop of the simple capillary vessel within.

Section 8.—Behind the ear-capsules the notochord has a diameter only two-fifths less than that of the hind-brain (Plate 13, fig. 5, *nc.*, C^3.); the investing mass (*iv.*) embraces it more below, and is rising, right and left, so as to form the lower half of the occipital arch (*e.o.*). Behind the gills, under the narrowed pharynx, the yolk-mass is now seen.

A vertical section of the head in one of the larger larvæ of this stage ($9\frac{1}{2}$ millims. long; Plate 14, fig. 1) shows the low position of all the parts of the fore-brain (C^1., C^{1a}., *inf.*), the forward position of the mid-brain (C^2.) and the huge size of the hind-brain (C^3.). The cavity of the fold of the mid-brain runs equally forwards and upwards, and the infundibulum (*inf.*) looks backwards and is quite distinct from the pituitary rudiment (*py*). The notochord (*nc.*) nearly reaches the latter, it is very large, arcuate, and bent downwards in front, showing no disposition to enter the fold of the mid-brain in the "post-clinoid" region. Two of the barbels (*bb.*) are seen to the right of the mid-line, and behind these we see the thick upper lip (*u.l.*) bounding the mouth (*m.*), which opens directly below the pituitary rudiment. It is a rather narrow passage, and has in its hinder boundary the thick lower lip (*l.l.*); in this the mandible (*mk.*) is seen at its ventral end, and the fore end of the pterygo-quadrate cartilage (*pg.q.*) is seen in the upper lip; these cartilages lie a little to the right of the section, which is exactly in the middle. In the pharynx, above and behind the oral chink, the first three clefts (cl^{1-3}.) come into view, below these part of the yolk (*y.*), and in front of the yolk the heart (*h.*).

Third Stage.—Larvæ of Acipenser ruthenus *from* $13\frac{1}{2}$ *millims. to* $14\frac{1}{2}$ *millims. in total length.*

Amongst the more advanced larvæ of this Fish only one reached the length of $14\frac{1}{2}$ millims., or $\frac{7}{12}$ of an inch; the next in size were 1 millim. shorter. One of these latter was made into horizontal sections, and others were *dissected;* the unique specimen was made into vertically-transverse sections.

The external appearance of these, next in size to the last, viz., $13\frac{1}{2}$ millims., is extremely Selachian (Plate 12, fig. 9, and Plate 13, figs. 8, 9). The tail is more than two-fifths of the whole length, the long arched notochord is ascending towards the upper part of the continuous azygous crest, and three-fifths of the caudal part of the vertical fin, above and below, is being marked off, ready to form the proper heterocercal tail-fin; the narrow crest above develops the "fulcra," and the wider part below the emarginate lobe. The high crest in front of the constriction becomes the backwardly-placed dorsal fin, and the deep crest, below, the anal fin; the ventral fins (*v.f.*) have a rudiment on each side as a thickening of the mesoblast of the abdominal wall close in front of the vent (*an.*); the pectoral fins (*p.f.*) are large and reniform. The region of the stomach still contains enough yolk to give it a swollen appearance; the intestinal region (*in.*) is narrow. The regions of the brain are still clearly seen from the outside; the eye ball (*e.*) is nearly perfect; the auditory capsules (*au.*) are seen as relatively small oval swellings.

The nasal capsules (*ol.*) have their opening almost subdivided into two by the descending flap. The suborbital fold of the face over the barbels is now large, and they (*bb.*) are long and finger-like. The lips (*u.l.*, *l.l.*) are now thick and rounded; the upper is emarginate in the middle, and the lower has a medium lobe; the teeth (*t.*) are now inside the lips. The spiracle is now a very small triangular hole under the auditory capsule (Plate 12, fig. 9, *cl*^{1}.); the operculum (*op.*) although very large and extensive does not cover the copious growth of young gills.

Dissections of the cranio-facial skeleton of a specimen $13\frac{1}{2}$ millims. long show, already, a well chondrified framework. The basis cranii, as seen from above (Plate 14, fig. 4), shows a structure very similar to that of an embryo of *Scyllium canicula* $1\frac{1}{3}$ inch long (see Zool. Trans., vol. x., plate 35, fig. 6), or more than twice as long as this embryo Sterlet.

The cartilage investing the notochord (Plate 14, fig. 4, *iv.*, *nc.*) is continued from the spinal into the cranial region with scarcely any change of form and consistence, a state of things quite like what I have found in the Selachians (*op. cit.*, plate 35, figs. 3, 5). In my preparation four pairs of spinal nerves were seen emerging behind the glossopharyngeal and vagus (Plate 14, fig. 4, *sp.n*$^{1-4}$., X., IX.). The bands of cartilage, whose size and form will be further shown in the sections (Plates 14 and 15) are confluent above for a short distance with the auditory capsules (*iv.*, *au.*) close behind the post-auditory nerves (IX., X.). In front of these conjugations there is a large open space, the "meatus internus" (VIII.), between the cartilage of the capsule and that of the skull floor; the latter widens a little in front of the meatus, then narrows a little to form the inner margin of the foramen ovale; outside this notch we see the huge Gasserian ganglion (V.). The investing mass then widens a little, and ascends a little; this wider ascending part is the "posterior clinoid" wall (*p.cl.*), and in it the notochord ends as a small rounded knob.

As in *Scyllium* (*op. cit.*, plate 35, fig. 6) the bands of cartilage then dip considerably, and these prochordal continuations (*tr.*) of the parachordal tracts, are wider than the roots they spring from. Instead of being wide apart, as in the Salmon, in *Lepi-*

dosteus, and in many types, they are really continuous under the pituitary body, as in *Scyllium* and *Raia*, forming a complete floor to the skull for some distance in front of the notochord and the post-clinoid wall; but this part has a small fontanelle immediately below the pituitary body (see fig. 1, *py.*). Opposite the optic nerves they part for a short distance, nearly meet again, and then diverge as two very short out-turned horns, the tips of which are the rudiments of the cornua trabeculæ (*c.tr.*). Between the Gasserian ganglion and the optic nerve (V., *e.*) the trabeculæ grow upwards, forming the rudiment of the alisphenoid (*al.s.*); the walls and roof, however, will be best studied in the sections.

In front of the eye-balls (*e.*), between them and the olfactory sacs (*ol.*), there is a definite antorbital cartilage growing outwards and a little forwards from the trabeculæ; this is the ethmo-palatine (*e.pa.*), a familiar element, which, here, is only semi-segmented from the trabecula. In Skates, as in Urodeles and Teleostei, it is distinct; but in Batrachia and Sharks it is not; it is well seen as a "process" in *Notidanus* (*Heptanchus* and *Hexanchus*, see GEGENBAUR'S 'Selachians,' plate 1, and also in embryos of *Scyllium*, T. Z. S., vol. 10, plate 37, fig. 1, *a.o.*). Here, it helps to form the back wall of the crypt in which the olfactory sac (*ol.*) is lodged; in these Fishes, as in *Lepidosteus* and the Teleostei, there is no distinct "paraneural" cartilage for the nasal pouch.

Leaving the walls and roof of the skull, until I come to the sections, I shall now describe the visceral arches, which are greatly developed since the last stage.

In a side view of these parts (Plate 13, fig. 11) I have shown the arches relative to the other cephalic structures, which are left in outline to give prominence to the view of the arches themselves.

In another figure (Plate 13, fig. 12) the arches are shown as seen from above, and in a third (Plate 14, fig. 5) they are shown as seen from below. The hyoid arch is twice as large as the others, and has undergone most segmentation and specialization; the branchial arches decrease in size and in the amount of their segmentation, from before, backwards.

The proper pedicle of the mandibular arch, in this extremely *hyostylic* type, is not only absent, but the stunted and forwardly projecting "pier" is much further from the normal point of attachment—under the outgoing fifth nerve—than in any of the Selachians. Each of these pterygo-quadrate bars (*pg.q.*) is a curved plate of cartilage, thick and bent forwards in its articular region, where it grows out over the temporal muscle, somewhat, to form the rudiment of the "orbitar process," and flat as it widens out in its pterygoid region.

The right and left plates meet each other for some distance in front, a notch separating their round ends; they are gently convex, above, and gently concave, below. These bars reach as far forwards as the antorbital region, but are embedded in the very loosely attached palatal skin, and are thus only slightly connected with the basis cranii. In the angle formed by their convergence a pyriform wedge of somewhat newer cartilage (*mt.pg′.*) has made its appearance since the last stage; this is, as far as I know, an unique structure, and but for this stage I should have suspected that this keystone piece had been formed by the fusion of a right and left nucleus of cartilage. Others form round it afterwards, but this is the first, as it is also the most important, morphologically, of the pieces that form the *hind palate* of the Sturgeon.

It would not have been difficult to have dealt with the paired pieces that appear afterwards (Plate 16, figs. 5 and 6), as they are manifestly the counterparts of the single or multiple "metapterygoid" cartilages of the Rays. In *Torpedo* (GEGENBAUR'S 'Selachians,' plate 13, fig. 3, and plate 20, fig. 1, Kr., *a, b,**) there are four of these cartilages on each side, but as a rule there is only one in that group, where it is a true metapterygoid segment of the pier, and not a mere "ray," as in the Shark. In Carinate Birds, where the skeleton reaches the utmost limits of specialization, remnants of Ichthyopsidan palatal cartilages reappear—ethmo-palatine, post-palatine, &c.; and in the Woodpeckers ("Picidæ") the palatine membrane bones are joined together under the basis cranii by an ossified cartilage, the medio-palatine (Trans. Linn. Soc., second series, "Zoology," vol. i., plates 1–5).*

MÜLLER simply calls the whole *compound* plate of the adult *Acipenser ruthenus* (*op. cit.*, plate 9, fig. 71, A, B) "unpaariger Gaumenknorpel," and the pterygo-quadrate bars "paariger Gaumenknorpel," so that the interpretation of these parts is left open. I shall merely, for the present, classify this plate with the free cartilages of the "Raiidæ," and call the median piece the "azygous metapterygoid" (*mt.pg′.*); the later pieces, right and left, will simply be called "paired metapterygoid segments" (*mt.pg″.*). In the adult (Plate 18, fig. 4) Mr. HOWES finds a row of four more much smaller segments along the mid line, in front of the main piece.†

The mandibles (*mn., mk.*) are about as long as the upper bars, but after forming a shallow articular cavity on their thick upper end they become rounded rods, and their position is almost transverse, as these dissections, and the transverse sections (Plate 15, fig. 3, *mk.*) demonstrate; they do not quite meet in the middle, at present.

The hyoid arch has five segments in it on each side, but, like the mandible, no basal segment below. The uppermost piece, the hyomandibular (Plate 13, figs. 11, 12, *hm.*) is by far the largest element in the whole series of arches; above, it is articulated by an oval condyle to the under surface of the pterotic ridge, under the horizontal canal ("tegmen tympani"), and below by a cylindroidal condyle to the symplectic (*sy.*).

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* The median cartilage seen in the "Myxinoids" (MÜLLER, 'Myxinoids," plate 3, figs. 2–5, U), the "Gaumenplatte"—which partially unites with the trabeculæ in the adult *Bdellostoma* and *Myxine*, and *early* and *totally* in *Petromyzon*, is not part of the palate, as MÜLLER'S term would suggest, and as I once thought, but is, as Mr. BALFOUR showed me, the "intertrabecula"—*i.e.*, its hinder or interorbital part.

† In the scanty living remnants of the "Chondrosteous Ganoids," only the "Acipenseridæ" show this peculiar structure; the "Polyodontidæ" show no trace of it (see MÜLLER'S 'Myxinoids,' plate 5, fig. 7, there called *Planirostra edentula*; TRAQUAIR, 'Ganoid Fishes of the British Carboniferous Formations,' Part I., "Palæoniscidæ," Palæont. Soc., 1877, plate 7, fig. 1; and BRIDGE, "On *Polyodon folium*," Phil. Trans., 1878, Part II., Plates 55–57, pp. 683–733.

Its form is like that of a phalangeal segment, it is curved backwards, gently, and has an enlargement there, above the middle, the rudiment of the "opercular process." Its axis is coincident with that of the symplectic and the mandible (*sy.*, *mn.*, *mk.*), together they form a crescentic series, passing downwards and gently forwards, and having the convex margin behind; they are bent inwards below (fig. 12). The curve forward, as well as downward, brings the lower end of the symplectic under the pituitary body; this cartilage (*sy.*) is only one-fourth the length of the upper piece (*hm.*), but it is as thick as its thickest part; it is scooped above for the hyomandibular, and rounded below, where it is tied, by ligaments, to the two elements of the first arch (*pg.q.*, *mn.*, *mk.*).

In the Teleostei the inter-hyal is articulated to the inside of the non-segmented cartilage uniting the hyo-mandibular and the symplectic; in the Sturgeon it is articulated to the inside of the distinct symplectic, rather below the middle—at least at this stage (Plate 13, fig. 11, *i.hy.*). The inter-hyal is a small piece of cartilage, wedge-shaped, with its narrow end upwards; its broad lower end articulates with the top of the cerato-hyal, which is bilobate above, the inter-hyal articulating with the larger upper lobe (fig. 12, *i.hy.*, *c.hy.*).

The cerato-hyal is a thickish, somewhat sigmoid bar, it passes inwards and forwards and nearly meets its fellow of the opposite side; there is no basi-hyal, but a semi-oval segment is formed out of the ventral end of each cerato-hyal; this is the hypo-hyal (*h.hy.*).

The branchial arches (Plate 13, figs. 11, 12, and Plate 14, fig. 5) are larger than the lower part of the hyoid at first, but they lessen backwards, so that the last is only one-fourth as solid as the first. The first three develop a hypo-branchial segment (*h.br.*) exactly like the hypo-hyal, but larger. The inturned dorsal end of all but the last is segmented off as an epi-branchial (*e.br.*), and its apex becomes distinct as a small pharyngo-branchial (*p.br.*) in the first four arches.

The basi-branchial (*b.br.*) is a thickish rod, rounded in front and compressed behind; it only carries the first three arches; the fourth and fifth meet below.

Bony scutes are now developing in the skin, and in the skin of the mouth and the mucous membrane of the palate and fauces several bony plates have appeared; all these are determinable.

In the upper and lower views (Plate 13, fig. 12, and Plate 14, fig. 5) these plates, *most of which are dentigerous*, are shown in their relation to the visceral arches and the largest of these bones; the maxillary and dentary—of one side—are shown separately (Plate 14, fig. 6, *mx.*, *d.*). There is no determinable premaxillary, which like the nasals, and vomers, only exists *as one or many* of a great number of generalised scutes that are to be found in those respective regions. The maxillary (fig. 5, *mx.*) runs across and meets its fellow in front of the pterygo-quadrate (*pg.q.*); it is a long, subarcuate bar, with thickened edges, and carrying five or six sharp, recurved teeth on its anterior third; it is somewhat notched and bent in its hinder third. The dentary (*d.*) is almost exactly like it, but gently bent in the opposite direction; its teeth look a little forwards and fit in between those of the upper jaw.

The palatine (Plate 14, fig. 5, *pa.*) is a styloid tract of bone binding on the external edge of the broad fore part of the suspensorium (*pg.q.*); it is edentulous. The pterygoid (fig. 5, *pg.*) carries teeth; it is a thin plate of bone, which lies inside the hinder convex edge of the cartilage.

Over the rudimentary "orbitar process" of the suspensorium a small scale of bone is visible, just as when the squamosal is formed over the quadrate in the Amphibia;* this is the preopercular (*p.op.*). In the roof and sides of the pharynx inside the first branchial arch there are three pairs of small, arcuate, dentigerous scales of bone (Plate 13, fig. 12, and Plate 14, fig. 5); these are "upper pharyngeal bones." Clamping the hinder part of the hyomandibular, its rudimentary "opercular process," the opercular bone (*op.*) is seen; it is a convex shell, with spurs in front and a sharp round margin behind. These bones are evidently all mere parostoses; the "ectosteal" sheaths of the visceral arches have not appeared yet; the cranium never acquires any well-grafted bony plates.

Third Stage (continued).—*Horizontal sections of one of the lesser larvæ of* Acipenser ruthenus, *at this stage*—13½ *millims. long.*

Section 1.—The uppermost of these sections (Plate 13, fig. 10) is through the hemispheres and the base of the mid-brain directly over the thalamencephalon (C^{1a}., C^{1}.), the eye-balls (*e.*), and the front and sides of the mouth (*m.*). The quadrate and pterygoid regions (*q.*, *pg.q.*) are severed, being cut through where there is a depression above; moreover, the broad pterygoid plate is obliquely placed, and thus this horizontal section shows its thickness, but not its breadth. Between the hooked fore ends of these cartilages a newer tract, the lozenge-shaped azygous metapterygoid (figs. 10 and 11, *mt.pg′.*), is seen. Behind the transversely oval quadrate region the lower part of the hyomandibular (*hm.*) is cut across; it is obliquely placed, and between the two sections of cartilage, on the outside, the lower part of the first cleft or spiracle (cl^{1}.) is laid open. Part of the "adductor mandibulæ" muscle (*ad.m.*) is seen outside the pterygo-quadrate bar, and the opercular fold of the first cleft is seen with its rough inner surface.

Section 2.—The next section, below the last, is through the whole length of the pterygo-quadrate bar (Plate 14, fig. 2, *pg.q.*), which is seen to be of almost uniform thickness, and of a sigmoid shape; the fore end is turned inwards, and the hind part outwards. Behind it the top of the articular region of the transverse mandible (*mk.*) was sliced off. The basis cranii (*tr.*) is cut through close in front of the post-pituitary wall and the notochord, where the trabeculæ are completely confluent; the large infundibulum (*inf.*) is severed at its "neck" (see Plate 14, fig. 1, *inf.*). Here, this process of the brain is above and in front of the distinct pituitary body (fig. 1, *py.*), and seems to be distinct from the fore-brain (C^1.); this, however, is due to the manner in which it has been cut through on its projecting part. In this section the rudimentary hemispheres (C^{1a}.) are cut through where they pass forward from the first cerebral vesicle (figs. 1 and 2, C^1., C^{1a}.). The small olfactory sacs (*ol.*) are cut across; they lie outside the junction of the hemispheres with the thalamencephalon; the eye-balls (*e.*) are cut through in the lower part.

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* The squamosal of the Amphibia has a large descending *preopercular process;* but there is no separate preopercular bone, such as Fishes possess.

Section 3.—The lowest section (Plate 14, fig. 3) is through the lower part of the curved head, and takes in the whole boundary of the mouth (*m.*; see also fig. 1). The olfactory capsules are removed, and the barbels (*bb.*) are cut across at their root; each, at present, contains a pith of true cartilage. The fore part of the pterygo-palatine and the quadrate region of the same (*pg.q.*, *q.*) are shown as severed, and the latter is seen as a transverse process, bulging below to form the hinge of the jaw. Behind the mouth (*m.*) the lower lip is seen with its teeth (*l.l.*, *t.*), and in the substance of this visceral fold the whole length of the mandibles (*mk.*) is shown, first thick, in the articular region, then slenderer, and again enlarging distally.

Third Stage (continued).—*Transversly vertical sections of the head of the largest larva of* Acipenser ruthenus; $14\frac{1}{2}$ *millims. long.*

Section 1.—The first section (Plate 14, fig. 7) is through the barbels (*bb.*) at right angles to the horizontal section just described (fig. 3), and also to the axis of the down-bent brain which is here developed into the rudimentary hemispheres (C^{1a}.). The plane of this section would form an angle of 45° with the plane of a section taken vertically through the hind part of the hind-brain, or through the front of the yolk mass and heart in such a larva as is shown in a longitudinally-vertical section (Plate 14, fig. 1, C^3., *y.*, *h.*). That, however, was a younger larva, and the head in this had straightened a little.

The hemispheres (fig. 7, C^{1a}.) are here shown to be, together, sub-oval in section, somewhat grooved above, and slightly protuberant below. The olfactory sacs (*ol.*) have no roof in front, and their floor is formed by the ethmo-palatine cartilage (*e.pa.*), which is distinct, in front, from the trabecula. The skin of the fore face, however, is developed into a valvular fold around the opening, and the mucous membrane is applied to the rising and falling of the structures that encompass it; there is no proper "paraneural" cartilage over the nasal capsule; in this, *Acipenser* agrees with *Lepidosteus* and the *Teleostei*, and differs from the Selachians, Cyclostomes, and Amphibia. Here the floor (*e.pa.*) is flat and distinct, and there is a considerable space between it and the trabecula (*tr.*) which is thick at its outer edge, and thins out towards its fellow of the opposite side, which it does not quite meet.

Section 2.—The next section (Plate 14, fig. 8) is through the widest part of the hemispheres (C^{1a}.) which are nearly oval in the section, from side to side, the top being convex as well as the bottom; this latter part is, however, narrower than the upper.

Here the roots of the front teeth are cut through in the base of the upper lip (*u.l.*, *t.*), and here the nasal pouch (*ol.*) has both roof and floor; the latter is continuous, now, with the trabecula (*tr.*) and bulges downwards, whilst the roof (*s.ob.*) is the fore end of a large tract of superorbital cartilage, which grows independently of the floor for some distance backwards—a very common thing in the Ichthyopsida.

This upper band is the lateral rudiment of the tegmen cranii, which in this type so soon covers in the great fontanelle and becomes so very massive; at present it is very similar to the palato-trabecular band (*e.pa.*, *tr.*) below; here the trabeculæ are wider apart than in the last section.

Section 3.—In this third pre-oral section (Plate 14, fig. 9) the projecting fore part of the mid-brain is seen to lie on the fore-brain (C^2., C^1.). This is through the back part of the olfactory pouch (*ol.*), which is becoming complicated; its palato-trabecular floor (*e.pa.*, *tr.*) is, here, at its widest and most solid part, and the superorbital roof over the nasal sac is narrower, and is sharp at its outer edge; the roots of the front teeth are still in view (*u.l.*, *t.*).

Section 4.—This is through the fore part of the eye-balls (Plate 14, fig. 10, *e.*); there is, here, a very solid lateral ethmoidal wall uniting the superorbital band and the trabecula (*s.ob.*, *tr.*); the latter is losing its palatine extension, outwards, and the two plates are still a good distance apart beneath the fore-brain (C^1.). Here the antorbital wall is cut through; it is membranous, at present.

Section 5.—In this section (Plate 14, fig. 11) the superorbital band (*s.ob.*) is simply a moiety of the tegmen cranii, for the projecting part is gone, and each band is creeping towards its fellow over the front of the mid-brain (C^2.). So also, below, the trabeculæ (*tr.*) have lost their palatine projection and they are creeping towards each other under the fore-brain (C^1.); these latter are the larger plates, and are rather indented below in the middle. Opposite the eye-ball, the front third of which is cut across, there is a small tract of cartilage cut through; it is rounded above and sharp below; this is a tract which runs backwards as far as to the Gasserian ganglion (see Plate 15, figs. 2, 3, *al.s.*, V.); it is orbito-sphenoidal here (*o.s.*), and alisphenoidal behind.

Section 6.—The lens of the eye (Plate 14, fig. 12, *e.*) is now reached, and the sclerotic is becoming cartilaginous; this section is very similar to the last, but the superorbital and orbito-sphenoidal bands (*s.ob.*, *o.s.*) are thinner, and the trabeculæ (*tr.*) are approximating; here the fold of the upper lip (*u.l.*) is cut through; it is seen to be folded off from the fore face, so that at this point it seems to be attached by a narrow isthmus; the next two figures will explain this (see also Plate 13, fig. 11).

Section 7.—This and the next (Plate 14, figs. 13, 14) are through the highest (or deepest) part of the brain, for here the bulging mid-brain (C^2.) lies right over the fore-brain (C^1.) in front of the infundibulum, which is cut through behind the second of these (Plate 14, fig. 1, *inf.*). Here the growing roof and walls of the orbital region of the skull are composed of *three* bands on each side (*s.ob.*, *o.s.*, *o.s′.*), for over the emerging optic nerve a lower tract of cartilage has appeared, which is thick on its inner face, growing into the sulcus between the swellings of the brain (C^2., C^1.). Here the large optic foramen is a "fenestra," as in Frogs; its upper margin is formed by this lower orbito-sphenoidal process (*o.s'.*), and its lower edge by the trabecula (*tr.*); this plate now touches its fellow of the opposite side. The upper lip (*u.l.*) is seen in all its depth here, and the teeth (*t.*) are in full view; they are looked at from behind.

Section 8.—In this slice (Plate 14, fig. 14) the hinder half of the eye-ball and optic nerve (*e.*, II.) is seen, and the brain (C^1., C^2.) is here at its greatest bulk. The trabeculæ (*tr.*) are confluent, are thicker, and rise more at the sides; this is close in front of the post-pituitary wall. The thick hind part of the orbito-sphenoid is now ready to become alisphenoidal; its two parts are molten together, and the upper tract or superorbital (*s.ob.*) is now thickening as it approaches the auditory sac, ready to become the postfrontal ("sphenotic") wing.

Close behind the upper lip (*u.l.*) two curved plates of cartilage are cut across; these are the pterygo-quadrate bars (*pg.q.*), they are thick outside, and bevelled where they meet.

Section 9.—In this section (Plate 15, fig. 1) the mid-brain (C^2.) is at its widest part and overlies the infundibular end of the fore-brain (*inf.*), to which the pituitary body (*py.*) is becoming attached; this is through the hinder part of the eye-balls (*e.*), and close in front of the Gasserian ganglion (see fig. 2, V.).

The superorbital band is now the "sphenotic" (*sp.o.*); below it there is a large membranous fenestra, and the short oval section of the narrow alisphenoidal band (*al.s.*). The trabeculæ (*tr.*) are apart again, where the pituitary body (*py.*) passes down; they are altogether narrower here, and are grooved by vessels below. Here the pterygo-quadrates (*pg.q.*) are cut through their middle part, and here, in the re-entering angle behind their upper junction, a considerable wedge of cartilage is seen; this is the azygous metapterygoid (*mt.pg'.*); it has a pair of smaller pieces (*mt.pg''.*) or lateral metapterygoids attached to it; underneath this arched palate a number of teeth (pterygoid teeth) are seen.

Section 10.—This section (Plate 15, fig. 2) escapes both the eye-balls and ear-sacs, and is through the body of the Gasserian ganglion (V.) and behind the pituitary body. Here the trabeculæ are confluent again in front of the low post-pituitary wall and the end of the notochord. The sphenotic cartilage (*sp.o.*) is now ear-shaped in section, convex outside, and has two sub-concave inner faces, one applied to the skull-wall and one hanging down free. The pterygo-quadrates (*pg.q.*) are cut across a little in front of the hinge of the lower jaw, they are acuminate-oval in section, with the point downwards. The lower lip with its teeth (*l.l.*, *t.*) is here displayed, and the large "adductor mandibulæ" (*ad.m.*) is cut across its belly.

Section 11.—This (Plate 15, fig. 3) is the first of the auditory sections; the capsules (*au.*) are severed in their antero-superior-angle, to which is attached the upper and lower processes of the sphenotic (*sp.o.*). Through the wall of the capsules the ampulla of the anterior canal (*a.s.c.*) is seen. The alisphenoid (*al.s.*) re-appears in this section, and is twice as large as in fig. 1; the Gasserian ganglion (V.) is here cut through behind, and lies within its proper boundaries, namely, with the alisphenoid (*al.s.*) above and the investing mass (*iv.*) below. This section is near the edge of the low post-clinoid wall (Plate 14, fig. 4, *p.cl.*) and the apex of the notochord (*nc.*), which here lies on the gently-scooped plate formed by fusion of the two parachordal bands (*iv.*). The pterygo-quadrate (*pg.q.*, *q.*) is cut across close in front of the hinge, above and below, for it appears in two parts on account of its curve downwards to form the hinge. Meckel's cartilages also (*mk.*) are cut through close to the hinge; they are placed across, behind the mouth, are gently arcuate, and slowly lessen towards the meeting point. Outside the hinge, an angle of the symplectic (*sy.*) has been cut off.

Section 12.—This section, half of which was drawn (Plate 15, fig. 4) like the last, is a *front view;* here the cavity of the auditory capsule is laid open, with the ampulla and part of the arch of the anterior semicircular canal (*a.s.c.*). The sphenotic cartilage is still seen above and below this front part of the capsule; from the capsule to the investing mass is the foramen ovale with the ganglion of the 5th (V.), perhaps also part of the "ganglion geniculatum," which belongs to the facial and auditory nerves (Plate 14, fig. 4. VII., VIII.); the notochord (*nc.*) still lies on the investing mass (*iv.*). The hinge of the lower jaw is seen from its front face; the rod of the mandible (*mk.*) was in the last section, and this shows the articular region. The quadrate end of the pterygo-quadrate (*q.c.*) is seen in its full size with its orbitar process (*or.p.*); outside the hinge the symplectic (*sy.*) is shown, and the hypo-hyal and half the cerato-hyal (*h.hy.*, *c.hy.*), and some of the teeth of the lower lip (*t.*), towards the mid line.

Section 13.—In this section (Plate 15, fig. 5) the hind-brain (C^3.) is becoming narrower, and each auditory capsule under the arch of the anterior canal (*a.s.c.*), is cut through so as to expose the ampulla of the horizontal canal (*h.s.c.*); above, there is a rudimentary tegmen cranii, continuous with the capsule and the hind part of the "sphenotic" tract. At this part the capsule, as is the rule in Fishes, is open towards the hind-brain, and in the fissure, below, the "ganglion geniculatum" (VII., VIII.) is shown. Here the capsules and the basal plates (*iv.*) are quite confluent, and the notochord (*nc.*) divides the latter, lying down between the two halves. The front face of this section was figured; here the massive symplectic (*sy.*) is shown in its front half, and a considerable portion of the cerato-hyal (*c.hy.*); the hypo-hyals are lost in this section, they were shown in the last (fig. 4); the basi-branchial, the first hypo-branchials, and part of the first cerato-branchials ($h.br^1$., $c.br^1$.) are shown here as well as the "protractor hyomandibularis" muscle (*pt.*, *hm.*). Teeth (*t.*) are seen as far down, inside the throat, as the first hypo-branchials.

Section 14.—In this section (Plate 15, fig. 6), the hind-brain (C^3.) is much smaller, but the auditory capsules are at their widest part; they are continuous with the thickening parachordals, which enclose an enlarging notochord (*iv.*, *nc.*). Below these parts and their underlying vessels, teeth (*t.*) are still seen. The super-occipital tegmen (*s.o.*) is growing inwards from the edge of the capsules, but these margins of the great fontanelle are nowhere closed in (from end to end) at present.

The arch of the anterior and horizontal canals, and the vestibule (*a.s.c*, *h.s.c.*, *vb.*) are laid open, and below, in the great open "meatus internus," the ganglion of the 7th and 8th nerves is still seen.

The hyomandibular (*hm.*) is here articulated by its convex head with the concave surface of the auditory capsule, and looks like the proximal segment of a large limb joined to its own limb-girdle. This is correlated with the downgrowth of the mouth, whose own arch has been carried far away from its cranial attachment. Here we see the symplectic, inter-hyal, and proximal part of the cerato-hyal (*sy.*, *i.hy.*, *c.hy.*); and behind these, near the middle, the lower part of the first and second branchial arches (*h.br*^{1}., *c.br.*^{2}, *h.br*^{2}., *b.br.*).

Section 15.—In this section (Plate 15, fig. 7) the posterior canal (*p.s.c.*) is laid open throughout its whole extent, and here the capsule (*au.*) is re-acquiring an inner wall. Below, it is joined to a very solid basal plate (*iv.*), right and left, the moieties of this plate clip the large notochord (*nc.*), which is only partly embraced by them.

Parts of the first and second branchial arches (*p.br*^{1}., *e.br*^{1}., *c.br*^{1}., *c.br*^{2}.) and some gill papillæ come into view here.

Section 16.—In this view (Plate 15, fig. 8) the back wall of the auditory capsule (*au.*) behind the posterior canal is shown; here the massive basal plates (*iv.*) are growing upwards to form the occipital arch (*e.o.*), and are separated from the auditory capsules by a large chink. Here the 9th and 10th nerves (IX., X.) are seen growing from the hind-brain (C^3.), forming their ganglia and giving off their trunks. Part of the gill arches (*br.*) and their gills are shown on each side of the pharynx.

Section 17.—Here we see (Plate 15, fig. 9) that the occipital arch (*e.o.*) is imperfect above; below, each mass of cartilage (*iv.*) cleaves closely to the huge notochord (*nc.*) the arch is produced into the angular processes on each side that project from the auditory capsules (see fig. 13).

Fourth Stage.—Young Sturgeons (Acipenser sturio), $7\frac{1}{4}$ *to* 8 *inches long.*

In this stage the Fish is completely metamorphosed, and the only important change which takes place afterwards is immense increase in size, and the addition of certain bony centres, both parosteal and ectosteal.

I have had no intermediate sizes between *Sterlets* 7 lines long and *Sturgeons* 7 inches long, but in *Lepidosteus*, another Ganoid (the subject of my next paper), two instructive stages come in at this point, and make the interpretation of this lower type of skull easy.

In larval *Lepidostei* the size of my largest larval Sterlets, namely, about 15 millims., the azygous intertrabecula has already filled in the space between the trabeculæ, in front; and in specimens already like the adult, and 1 inch long, the trabeculæ have developed their cornua, and the intertrabecula has shot forwards as a long pre-cornual rostrum. At that stage the endocranium of *Lepidosteus* is extremely Acipenserine, and explains, and is explained by, the skull of this stage in the young Sturgeon.

A. *Ectocranium of the young Sturgeon* (Acipenser sturio).

In this stage there are scarcely any proper "parostoses" and very few "endostoses," but the head is well covered with "dermostoses" that are simply the ordinary Ganoid scutes (Plate 15, figs. 10–12), brought more or less into relation, both in *form* and in *number*, with the underlying endo-skeletal structures.

Of course, the homology of these scarcely altered superficial scutes with the special deep laminæ of bone that are so completely dominated by the endocranium in higher types, is imperfect and partial. These scutes present us with *too much* or *too little* when we are looking for the normal, highly specialised "investing bones" of the higher types; their inner layer, only, can correspond with those bones, and that is but imperfectly related to the parts within. Moreover, it is only in certain regions that any strict comparison can be made: this is in those cases where some unusually large scute has starved out its neighbours and has become the roof or wall of some particular part of the skull or face. Nevertheless, for the sake of uniformity of language, I shall call that scute which more perfectly than any other covers the nasal capsule, the *nasal* (*n'.*); the large plates over the hemispheres, the *frontals* (*f'.*); those over the mid-brain, the *parietals* (*p'.*); and those over the auditory ledge, the *squamosals* (*sq'.*).

In the face it will not be difficult to seize upon the true meaning of certain ichthyic bones, namely, the "operculars;" and the "splints" (parostoses) that are applied to the highly specialised pterygo-quadrate and mandibular apparatus will also, by comparison with like parts in other Ganoids, and in the *Teleostei*, be interpretable.

The round swollen head of the larva (Plate 12) is now changed into a long wedge-shaped recurved rostral structure (Plate 15, figs. 10–12), and this structure is invested now by solid ganoid plates in great number; but only certain of these can be pitched upon as deserving a special name.* The number of bones covering the snout is very great; none of these can be called "premaxillary," only one can even by courtesy be called "nasal" (figs. 10, 11, *n'.*), whilst below (fig. 12) several bones contend for the name of "vomer." The eye is protected by supra-, post-, and sub-orbitals (*s.ob.*, *pt.ob.*, *su.ob.*); the post-orbitals run back and become temporal scutes, the chief of which is called the squamosal (*sq'.*). A very fine scute lies over the opercular region and is the true opercular (*op.*); under it there are two rugged, squarish plates—these are the sub-opercular and inter-opercular (*s.op.*, *i.op.*); the pre-opercular (Plate 16, fig. 1, *p.op.*), as in *Lepidosteus*, is a very small "parostosis," applied to the side of the quadrate region of the suspensorium.

The orbital rim and eye (fig. 10) are small; the nasal pouch has a small upper, and a large lower opening; these openings are obliquely placed, so that the upper is also the foremost space. These capsules keep close to the antorbital region; in *Lepidosteus* they are carried to the end of the long beak. Behind the main post-orbital, and at the antero-superior angle of the opercular, the small spiracle (cl^1.) is seen; inside the opercular the *half*-gill of the hyoid arch may be seen, transferred from its own arch, now required for suspensory purposes, and attached to the inner face of a specialised ganoid scute. The mouth and lips (fig. 12, *m.*, *u.l.*, *l.l.*) are now curiously modified, and this structure is extremely protusible; the opening is transverse, and crescentic when closed.

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* I must refer my readers to the views of those excellent "experts" whose works are referred to in the Bibliographical List, especially to those of Professors HUXLEY, TRAQUAIR, and BRIDGE.

The barbels (figs. 10, 12, *bb.*) are long and slender, now; they have lost their cartilaginous pith.

I shall describe the oral parostoses with the cartilages to which they are attached.

B. *Endocranium of young Sturgeons from 7¼ to 8 inches long.*

When the superficial bones have been removed we find an extremely solid chondrocranium underneath (Plate 15, fig. 13, and Plate 16, figs. 1–4). In the last stage there was a continuous membranous fontanelle along the whole top of the head, the orbitosphenoidal region was partly membranous, right and left, a small fontanelle existed under the pituitary body, and an open notch in front between the trabeculæ.

Now, the only membranous space is a small trilobate supraoccipital fontanelle (Plate 15, fig. 13, *s.o.*, *fo.*), not over the proper brain cavity but in an extension of the chondrocranium over the fore part of the spinal region.

Everywhere there is the same intense hypertrophy of the hyaline cartilage, and in no part of the cranium, proper, nor in the auditory capsules, do true "ectosteal" plates graft themselves upon the cartilage—even in very old individuals; moreover there is no *calcification* of the surface-cartilage, such as is seen in the Selachians. Even now, in these young specimens, the actual size of the brain and brain-cavity is extremely small (Plate 16, fig. 2) in proportion to the size of the skull; which, measured to the end of the rostrum, is three times the length of the cranial cavity.

Here we see the permanence of the early "mesocephalic flexure;" for besides the sudden loop formed by the mid-brain, represented now by the post-pituitary chink, which looks forwards, the solid ethmoidal region of the skull is bent gently, but steadily, downwards, before it rises to form the recurved rostrum. The orbits are very large, out of all proportion to the small, thick, cartilaginous sclerotics; the nasal capsules (*ol.*) are set in the sides of the hind part of the huge rostrum, the ethmo-palatine or antorbital wings of which are thick and twice swollen. The rostrum is composed of three tracts, answering to the three offshoots of cartilage that have grown so rapidly since the last stage from the end of the primary chondrocranial floor-bands—the short *prochordal* trabeculæ. Seen from above (Plate 15, fig. 13) and in section (Plate 16, fig. 2, *c.tr.*, *i.tr.*) the rostrum is convex at its edges, and gently concave in the middle; but below (Plate 16, figs. 3, 4) it is like a sagittate leaf—thick and succulent—with a very solid convex mid-rib and thickened margins; here the margins are the cornua trabeculæ and the mid-rib the intertrabecula (*c.tr.*, *i.tr.*). The deepest part of the skull is in the presphenoidal region, close behind the huge aliethmoidal and ethmo-palatine outgrowths; (compare in Plate 16, figs. 1–4, *al.e.*, *e.pa.*, *tr.*, *i.tr.*).

Here the deep middle part is formed by the hinder intercalating part of the huge intertrabecula (*i.tr.*), and the thick suborbital ledges, close beneath the optic nerves (II.), are expansions of the trabeculæ (*tr.*) behind their cornua (*c.tr.*). In the sectional view (Plate 16, fig. 2) a peninsula of cartilage is almost insulated, below, and the last two of the vomerine series (Plate 16, fig. 3) in front of the fore part of the parasphenoid (*pa.s.*), behind, have crept into the chinks of the cartilaginous mass. A deep, rounded notch, right and left, separates the antorbital from the postorbital (sphenotic) masses of cartilage, which are thick, round, leafy plates turning their convex margin the one to the other (Plate 15, fig. 13; and Plate 16, figs. 1, 3, and 4, *al.e.*, *sp.o.*).

Here, again, in front of and around the auditory capsules the cartilage has grown freely, yet not, at present, hiding the form of the imbedded labyrinths in the lateral and upper views (Plate 16, fig. 1, and Plate 15, fig. 13, *a.s.c.*, *h.s.c.*, *p.s.c.*); and here, in the midst of all this profusion of cartilage, there is an open "aqueduct" (*aq.v.*) between the anterior and horizontal canals.*

The trilobate fontanelle has a cranio-spinal position (Plate 15, fig. 13, *fo.*), and behind it the cartilage runs, without division, along the spinal roof as a stout rounded process, whilst on each side there is a longer, arcuate, diverging process, growing from the epiotic and pterotic regions; the paired processes reach almost as far back as the branchial arches; they end over the fourth. Even the segmentation of the skull from the spine is absent, as it was in the larva (Plate 14, fig. 4); and the section (Plate 16, fig. 2, $sp.n^{1-6}$.) shows the exit of the first six of the spinal nerves.

However we may interpret this continuity of the chondrocranium with the spine, it is a very important thing to note it fairly down. The general form of the chondrocranium as seen from above is like that of two broadly sagittate leaves, set end to end by their broad leaf-stalks—the narrower orbital region; but the hinder half, as we have seen, is trilobate, and not simple at its free end. The arched canals (Plate 15, fig. 13, *a.s.c.*, *h.s.c.*, *p.s.c.*) are midway between the sphenotic lobes and the roots of the paired cranio-spinal processes (*c.s.p.*), and the small aqueduct opens in the hollow formed by the arches. They project above the large lateral "eave" (Plate 16, fig. 1); to the front third of this outer projection the hyomandibular (*hm.*) is articulated by its convex condyle. Looked at laterally, the skull shows the 5th (V.) nerve emerging in front of the hyomandibular, the 7th (VII.) inside it, and the 9th and 10th (IX., X.) further back.

The hind part of the basis cranii is strongly under-floored by the huge parasphenoid (Plate 16, figs. 2 and 3, *pa.s.*), in section (Plate 16, fig. 2) it is only seen as far back as the 2nd spinal nerve, but beneath (Plate 16, fig. 3) its forks are shown to pass far back as to about the 10th; behind the parasphenoid the neuro-central cartilages appear right and left of the notochord. That rod (*nc.*) is invested with a thick mesoblastic sheath of true cartilage, which, as the sections show, is thoroughly confluent with the parachordals in the basis cranii. In front (Plate 16, fig. 2, *nc.*, *p.cl.*, *py.*), it runs up nearly to the pituitary space, but does not ascend into the post-clinoid wall—an arched and almost horizontal plate of cartilage, the *true organic end* of the skeletal axis ; all the rest, to the end of the snout, is formed of special outgrowths from the fore end of the basal plates. The relative size and thickness of the various parts of the chondrocranium will be shown afterwards in the description of the sections.

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* This passage, which I find in *Siren lacertina*, has been described and figured in *Polyodon*, by BRIDGE (Phil. Trans., 1878, Part 2, Plate 56, fig. 5, *f.g.*, *p.f.*, p. 699).

C. *Visceral arches of a young Sturgeon 7¼ inches long.*

There is no *distinct* rudiment of any arch in front of the mandibular, with its extended and complex pterygo-quadrate "pier;" the rudiment of the ethmo-palatine cartilage only exists as an extension of the aliethmoidal mass (Plate 16, fig. 1, *al.e.*). The pterygo-quadrate plate (Plate 16, figs. 1, 4, 5, 6, *pg.q.*) lies in an almost horizontal plane, and at a very variable distance from the basis cranii. The right and left plates meet by their extensive straight upper edge ; then they curve outwards and backwards ; their fore margin is rounded, their inferior edge concave, and their hinder edge is sinuous and notched. Outside the rounded condyle (*q.*) there is a leafy growth, imperfectly adze-shaped, which passes outside the adductor mandibulæ ; this is the "orbitar process" (Plate 16, figs. 1, 7, *or.p.*). Three-fourths of their inner face is invested by the pterygoid bone (*pg.*), a thickish plate with a deep gap in its blade, in front ; the front third of the cartilage is bare, and its hinder margin has suffered absorption—not direct ossification, through the pressure of the pterygoid bone. A sharp style of bone, with its broad end in front, lies along the concave antero-inferior margin ; this is the palatine (*pa.*)—a mere parostosis.

A larger bone curves round the front of each plate, where the two sides meet—both bone and cartilage—and then runs backwards, outside the adductor mandibulæ, and is attached to the outer face of the orbitar process ; this is the maxillary (*mx.*). Mounted on the hind part of the maxillary, and at right angles with it, there is a little triangular bone, with its apex upwards ; it binds against the ribbed outer edge of the quadrate region of the cartilage ; this is the pre-opercular (*p.op.*), smaller, here, than in *Lepidosteus.*

The mandible (*mk.*) is shorter than the forwardly extended pier, it is like that of a Tadpole, having a thick articular region, a hollow for the quadrate, a rounded angular process, and a short terete main rod ; a flat dentary bone (*d.*) invests its outer surface, which has a similar outline to the rod.

The most remarkable part of this apparatus, however, is the *common compound* (tesselated) "metapterygoid" region. At first (Plate 13, fig. 10) the main middle piece, only, was present ; then a right and left segment appeared (Plate 15, fig. 1) ; now (Plate 16, figs. 1, 5, 6, *mt.pg'.*, *mt.pg''.*) there are fifteen. These have a general symmetry, the large median piece being sub-oval and the large lateral pieces wedge-shaped; but the number of the lesser segments varies, and the outermost, on the left side, is only half separated. In front, the edge is turned downwards (Plate 16, fig. 6), and here a second median cartilage appears, with three lesser segments on the right, and two on the left side. The whole of this "unpaariger Gaumenknorpel" (MÜLLER) is a gently convexo-concave, two-winged structure, which finishes the hard roof of this remarkable tubular protractile mouth.

The uppermost segment of the hyoid arch (*hm.*) is nearly twice the height of the ascending, but arrested, quadrate region; it is the largest of five "internodes" in this double-sized, forked arch, with its double function. The head of the hyomandibular is rounded (Plate 15, fig. 13, and Plate 16, figs. 1, 5, *hm.*); it curves backwards, and sends from its lower two-thirds a large thick flange, the opercular process; this causes the width of the bar to be more than doubled. Its lower condyle is cylindroidal, and the concavity on the next joint answers to it, so that it is like the humero-ulnar joint of a Mammal. Just the neck of the bar is ringed with an *ectosteal* deposit. The next segment, or symplectic (*sy.*), is scarcely half the bulk and length of the last, and its shape is different; it is a phalangiform cartilage, hinged to the hyomandibular, above, tied by ligament to the quadrate and angle of the jaw, antero-inferiorly, and having a little concave facet inside its upper part for the inter-hyal (*i.hy.*), a very small subquadrate segment.

This small *secondary* suspensorium carries the lower part of the arch so that it lies inside the upper; it is composed of two segments, for the distal fifth is segmented off; the main piece is the phalangiform cerato-hyal (Plate 16, figs. 1, 4, 5, *c.hy.*). The middle third is ossified; the distinct piece at the end, the semi-oval hypo-hyal (*h.hy.*), is soft; they meet by their narrow rounded ends, without the intervention of a basi-hyal.

The rest of the arches—the five branchials (Plate 15, fig. 13, and Plate 16, figs. 1, 4, 5)—are very uniform, very solid, grooved on their outer faces for the branchial vessels, and are quite unossified at present, and at present they have only one common basal bar, the basi-branchial (*b.br.*), which does not reach the last arch. They form, below (Plate 16, fig. 4), a very regular series with the lower part of the hyoid arch; they are, in their lower part, larger than the cerato-hyal at first, and then lessen, backwards.

Only the first three have hypo-branchial segments (*h.br.*); these are larger than the hypo-hyal. The upper part of each arch is shorter than the lower; in the first four there is a single small ear-shaped pharyngo-branchial (*p.br.*); thus the last arch has only one piece on each side, and the last but one, three. The segmentation of the hyoid arch is thus seen to be very different from that of a typical branchial, which has no *interbranchial* piece, and the upper part of which is directly superimposed on the lower; moreover, I look upon the hyomandibular and symplectic as a divided *epi-hyal*, with no *pharyngo-hyal*, above.

D. *Transversely-vertical sections of the head of a young Sturgeon* 8 *inches in length.*

These sections show much clearer signs of the composition of the great ethmo-nasal tract than can be got from the outside views of dissected skulls; moreover, they display the curiously cancellated and burrowed condition of the very hard hyaline cartilage. There is generally some fatty tissue imbedding the vessels that run in these burrows and spaces.

Section 1.—The first of these (Plate 15, fig. 14) is through the end of the beak, and the "prenasal rostrum" or intertrabecula (*i.tr.*) is depressed or spindle-shaped in section; rough dermal bones are seen surrounding it.

Section 2.—In the next (Plate 15, fig. 15) the cornua trabeculæ (*c.tr.*) are cut through at their fore end; they are depressed at this part, sub-concave above, rounded below, and attached by a narrow isthmus to the top of the intertrabecula (*i.tr.*) which is sub-triangular here, and dilated and convex below. A rounded chink separates these three pre-cranial elements almost to their roots. The dermal bones form a flat upper, a round lower surface, and a lobate side.

Section 3.—Here (Plate 15, fig. 16) the three elements have become fused together, but the burrowing vessels and the fatty tissue show the original line of union of the parts. The form of the triple rostrum is, above, concave at the middle, and convex at the sides; below, concave laterally, and convex at the middle.

The dermal covering takes the same form, but the sides are notched; here, as in the last, a sub-marginal groove, right and left, appears, below.

Section 4.—Here (Plate 16, fig. 8) the enlarging beak shows a more rounded cartilaginous pith, but is very similar to the last; this is more than half way to the nasal capsule, behind.

Section 5.—In this (Plate 16, fig. 9) the rostrum, close in front of the nasal capsules, has become multiangular; the cornua trabeculæ (*c.tr.*) are again united merely by an isthmus to the intertrabecula mass; they are thick, almost bilobate, and descend obliquely; they are separated by a semicircular notch, laterally, from the middle part (*i.tr.*); below, also, a similar concavity is seen, right and left, but twice as large as those on the sides. Between the latter the intertrabecula is a rounded beam; above, it is concave, and on each side has a lateral angle which bounds the lateral notch. The bones outside carry out this ridged form; below, a thick squarish mass of bone is seen, which is one of the vomerine series (*v.*).

Section 6.—Here (Plate 16, fig. 10) the olfactory sacs (*ol.*) are cut through their middle, and here the angulation of the parts is intensified; the cornua trabeculæ (*c.tr.*) are now large wings, pedate at their free ends, and are separated laterally from the intertrabecular mass (*i.tr.*) by a very large semi-circular notch, in which the nasal capsule lies. Here the intertrabecula forms a roof over the sac, right and left, and it is very hollow above; below it is burrowed by a vomerine scute (*v.*), and is becoming narrower, and more like a distinct beam; here also the outer bones do but fringe and enlarge the section.

Section 7.—In this (Plate 16, fig. 11) section the antorbital wall is exposed, behind the nasal sacs; that wall is composed of the aliethmoids (*al.e.*) and the ethmo-palatines (*e.pa.*). Here the paired cartilages are the trabeculæ (*tr.*) for this section is behind the cornua; but the intertrabecula (*i.tr.*) is very large from top to bottom. The middle region of the intertrabecula is the perpendicular ethmoid (*p.e.*), on each side of which the olfactory nerves (I.) are escaping; its alæ above are becoming the superorbital bands (*s.ob.*), whilst below, its huge beam is burrowed by a vomerine bone (*v.*).

Section 8.—In this section (Plate 16, fig. 12) the eye-ball (*e.*) is just caught, and the cranial cavity laid open at the olfactory foramina, with their nerves (I.); the rest is very similar to the last section; in both, dermal bones defend the sides of the face, and above, the frontal scute is cut through.

Section 9.—This (Plate 17, fig. 1) is through the middle of the orbit, where the superorbital band (*s.ob.*) narrows in; here the supercranial valley is very large and deep, and the section of the skull has the shape of an Ox's face. In this young specimen the chondrocranium is very massive, and the pyriform cavity for the hemispheres (C^{1a}.). is only one-fourth as large as the section itself, which is widest below, and rather pinched in the middle. The skull, below the cranial cavity, is as deep as the cavity, and is burrowed, below, by the splintery fore end of the huge parasphenoid (*pa.s.*); the swelling cartilage on each side belongs to the trabeculæ (*tr.*) the middle part to the intertrabecula (*i.tr.*). The orbital muscles (*or.m.*) are planted in chinks of the basal mass.

This section is in front of the mouth and through the front of the upper lip (*u.l.*); there is, here, a crescentic cavity, with the horns below.

Section 10.—In the next section (Plate 17, fig. 2) the small eye-ball is just missed, but the optic nerves (II.) are seen emerging from the brain (C^{1a}.). Here the cavity is something like an hour-glass, being as wide below as above; the tegmen cranii (*t.cr.*) is twice as thick as in the last, and the basal mass (*tr.*, *i.tr.*) only half as thick, for this is behind the lobes that envelop the parasphenoid (see Plate 16, figs. 1–4).

Here the superorbital bands are wider and more solid, and externally, a new plate of cartilage has come in, on each side from the eaves of the hind skull; this is the sphenotic (*post-frontal*) lamina (*sp.o.*). The solid basal plate is grooved sub-laterally and in the middle, and the parasphenoid (*pa.s.*) fitting to these sinuosities is, in section, like a stretched bow.

Below the base of the skull there is a quantity of very lax tissue, permitting the greatest freedom to the movements of the protrusible oral apparatus, and below this we see the large arched mass of the "adductor mandibulæ" muscle (*ad.m.*) on each side. Below the muscles, the fore part of the pterygo-quadrate cartilages (*pg.q.*) are cut through, and they are flanked by the maxillary and palatine bones (*mx.*, *pa.*) the former outside, and the latter beneath.

The mouth is nearly closed, the upper and lower lips, as in the next section (*u.l.*, *l.l.*), are seen with their rugæ, but without their teeth, and in the lower lip the mandibles (*mk.*) are just brought into view.

The parietal scutes were cut through above, and post- and sub-orbitals were seen, at the sides, and below, but not figured.

Section 11.—In this section (Plate 17, fig. 3) the sphenotic lamina (*sp.o.*) is thicker, but is still separated by a notch from the tegmen cranii (*t.cr.*); that part is thinner in the middle, and deeply concave. Here the cranial cavity has widened, for the section is close in front of the mid-brain. The section, now, is that of an Ox face with drooping horns, and the muzzle is now narrower around and under the infundibulum (*inf.*). On account of the very small size of the eye-ball (Plate 17, fig. 1, *e.*) *four* of these sections are between the eye and the ear; in this the orbital wall (*o.s.*), which is extremely thick, is twice scooped for the orbital muscles and the fatty masses that occupy the orbit—much too large for the eye-ball. Here the intertrabecula has died out, and the trabeculæ (*tr.*) have united directly with each other, and the plate thus formed has lost half the thickness seen in the last section (Plate 17, fig. 2, *tr.*, *i.tr.*).

The primary form of the trabeculæ is still seen at the sides, below, and the whole of the basal plate is crenate, making the parasphenoid (*pa.s.*) assume an undulated form as it follows the risings and fallings of the cartilage. In this section the oral apparatus is cut through the middle, and the opening itself is crescentic below, with a notched upper outline; this is due to the projection right and left of the pterygo-quadrates (*pg.q.*) and their investing bones, the palatines (*pa.*, *pg.*).

Here, the cartilages covered by the large adductor muscles (*ad.m.*) are sigmoid in section, and besides the outer bones, the pterygoids (*pg.*) now show themselves, both as an inner and an outer section, being through their forks. The quadrate region, with its orbitar process (*q.*, *or.p.*) is separate, now, and over it is the maxillary (*mx.*); to the quadrate the mandible (*mk.*) is articulated, it then passes almost directly inwards towards its fellow, which it does not quite meet; the dentary (*d.*) is seen flanking the Meckelian rod.

Section 12.—In this section (Plate 17, fig. 4) the cranial cavity is Y-shaped, and contains the front of the mid-brain above the fore-brain (C^1.), with the infundibulum passing into the pituitary body (*py.*). Here, in front of the post-clinoid wall, the alisphenoidal region (*al.s.*) is extremely thick—twice as thick as the tegmen (*t.cr.*) and the continuous sphenotic wings (*sp.o.*).

In this section the parts round the mouth are like those of the last, but the foremost azygous metapterygoid (*mt.pg'.*) comes into view, and the mandibles (*mk.*) are cut away, distally.

Section 13.—This (Plate 17, fig. 5) is through the middle of the mid-brain (C^2.), and the back of the pituitary body (*py.*). The post-clinoid wall, which is an oblique shelf running forwards and a little upwards (Plate 16, fig. 2, *p.cl.*), is here cut through obliquely so as to appear thicker than it is actually. This section seems to show the pituitary body as lying in a separate and very solid box, for the rest of the skull is quite distinct from it, above. This arises from the fact that the foramina ovalia are here, with the trigeminal nerves (V.); the alisphenoidal wall, the roof, and the thickened sphenotics are seen in the upper part of the section. The parasphenoid (*pa.s.*) curls round the flatter base, the parietal (*pg.* by mistake for *p′.*) and post-orbtial scutes were cut through. Below, the adductor mandibulæ muscles (*ad.m.*), the pterygoid cartilages and bones (*pg.q.*, *pg. q.*, read *pg.*) are severed, and the latter are in one piece, for this is behind their great notch. The smaller azygous metapterygoid (*mt.pg′.*) is seen; the extreme angle of the quadrate (*q.c.*); and the hinge and part of the shaft of the mandible (*mk.*) through the dentary and the pre-opercular bones.

Section 14.—Here (Plate 16, fig. 6) the breach in the lower part of the walls is repaired, for this is behind the foramina ovalia, and through the fore part of the auditory capsules (*au.*); the ampulla and front part of the arch of the anterior canal (*a.s.c.*) are laid open. The roof is thicker and flatter; the sphenotic wings (*sp.o.*) are still large and thick; the cavity of the skull is single, narrower, and is half a long ellipse in shape, the angles of the broad upper end being rounded. Here the notochord is not seen, it has retreated too far backwards, but the parachordal mass (*iv.*) is hugely thick, and is wider; it is close behind the post-clinoid wall. The fore part of the hind-brain (C^3.) is cut across, where it is giving off the facial nerves (VII.), close behind the roots of the trigeminals (fig. 5, V.). The last section was cut through the interspace between the pterygoid band and the orbitar process (fig. 5, *pg.q.*, *q.c.*; and see also Plate 16, figs. 1 and 7); here the back of the suspensorium (*q.c.*) is cut through, at the hind part of the maxillary bone and the orbitar process (*mx.*, *or.p.*). A notch seen in the hind margin of the pterygo-quadrate cartilage (Plate 16, fig. 5, *pg.q.c.*) is cut across here, and the median part of the cartilage crops up, again, right and left; between these the main azygous metapterygoid (*mt.pg′.*) is seen. The angle of the mandible (*mk.*) and the end of the dentary (*d.*) are also cut across.

Section 15.—In this section (Plate 17, fig. 7) the chondrocranium attains its greatest solidity, and here the small cranial notochord (*nc.*) has its point cut through. The sphenotic has given place to the "pterotic" region, the arch of the anterior and the ampulla of the horizontal canals (*a.s.c.*, *h.s.c.*) are exposed, and so also is the auditory nerve (VIII.), as it arises in the hind-brain (C^3.) and runs through the meatus internus into the vestibule (*vb.*). Here the tegmen cranii is thick and concave, and the pterotic expansions (*pt.o.*) are very rough and lobulate. Under the bulging sinuous parachordal mass the parasphenoid (*pa.s.*) has become very wide, and now sends down its free edges. Under the capsules the "protractor hyomandibularis" muscles (*pt.hm.*) are seen as huge triangular masses, and the "adductor mandibulæ" muscles (*ad.m.*) are cut through in their hinder part. A little of the pterygoid region (*pg.q.*) and the hind angle of the quadrate (*q.c.*) come into view, and between the former the median and main lateral metapterygoids (*mt.pg′.*, *mt.pg″.*). The deepest part of the pterygoid bone (*pg.*) is seen flanking the inner face of the suspensorium. Under the quadrate a new and thick cartilage appears; this is the symplectic (*sy.*), it is crescentic in this, its front edge (lower end), and between it and the quadrate there is a small nucleus of cartilage which will be described in the next stage—it is a "suspensorial ray" (*sp.r.*). The chief scutes seen are the parietals; the squamosal, the opercular, and the sub-opercular, also, may be traced at this point, but are not figured.

Section 16.—In this slice (Plate 17, fig. 8) we see several new parts come into view below; above, the skull is altogether flatter, and the supercranial valley narrower. The auditory nerve (VIII.) is still shown, also the arch of the anterior and horizontal canals (see also fig. 7, *a.s.c.*, *h.s.c*). The basal plate (*iv.*, *nc.*) is very irregular, as it passes into the auditory capsules, right and left; the parasphenoid (*pa.s.*) partakes of this irregularity, and here its decurved wings are larger. The compound meta-pterygoid plate is composed at this hind part of a large middle, two large lateral, and two lesser intervening plates (*mt.pg'.*, *mt.pg''.*). The top of the hyomandibular is just seen and most of its great protractor muscle (*pr.hm.*); below, the symplectic is cut through from top to bottom. Below the wide oral cavity the cerato-hyal (*c.hy.*) with a thin ectosteal coating, the hypo-hyals (*h.hy.*), the fore part of the basi-branchial (*b.br.*), and the fore end of the first hypo-branchials ($h.br^1.$) are also seen. The scutes are very similar to those of the last.

Section 17.—In this (Plate 17, fig. 9), the cranial valley is seen to be wider again; the auditory sacs are cut through where the posterior canal (*p.s.c.*) is descending, and the whole structure is lessened laterally; a thin pterotic eave (*sp.o.*, read *pt.o.*) grows down on each side. Here the hind-brain ($C^3.$) is less, and it is giving off the roots of the large vagus nerve (X.) whose ganglion root and stem are laid bare. Here we still see that the auditory sacs are open to the skull (or only closed by membrane), and here the sacculus (*s.*) is seen hanging from the rest of the vestibule (*vb.*). The notochord (*nc.*) is becoming larger, and the thick curved basal plate (*iv.*) is here separate from the auditory capsules; below it the parasphenoid (*pa.s.*) is thick, and at its notched sides we see a pharyngo-branchial (*p.br.*) and outside this part of the gills (*g.*). The lower part of the hyomandibular, part of the symplectic, inter-hyal, and cerato-hyal (*hm.*, *sy.*, *i.hy.*, *c.hy.*) are cut across; and near the mid line the basi-branchial, the first cerato- and hypo-branchials, and the second hypo-branchials (*b.br.*, $c.br^1.$, $h.br^1.$, $h.br^2.$).

Section 18.—In this section (Plate 17, fig. 10) the post-auditory fontanelle is just missed, and we see that it must answer to the interspace between the supra-occipital and first vertebral arch; for the chondrocranium (see Plate 15, fig. 13) sends its copious growths over the proper spinal region. The cavity here is very large, this being, indeed, the "foramen magnum," and the ex-occipital walls are very thick, and almost vertical. The hinder part of the vagus nerve (X.) is still seen in the chink which separates—up to this point the sides form the base, which is increasingly massive, but has not a very large notochordal section (*nc.*) in it. The parasphenoid (*pa.s.*) is beginning to break up at the middle; outside it a pharnygo-branchial is seen (*p.br.*), and outside that the most backwardly projecting part of the massive hyomandibular (*hm.*).

Section 19.—The last of this series (Plate 17, fig. 11) is post-cranial; the ganglion of a spinal nerve (*sp.n.*) is cut across; the medulla spinalis (*my.*) is seen to be very small in proportion to the spinal cavity, and the notochord (*nc.*) is twice as large as in the skull, and reaches the theca vertebralis. The parasphenoid (*pa.s.*) is in two parts (see Plate 16, fig. 3); the hind and transverse processes of the undivided vertebral tract are very large, and are covered with large scutes. This section is behind the operculum (Plate 15, fig. 10, *op.*), and through one of the hinder branchial arches; in the general branchial recess, the pharyngo-, epi-, and part of the cerato-branchials (*p.br.*, *c.br.*, *e.br.*) of a middle gill arch, and some of the gills, are shown.

Behind this part the notochord becomes thicker, and its chondrified sheath more distinct; the forked parasphenoid dies out, and the *neuro-central* cartilages appear (Plate 16, fig. 3).

Fifth Stage.—Adult Sturgeon (Acipenser sturio).*

The Sturgeons dissected for this stage were about 4 or 5 feet long, such as are most commonly seen in the markets; my last stage is from a much larger specimen, but not from the largest. Dr. GÜNTHER ('The Study of Fishes,' p. 362) says that this species attains a length of 18 feet.

The swelling of the fore part of the trabeculæ and intertrabecula in the orbital region becomes very remarkable, and the cartilage becomes subdivided into an upper and a lower stratum; between these the parasphenoid (Plate 18, fig. 3, *tr.*, *i.tr.*, *pa.s.*) grows as a thin dentate lamina, with one very long median spike. The clavate swellings of the trabeculæ bind on the sides of the exposed part of the parasphenoid, and the *second floor* of cartilage in the middle (*i.tr.*) runs up to the base of the huge rostrum, appearing below as a rounded elevation, separated from the lateral lobes by a deep sulcus, which is filled with fatty tissue. In the fatty tissue outside the paired lobes of cartilage there is a series of four thin cartilaginous plates (*e.pa'.*); they are outside the trabeculæ, and behind the ethmo-palatine boundary of the nasal sac, and probably represent, partly, as Mr. HOWES suggests, the free ethmo-palatines of other kinds; in Siredon I have shown that there are two of those cartilages on each side (Phil. Trans., 1877, Plate 24, figs. 1–3, *e.pa.*, *pt.pa.*)—an "ethmo-palatine" and a "post-palatine."†

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* The dissections, figures, and notes for this stage were made for me by my friend Mr. GEORGE HOWES, Professor HUXLEY'S talented Demonstrator.

† I wish to put all these *pre-oral* growths together for comparison, in hope of obtaining a clue to their real nature. Mr. BALFOUR'S writings have made me very doubtful of the view that, at one time, seemed to me to be almost demonstrable—viz., that these antorbital cartilages were in reality the rudimentary upper part of a *pre-mandibular visceral arch*, but their position, in front of the hypoblast, makes their determination as difficult as that of the *pro-chordal* part of the basis cranii.

The most important further modifications to be seen in these large individuals, are in the visceral arches.

There are seven arches, and the whole series is displayed in a side view (Plate 18, fig. 5) so as to show their relative size, and their varied modification; in that figure the inferior part of the hyoid arch is shown as drawn backwards towards the first basi-branchial (*c.hy.*, *b.hy.*, *b.br*^{1}.); in reality it runs across to meet its fellow in the floor of the mouth (fig. 10, *c.hy.*, *h.hy.*).

The two first arches, the mandibular and hyoid, are intensely specialised; the other five are very similar to those of the Elasmobranchs, save that they have some of the larger bars partly ossified; and are still more like the branchial arches of Holostean Ganoids and Teleosteans. The upper elements of the mandibular arch, the right and left "suspensoria," are transformed into the oblique convex roof of the very mobile protrusible mouth, and the free mandibles are made to be antagonistic to the antero-inferior part of this complex structure. Each cartilaginous suspensorium is a broadly falcate plate, whose arched upper border comes in contact with that of its fellow in front and above; below and behind, it suddenly bends forwards and becomes very solid, to form the quadrate condyle (figs. 4, 5, *q.c.*). The whole plate is gently and sinuously convex above and concave below; behind, over the hinge, it thickens into a rib-like enlargement, and grows outwards and forwards as a superficial "orbitar process" (*or.p.*). The upper margin is arched regularly until near the hinge, and then is hollowed a little; the lower margin is twice-notched and concave. The bones applied to these pterygo-quadrate plates have not caused the absorption of the cartilage in this stage, which is *relatively* more extensive than in the last stage, in which the hind margin was notched. The outermost bones are the largest; they are the maxillaries (*mx.*), they are strongly bowed, dilated most where they meet in front of the cartilaginous plates, and to a lesser degree where they bind upon the orbitar processes behind. A large space for each "adductor mandibulæ" muscle exists between the maxillaries and the pterygo-quadrate cartilages. Another superficial bone stands upright on the hind end of the maxillary; it is a high triangle, and its base is below. It binds on the thick, ribbed, outer edge of the suspensorium in its quadrate region; this is the small "pre-opercular" (*p.op.*), whose almost equally small counterpart exists in *Lepidosteus*. Along the lower edge of the cartilage, in front, a very jagged little bony scale is seen, overlapped by the fore end of the maxillary. This is the mesopterygoid (*ms.pg.*); it was not a separate bone in the last stage. Behind it there is a small style, with its sharp end behind, reaching to the end of the foremost and larger notch; this is the palatine (*pa.*). Inside the plate (*pg.q.*) there is a large parostosis, the pterygoid; it lines all the cartilage except a falcate tract in front and above; this causes it to be sharply notched in front; this form is well shown by the shading of the cartilage (Plate 18, fig. 4).

The mandible or Meckelian rod (Plate 18, figs. 5 and 8, *mk.*) is a very short stout cartilage like that of a Tadpole; the condyle is convexo-concave, and the angular process is large and projecting. That is the only part not covered, on the outside, by the flattish but thick dentary bone (*d.*). On the inside (Plate 18, fig. 8, *d.*) it shows a large crescent of bone, where the rod becomes slenderer in the middle. Behind that part, on an apophysis of the cartilage, a small oblong second bone appears. This is the coronoid (*cr.*); its direction is backwards and a little downwards. The distal end of the rod is thick and massive, and is united to its fellow by ligamentous fibres.

A small squarish suspensorial ray is seen behind the angle of the mandible (Plate 18, fig. 5, *sp.r.*); it is far below the usual place for the spiracular cartilage of a Shark. The other half of the general roof of the mouth is fan-shaped (Plate 18, figs. 4 and 5, *mt.pg′.*, *mt.pg″.*), the narrow handle running in between the two pterygo-quadrates, and the outspread part growing round their convex hind margin. Thus this complex plate has two convex edges behind and two concave edges in front; then it runs to a sharp point between the symmetrical plates. The earliest pieces of this patchwork are still the largest; they are the main azygous plate and the main symmetrical plates (*mt.pg′.*, *mt.pg″.*); the single piece is like the bowl of a spoon, but longer; the other two are roughly three-cornered. Outside, between the latter and the quadrate region of the paired plate (*q.c.*) there is a much smaller but tolerably constant piece on each side. Behind, between the three main patches, there are three or four on each side, inconstant in number and form; and in front, in the interspace between the three main patches and the pterygo-quadrates, there is a patch or two on each side, and then a single row of four or five, lessening forwards. These patches are all set in one common web of fibrous tissue, so as to look, in a rough dissection, like one unpaired hard-palate plate.

Half this complex palate of pterygo-quadrates and metapterygoids is nearly equal in length and width, but far inferior in thickness, to a single hyomandibular (Plate 18, fig. 5, *hm.*). That segment has no "serial homologue" either before or behind it, for the subdivision of the upper part of the visceral arches is different in the mandibular, hyoid, and branchial arches. The hyomandibular is not the *uppermost* segment of a normal visceral (*branchial*) arch, for there is no pharyngo-hyal in this type. Nor does it correspond to more than the upper two-thirds of an epi-branchial; the lower third is the separate symplectic (*sy.*). Neither does it harmonise with the pier of the arch in front, which is segmented quite after another fashion; there, the part answering to the symplectic is the pterygoid foregrowth of the suspensorium, whilst the part which should correspond to the head of the hyomandibular is partly continuous with the head of the pier of the opposite side, and largely broken up into a tesselated pavement of irregular segments. The *upper* "epi-hyal" (*hm.*) is, above, a normal phalangiform segment, but below and behind it expands into a huge pedate plate; the "toe" below, is tied to the next segment, the "heel" behind, carrying the "opercular," and answering to the opercular process in Teleostei; this convex pedate slab is almost entirely unossified.

The neck and shoulders of the hyomandibular are ossified, so as to leave the round convex head and expanded lower part unchanged. The osseous shaft is hour-glass-shaped, very thick, and leaving only a small core of cartilage unossified (Plate 18, figs. 5–7, *hm.*); this bony tract is very angular, in section, and somewhat four-cornered. On the outside of the bony sheath, embedded in the periosteum, there are simple and forked cartilages (Plate 18, figs. 5, 6, 7, *hm.r.*), evidently rudimentary "branchial-rays;" as seen in the sections (figs. 6, 7) they are very solid, "and usually exist upon (*a*) the anterior, and (*b*) the outer face only; in one specimen, however, (fig. 7) they also occurred upon the anterior and inner" (HOWES); they are very variable as these sections show.

The "toe" of the hyomandibular (*hm.*) rests upon the upper face of the upper end of the next segment, the symplectic or lower epi-hyal (Plate 18, fig. 5, *sy.*); the two are attached together by a strong fibrous joint. This distal segment of the hyoid "pier" is half the length and half the medium width of the upper piece; it is phalangiform, straight, and thick. Distally it is attached by a strong fibrous joint to the quadrate region (*q.c.*), which it carries; proximally, at its proper upper end it is articulated to the inter-hyal (*i.hy.*); it is wholly unossified.

The intercalary segment, or "inter-hyal" (*i.hy.*) is a sub-cubical, lobulated mass of cartilage, interposed between the symplectic and the lower element of the hyoid arch, the cerato-hyal (*c.hy.*). This latter segment in the undisturbed condition of the parts runs across and a little forwards under the throat; it is about the size of the symplectic, but more slender and has a narrow waisted ectosteal tract a little below its middle. This bone is nearly half the height of the bar, is pinched and angular, has an unossified pith and extra-hyal cartilages (Plate 18, fig. 9, *c.hy.*, *c.hy.r.*), like the hyomandibular.

There is a thick wedge of cartilage segmented off from the cerato-hyal, the hypo-hyal (*h.hy.*); it is attached by fibrous tissue to its fellow, the related ends being narrow, and also to the fore end of the first basi-branchial (Plate 18, figs. 5 and 10, *h.hy.*, $b.br^1$.).

The thick, solid, strongly bent branchial arches (Plate 18, figs. 5 and 10) lessen and become simpler from before backwards. The joint between the main upper and lower segments (*e.br.*, *c.br.*) is a well formed "elbow," the upper part is a little less than the lower. Only the first and second branchial arches are perfect, and they have acquired an additional pharyngo-branchial (fig. 5), and these small segments rest on short forks of the epi-branchial (*e.br.*). There is only one pharygno-branchial on the third arch ($p.br^3$.), and that on the fourth has (evidently) become fused with the epi-branchial ($e.br^4$.). The fifth arch is in one piece, it is a cerato-branchial (fig. 10, $c.br^5$.), and like the four cerato-branchials in front of it, has a bony tract in its middle part. Only the first and second epi-branchials ($e.br^1$., $e.br^2$.) have ectosteal tracts. On the first of these there are at least two imperfect "rays" (*br.r.*), as on the arches in front. The lower part of each arch is bowed outwards, the upper is sigmoid, and they are very angular, especially where the ectosteal tract exists. Only the first three have hypobranchials (Plate 18, figs. 5 and 10, *h.br*$^{1-3}$.); these begin much larger than the hypohyal and then lessen backwards; they are flat, finger-shaped, and not ossified.

Another addition of cartilaginous segments has taken place below; in the young Sturgeons (Plate 14, fig. 5, and Plate 16, fig. 5) there was only one basi-branchial carrying the three first arches. Now, *three* new smaller segments have appeared (*b.br*$^{4-6}$.), the last of these is wedge-shaped, and belongs to a suppressed arch, for it passes behind the pedate end of the arrested fifth arch; the foremost piece belongs to three arches.

These piers are compressed, and the first, which belongs to *three* arches (*b.br*$^{1-3}$.), is nodose.

Sixth Stage.—Skull of an unusually large Sturgeon* (Acipenser sturio) (*in the Hunterian Museum*).

This very valuable specimen of a much older individual, shows some very important modifications, which have, as it were, *an upward look* towards the Holostean Ganoids and the Teleostei.

In the last stage the pterygo-quadrate cartilages were even more perfect than in the young of 7 or 8 inches in length. In this specimen, however, the "parostoses" have caused a considerable amount of absorption of the hyaline cartilage, just as the dentary, normally, causes MECKEL'S cartilage to shrink and even become absorbed; but I cannot find any evidence of a direct *ectosteal* relation of the bone to the cartilage. The pterygoid bone (Plate 17, figs. 12, 13, *pg.*) gets over to the *outer* side over the convex inner margin, and causes the absorption of the cartilage in that region. Moreover, the jagged bony tract which represents the mesopterygoid, and which in the last was at the antero-inferior edge of the cartilage, is now (Plate 17, figs. 12 and 13, *ms.pg.*) a sharp wedge of bone filling up the space between the forks of the pterygoid on the inner side (fig. 13), and is seen as an oval plate of bone nearer the top than the bottom, on the outer side (fig. 12, *ms.pg.*). These differences are not all due to *age;* there is a considerable amount of variation in individuals of the same age in these types, where the sub-cutaneous and sub-mucous bony tracts are but *deeper scutes*, and where the chondro-skeletal regions are so generalised and, as it were, *hypertrophied.*

With regard to the *buried scutes*, we see a right and left variation in this same specimen (Plate 18, figs. 1 and 2, the *right* and *left* orbital region in this large skull).

Here there are no proper *ectosteal* prefrontals (ecto-ethmoids), orbito-sphenoids, alisphenoids, or prootics; but, since the last stage, parosteal tracts have appeared in all those places; they are *deep* "scutes," not *shallow* endo-skeletal elements, and are the prophecy, so to speak, of the special "ectostoses" seen in the Holostean Ganoids and the Teleostei, and in all the types above them. Round the thick antorbital

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* Even this specimen was not half the size this species sometimes attains to.

(ecto-ethmoidal) mass of cartilage I find a thin but rough irregular layer of bone, just where the Teleostei have their proper ecto-ethmoid (Plate 18, figs. 1 and 2, *e.eth'.*). Also over the optic foramen (II.) there is a very large splint (*os'.*) applied to the orbital cartilage; it runs upwards and forwards. On the *right side only* (fig. 1, *al.s'.*) there is a much smaller plate in the alisphenoidal region, and *behind* and partly *round* the foramen ovale (V.) a thin semi-annular plate (*pro'.*) representing the prootic of the higher types. But no direct grafting of bone on the cartilage can be seen, and the *affinity* of the bone for the cartilage or, *vice versâ*, of the cartilage for the bone, is here extremely feeble; there is no material interaction; the co-ordinating force, however, has produced a plate of the proper form, and put it ready for use in the proper place.

Summary and comparison with other types.

It is evident that we have in the Sturgeon a form which is practically intermediate between the Selachians and the Osseous Ganoids (Holostei); the form of the larvæ (Plate 12) suggests this view at once. I must again refer the reader to the researches of SALENSKY and BALFOUR on the embryology of this type; my own recapitulation and comparison must be confined to the cephalic skeleton.

A. *The primordial skull.*

We saw that in the larvæ of *Acipenser ruthenus* only about a third of an inch in length ($9\frac{1}{2}$ millims.) the "embryonic cartilage" had largely become "hyaline;" that the foundations of the cranium were laid, and the visceral arches were differentiated and becoming quite solid. Here, it would seem, that in so small a larva of so large a Fish—and that Fish lying at the base of the great *archaic* group of the Ganoids—we have a good chance of seeing the primordial vertebrate skull in its utmost simplicity. The fact is, that we have *a confusingly simple* state of things.

In the fore part of the spine, as well as in the whole basi-cranial region, the paired skeletal tracts that lie right and left of the *mesoblastic sheath* of the notochord, the hardening cartilage shows no signs of segmentation or *intercalary vertebral subdivision;* this is just like what occurs in the Selachians (Trans. Zool. Soc., vol. x., plate 35). Moreover, this Chondrosteous Ganoid remains in this condition, as far as I can find, throughout life, and does not acquire the occipito-cervical articulation, so well known in the Selachians, but which in them is a secondary modification of the parts. By careful comparison of all the facts I have been able to gather by observation of many types and at many stages, I cannot help coming to the conclusion that the *normal intercalary vertebral segmentation* with which we are so familiar in the post-cephalic region of the Vertebrata, generally, is *a comparatively late and secondary specialization* in the evolution of this, the highest, group of animals.

If Mr. BALFOUR'S suggestion ('Comp. Embryol.,' vol. ii., p. 366) be true, viz.: that the fore-brain with its special optic and olfactory outgrowths is a sort of outgrowth or addition, itself, to the neural axis, then we have to reconsider the meaning of any and every part of the cranial skeleton which may be formed in front of the notochord and of the overlying and overfolded mid-brain.

Thus all the *prochordal* cranial, and all the *pre-mandibular* facial growths of the skeletal cartilage have to be subjected, again, to the severest morphological inquisition under that light—the light of Embryology—which alone can make manifest their true meaning.

If the notochord be the true organic axis of the skeleton of the animal, and if the visceral (or branchial) arches were only developed in relation to the hypoblastic branchial outgrowths of the dilated respiratory pharynx, then it seems to be necessary that we should consider all the skeletal parts in front of those structures as superadded specializations of, or "outgrowths" from, the front end of the proper Vertebrate form.*

In certain types, namely, the Lamprey, the Selachians, the Urodela, and the Anura, the trabeculæ are chondrified before the hinder or parachordal tracts. Nevertheless, the posterior part of the trabeculæ in them is parachordal; the rest, up to the "atlantal" region, may afterwards chondrify separately, as in the Urodela, or continuously, as in the others.

In *Acipenser*, *Lepidosteus*, and *Salmo* I find no difference of time in this matter, and should therefore be inclined to look upon the earlier growth of the trabeculæ as due to the special weight and pressure of the fore-brain in those cases, and as a non-essential modification, just as I consider the later segmentation of the trabeculæ from the investing mass in *Salmo* and *Chelone*, and to a less extent in Crocodiles and Birds, as a non-essential specialization.

Yet the trabeculæ are as truly part of the proper mesoblastic axial skeleton as the fore-brain is part of the proper epiblastic neural axis.

The whole of the *pre-cerebral* tracts of the skull, namely, the cornua trabeculæ and the greater part of the intertrabecula—all of it that lies in front of the exit of the olfactory nerves—I should consider to be mere *outgrowths* or "apophyses" of the cranial skeleton.

The axial skeleton is more aborted in front than the axial nervous system, even if the fore-brain is reckoned as an additional part, for the mid-brain bends completely over upon itself, and the notochord only partially. Yet, as I have shown in *Chelone viridis*, its cartilaginous mesoblastic sheath is continued downwards as solid cartilage below and in front of the end of the notochord. I am inclined to think that the intertrabecula is a breaking out again of that mesoblastic tract (a sort of cranial "spadix") but with its continuity, for a short space, interrupted.

The ventral part of the primordial skull presents as many difficulties as the dorsal; the post-oral visceral (or branchial) arches, which may be both *superficial and deep*, have to be compared with such facial growths as may exist in front of the mouth, and these, if possible, have their relation to the post-cephalic arches determined.

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* For my own part, I am quite content that this should be so, albeit my own descriptive language will have to undergo a considerable amount of evolutional modification, and many things that seemed to me, once, to be clear and certain, made dark and uncertain.

Long ago, to those who knew nothing of Embryology, all this seemed to be easy enough; now, with all the new, increased light from that source, the problem has become extremely difficult, and is certainly not solved, as yet.

I do not see that the superficial cartilages that surround the mouth have any right to be compared, serially, to the arches of the pharynx or of the chest; they appear to me to be the most *archaic* structures in the skeleton—"barbels," "labial cartilages," "nasal valves"—all these appear to me to be lineally descended from the inner supporting tracts of tissue of oral palpi, such as are met with in so many of the aquatic *Invertebrata*. The investing bones of the face may be grafted on such cartilages, but the two things are quite different in their nature. The condition of the visceral arches in this type, both in the larva and in the adult, has led me to reconsider the whole question of the nature of these arches.

There are several things to be considered at the outset, before a comparison is made of the skeleton of the throat, the branchial skeleton, and the skeleton of the chest. The post-auditory part of the cranium has manifestly undergone *secular* shortening, so that the pharyngeal or ventral region belonging to it often extends under the *twice-segmented* spine, whose fore part is, so to speak, intercalary or superadded, and does not correspond with the arches beneath it, which often extend backwards for some distance.

At one time this appeared to me to be an explanation of the fact that the inner (or proper) branchial arches of Fishes are developed beneath the fore part of the spine and the hind part of the basis cranii, whilst the mandibular and hyoid arches, the first and second of the branchial category, often fix themselves to the basal plates of the cranium.

I now strongly suspect this view of the matter to have been a mistaken one; and that it is the *abnormal size* and *special modification* of the mandibular and hyoid arches that make it necessary for those arches to seize hold of swinging points above their normal dorsal region or apex.

I think that the figures I have given of the visceral arches in the larval Sterlet (Plate 13, figs. 6 and 11) will make my meaning plain. Here only one arch is attached to the edge of so much of the parachordal plate as may creep under the auditory capsule; all the other arches are fairly under the head and forepart of the neck.

This normally inferior position of the visceral arches is best seen however in the early embryo of the Skate (see in "Pristiurus," Trans. Zool. Soc., vol. x., plate 35, fig. 4), where the metapterygoid, hyomandibular, and all the pharyngo-branchials, are shown *in situ*, after the whole cranium had been removed from above.

The sub-division of the pleuro-peritoneal cavity by the hypoblastic branchial pouches in the early embryo, the rapid closure of those cavities (the "head-cavities"), and the relation of the skeletal bars of the pharynx and mouth, whether deep or superficial—all these things have to be carefully reflected upon before any classification of these ventral arches can be made.

It seems to me that a cartilaginous arch developed *inside* a recently-closed "head-cavity" must be a part of the "splanchno-pleure," and that a cartilage developed outside such a recently-closed cavity must belong to the "somato-pleure." *

The *ribs* belong to the outer lamina of the body-wall of the embryo, so also, it appears to me, do the "extra-branchials" of the Lamprey, the Tadpole, and the Shark.

If all this be true, the normal "intra-branchial" arches have no counterparts whatever in the skeleton of the trunk; they culminate in the class of Fishes, and are imperfectly developed, not only in the Abranchiata, but also in the degenerate Marsipobranchii, and the metabolic Amphibia Anura.

The extra-branchials of some of these latter types, and of the Sharks, have some right and title to be classified as a sort of cephalic ribs, but perhaps that claim had better not be pressed for the present; it is safer for the Morphologist to keep certain things in solution, when any doubt remains, than to crystallize them into what may turn out to be obstructive error.

In the metamorphosis of the larval Sturgeon the additional parts are easily explained, and are, for the most part, due to mere increase of certain tracts of tissue, and super-additions of secondary nuclei of cartilage, and of various centres of ossification.

Moreover, the fact that the dermal scutes are largely dominated by the cartilaginous endoskeletal structures of the cranium and pharynx, however interesting from one point of view, is not of any great fundamental importance.

With regard to the great "shoots" of cartilage that grow out, forwards, from the primary basis-cranii, these are parts that undergo a most extraordinary amount of modification in various types; they are specialised superadditions to the essential skull, of great importance in *Taxonomy*, but of little account in that which is fundamental in *Morphology*.

Comparison with Polyodon.

The skull of the other principal existing Chondrosteous Ganoid type comes singularly near to that of the Acipenserine skull, and in some things is curiously unlike it. Near as *Polyodon* approaches in the structure of its skull to *Acipenser*, it differs in having no complex metapterygoid plate, in the feebler ossification of its visceral arches, and in the presence of three pairs of "endosteal" centres not to be found in the Sturgeon. The anterior palatine ectostosis placed just where both the mesopterygoid and palatine plates meet in the Sturgeon, is probably the true homologue of the palatine bones of the Holostean Ganoids; the presence of a small prootic and opisthotic brings us nearer in this case to those higher types. The much larger and more perfect "orbitar process" shows in Polyodon a nearer relationship to the larval Frog; the absence of a metapterygoid plate brings it nearer to the Sharks, and the three pairs of *ectosteal* centres to the Holostean Ganoids. (See BRIDGE, Phil. Trans., 1878, Plates 55–57.)

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* The branchial artery lies *inside* the head cavity, and afterwards, when the branchial arch is developed, runs up a groove on the *outside* of it (see BALFOUR'S 'Elasmobranchs,' plate 14, fig. 13, *a*, p. 208; and 'Comp. Embryol.,' vol. ii., p. 472, fig. 328).

Comparsion with the Selachians.

The development of the basis-cranii and cranial walls in *Acipenser* is very similar to what is seen in the Selachians; and the after modifications are essentially alike, except that in the Sturgeon the cartilage is very massive, and the occipito-cervical articulation is not formed. The separation of a large symplectic, and a lesser interhyal segment, the complex metapterygoid plate, and the partial ossification of the visceral arches, are all modifications that separate this from the Selachian types.

Comparison with the Holostean Ganoids.

Most of the peculiarities which distinguish the skull in that group from that of a Sturgeon are so many steps in the direction of the Teleostean skull. To say nothing of the lighter build of the chondrocranium, the numerous ectosteal patches in the skull wall, the intenser ossification of the visceral arches, and the absence of a distinct symplectic cartilage (it may exist as a separate bony centre, as in the Teleosteans), all these things show how far the Holostei have become specialized beyond the chondrosteous *Acipenser*. An important modification occurs early in *Lepidosteus*, namely, that the palato-quadrate is continuous, at first, with the trabecula; the intertrabecula also appears earlier. I shall compare the skull of the Ganoids, generally, with that of the Teleostei, in my next paper, which will treat of the skull of *Lepidosteus*.

Explanation of the Plates.

Plate.	Fig.	Stage.		Number of times magnified.
12	1	1	Larva (recently hatched) of *Acipenser ruthenus*, $5\frac{1}{2}$ millims. long	17
,,	2	1	Larva of same, $6\frac{1}{2}$ millims. long	15
,,	3	1	Under view of head of same specimen	15
,,	4	2	Head of larva of same, $8\frac{1}{2}$ millims. long.; side view	15
,,	5	2	The same head; lower view	15
,,	6	2	The same head; upper view	15
,,	7	2	A larger larva of same, $9\frac{1}{2}$ millims. long	10
,,	8	2	Head of same larva; upper view	10
,,	9	3	A still larger larva of same, $13\frac{1}{2}$ millims. long . .	15
,,	10	2	First of a series of transversely vertical sections of a larva of *Acipenser ruthenus*, $9\frac{1}{2}$ millims. long .	40
,,	11	2	Second section of same	40
,,	12	2	Third section of same	40
13	1	2	Fourth section of same	40
,,	2	2	Fifth section of same	40
,,	3	2	Sixth section of same	40
,,	4	2	Seventh section of same	40
,,	5	2	Eighth section of same	40
,,	6	2	Side view of dissected head of another larva, $9\frac{1}{2}$ millims. long	$26\frac{2}{3}$
,,	7	2	Under view of head of larva of *Acipenser ruthenus*, $9\frac{1}{2}$ millims. long	14
,,	8	3	Head of a larger larva of same, $13\frac{1}{2}$ millims. long; under view	14
,,	9	3	The same object; upper view	14
,,	10	3	Horizontal section of head of a larva of the same, $13\frac{1}{2}$ millims. long	40
,,	11	3	Larva of *Acipenser ruthenus*, $13\frac{1}{2}$ millims. long; side view of dissected head	20
,,	12	3	Visceral arches of the same head; upper view . .	20

Plate.	Fig.	Stage.		Number of times magnified.
14	1	2	Larva of *Acipenser ruthenus*, 9½ millims. long; vertical section of head	26
,,	2	3	Second horizontal section of larva of same, 13½ millims. long (see Plate 13, fig. 10).	40
,,	3	3	Third section of same head	40
,,	4	3	Same head; basis cranii, upper view	20
,,	5	3	Same dissection as last; lower view	20
,,	6	3	Maxillary and dentary of same dissection; side view	53
,,	7	3	First of a series of sections of the largest larva of *Acipenser ruthenus*, 14½ millims. long	30
,,	8	3	Second section of same	30
,,	9	3	Third section of same	30
,,	10	3	Fourth section of same	30
,,	11	3	Fifth section of same	30
,,	12	3	Sixth section of same	30
,,	13	3	Seventh section of same	30
,,	14	3	Eighth section of same	30
15	1	3	(Same sections as on last Plate, continued.) Ninth section of same	30
,,	2	3	Tenth section of same	30
,,	3	3	Eleventh section of same	30
,,	4	3	Twelfth section of same	30
,,	5	3	Thirteenth section of same	30
,,	6	3	Fourteenth section of same	30
,,	7	3	Fifteenth section of same	30
,,	8	3	Sixteenth section of same	30
,,	9	3	Seventeenth section of same	30
,,	10	4	Head of a young specimen of *Acipenser sturio*, 8 inches long; side view	1⅓
,,	11	4	The same; upper view.	1⅓
,,	12	4	The same; lower view.	1⅓

Plate.	Fig.	Stage.		Number of times magnified.
15	13	4	Dissection of head of same; upper view	$2\frac{2}{3}$
„	14	4	First of a series of sections of same	8
„	15	4	Second section of same	8
„	16	4	Third section of same	8
16	1	4	Dissection of head of a younger specimen of same, $7\frac{1}{4}$ inches long; side view	$2\frac{2}{3}$
„	2	4	A vertical section of the head of another specimen, 8 inches long	$2\frac{2}{3}$
„	3	4	Dissected skull of young *Acipenser sturio,* 8 inches long; lower view.	$2\frac{2}{3}$
„	4	4	The same; and same view, with visceral arches attached	$2\frac{2}{3}$
„	5	4	Visceral arches of same; upper view	$2\frac{2}{3}$
„	6	4	Fore part of same object; lower view	$2\frac{2}{3}$
„	7	4	Part of same as last; inner view	8
„	8	4	Fourth section of same (see Plate 15, figs. 14–16) .	4
„	9	4	Fifth section of same	4
„	10	4	Sixth section of same	4
„	11	4	Seventh section of same	4
„	12	4	Eighth section of same	4
17	1	4	(Continuation of sections of head of *Acipenser sturio,* 8 inches long.) Ninth section of same .	4
„	2	4	Tenth section of same	4
„	3	4	Eleventh section of same	4
„	4	4	Twelfth section of same	4
„	5	4	Thirteenth section of same	4
„	6	4	Fourteenth section of same	4
„	7	4	Fifteenth section of same.	4
„	8	4	Sixteenth section of same	4
„	9	4	Seventeenth section of same.	4
„	10	4	Eighteenth section of same	4
„	11	4	Nineteenth section of same	4

Plate.	Fig.	Stage.		Number of times magnified.
17	12	6	Pterygo-quadrate plate of old individual of *Acipenser sturio;* outer view . . .	$\frac{1}{3}$ natural size.
,,	13	6	The same; inner view	$\frac{1}{3}$ natural size.
18	1	6	Right orbital region of old specimen . .	$\frac{1}{2}$ natural size.
,,	2	6	Left orbital region of old specimen . .	$\frac{1}{2}$ natural size.
,,	3	5	Basis cranii, orbital region, in adult *Acipenser sturio*	Slightly enlarged.
,,	4	5	Palato-quadrate apparatus of same, from above	Slightly enlarged.
,,	5	5	Visceral arches of same, side view . . .	$\frac{3}{4}$ natural size.
,,	6	5	The same; section of hyomandibular . .	Slightly enlarged.
,,	7	5	The same; section in another specimen .	Slightly enlarged.
,,	8	5	Mandible of same; inner view	Slightly enlarged.
,,	9	5	Section of cerato-hyal of same	Slightly enlarged.
,,	10	5	Visceral arches of same, from below . .	Slightly enlarged.

List of Abbreviations.

The Roman figures indicate nerves or their foramina.

ad.m.	Adductor mandibulæ.
al.s.	Alisphenoidal region.
al.s′.	Alisphenoidal bony plate.
an.	Anus.
aq.v.	Aqueduct of vestibule.
a.s.c.	Anterior semicircular canal.
au.	Auditory capsule.
bb.	Barbel.
b.br.	Basi-branchial.
C^1.	Fore-brain.
C^2.	Mid-brain.
C^3.	Hind-brain.
d.	Dentary.
e.	Eye-ball.
e.br.	Epi-branchial.
e.eth′.	Ethmoidal bony plate.
e.o.	Ex-occipital region.
e.pa.	Ethmo-palatine.
e.pa′.	Extra ethmo-palatine.
f′.	Frontal bony scute.
fo.	Fontanelle.
g.	Gills.
h.	Heart.
h.br.	Hypo-branchial.
h.hy.	Hypo-hyal.
hm.	Hyomandibular.
hm.r.	Hyomandibular ray.
h.s.c.	Horizontal semicircular canal.
hy.	Hyoid fold.
i.hy.	Inter-hyal.
i.tr.	Intertrabecula.
iv.	Investing mass.
l.	Liver.
l.l.	Lower lip.
m.	Mouth.
mk.	Meckel's cartilage.
mn.	Mandibular fold or bar.
ms.pg.	Mesopterygoid.
mt.pg′.	Median metapterygoids.
mt.pg″.	Lateral metapterygoids.
mx.	Maxillary.
nc.	Notochord.
ol.	Olfactory capsule.
op.	Opercular scute.
o.s′.	Orbito-sphenoidal bony plate.
p′.	Parietal scute.
pa.	Palatine bone.
pa.s.	Parasphenoid.
p.br.	Pharyngo-branchial.
p.f.	Pectoral fin.
pg.	Pterygoid bone.
pg.q.	Pterygo-quadrate.
px.	Pharynx.
p.op.	Pre-opercular bone.
pr.o′.	Prootic bony plate.
p.s.c.	Posterior semicircular canal.
pt.hm.	Protractor hyomandibularis.
pt.ob.	Postorbital scute.
q.c.	Quadrate condyle.
s.ob.	Superorbital scute.
sp.	Spiracle.
sp.r.	Spiracular ray.
sq′.	Squamosal scute.
su.ob.	Suborbital scute.
sy.	Symplectic.
t.	Teeth.
t.cr.	Tegmen cranii.
tr.	Trabeculæ.
u.l.	Upper lip.
v.	Vomerine bone.
vb.	Vestibule.
v.f.	Ventral fin.
y.st.	Yolk-mass and stomach.

IV. *On the Stresses caused in the Interior of the Earth by the Weight of Continents and Mountains.*

By G. H. Darwin, *F.R.S.*

Received June 11,—Read June 16, 1881.

[Plates 19, 20.]

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In this paper I have considered the subject of the solidity and strength of the materials of which the earth is formed, from a point of view from which it does not seem to have been hitherto discussed.

The first part of the paper is entirely devoted to a mathematical investigation, based upon a well-known paper of Sir William Thomson's. The second part consists of a summary and discussion of the preceding work. In this I have tried, as far as possible, to avoid mathematics, and I hope that a considerable part of it may prove intelligible to the non-mathematical reader.

I.

THE MATHEMATICAL INVESTIGATION.

§ 1. *On the state of internal stress of a strained elastic sphere.*

Let there be a homogeneous elastic sphere, for which $\omega-\frac{1}{3}\upsilon$ is the modulus of compressibility (or incompressibility, as I shall call it) and υ the rigidity.* Take the centre of the sphere as the origin for a set of rectangular axes x, y, z. Let the sphere be subjected to no surface stresses, let it be devoid of gravitation, but subject to internal force such that the force acting on a unit volume of the elastic solid is expressible by a gravitation potential W_i, a solid spherical harmonic of the i^{th} degree of the coordinates x, y, z.

Let w be the density of the elastic solid, a the radius of the sphere, and r the radius vector of any point measured from the centre of the sphere.

Sir WILLIAM THOMSON has investigated the state of internal strain produced under the conditions above described. If α, β, γ be the displacements his solution is as follows :—

$$\left.\begin{array}{l}
\alpha=(E_i a^2-F_i r^2)\dfrac{dW_i}{dx}-G_i r^{2i+3}\dfrac{d}{dx}(W_i r^{-2i-1}) \\
\text{where} \\
E_i=\dfrac{i[(i+2)\omega-\upsilon]}{2(i-1)\upsilon\{[2(i+1)^2+1]\omega-(2i+1)\upsilon\}} \\
F_i=\dfrac{(i+1)(2i+3)\omega-(2i+1)\upsilon}{2(2i+1)\upsilon\{[2(i+1)^2+1]\omega-(2i+1)\upsilon\}} \\
G_i=\dfrac{i\omega}{(2i+1)\upsilon\{[2(i+1)^2+1]\omega-(2i+1)\upsilon\}}
\end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and similar expressions for β and γ.†

Now let P, Q, R, S, T, U be the six stresses, across three planes mutually at right angles at the point x, y, z, estimated as is usual in works on the theory of elasticity.

Let P, Q, R be tractions and not pressures, and let p be the hydrostatic pressure at the point x, y, z.

Then P+Q+R being an invariant of the stress quadric, we have,

$$p=-\tfrac{1}{3}(\mathrm{P}+\mathrm{Q}+\mathrm{R})$$

if $\delta=\frac{d\alpha}{dx}+\frac{d\beta}{dy}+\frac{d\gamma}{dz}$, so that δ is the dilatation, then according to the usual formulas,‡

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* The phraseology adopted by THOMSON and TAIT (first edition) and others seems a little unfortunate. One might be inclined to suppose that compressibility and rigidity were things of the same nature; but rigidity and the reciprocal of compressibility are of the same kind. If one may give exact meanings to old words of somewhat general meanings, then one may pair together compressibility and "pliancy," and call the moduli for the two sorts of elasticity the "incompressibility" and rigidity.

† THOMSON and TAIT'S 'Nat. Phil.,' § 834, (8) and (9); or Phil. Trans., 1863, p. 573.

‡ THOMSON and TAIT'S 'Nat. Phil.,' § 693.

$$\mathrm{P}=(\omega-\upsilon)\delta+2\upsilon\frac{d\alpha}{dx}$$

$$\mathrm{T}=\upsilon\left(\frac{d\alpha}{dz}+\frac{d\gamma}{dx}\right)$$

and the other four stresses are expressed from these by cyclic changes of P, Q, R; S, T, U; α, β, γ; x, y, z.

The first task is to find p.

Now by adding P, Q, R together we have,

$$p=-(\omega-\tfrac{1}{3}\upsilon)\delta$$

We must now find δ from (1).

By differentiation

$$\frac{d\alpha}{dx}=(E_ia^2-F_ir^2)\frac{d^2W_i}{dx^2}-2F_ix\frac{dW_i}{dx}-G_ir^{2i+3}\frac{d^2}{dx^2}(W_ir^{-2i-1})-(2i+3)G_ir^{2i+1}x\frac{d}{dx}(W_ir^{-2i-1})$$

and similar expressions for $d\beta/dy$, $d\gamma/dz$.

Now W_i, W_ir^{-2i-1} are spherical harmonics of degrees i, $-i-1$, and are also homogeneous functions of the same degrees. If therefore we add the three expressions together, and note the properties of harmonics and of homogeneous functions, we have

$$\delta=-2iF_iW_i+(2i+3)(i+1)G_iW_i$$

Omitting for brevity that part of the divisors in the expressions for F and G which is common to both,

$$\begin{aligned}-2iF_i+(2i+3)(i+1)G_i&=-i(i+1)(2i+3)\omega+i(2i+1)\upsilon+(2i+3)(i+1)i\omega\\&=\quad i(2i+1)\upsilon\end{aligned}$$

and we have, on introducing the omitted denominator,

$$\delta=\frac{i}{[2(i+1)^2+1]\omega-(2i+1)\upsilon}W_i$$

And

$$p=\frac{-i(\omega-\frac{1}{3}\upsilon)}{[2(i+1)^2+1]\omega-(2i+1)\upsilon}W_i$$

Throughout the rest of this paper (excepting in § 10) the elastic sphere will be treated as incompressible, so that ω is to be considered as infinitely large compared with υ.

Henceforth I write

$$I=2(i+1)^2+1 \quad \text{. . . . .} \quad (2)$$

and when ω is infinite compared with υ, we have,

$$p=-\frac{i}{I}W_i \quad \text{. . . . .} \quad (3)$$

Also we may put

$$\left.\begin{aligned} \mathrm{P} &= -p + 2v\frac{d\alpha}{dx} \\ \mathrm{T} &= v\left(\frac{d\alpha}{dz}+\frac{d\gamma}{dx}\right) \end{aligned}\right\} \quad \ldots \ldots \ldots \quad (4)$$

And on putting ω infinite in (1) we have

$$\left.\begin{aligned} &\alpha = \frac{1}{Iv}\left[\left\{\frac{i(i+2)}{2(i-1)}a^2 - \frac{(i+1)(2i+3)}{2(2i+1)}r^2\right\}\frac{dW_i}{dx} - \frac{i}{2i+1}r^{2i+3}\frac{d}{dx}(W_i r^{-2i-1})\right] \\ &\text{and symmetrical expressions for } \beta \text{ and } \gamma. \end{aligned}\right\} \quad \ldots \quad (5)$$

The hydrostatic pressure might have been found from this general solution for the case of incompressibility, but in order to do so it would have been necessary to go back to the equations of equilibrium of the solid, and I prefer to deduce it from Sir WILLIAM THOMSON'S solution in the more general case.

Since

$$r^{2i+3}\frac{d}{dx}(W_i r^{-2i-1}) = -(2i+1)xW_i + r^2\frac{dW_i}{dx}$$

and since

$$(i+1)(2i+3)+2i=(2i+1)(i+3)$$

we may write α as follows :—

$$\alpha = \frac{1}{2Iv}\left[\left\{\frac{i(i+2)}{i-1}a^2-(i+3)r^2\right\}\frac{dW_i}{dx}+2ixW_i\right] \quad \ldots \ldots \quad (6)$$

In order to find the stresses P, Q, &c., we must now evaluate $\frac{d\alpha}{dx}$, $\frac{d\alpha}{dz}$, $\frac{d\gamma}{dx}$, &c. Differentiating (6) with regard to x,—

$$2Iv\frac{d\alpha}{dx} = \left\{\frac{i(i+2)}{i-1}a^2-(i+3)r^2\right\}\frac{d^2W_i}{dx^2} - 6x\frac{dW_i}{dx} + 2iW_i \quad \ldots \ldots \quad (7)$$

Differentiating with regard to z,—

$$2Iv\frac{d\alpha}{dz} = \left\{\frac{i(i+2)}{i-1}a^2-(i+3)r^2\right\}\frac{d^2W_i}{dxdz} + 2\left\{ix\frac{dW_i}{dz}-(i+3)z\frac{dW_i}{dx}\right\} \quad \ldots \quad (8)$$

and by symmetry

$$2Iv\frac{d\gamma}{dx} = \left\{\frac{i(i+2)}{i-1}a^2-(i+3)r^2\right\}\frac{d^2W_i}{dxdz} + 2\left\{iz\frac{dW_i}{dx}-(i+3)x\frac{dW_i}{dz}\right\} \quad \ldots \quad (9)$$

Adding (8) and (9) together and dividing by 2, we have

$$Iv\left(\frac{d\alpha}{dz}+\frac{d\gamma}{dx}\right) = \left\{\frac{i(i+2)}{i-1}a^2-(i+3)r^2\right\}\frac{d^2W_i}{dxdz} - 3\left\{x\frac{dW_i}{dz}+z\frac{dW_i}{dx}\right\} \quad \ldots \quad (10)$$

Hence from (3) (4) (7) and (10) we have,

$$\left.\begin{aligned}IP&=\left\{\frac{i(i+2)}{i-1}a^2-(i+3)r^2\right\}\frac{d^2W_i}{dx^2}-6x\frac{dW_i}{dx}+3iW_i\\ IT&=\left\{\frac{i(i+2)}{i-1}a^2-(i+3)r^2\right\}\frac{d^2W_i}{dxdz}-3\left(x\frac{dW_i}{dz}+z\frac{dW_i}{dx}\right)\end{aligned}\right\}\quad .\ .\ .\ .\quad (14)$$

where $I=2(i+1)^2+1$.

The expressions for Q, R, S, U may be written down from these by means of cyclic changes of the symbols.

These are the required expressions for the stresses at any point in the interior of the sphere.

In order to find the magnitude and direction of the principal stress-axes at any point it would be necessary to solve a cubic equation. The solution of this equation appears to be difficult, but the special case in which it reduces to a quadratic equation will fortunately give adequate results. It may be seen from considerations of symmetry that if W_i be a zonal harmonic, two of the principal stress-axes lie in a meridional plane and the third is perpendicular thereto. Moreover the greatest and least stress-axes are those which lie in that plane, and the mean stress-axis is that which is perpendicular thereto. If this is not obvious to the reader at present, it will become so later.

I shall therefore take W_i to be a zonal harmonic, and as the future developments will be by means of series (which though finite will be long for the higher orders of harmonics) I shall attend more especially to the equatorial regions of the sphere.

§ 2. *The determination of the stresses when the disturbing potential is an even zonal harmonic.*

If θ be colatitude the expression for a zonal surface harmonic or LEGENDRE'S function of order i is

$$\cos^i\theta-\frac{i(i-1)}{4.(1\,!)^2}\cos^{i-2}\theta\sin^2\theta+\frac{i(i-1)(i-2)(i-3)}{4^2(2\,!)^2}\cos^{i-4}\theta\sin^4\theta-\ldots$$

or if we begin by the other end of the series, and take i as an even number, the expression is

$$(-)^{\frac{1}{2}i}\frac{i\,!}{2^i\{\frac{1}{2}i\,!\}^2}\left[\sin^i\theta-\frac{i^2}{2\,!}\sin^{i-2}\theta\cos^2\theta+\frac{i^2(i-2)^2}{4\,!}\sin^{i-4}\theta\cos^4\theta-\ldots\right]$$

This latter is the appropriate form when we wish to consider especially the equatorial regions, because $\cos\theta$ is small for that part of the sphere.

There is of course a similar formula when i is odd, but of this I shall make no use.

Now let $\rho^2=y^2+z^2$, so that $\sin\theta=\rho/r$, $\cos\theta=z/r$.

Then we may put

$$\mathrm{W}_i=\rho^i-\frac{i^2}{2!}\rho^{i-2}z^2+\frac{i^2(i-2)^2}{4!}\rho^{i-4}z^4-\frac{i^2(i-2)^2(i-4)^2}{6!}\rho^{i-6}z^6+\ldots \qquad (12)$$

W_i is a solid zonal harmonic of degree i; but $r^{-i}W_i$ requires multiplication by a factor $(-)^{\frac{1}{2}i}i!/2^i\{\frac{1}{2}i!\}^2$ in order to make it a LEGENDRE'S function.

The factors by which W_i must be deemed to be multiplied in order that it may be a potential, will be dropped for the present, to be inserted later. Or we may, if we like, suppose that the units of length or of time are so chosen as to make the factor equal to unity.

Now let

$$\beta_0=1,\ \beta_2=\frac{i^2}{2!},\ \beta_4=\frac{i^2(i-2)^2}{4!},\ \beta_6=\frac{i^2(i-2)^2(i-4)^2}{6!}\ \&\mathrm{c.} \qquad (13)$$

Then, dropping the suffix to W for brevity, we may write

$$W=\beta_0\rho^i-\beta_2\rho^{i-2}z^2+\beta_4\rho^{i-4}z^4-\beta_6\rho^{i-6}z^6+\ldots \qquad (14)$$

I shall now find P, Q, R, T at any point in the meridional plane which is determined by $y=0$.

In evaluating the first differential coefficients of W we must not put $y=0$, in as far as these coefficients are a first step towards the determination of the second differential coefficients. But in as far as these first coefficients are directly involved in the expressions for P, Q, R, and T, and in the second coefficients in the same expressions, we may put $y=0$, and thus write x in place of ρ.

$$\rho\frac{d\rho}{dx}=x,\ \rho\frac{d\rho}{dy}=y,\ \frac{d\rho}{dz}=0,\ \text{since } \rho^2=x^2+y^2.$$

Then

$$\begin{aligned}\frac{dW}{dx}&=x[i\beta_0\rho^{i-2}-(i-2)\beta_2\rho^{i-4}z^2+(i-4)\beta_4\rho^{i-6}z^4-\ldots]\\ \frac{dW}{dy}&=y[\text{same series}\qquad\qquad\qquad\qquad]\\ \frac{dW}{dz}&=z[\qquad -2\beta_2\rho^{i-2}+4\beta_4\rho^{i-4}z^2-6\beta_6\rho^{i-6}z^4+\ldots]\end{aligned}$$

In differentiating a second time we may treat ρ as identical with x, because y is to be put equal to zero. Thus

$$\left.\begin{aligned}\frac{d^2W}{dx^2}&=i(i-1)\beta_0x^{i-2}-(i-2)(i-3)\beta_2x^{i-4}z^2+(i-4)(i-5)\beta_4x^{i-6}z^4-\ldots\\ \frac{d^2W}{dy^2}&=i\beta_0x^{i-2}-(i-2)\beta_2x^{i-4}z^2+(i-4)\beta_4x^{i-6}z^4-\ldots\\ \frac{d^2W}{dz^2}&=-1.2\beta_2x^{i-2}+3.4\beta_4x^{i-4}z^2-5.6\beta_6x^{i-6}z^4+\ldots\end{aligned}\right\} . \qquad (15)$$

$$\left.\begin{aligned}&\frac{d^2W}{dxdz}=xz[-2(i-2)\beta_2x^{i-4}+4(i-4)\beta_4x^{i-6}z^2-6(i-6)\beta_6x^{i-8}z^4+\ldots]\\&\frac{d^2W}{dxdy}=0,\qquad\frac{d^2W}{dydz}=0\end{aligned}\right\}\quad .\quad(16)$$

Also treating ρ as identical with x, and putting $y=0$,

$$\left.\begin{aligned}&x\frac{dW}{dx}=i\beta_0x^i-(i-2)\beta_2x^{i-2}z^2+(i-4)\beta_4x^{i-4}z^4-\ldots\\&y\frac{dW}{dy}=0\\&z\frac{dW}{dz}=-2\beta_2x^{i-2}z^2+4\beta_4x^{i-4}z^4-6\beta_6x^{i-6}z^6+\ldots\end{aligned}\right\}\quad .\ .\ .\ .\ .\quad(17)$$

$$\left.\begin{aligned}&\left(z\frac{dW}{dx}+x\frac{dW}{dz}\right)=xz\{(i\beta_0-2\beta_2)x^{i-2}-[(i-2)\beta_2-4\beta_4]x^{i-4}z^2\\&\qquad\qquad\qquad\qquad+[(i-4)\beta_4-6\beta_6]x^{i-6}z^4-\ldots\}\\&\left(y\frac{dW}{dx}+x\frac{dW}{dy}\right)=0,\qquad\left(y\frac{dW}{dz}+z\frac{dW}{dy}\right)=0\end{aligned}\right\}\quad .\quad(18)$$

These various results have now to be introduced into the expressions (11) for P, Q, R, S, T, U.

In performing these operations it will be convenient to write J for $i(i+2)/(i-1)$. Also $r^2=\rho^2+z^2=x^2+z^2$, when $y=0$.

From these formulas we see that S=0, U=0; which shows that a meridional plane is one of the three principal planes, a result already observed from principles of symmetry.

Now

$$\left.\begin{aligned}&r^2\frac{d^2W}{dx^2}=i(i-1)\beta_0x^i+[i(i-1)\beta_0-(i-2)(i-3)\beta_2]x^{i-2}z^2\\&\qquad\qquad-[(i-2)(i-3)\beta_2-(i-4)(i-5)\beta_4]x^{i-4}z^4+\ldots\\&r^2\frac{d^2W}{dy^2}=i\beta_0x^i+[i\beta_0-(i-2)\beta_2]x^{i-2}z^2-[(i-2)\beta_2-(i-4)\beta_4]x^{i-4}z^4+\ldots\\&r^2\frac{d^2W}{dz^2}=-1.2\beta_2x^i-[1.2\beta_2-3.4\beta_4]x^{i-2}z^2+[3.4\beta_4-5.6\beta_6]x^{i-4}z^4-\ldots\end{aligned}\right\}\quad(19)$$

$$\left.\begin{aligned}&-2x\frac{dW}{dx}+iW=-i\beta_0x^i+(i-4)\beta_2x^{i-2}z^2-(i-8)\beta_4x^{i-4}z^4+\ldots\\&-2y\frac{dW}{dy}+iW=i\beta_0x^i-i\beta_2x^{i-2}z^2+i\beta_4x^{i-4}z^4-\ldots\\&-2z\frac{dW}{dz}+iW=i\beta_0x^i-(i-4)\beta_2x^{i-2}z^2+(i-8)\beta_4x^{i-4}z^4-\ldots\end{aligned}\right\}\quad .\ .\quad(20)$$

$$
\begin{aligned}
r^2\frac{d^2W}{dxdz}=xz\{ & -2(i-2)\beta_2x^{i-2}-[2(i-2)\beta_2-4(i-4)\beta_4]x^{i-4}z^2 \\
& +[4(i-4)\beta_4-6(i-6)\beta_6]x^{i-6}z^6-\ldots\} \quad . \quad . \quad (21)
\end{aligned}
$$

Then multiplying (19) by $-(i+3)$, (20) by 3, and (15) by Ja^2, and adding them each to each, we get the expressions for P, Q, R.

Also multiplying (21) by $-(i+3)$, (18) by -3, and (16) by Ja^2 and adding, we get the expression for T. The results are

$$
\begin{aligned}
I\mathrm{P}= & -[(i+3)i(i-1)+3i]\beta_0x^i \\
& \qquad +[\{(i+3)(i-2)(i-3)+3(i-4)\}\beta_2-(i+3)i(i-1)\beta_0]x^{i-2}z^2 \\
& -[\{(i+3)(i-4)(i-5)+3(i-8)\}\beta_4-(i+3)(i-2)(i-3)\beta_2]x^{i-4}z^4 \\
& +[\{i+3)(i-6)(i-7)+3(i-12)\}\beta_6-(i+3)(i-4)(i-5)\beta_4]x^{i-6}z^6-\ldots \\
& +Ja^2[i(i-1)\beta_0x^{i-2}-(i-2)(i-3)\beta_2x^{i-4}z^2+(i-4)(i-5)\beta_4x^{i-6}z^4-\ldots]
\end{aligned}
$$

$$
\begin{aligned}
I\mathrm{R}= & [(i+3).1.2\beta_2+3i\beta_0]x^i-[(i+3).3.4\beta_4-\{(i+3).1.2-3(i-4)\}\beta_2]x^{i-2}z^2 \\
& +[(i+3).5.6\beta_6-\{(i+3).3.4-3(i-8)\}\beta_4]x^{i-4}z^4 \\
& -[(i+3).7.8\beta_8-\{(i+3).5.6-3(i-12)\beta_6]x^{i-6}z^6+\ldots \\
& -Ja^2[1.2\beta_2x^{i-2}-3.4\beta_4x^{i-4}z^2+5.6\beta_6x^{i-6}z^4-\ldots]
\end{aligned}
$$

$$
\begin{aligned}
I\mathrm{Q}= & -[(i+3)i-3i]\beta_0x^i+[\{(i+3)(i-2)-3i\}\beta_2-(i+3)i\beta_0]x^{i-2}z^2 \\
& -[\{(i+3)(i-4)-3i\}\beta_4-(i+3)(i-2)\beta_2]x^{i-4}z^4 \\
& +[\{(i+3)(i-6)-3i\}\beta_6-(i+3)(i-4)\beta_4]x^{i-6}z^6-\ldots \\
& +Ja^2[ix^{i-2}-(i-2)\beta_2x^{i-4}z^2+(i-4)\beta_4x^{i-6}z^4-\ldots]
\end{aligned}
$$

$$
\begin{aligned}
\frac{I\mathrm{T}}{xz}= & [\{(i+3)2(i-2)+3.2\}\beta_2-3i\beta_0]x^{i-2} \\
& -[\{(i+3)4(i-4)+3.4\}\beta_4-\{(i+3)2(i-2)+3(i-2)\}\beta_2]x^{i-4}z^2 \\
& +[\{(i+3)6(i-6)+3.6\}\beta_6-\{(i+3)4(i-4)+3(i-4)\}\beta_4]x^{i-6}z^4 \\
& -[\{(i+3)8(i-8)+3.8\}\beta_8-\{(i+3)6(i-6)+3(i-6)\}\beta_6]x^{i-6}z^6+\ldots \\
& -Ja^2[2(i-2)\beta_2x^{i-4}-4(i-4)\beta_4x^{i-6}z^2+6(i-6)\beta_6x^{i-8}z^4-\ldots]
\end{aligned}
$$

The general law of formation of the successive coefficients is obvious, and it is easy to write down the general term in each of the eight series involved in these four expressions; the best way indeed of obtaining the formulas given below is to write down and transform the general term.

The semi-polar coordinates used hitherto are not so convenient as true polar coordinates; I therefore substitute r, radius vector, and l, latitude, for the x, z system, and putting $x=r\cos l$, $z=r\sin l$ write

$$\left.\begin{aligned}
\mathrm{P}&=r^i\cos^i l(A_0+A_2\tan^2 l+A_4\tan^4 l+\ldots)\\
&\qquad+a^2r^{i-2}\cos^{i-2}l(B_0+B_2\tan^2 l+B_4\tan^4 l+\ldots)\\
\mathrm{R}&=r^i\cos^i l(C_0+C_2\tan^2 l+C_4\tan^4 l+\ldots)\\
&\qquad+a^2r^{i-2}\cos^{i-2}l(D_0+D_2\tan^2 l+D_4\tan^4 l+\ldots)\\
\mathrm{T}&=r^i\sin l\cos^{i-1}l(E_0+E_2\tan^2 l+E_4\tan^4 l+\ldots)\\
&\qquad+a^2r^{i-2}\sin l\cos^{i-3}l(F_0+F_2\tan^2 l+F_4\tan^4 l+\ldots)\\
\mathrm{Q}&=r^i\cos^i l(G_0+G_2\tan^2 l+G_4\tan^4 l+\ldots)\\
&\qquad+a^2r^{i-2}\cos^{i-2}l(H_0+H_2\tan^2 l+H_4\tan^4 l+\ldots)
\end{aligned}\right\}\quad(22)$$

Then introducing for J and for the β's their values in terms of i, I find that the coefficients A, B, &c., are reducible to the forms given in the following equations:—

$$\left.\begin{aligned}
IA_0&=-\frac{1}{0!}\{i(i+2)(i-0)-3.0.(i+1)\}+0.(i+3)(i+2)(i+1)=-i^2(i+2)\\
IA_2&=\frac{i^2}{2!}\{i(i-0)(i-2)-3.2(i-1)\}-\frac{1}{0!}(i+3)(i+0)(i-1)\\
IA_4&=-\frac{i^2(i-2)^2}{4!}\{i(i-2)(i-4)-3.4(i-3)\}+\frac{i^2}{2!}(i+3)(i-2)(i-3)\\
IA_6&=\frac{i^2(i-2)^2(i-6)^2}{6!}\{i(i-4)(i-6)-3.6(i-5)\}-\frac{i^2(i-2)^2}{4!}(i+3)(i-4)(i-5)\\
\&\text{c.}&=\&\text{c.}\\
IB_0&=\frac{i(i+2)}{i-1}\,\frac{1}{0!}i(i-1)\\
IB_2&=-\frac{i(i+2)}{i-1}\,\frac{i^2}{2!}(i-2)(i-3)\\
IB_4&=\frac{i(i+2)}{i-1}\,\frac{i^2(i-2)^2}{4!}(i-4)(i-5)\\
\&\text{c.}&=\&\text{c.}
\end{aligned}\right\}\quad(23)$$

$$\left.\begin{aligned}
IC_0&=\frac{1}{0!}\{(i+3)i^2-[i(1(-2)-1)+3.0.1]\}=i[(i+1)(i+2)+1]\\
IC_2&=-\frac{i^2}{2!}\{(i+3)(i-2)^2-[i(3.0-1)+3.2.3]\}\\
IC_4&=\frac{i^2(i-2)^2}{4!}\{(i+3)(i-4)^2-[i(5.2-1)+3.4.5]\}\\
IC_6&=-\frac{i^2(i-2)^2(i-4)^2}{6!}\{(i+3)(i-6)^2-[i(7.4-1)+3.6.7]\}\\
\&\text{c.}&=\&\text{c.}\\
ID_0&=-\frac{i(i+2)}{i-1}\,\frac{i^2}{0!}\\
ID_2&=\frac{i(i+2)}{i-1}\,\frac{i^2(i-2)^2}{2!}\\
ID_4&=-\frac{i(i+2)}{i-1}\,\frac{i^2(i-2)^2(i-4)^2}{4!}\\
\&\text{c.}&=\&\text{c.}
\end{aligned}\right\}\quad(24)$$

$$\left.\begin{aligned}
IE_0&=-\frac{1}{1!}i[1(3i+3)-i(i-0)(i+1)]=i(i+1)(i^2\quad 3)\\
IE_2&=\frac{i^2}{3!}(i-2)[3(5i+3)-i(i-2)(i-1)]\\
IE_4&=-\frac{i^2(i-2)^2}{5!}(i-4)[5(7i+3)-i(i-4)(i-3)]\\
IE_6&=\frac{i^2(i-2)^2(i-4)^2}{7!}(i-6)[7(9i+3)-i(i-6)(i-5)]\\
\&c.&=\&c.\\
IF_0&=-\frac{i^3(i^2-4)}{i-1}\\
IF_2&=\frac{i^3(i^2-4)}{i-1}\,\frac{(i-2)(i-4)}{3!}\\
IF_4&=-\frac{i^3(i^2-4)}{i-1}\,\frac{(i-2)(i-4)^2(i-6)}{5!}\\
IF_6&=\frac{i^3(i^2-4)}{i-1}\,\frac{(i-2)(i-4)^2(i-6)^2(i-8)}{7!}\\
\&c.&=\&c.
\end{aligned}\right\}\quad . \quad . \quad . \quad (25)$$

$$\left.\begin{aligned}
IG_0&=-\frac{1}{0!}\{i(i-0)-3.0\}+0.(i+3)(i+2)=-i^2\\
IG_2&=\frac{i^2}{2!}\{i(i-2)-3.2\}-\frac{1}{0!}(i+3)i\\
IG_4&=-\frac{i^2(i-2)^2}{4!}\{i(i-4)-3.4\}+\frac{i^2}{2!}(i+3)(i-2)\\
\&c.&=\&c.\\
IH_0&=\frac{i(i+2)}{i-1}\,\frac{1}{0!}i\\
IH_2&=-\frac{i(i+2)}{i-1}\,\frac{i^2}{2!}(i-2)\\
IH_4&=\frac{i(i+2)}{i-1}\,\frac{i^2(i-2)^2}{4!}(i-4)\\
\&c.&=\&c.
\end{aligned}\right\}\quad . \quad . \quad (26)$$

These sets of coefficients are all written down in such a form that the laws of their formation are obvious, and the general terms may easily be found. I have computed their values from these formulas for the even zonal harmonics of orders 2, 4, 6, 8, 10, 12; the results are given in the following tables both in the form of fractions and of decimals approximately equal to those fractions.

The *G*'s and *H*'s were not computed because their values were not required for subsequent operations.

TABLE I.—The coefficients for expressing the stress P.

i	A_0	A_2	A_4	A_6	B_0	B_2	B_4	B_6
2	$-\frac{16}{19}$ $-{\cdot}8421$	$-\frac{22}{19}$ $-1{\cdot}1579$			$+\frac{16}{19}$ $+{\cdot}8421$			
4	$-\frac{32}{17}$ $-1{\cdot}8824$	$+\frac{28}{51}$ $+{\cdot}5490$	$+\frac{48}{17}$ $+2{\cdot}8235$		$+\frac{32}{17}$ $+1{\cdot}8824$	$-\frac{128}{51}$ $-2{\cdot}5098$		
6	$-\frac{32}{11}$ $-2{\cdot}9091$	$+18$ $+18{\cdot}0000$	$+\frac{184}{11}$ $+16{\cdot}7273$	$-\frac{272}{55}$ $-4{\cdot}9455$	$+\frac{32}{11}$ $+2{\cdot}9091$	$-\frac{1152}{55}$ $-20{\cdot}9455$	$+\frac{256}{55}$ $+4{\cdot}6545$	
8	$-\frac{640}{163}$ $-3{\cdot}9264$	$+\frac{10328}{163}$ $+63{\cdot}3620$	$-\frac{2112}{163}$ $-12{\cdot}9571$	$-\frac{12160}{163}$ $-74{\cdot}6012$	$+\frac{640}{163}$ $+3{\cdot}9264$	$-\frac{76800}{1141}$ $-67{\cdot}3094$	$+\frac{92160}{1141}$ $+80{\cdot}7713$	$-\frac{8192}{1141}$ $-7{\cdot}1797$
10	$-\frac{400}{81}$ $-4{\cdot}9383$	$+\frac{36130}{243}$ $+148{\cdot}6831$	$-\frac{69200}{243}$ $-284{\cdot}7737$	$-\frac{56000}{243}$ $-230{\cdot}4527$	$+\frac{400}{81}$ $+4{\cdot}9383$	$-\frac{112000}{729}$ $-153{\cdot}6352$	$+\frac{320000}{729}$ $+438{\cdot}9561$	$-\frac{51200}{243}$ $-210{\cdot}6996$
12	$-\frac{672}{113}$ $-5{\cdot}9469$	$+\frac{32316}{113}$ $+285{\cdot}9823$	$-\frac{138000}{113}$ $-1221{\cdot}2389$	$+\frac{24000}{113}$ $+212{\cdot}3894$	$+\frac{672}{113}$ $+5{\cdot}9469$	$-\frac{362880}{1243}$ $-291{\cdot}9389$	$+\frac{1881600}{1243}$ $+1513{\cdot}7570$	$-\frac{2150400}{1243}$ $-1730{\cdot}0080$

TABLE II.—The coefficients for expressing the stress R.

i	C_0	C_2	C_4	C_6	D_0	D_2	D_4	D_6
2	$+\frac{26}{19}$ $+1{\cdot}3684$	$+\frac{32}{19}$ $+1{\cdot}6842$			$-\frac{32}{19}$ $-1{\cdot}6842$			
4	$+\frac{124}{51}$ $+2{\cdot}4314$	$-\frac{112}{51}$ $-2{\cdot}1961$	$-\frac{256}{51}$ $-5{\cdot}0196$		$-\frac{128}{51}$ $-2{\cdot}5098$	$+\frac{256}{51}$ $+5{\cdot}0196$		
6	$+\frac{38}{11}$ $+3{\cdot}4545$	-24 $-24{\cdot}0000$	$-\frac{208}{11}$ $-18{\cdot}9091$	$+\frac{512}{55}$ $+9{\cdot}3091$	$-\frac{192}{55}$ $-3{\cdot}4909$	$+\frac{1536}{55}$ $+27{\cdot}9273$	$-\frac{512}{55}$ $-9{\cdot}3091$	
8	$+\frac{728}{163}$ $+4{\cdot}4663$	$-\frac{12352}{163}$ $-75{\cdot}7791$	$+\frac{4224}{163}$ $+25{\cdot}9141$	$+\frac{76288}{815}$ $+93{\cdot}6049$	$-\frac{5120}{1141}$ $-4{\cdot}4873$	$+\frac{92160}{1141}$ $+80{\cdot}7713$	$-\frac{122880}{1141}$ $-107{\cdot}6950$	$+\frac{16384}{1141}$ $+14{\cdot}3593$
10	$+\frac{1330}{243}$ $+5{\cdot}4733$	$-\frac{41200}{243}$ $-169{\cdot}5473$	$+\frac{84800}{243}$ $+348{\cdot}9712$	$+\frac{60160}{243}$ $+247{\cdot}5720$	$-\frac{4000}{729}$ $-5{\cdot}4870$	$+\frac{128000}{729}$ $+175{\cdot}5830$	$-\frac{384000}{729}$ $-526{\cdot}7490$	$+\frac{204800}{729}$ $+280{\cdot}9328$
12	$+\frac{732}{113}$ $+6{\cdot}4779$	$-\frac{35856}{113}$ $-317{\cdot}3097$	$+\frac{158400}{113}$ $+1401{\cdot}7700$	$-\frac{38400}{113}$ $-339{\cdot}8230$	$-\frac{8064}{1243}$ $-6{\cdot}4875$	$+\frac{403200}{1243}$ $+324{\cdot}3765$	$-\frac{2150400}{1243}$ $-1730{\cdot}0080$	$+\frac{2580480}{1243}$ $+2076{\cdot}0097$

TABLE III.—The coefficients for expressing the stress T.

i	E_0	E_2	E_4	E_6	F_0	F_2	F_4	F_6
2	$+\frac{6}{19}$ +·3158							
4	$+\frac{260}{51}$ +5·0980	$+\frac{80}{17}$ +4·7059			$-\frac{256}{51}$ −5·0196			
6	+14 +14·0000	$-\frac{56}{11}$ −5·0909	$-\frac{1008}{55}$ −18·3273		$-\frac{768}{55}$ −13·9636	$+\frac{1024}{55}$ +18·6182		
8	$+\frac{4392}{163}$ +26·9448	$-\frac{13248}{163}$ −81·2761	$-\frac{10368}{163}$ −63·6074	$+\frac{244224}{5705}$ +42·8088	$-\frac{30720}{1141}$ −26·9238	$+\frac{122880}{1141}$ +107·6950	$-\frac{49152}{1141}$ −43·0780	
10	$+\frac{10670}{243}$ +43·9095	$-\frac{74800}{243}$ −307·8189	$+\frac{17600}{243}$ +72·4280	$+\frac{577280}{1701}$ +339·3769	$-\frac{32000}{729}$ −43·8958	$+\frac{256000}{729}$ +351·1660	$-\frac{102400}{243}$ −421·3992	$+\frac{409600}{5103}$ +80·2664
12	$+\frac{7332}{113}$ +64·8850	$-\frac{90480}{113}$ −800·7089	$+\frac{137280}{113}$ +1214·8673	$+\frac{99840}{113}$ +883·5398	$-\frac{80640}{1243}$ −64·8753	$+\frac{1075200}{1243}$ +865·0040	$-\frac{2580480}{1243}$ −2076·0097	$+\frac{1474560}{1243}$ +1186·2912

If W be a 2nd, 4th, or 6th harmonic these tables give the complete expressions for P, R, and T; if W be an 8th harmonic the only further coefficients required are A_8 and C_8.

For the cases of the 10th and 12th harmonics the values in the tables are sufficient to give the stresses approximately over a wide equatorial belt, because the series for P, R, T proceed by powers of the tangent of the latitude, and the omitted terms involve high powers of that tangent. It would hardly be safe however to apply the formula—at least as regards the 12th harmonic—for latitudes greater than 15°, because the coefficients are large.

§ 3. *On the direction and magnitude of the principal stresses in a strained elastic solid.*

Let P, Q, R, S, T, U specify the stresses in a homogeneously stressed and strained elastic solid. Let l, m, n be the direction cosines of a principal stress axis.

The consideration, that at the extremity of a principal axis the normal to the stress quadric is coincident with the radius vector, gives the equations

$$(\mathrm{P}-\lambda)l+\mathrm{U}m+\mathrm{T}n=0$$
$$\mathrm{U}l+(\mathrm{Q}-\lambda)m+\mathrm{S}n=0$$
$$\mathrm{T}l+\mathrm{S}m+(\mathrm{R}-\lambda)n=0$$

These equations lead to the discriminating cubic for the determination of λ, and the solution for l, m, n is then

$$\frac{l^2}{(\mathrm{Q}-\lambda)(\mathrm{R}-\lambda)-\mathrm{S}^2}=\frac{m^2}{(\mathrm{P}-\lambda)(\mathrm{R}-\lambda)-\mathrm{T}^2}=\frac{n^2}{(\mathrm{P}-\lambda)(\mathrm{Q}-\lambda)-\mathrm{U}^2}$$

In the case considered in the preceding sections S and U vanish, and the cubic reduces to the quadratic

$$(P-\lambda)(R-\lambda)-T^2=0$$

of which the solution is

$$2\lambda=P+R\pm\sqrt{(P-R)^2+4T^2}$$

m is obviously zero and l, n are determinable from

$$l^2(P-\lambda)=n^2(R-\lambda)$$

Let

$$l=\cos\vartheta, \quad n=\sin\vartheta$$

Then it is easily proved that

$$\cot 2\vartheta=\frac{P-R}{2T} \quad \dots\dots\dots\dots \quad (27)$$

This equation gives the directions of the principal stress-axes.

The two principal stresses N_1, N_3 are the two values of λ, so that

$$\left.\begin{aligned} N_1&=\tfrac{1}{2}(P+R)+\tfrac{1}{2}\sqrt{(P-R)^2+4T^2} \\ N_3&=\tfrac{1}{2}(P+R)-\tfrac{1}{2}\sqrt{(P-R)^2+4T^2} \end{aligned}\right\} \quad \dots\dots\dots \quad (28)$$

and the third principal stress, which we suppose intermediate in value between N_1 and N_3, is of course Q.

When an elastic solid is in a state of stress it is supposed, in all probability with justice, that the tendency of the solid to rupture at any point is to be estimated by the form of the stress quadric. At any rate the hypothesis is here adopted that the tendency to break is to be estimated by the difference between the greatest and least principal stresses. For the sake of brevity I shall refer to the difference between the greatest and least principal stresses as "the stress-difference." This quantity I shall find it convenient to indicate by Δ.

We may also look at the subject from another point of view:—It is a well-known theorem in the theory of elastic solids that the greatest shearing stress at any point is equal to a half of the stress-difference. It is difficult to conceive any mode in which an elastic solid can rupture except by shearing, and hence it appears that the greatest shearing stress is a proper measure of the tendency to break. This measure of tendency to break is exactly one-half of the stress-difference, and it is therefore a matter of indifference whether we take greatest shearing stress or stress-difference. For the sake of comparison with experimental results as to the stresses under which wires and rods of various materials will break and crush, I have found it more convenient to use stress-difference throughout; but the results may all be reduced to shearing stresses by merely halving the numbers given.

From (28) we have then

$$\Delta=\sqrt{(P-R)^2+4T^2} \quad \dots\dots\dots\dots \quad (29)$$

and the greatest shearing stress at the same point is $\frac{1}{2}\Delta$.

§ 4. *The application of previous analysis to the determination of the stresses produced by the weight of superficial inequalities.*

I have in a previous paper shown how Sir WILLIAM THOMSON'S solution for the state of internal strain of an elastic sphere subject to bodily forces, but not acted on by any surface forces, is to be adapted to the case of a spheroid (whose small inequalities are expressed as surface harmonics) of homogeneous elastic matter, endued with the power of mutual gravitation.* THOMSON'S solution is of course directly applicable for finding the state of strain due to a true external force, such as the tide-generating influence of the moon, but this forms only a part of the complete solution when the sphere has the power of gravitation. He introduced the effects of gravitation synthetically, but for my own purposes I prefer the analytical method pursued in my paper above referred to.

Suppose that $r=a+\sigma_i$ be the equation to an harmonic spheroid of the i^{th} order, forming inequalities on the surface of the sphere, whose density is w.

Then the causes producing a state of stress and strain in the mean sphere of radius a are, first a normal traction per unit area of the surface of the sphere equal to $-gw\sigma_i$, when g is the value of gravity, and secondly the attraction of the inequalities σ_i, acting throughout the whole sphere.

The first of these causes (viz. : the weight of the mountains or continents) is shown in my paper to produce the same state of strain as would be produced in the sphere, now free from surface action, by bodily forces corresponding with a potential $-gw(r/a)^i\sigma_i$.

As regards the second of these causes (viz. : the attraction of the mountains or continents), the potential of the layer of matter σ_i on any internal point, estimated per unit volume, is $3gw(r/a)^i\sigma_i/(2i+1)$.†

Then adding these two potentials together, we see that the surface inequalities σ_i produce the same state of strain as would be caused by the bodily forces due to a potential $-2(i-1)gw(r/a)^i\sigma_i/(2i+1)$, and the surface of the sphere is now subjected to no forces.

This expression is a solid harmonic of the i^{th} degree, and therefore the analytical results of the preceding sections are directly available for finding the state of stress due to continents and mountain chains.

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* "On the Bodily Tides of Viscous and Semi-elastic Spheroids, &c.," Phil. Trans., Part I., 1879, p. 1 (see § 2). This paper treats of the state of flow of a viscous sphere, but the problem is exactly the same as that concerning elasticity here considered. It is easy to see that if a viscous sphere be deformed into the shape of a zonal harmonic, the flow of the fluid must be meridional, and from this we may conclude that in the elastic sphere the plane of greatest and least principal stresses is also meridional. This has been already assumed to be the case in the present paper.

† If we could suppose a sphere to have homogeneous elasticity but heterogeneous density, this manner of building up the effective disturbing potential would have to be somewhat modified. Such an hypothesis is somewhat absurd, and I shall regard the sphere as homogeneous. In application to the case of the earth I shall however pay attention to the smaller density of the superficial layers by halving the height of the actual continents and the depth of the actual seas.

We must in fact put

$$W_i = -\frac{2(i-1)}{2i+1} gw\left(\frac{r}{a}\right)^i \sigma_i$$

Now in the previous developments the factors involving g, w, &c., have been omitted and W_i has been put equal to a zonal harmonic which had the value unity at the equator.

If we write

$$\varsigma_i = \sin^i\theta - \frac{i^2}{2\,!}\sin^{i-2}\theta\cos^2\theta + \&c. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (30)$$

where θ is the colatitude, and put h as the height above the mean sphere of the elevation at the equator, then $h\varsigma_i = \sigma_i$ and

$$W_i = -\frac{2(i-1)}{2i+1}\frac{gwh}{a^i}(r^i\varsigma_i) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (31)$$

W_i was in (12) put equal to $r^i\varsigma_i$.

Thus in order to apply the preceding results to finding the stresses caused in a sphere, possessing the power of gravitation, by the weight and attraction of surface inequalities expressed by

$$r = a + h\varsigma_i \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (32)$$

we must multiply the preceding results for P, R, T, Q by

$$-\frac{2(i-1)}{2i+1}\frac{gwh}{a^i} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (33)$$

§ 5. *The state of stress due to ellipticity of figure or to tide-generating forces.*

When the effective disturbing potential W_i is a solid harmonic of the second degree the solution found above will give the stresses caused by oblateness or prolateness of the spheroid. It will of course also serve for the case of a rotating spheroid with more or less oblateness than is appropriate for the equilibrium figure.

When an elastic sphere is under the action of tide-generating forces the disturbing potential is a solid harmonic of the second degree, and therefore the present solution will apply to this case also.

The formula for the stress-difference admits of reduction to a simple form when $i=2$.

On substituting colatitude θ for latitude l, (22) gives

$$\begin{aligned} \mathrm{P}-\mathrm{R} &= r^2\sin^2\theta[(A_0-C_0)+(A_2-C_2)\cot^2\theta]+a^2(B_0-D_0) \\ \mathrm{T} &= r^2\sin\theta\cos\theta(E_0) \end{aligned}$$

Then substituting for A, B, C, D, E their values from Tables I., II., III., and making some simple reductions, we find

$$\left.\begin{aligned}\mathrm{P}-\mathrm{R}&=\tfrac{6}{19}[8(a^2-r^2)-r^2\cos 2\theta]\\ 2\mathrm{T}&=\tfrac{6}{19}r^2\sin 2\theta\end{aligned}\right\} \quad \ldots\ldots\ldots \quad (34)$$

Therefore by (29) in the present case, the stress-difference

$$\Delta=\tfrac{6}{19}\sqrt{64(a^2-r^2)^2+r^4-16r^2(a^2-r^2)\cos 2\theta} \quad \ldots\ldots\ldots \quad (35)$$

In order to find the actual value of Δ in any special case we shall have to multiply (35) by appropriate factors; the factors will be determined below. For the present it will be more convenient to omit the factor $\tfrac{6}{19}$, and to reintroduce it along with these other factors.

The formula (35) enables us to determine the distribution of stress-difference throughout the sphere in the cases to which this section applies.

The curves of equal stress-difference are given by the equation

$$64(a^2-r^2)+r^4-16r^2(a^2-r^2)\cos 2\theta=\text{a constant}$$

The stress-difference at any point on any equatorial radius (for which $\theta=\tfrac{1}{2}\pi$) is clearly given by $8a^2-7r^2$, and along the polar radius (for which $\theta=0$) by $8a^2-9r^2$.

From this result it is clear that the stress-difference vanishes at that point on the polar radius for which $r=\tfrac{2}{3}a\sqrt{2}=\cdot 9425a$; this is the only point within the whole spheroid for which it vanishes.

When $r=a$ the stress-difference is equal to a^2, from which we obtain the remarkable result that the stress-difference is constant all over the surface. When $r=0$, it is equal to $8a^2$, which is eight times the surface value.

By means of arithmetical calculation and graphical interpolation I have drawn fig. 1; it shows the curves of equal stress-difference throughout a meridional section of the spheroid. The numbers written on the curves represent the values of stress-difference when the radius of the sphere is unity and when the factors above referred to are omitted (see Plate 19, fig. 1).

The point marked 0 is that in which the stress-difference vanishes. Round this point are drawn two dotted curves along which the stress-differences are $\tfrac{1}{2}$ and $\tfrac{3}{4}$ respectively. The remainder of the curves are drawn for equidistant values of stress-difference, and are marked 1, 2, 3 . . . 8. The curve marked 1 is singular, for the whole of the surface forms one branch of it, whilst there is another branch which runs below the surface from the polar axis and then rises to the surface at the point where $\cos 2\theta=-\tfrac{1}{8}$, that is to say, in lat. 41° 25′. Near the centre the curves are approximately circular, and they become somewhat like ellipses as we recede from the centre.

If this figure be made to rotate about the polar axis, the several curves will of course generate the surfaces of equal stress-difference throughout the sphere.

Writing ϑ for the inclination of one of the principal axes to the equator, we have by means of the formula (27)

$$\cot 2\vartheta = \frac{\mathrm{P}-\mathrm{R}}{2\mathrm{T}} = 8\left\{\left(\frac{a}{r}\right)^2 - 1\right\} \operatorname{cosec} 2\theta - \cot 2\theta$$

It would be easy to trace out the changes of direction of the principal stress-axes throughout the sphere, but I will only now make the observation that all over the surface they are parallel to and perpendicular to the surface, and that at the centre they are polar and equatorial, the stress-quadric being of course an ellipsoid of revolution.

We have next to find the actual amount of stress-difference which arises from given ellipticity of form of the spheroid. Putting $i=2$ in (30) we have

$$\varsigma_i = \sin^2\theta - 2\cos^2\theta = 3[\tfrac{1}{3} - \cos^2\theta]$$

The equation to the spheroid is

$$\begin{aligned} r &= a + h\varsigma_i \\ &= a\left[1 + 3\frac{h}{a}(\tfrac{1}{3} - \cos^2\theta)\right] = a[1 + e(\tfrac{1}{3} - \cos^2\theta)] \end{aligned}$$

Thus $3h/a$ is the ellipticity of the spheroid, which we may put equal to e.

Then it was shown in (33) § 4 that the results for the stresses P, Q, R, &c., are to be multiplied by $-\frac{2}{5}gwh/a^2$, and this will of course be also the factor for the stress-difference Δ.

Then substituting e for $3h/a$, and introducing the factor $\frac{6}{19}$, which has been omitted in considering the distribution of stress within the spheroid, we see that ellipticity e gives a stress-difference represented by

$$\Delta = -\frac{4e}{95} gwa \sqrt{64\left(1 - \left(\frac{r}{a}\right)^2\right)^2 + \left(\frac{r}{a}\right)^4 - 16\left(\frac{r}{a}\right)^2\left(1 - \left(\frac{r}{a}\right)^2\right) \cos 2\theta}$$

If we estimate the forces in gravitation units the factor g must be omitted.

The expression under the square-root sign is equal to unity at the surface, and to 8 at the centre.

Thus the stress-differences, in gravitation units of force, at the surface and at the centre are $\frac{4}{95}ewa$ and $\frac{32}{95}ewa$ respectively.

To apply this to the case of the earth, take $a = 637 \times 10^6$ c.m., and $w = 5{\cdot}66$, and we find the surface and central stress-differences to be respectively $152e$ and $1214e$ metric tonnes per square centimeter.

If these numbers be multiplied by 6·34, we get the same result expressed in tons per square inch. Thus in British units these two stress-differences are $926e$ and $7698e$.

If then the ellipticity e be $\frac{1}{1000}$th, the surface and central stress-differences will be nearly 1 ton and nearly 8 tons to the square inch respectively.

From the Table VII. in § 9 it will appear that cast brass ruptures with a stress-difference of about 8 tons to the square inch.

Thus a spheroid, made of material as strong as brass, and of the same dimensions and density as the earth, would only just support an excess or deficiency of ellipticity equal to $\frac{1}{1000}$th, above or below the equilibrium ellipticity adapted for its rotation.

The following is a second example:—If the homogeneous earth (with ellipticity $\frac{1}{232}$) were to stop rotating, the stress-difference at the centre would be 33 tons per square inch.

Now suppose the cause of internal stress to be the moon's tide-generating influence, and let $m=$ moon's mass, and $c=$ moon's distance.

Then the potential under which the earth is stressed is $-\frac{3}{2}(m/c^3)(\frac{1}{3}-\cos^2\theta)wr^2$, or according to the notation of § 4 $-\frac{1}{2}(m/c^3)wr^2\mathrm{s}_2$.

If we took into account the elastic yielding of the earth and the weight and attraction of the tidal protuberance, this potential would have to be diminished. To estimate the diminution we must of course know the amount of elastic yielding, but as there is no means of approximating thereto, it will be left out of account.

Then it is obvious that the factor by which Δ, as given in (35), must be multiplied in order to give the stress-difference is $\frac{1}{2}mw/c^3$. Thus the surface stress-difference is $\frac{6}{19}\cdot\frac{1}{2}(m/c^3)wa^2$ in absolute force units.

Then putting M for the earth's mass, and dividing by gravity, we have $\frac{3}{19}(ma^3/Mc^3)wa$ as the surface value of Δ in gravitation units. The central value of Δ is of course eight times as great.

With the numerical data used above, $wa=3605$ metric tonnes per square c.m., and $m/M=\frac{1}{82}$, $a/c=\frac{1}{60}$, whence the surface stress-difference is 32 grammes, and the central stress-difference 257 grammes per square centimeter.

But this conclusion is erroneous for the following reason. If we suppose the moon to revolve in the terrestrial equator, and imagine that the meridian from which longitudes are measured is the meridian in which the moon stands at the instant under consideration, then the tide-generating potential is $-\frac{3}{2}(m/c^3)r^2[\frac{1}{3}-\sin^2\theta\cos^2\phi]$; this expression may be written $\frac{3}{4}(m/c^3)r^2(\frac{1}{3}-\cos^2\theta)+\frac{3}{4}(m/c^3)r^2\sin^2\theta\cos 2\phi$. The former of these terms produces a permanent increase of the earth's ellipticity, and is confused and lost in the ellipticity due to terrestrial rotation, and can produce no stress in the earth. The second term is the true tide-generating potential, but it is a sectorial harmonic, and I have failed to treat such cases. Now the first of these terms causes ellipticity in a homogeneous earth equal to $(\frac{5}{2}a/g)(\frac{3}{4}m/c^3)$ according to the equilibrium tide-theory. This ellipticity is equal to $\cdot1039\times10^{-6}$, an excessively small quantity. If however this permanent ellipticity does not exist (and the above investigation in reality presumes it not to exist), then there will be a superficial stress-difference equal to $152\times\cdot 1039\times 10^{-6}$ metric tonnes per square centimeter, and a central stress-difference of eight times as much.

Since a metric tonne is a million grammes this surface stress-difference is 16 grammes, and the central 128 grammes per square centimeter. These stress-differences are exactly the halves of those which have been computed above. Thus the remaining stress-difference which is due to the moon's tide-generating influence is 16 grammes at the surface and 128 grammes at the centre per square centimeter.

A flaw in this reasoning is that stress-difference is a non-linear function of the stresses, and therefore the stress-difference arising from the sum of two sets of bodily stresses is not the sum of their separate stress-differences.

I conceive however that the above conclusion is not likely to be much wrong.

These stresses are very small compared with those arising from the weights of mountains and continents as computed below, nevertheless they are so considerable that we can understand the enormous rigidity which Sir WILLIAM THOMSON has shown that the earth must possess in order to resist considerable tidal deformations of its mass.

§ 6. *On the stresses due to a series of parallel mountain chains.*

Having considered the case of the second harmonic, I now pass to the other extreme and suppose the order of harmonics i to be infinitely great, whilst the radius of the sphere is also infinitely great.

The equatorial belt now becomes infinitely wide, and the surface inequalities consist of a number of parallel simple harmonic mountains and valleys.

If i be infinitely large, we have from (12)

$$W_i=\rho^i\left[1-\frac{1}{2!}\left(\frac{iz}{\rho}\right)^2+\frac{1}{4!}\left(\frac{iz}{\rho}\right)^4-\&c.\right]$$

Now let ξ be the depth below the surface of the point indicated in the sphere (now infinitely large) by x, y, z.

As the formulas given above apply to the meridional plane for which $y=0$, we have $\rho=a-\xi$.

Now let $b=a/i$, then when both i and a are infinite

$$\rho^i=a^i\left(1-\frac{\xi}{a}\right)^i=a^i\epsilon^{-\xi/b}$$

and since in the limit $\rho/i=a/i=b$,

$$W_i=a^i\epsilon^{-\xi/b}\left(1-\frac{1}{2!}\frac{z^2}{b^2}+\frac{1}{4!}\frac{z^4}{b^4}-\&c.\right)$$

$$=(a^i)\epsilon^{-\xi/b}\cos\frac{z}{b}$$

This expression for W involves the infinite factor a^i, and in order to get rid of it we must now consider the factor by which it is to be multiplied, in introducing the height of the mountains and gravity.

This factor is computed in § 4; it is there shown that if $r=a+h\varsigma_i$ be a harmonic spheroid, the factor is $-2(i-1)gwh/(2i+1)a^i$.

Now if the harmonic i be of an infinitely high order, ς_i becomes simply $\cos z/b$, and the equation to the surface is

$$\xi=-h\cos\frac{z}{b}$$

ξ being measured downwards. Thus the harmonic spheroid $h\varsigma_i$ now represents a series of parallel harmonic mountains and valleys of height and depth h, and wave-length $2\pi b$.

The factor becomes $-gwh/a^i$, when i is infinite.

Thus the effective disturbing potential W, which is competent to produce the same state of stress and strain as the weight of the mountains and valleys, is given by

$$W=-gwh\epsilon^{-\xi/b}\cos\frac{z}{b} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (36)$$

Now revert to the expressions (11) for the stresses.

When i is infinite $I=2i^2$, and they become, on changing x into $(a-\xi)$

$$\mathrm{P}=\frac{1}{2i}(a^2-r^2)\frac{d^2W}{d\xi^2}+\frac{6(a-\xi)}{2i^2}\frac{dW}{d\xi}+\frac{3}{2i}W_i$$

$$\mathrm{T}=-\frac{1}{2i}(a^2-r^2)\frac{d^2W}{d\xi dz}-\frac{3}{2i^2}\left((a-\xi)\frac{dW}{dz}-z\frac{dW}{d\xi}\right)$$

Now as shown above $a^2-r^2=2a\xi$, and $a/i=b$ in the limit; making these substitutions, and dropping the terms which become infinitely small when i is infinite, we have

$$\left.\begin{array}{l}\mathrm{P}=\xi b\dfrac{d^2W}{d\xi^2},\ \mathrm{T}=-\xi b\dfrac{d^2W}{d\xi dz}\\[2ex] \mathrm{R}=\xi b\dfrac{d^2W}{dz^2},\ \mathrm{Q}=0\end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (37)$$

and by a similar process

Then from (36) and (37) we have

$$\left.\begin{array}{l}\mathrm{P}=-gwh\dfrac{\xi}{b}\epsilon^{-\xi/b}\cos\dfrac{z}{b}\\[2ex] \mathrm{R}=\ \ gwh\dfrac{\xi}{b}\epsilon^{-\xi/b}\cos\dfrac{z}{b}\\[2ex] \mathrm{T}=\ \ gwh\dfrac{\xi}{b}\epsilon^{-\xi/b}\sin\dfrac{z}{b}\end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (38)$$

Since the stress-difference

$$\Delta=\sqrt{(\mathrm{P}-\mathrm{R})^2+4\mathrm{T}^2}$$

we have

$$\Delta=2gwh\frac{\xi}{b}\epsilon^{-\xi/b}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (39)$$

The directions of the stress-axes are given by

$$\cot 2\vartheta = \frac{P-R}{2T} = \cot\frac{z}{b}$$

so that

$$\vartheta = \tfrac{1}{2}\frac{z}{b} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (40)$$

Equation (39) gives the stress-difference at a depth ξ below the mean surface, and is very remarkable as showing that the stress-difference depends on depth below the mean horizontal surface and not at all on the position of the point considered with reference to the ridges and furrows.

Equation (40) shows that if we travel along uniformly horizontally through the solid perpendicular to the ridges, the stress-axes revolve with a uniform angular velocity.

They are vertical and horizontal when we are under a ridge, and they have turned through a right angle and are again vertical and horizontal by the time we have arrived under a furrow.

Since the function $x\epsilon^{-x}$ is a maximum when $x=1$, the stress-difference Δ is a maximum when $\xi=b$,—that is to say, at a depth equal to $1/2\pi$ of the wave-length—and is then equal to $2gwh\epsilon^{-1}$ or in gravitation units of force to $\cdot 736\, wh$. It is interesting to notice that the value of this maximum depends only on the height and density of the mountains, and does not involve the distance from crest to crest. The depth at which this maximum is reached depends of course on the wave-length.

Plate 19, fig. 2, shows the distribution of stress-difference, the vertical ordinates represent stress-difference, and the horizontal ones depth below the surface. The numbers written on the horizontal axis are multiples of b; the distance OL on this scale is equal to 6·28, and is therefore equal to the wave-length from crest to crest, and the distance OH is the semi-wave-length from crest to furrow.

In the case of terrestrial mountains w is about 2·8, and if we suppose h to be 2000 meters, or a little over 6000 feet, we have the case of a series of lofty mountain chains—for it must be remembered that the valleys run down to 2000 meters below the mean surface, so that the mountains are some 13,000 feet above the valley-bottoms.

Then $h=2\times 10^5$, $w=2\cdot 8$, and the maximum stress-difference is

$$\cdot 736\times 2\cdot 8\times 2\times 10^5 = \cdot 412\times 10^6 \text{ grammes per square centimeter.}$$

This stress-difference is, in British measure, 2·6 tons per square inch.

If we suppose (as is not unreasonable) that it is 314 miles from crest to crest of the mountains, then the maximum stress will be reached at 50 miles below the surface.

From Table VII., § 9, it will be seen that if the materials of the earth at this depth of 50 miles had only as much tenacity as sheet lead, the mountain chains would sink down, but they would just be supported if the tenacity were equal to that of cast tin.

§ 7. *On the stresses due to the even zonal harmonic inequalities.*

Having considered the two extreme cases where i is 2, and infinity, I pass now to the intermediate ones. As the odd zonal harmonics were not required for the investigation in the following section I have only worked out in detail the even ones.

The surface of the sphere is now corrugated by a series of undulations parallel to the equator, and the altitude of the corrugations increases towards the poles. The form of the undulation in the neighbourhood of the equator is exhibited in Plate 19, fig. 3.

The stress-difference is as before given by

$$\Delta=\sqrt{(\mathrm{P}-\mathrm{R})^2+4\mathrm{T}^2}$$

To form this expression the series in (22) for R must be subtracted from the series for P. Since the *C*'s and *D*'s of Table II. have always the opposite signs from the *A*'s and *B*'s of Table I., this algebraic subtraction becomes a numerical addition of the numbers in these two tables.

The results are given in the following table.

Table IV.—The coefficients for expressing P−R.

i	A_0-C_0	A_2-C_2	A_4-C_4	A_6-C_6	B_0-D_0	B_2-D_2	B_4-D_4	B_6-D_6
2	−2·2105	−2·8421			+2·5263			
4	−4·3137	+2·7451	+7·8431		+4·3922	−7·5294		
6	−6·3636	+42	+35·6364	−14·2545	+6·4	−48·8727	+13·9636	
8	−8·3926	+139·1411	−38·8712	−168·2061	+8·4137	−148·0806	+188·4663	−21·5390
10	−10·4115	+318·2304	−633·7449	−478·0247	+10·4252	−329·2182	+965·7051	−491·6324
12	−12·425	+603·292	−2623·009	+552·212	+12·434	−616·315	+3243·765	−3806·018

Then we have

$$\begin{aligned}\mathrm{P}-\mathrm{R}=r^i\cos^i l[(A_0-C_0)+(A_2-C_2)\tan^2 l+\ldots]\\ +a^2r^{i-2}\cos^{i-2} l[(B_0-D_0)+(B_2-D_2)\tan^2 l+\ldots]\end{aligned}$$

The materials for computing T have been already given in Table III.

The series for P−R and for 2T should now be squared and added together, but the result would be so complex that it is not worth while to proceed algebraically.

At the equator T=0, and Δ=P−R, and P−R reduces to only two terms, whatever be the order of harmonic.

By reference to (23) and (24) we see that at the equator

$$\Delta=\frac{ir^{i-2}}{2(i+1)^2+1}\left[\frac{i(i+2)(2i-1)}{i-1}a^2-(i+1)(2i+3)r^2\right]$$

or

$$=\frac{i(i+1)(2i+3)a^2r^{i-2}}{(i+1)(2i+3)-i}\left[1-\left(\frac{r}{a}\right)^2+\frac{3}{(i^2-1)(2i+3)}\right] \quad . \quad . \quad . \quad . \quad . \quad (41)$$

This value for Δ requires of course multiplication by appropriate factors involving the height of the continents and gravity.

Even when i is no larger than 6, (41) differs but little from $ir^{i-2}(a^2-r^2)$, at least for values of r not very nearly equal to a.

Δ clearly reaches a maximum when

$$\left(\frac{r}{a}\right)^2=\frac{i-2}{i}\left\{1+\frac{3}{(i^2-1)(2i+3)}\right\}$$

For large values of i this maximum is nearly equal to $2\{(i-2)/i\}^{\frac{1}{2}i-1}a^i$

From these formulas the following results are easily obtained.

TABLE V. (a).

$i=$	2	4	6	8	10	12
Maximum value of Δ . . .	2·526	1·118	·959	·894	·859	·836
Value of r/a when Δ is max. .	0	·714	·819	·867	·895	·913

Plate 20, fig. 4, shows graphically the law of diminution of stress-difference for the even zonal harmonics, the vertical ordinates representing stress-difference and the horizontal ones the distances from the surface or from the centre of the globe.

In order to find a numerical value of these maximum stress-differences which shall be intelligible according to ordinary mechanical ideas, I will suppose the height of each of the harmonics at the equator to be 1500 meters. On account of the small density of the superficial layers in the earth, this is very nearly the same as supposing that in the earth the maximum height of the continents above, and the maximum depth of the depressions below the mean level of the earth are each about 3000 meters. In the summary at the end I shall show that there is reason to believe that this is about the magnitude of terrestrial inequalities.

Then by (33) we have to multiply the maximum stress-differences in the above table by $2(i-1)wh/(2i+1)$, in order to obtain the stress-differences for the supposed continents in grammes or tonnes per square centimeter.

Now $w=5{\cdot}66$ and $h=1{\cdot}5\times10^5$ according to the above hypothesis as to height of continent; and the coefficient in i is of course different for each harmonic.

By performing these multiplications I find the following results.

TABLE V. (*b*).—Maximum stress-differences due to harmonic continents and seas.

Order of harmonic.	2	4	6	8	10	12
Max. stress-difference, in metric tonnes per sq. c.m. due to continents 1500 meters high	·858	·633	·626	·625	·625	·625
Ditto in British tons per sq. inch, for same continents.	5·43	4·01	3·97	3·96	3·96	3·96
Depth in British miles at which this stress is attained	Centre of earth	1146	725	532	420	347

N.B.—The continents referred to are supposed to be of the earth's mean density and are equivalent to actual continents of double the height.

Thus far we have only considered the stress-differences at the equator immediately underneath the centres of the continents, but we must now see how they differ as the latitude of the place of observation increases. In order to attain this result a good deal of computation was necessary.

For this purpose I calculated P—R and 2T for a number of points and found the square root of the sum of these squares. As the computations were laborious, and as the results given in the following table are amply sufficient for the purpose in hand, I did not think it worth while to trace the changes to a greater depth than $r=\cdot 7$. Moreover the correctness of the last significant figures given cannot be guaranteed, although I believe that it is correct in most cases.

TABLE VI.—Showing the stress-difference due to the several harmonic inequalities at various depths and in various latitudes.

i		Equator.	Lat. 6°.	Lat. 12°.	i		Equator.	Lat. 6°.	Lat. 12°.
2	$r=1.$	·316	·316	·316	8	$r=1.$	·021	·015	·000
	$r=\cdot 9$	·736	·732	·721		$r=\cdot 9$	·859	·853	·853
	$r=\cdot 8$	1·112	1·108	1·097		$r=\cdot 8$	·798	·795	·797
	$r=\cdot 7$	1·443	1·440	1·431		$r=\cdot 7$	·506	·505	·507
4	$r=1.$	·079	·074	·061	10	$r=1.$	·014	·008	·007
	$r=\cdot 9$	·727	·719	·700		$r=\cdot 9$	·857	·854	·860
	$r=\cdot 8$	1·044	1·038	1·025		$r=\cdot 8$	·631	·630	·635
	$r=\cdot 7$	1·116	1·113	1·104		$r=\cdot 7$	·307	·307	·309
6	$r=1.$	·036	·031	·016	12	$r=1.$	·010	·003	·019
	$r=\cdot 9$	·817	·810	·800		$r=\cdot 9$	·827	·824	·835
	$r=\cdot 8$	·953	·949	·945		$r=\cdot 8$	·481	·481	·486
	$r=\cdot 7$	·788	·786	·785		$r=\cdot 7$	·179	·179	·181

The numbers given in the column marked "equator" might be computed from (41), and are those exhibited graphically in fig. 4; they are here given as a means of comparison with the numbers corresponding to latitudes 6° and 12°.

The result to be deduced from this table is that the lines of equal stress-difference are very nearly parallel with the surface, and that it is for all practical purposes sufficient to know the stress-difference immediately under the centre of the continents.

We have already seen in § 6 that for harmonics of infinitely high orders the lines of equal stress-difference are rigorously parallel with the mean surface.

§ 8. *On the stresses due to the weight of an equatorial continent.*

The actual continents and seas on the earth's surface have not got quite the regular wavy character of the elevations and depressions which have been treated hitherto. The subject of the present section possesses therefore a peculiar interest for the purpose of application to the earth. Had I however foreseen, at the beginning, the direction which the results of this whole investigation would take, it is probable that I might not have carried out the long computations which were required for discussing the case of an isolated continent. But now that that end has been reached, it seems worth while to place the results on record.

The function exp.$[-\cos^2\theta/\sin^2\alpha]$ (where θ is colatitude) obviously represents an equatorial belt of elevation, and I therefore chose it as the form of the required equatorial continent. This function has to be expanded in a series of zonal harmonics in order to apply the analytical solutions for the stresses produced by the weight of the continent.

It is obvious by inspection that the odd zonal harmonics can take no part in the representation of the function, and it was on this account that I have above only worked out the cases of the even zonal harmonics.

The multiplication of this function by the successive LEGENDRE'S functions, and integration over the surface of the sphere, are operations algebraically tedious, and wholly uninteresting, and I therefore simply give the results.

I find then that if $\alpha=10°$, and

$$\mathfrak{s}_i = \sin^i\theta - \frac{i^2}{2!}\sin^{i-2}\theta\cos^2\theta + \frac{i^2(i-2)^2}{4!}\sin^{i-4}\theta\cos^4\theta - \&c.$$

Then

$$\left.\begin{array}{l} 2\epsilon^{-\cos^2\theta/\sin^2\alpha} - \beta_0 = \beta_2\mathfrak{s}_2 + \beta_4\mathfrak{s}_4 + \beta_6\mathfrak{s}_6 + \beta_8\mathfrak{s}_8 + \beta_{10}\mathfrak{s}_{10} + \beta_{12}\mathfrak{s}_{12} + \dots \\ \text{where} \\ \beta_0 = \cdot 3078,\ \beta_2 = \cdot 3673,\ \beta_4 = \cdot 3339,\ \beta_6 = \cdot 2829,\ \beta_8 = \cdot 2252,\ \beta_{10} = \cdot 1688, \\ \quad \beta_{12} = \cdot 1193 \end{array}\right\} \quad . \qquad (42)$$

β_0 is put on the left-hand side in order that the mean value of the function may be zero. The β's obviously decrease very slowly, and as I stop with the 12th harmonic, the representation of the function is very imperfect.

Plate 20, fig. 5, illustrates the results of the representation, the portion of a circle marked "mean level of earth" represents a meridional section of the earth; the dotted curve marked "inequality to be represented" shows the true value of the function $2\exp.[-\cos^2\theta/\sin^2\alpha]-\beta_0$; the curve marked "representation" shows the right-hand side of (42) stopping with the 12th harmonic; the second curve is the same without the 2nd harmonic constituent $\beta_2\varsigma_2$, and it is introduced for the reasons explained in the discussion and summary at the end.

The equatorial value of the exponential function is 1·792, that of the "representation" is 1·497, and that of "the representation without the 2nd harmonic" is 1·130.

The polar value of the exponential is $-·3078$, that of the "representation" is $-·0830$, and that of "the representation without 2nd harmonic" is $+·6516$. This latter function thus gives us an equatorial continent and a pair of polar continents of less height.

The extreme difference of height in the "representation" between the elevation at the equator and the depression at the pole is $(1·497+·083)h$ or $1·58h$. I do not exactly know the extreme difference in the case where the 2nd harmonic is omitted, because I have not traced the inequality throughout, but it is probably about 1·3 or $1·4h$.

Now let Δ_i be the numerical value, as computed for § 7, of the stress-difference due to the harmonic spheroid ς_i. Then it has been shown above that the stress-difference due to the spheroid whose equation is $r=a+h\varsigma_i$ is $-2(i-1)gwh\Delta_i/(2i+1)$.

Now stress-difference is a non-linear function of the component stresses P, R, T, and therefore the stress-difference due to a compound harmonic spheroid is not in general the sum of the stress-differences due to the constituent harmonic spheroids. At any point, however, where the principal stress-axes are all coincident in direction and where all the greater stress-axes coincide, and not a greater with a less, and where $T=0$, the stress-difference is linear and is the sum of the constituent stress-differences. This is the case at the equator for the present equatorial continent.

Hence, if Δ be the stress-difference at the equator due to the spheroid,

$$r=a+h(\beta_2\varsigma_2+\beta_4\varsigma_4+\ldots+\beta_{12}\varsigma_{12})$$

We have

$$\Delta=-gwh\left[\tfrac{2}{5}\beta_2\Delta_2+\tfrac{6}{9}\beta_4\Delta_4+\tfrac{10}{13}\beta_6\Delta_6+\tfrac{14}{17}\beta_8\Delta_8+\tfrac{18}{21}\beta_{10}\Delta_{10}+\tfrac{22}{25}\beta_{12}\Delta_{12}\right] \quad . \quad . \quad . \quad (43)$$

In this formula the Δ_i's are the numbers which were computed for drawing Plate 20, fig. 4, from the formula (41), namely

$$\Delta_i=-\frac{i(i+1)(2i+3)}{2(i+1)^2+1}\left(\frac{r}{a}\right)^{i-2}\left[1-\left(\frac{r}{a}\right)^2+\frac{3}{(i^2-1)(2i+3)}\right]$$

By using these computations I have drawn Plate 20, fig. 6. The vertical ordinates are $-\Delta\div gwh$, and the horizontal are the distances from surface or centre of the sphere.

The maxima in the two curves are merely found graphically, and the distances where the maxima are reached (viz.: 660 and 590 miles from the surface) are written down on the supposition that the radius of the sphere is 4000 miles.

In the discussion in the second part of this paper, I have endeavoured to make an estimate of the proper elevation to attribute to these isolated continents; so as to make the case, as nearly as may be, analogous to the earth.

Although it appears impossible to make an accurate estimate, I conclude that it will not be excessive if we assume that the greatest difference of height, between the highest point in the equatorial elevation and the approximately spherical remainder of the globe, is 2000 meters.

Accordingly for the representation I put $1{\cdot}58h=2000$, and for the second curve $1{\cdot}4h=2000$; these give $h=1{\cdot}27\times10^5$ c.m. and $h=1{\cdot}4\times10^5$ c.m. respectively.

Taking $w=5{\cdot}66$, then for the representation we have $wh={\cdot}72$ tonnes per square centimeter, and for the other curve $wh={\cdot}79$ of the same units. The maximum stress-differences are ${\cdot}91wh$ and ${\cdot}76wh$ respectively.

Therefore for the equatorial table-land (called above the representation) we have a maximum stress-difference of ·66 metric tonnes per square c.m. or 4·1 British tons per square inch; and for the equatorial table-land balanced by a pair of polar continents (2nd harmonic omitted) we have a maximum stress-difference of ·60 tonne per square c.m. or 3·8 tons per square inch.

I therefore conclude that our great continental plateaus produce a stress-difference of about 4 tons per square inch at a depth of 600 or 700 miles from the earth's surface.

§ 9. *On the strength of various substances.*

In order to have a proper comprehension of the strength which the earth's mass must possess in order to resist the tendency to rupture, produced by the unequal distribution of weights on the surface, it is necessary to consider the results of experiments.

Rankine[*] gives a large number of results obtained by various experimenters, and Sir William Thomson also gives similar tables in his article on 'Elasticity.'[†]

Amongst other constants Sir William gives Young's modulus and the greatest elastic extension. If the materials of a wire remain perfectly elastic when the wire is just on the point of breaking under tension, then the product of Young's modulus into the greatest elastic extension should be equal to what is called the tenacity, which is defined as the breaking tension per square centimeter of the area of the wire.

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[*] 'Useful Rules and Tables:' Griffin, London, 1873, p. 191, *et seq.*

[†] 'Elasticity:' Black, Edinburgh, 1878. This is the article from the 'Encyclopedia Britannica,' but it is also published as a separate work.

If however a permanent set begins to take place before the wire breaks, this product should be less than the tenacity. I do not see how it can ever be greater, unless there be a marked departure from HOOK's law "ut tensio sic vis;" or different sets of experiments with the same class of material might make it seem greater. In some of the results given by Sir WILLIAM THOMSON the product of modulus and elastic extension is however greater than tenacity.

Ordinary experience would lead one to suppose that such materials as lead and copper would undergo a considerable stress beyond the limits of perfect elasticity, before breaking. It is surprising therefore to see how nearly identical this product is to the tenacity—indeed in the case of lead absolutely identical, as may be seen in the table below.

With regard to the earth we require to know what is the limiting stress-difference under which a material takes permanent set or begins to flow, rather than the stress-difference under which it breaks; for if the materials of the earth were to begin to flow, the continents would sink down and the sea bottoms rise up.

It will be seen from the definition of tenacity given above that it is the rupturing stress-difference for tensional stresses. There is no word specially applied to rupturing stress-difference under pressure.

I am inclined to think that for the purposes of this investigation these tables in most cases rate the strength of materials somewhat too highly; for it seems probable that a permanent set would be taken, if a material were subjected for a long time to a stress-difference, which is a considerable fraction of the limiting value. We are likely to know more on this point in some years time when the wires hung by Sir WILLIAM THOMSON in the tower of Glasgow University have been subjected to several years of tension. However this may be I give the results of some of the experiments as collected and quoted by Sir WILLIAM THOMSON and the late Professor RANKINE. The first table of tenacity, except the results denoted by the letter R, are taken from Sir WILLIAM THOMSON. The second table of crushing stress-difference is taken entirely from RANKINE. The multiplications and reductions to different units I have done myself.

TABLE VII.—Limiting stress-difference.

Produced by tension.				Produced by crushing.		
Material.	Breaking stress-difference in metric tonnes per square centimeter.	Stress-difference at which permanent set begins in—		Material.	Breaking stress-difference in—	
		Metric tonnes per square centimeter.	British tons per square inch.		Metric tonnes per square centimeter.	British tons per square inch.
Sheet lead. . . .	·23	·23	1·46	Strong red brick .	·077	·49
Cast tin	·416	·417	2·64	Strong sandstone .	·39	2·45
,, ...	·325 (R)	. .	2·06 (R)			
Wood (ash) . . .	1·20	1·20	7·61	(F) Strong limestone	·60	3·80
Cast brass. . . .	1·27	1·27	8·05	Marble	·39	2·45
,, iron	·94 to 2·04	1·14 to 1·87	7·23 to 11·86	Granite	·39 to ·77	2·45 to 4·91
Drawn copper . .	4·10	4·00	25·36	(F) Granite (Mount Sorrel)	·905	5·74
English steel pianoforte wire . . .	23·62	23·56	149·6	(F) Grauwacke. .	1·19	7·54
(R) Brick, cement .	·020 to ·021	. .	·125 to ·134	Ash (along the grain)	·63	4·02
(R) Glass	·66	. .	4·20	Cast brass . . .	·73	4·60
(R) Slate	·68 to ·90	. .	4·3 to 5·7	Wrought iron . .	2·52 to 2·84	16 to 18

NOTE.—The second and third columns give the product of YOUNG'S modulus into greatest elastic extension, and this should give the stress-difference when permanent set begins. RANKINE does not give the data for this quantity, but the breaking stress-difference is given in both metric and British units, the latter being in the third column.

In the second half of the table the results marked F are from Sir WILLIAM FAIRBAIRN'S experiments.

The only cases in these two tables in which we have the opportunity of comparing the strength for resisting the stress-difference, when produced in the two manners, is for the materials cast brass and ash; in both cases we see that the substance is considerably weaker for crushing than for tension.

I should be inclined to suppose that the crushing strength is more nearly the datum we require for the case of the stresses in the earth.

In the first half of the table we probably see the effect of permanent set in the cases of copper and pianoforte wire (compare 4·00 with 4·10, and 23·56 with 23·62), but it is surprising that the contrast between the two columns is not more marked.

§ 10. *On the case when the elastic solid is compressible.*

It appears desirable to know how far the results of the preceding investigation may differ, if the elastic solid be compressible. According to the views of Dr. RITTER, referred to in the summary, this must be largely the case.

As I did not examine this point until after the above was completed, it seemed preferable to make a fresh beginning, rather than to modify the whole investigation.

The process is however so closely analogous, that it presents no difficulty, and may be dismissed shortly. I shall accordingly follow closely the processes of §§ 1 and 2.

If a solid be very compressible it takes a comparatively small hydrostatic pressure to produce a given amount of compression ; that is to say, although the compressibility is large, the modulus of compressibility is small compared with that of rigidity. The modulus of compressibility I shall call the incompressibility. In the preceding investigation the converse was the case, for the incompressibility was infinite compared with the rigidity. By the definitions of ω and υ in § 1, the incompressibility

$$k=\omega-\tfrac{1}{3}\upsilon.$$

It will be found convenient to use k and ω as the two moduli, which define the nature of the elastic solid.

In the denominators in E_i, F_i, G_i of (1), the expression $(2(i+1)^2+1)\omega-(2i+1)\upsilon$ occurs, this I shall call K, in analogy with I.

Then

$$K=2i(i-1)\omega+3(2i+1)k$$

If we develop the last differential coefficient in the expression (1) for α, we find

$$2\upsilon K\alpha=3k\left(\frac{i}{i-1}a^2-r^2\right)\frac{dW}{dx}+i\omega\left[(a^2-r^2)\frac{dW}{dx}+2xW\right] \quad . \quad . \quad . \quad . \quad (44)$$

Also

$$K\delta=iW \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (45)$$

and

$$\left.\begin{array}{l} \mathrm{P}=(3k-2\omega)\delta+2\upsilon\dfrac{d\alpha}{dx} \\ \mathrm{T}=\upsilon\left(\dfrac{d\alpha}{dz}+\dfrac{d\gamma}{dx}\right) \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (46)$$

Differentiating (44) with regard to x, and substituting in (46) we find

$$K\mathrm{P}=3k\left[\left(\frac{i}{i-1}a^2-r^2\right)\frac{d^2W}{dx^2}-2x\frac{dW}{dx}+iW\right]+i\omega(a^2-r^2)\frac{d^2W}{dx^2} \quad . \quad . \quad . \quad (47)$$

Again, differentiating (44) with regard to z, writing down $d\gamma/dx$ by symmetry, and adding the two together, we have from (46)

$$K\mathrm{T}=3k\left[\left(\frac{i}{i-1}a^2-r^2\right)\frac{d^2W}{dxdz}-z\frac{dW}{dx}-x\frac{dW}{dz}\right]+i\omega(a^2-r^2)\frac{d^2W}{dxdz} \quad . \quad . \quad . \quad (48)$$

From (47) and (48) the other stresses may be written down by cyclic changes of letters.

Now let us suppose that the incompressibility is small compared with the rigidity.

Then ω becomes nearly equal to $\frac{1}{3}v$, and k is small compared with v and ω. Also K becomes nearly equal to $2i(i-1)\omega$. Hence in this case

$$\left.\begin{aligned} \mathrm{P}&=\frac{1}{2(i-1)}(a^2-r^2)\frac{d^2W}{dx^2} \\ \mathrm{T}&=\frac{1}{2(i-1)}(a^2-r^2)\frac{d^2W}{dxdz} \end{aligned}\right\} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (49)$$

Now if we suppose W to be a zonal harmonic of degree i, and only consider the state of stress at the equator, immediately underneath the centre of the elevation, then T is zero and by (15)

$$\mathrm{P}=\frac{i(i-1)}{2(i-1)}(a^2-r^2)r^{i-2}$$

$$\mathrm{R}=-\frac{i^2}{2(i-1)}(a^2-r^2)r^{i-2}$$

And

$$\Delta=\mathrm{P}-\mathrm{R}=\frac{i(2i-1)}{2(i-1)}(a^2-r^2)r^{i-2} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (50)$$

If i be large $\Delta=i(a^2-r^2)r^{i-2}$.

In § 7 we have found this identical result, under the like conditions, when the solid is incompressible.

Now take the case of the 2nd harmonic, so that

$$W=r^2(\tfrac{1}{3}-\cos^2\theta)=\tfrac{1}{3}(x^2+y^2-2z^2)$$

And we have

$$\begin{aligned} \mathrm{P}&=\ \ \tfrac{1}{3}(a^2-r^2) \\ \mathrm{Q}&=\ \ \tfrac{1}{3}(a^2-r^2) \\ \mathrm{R}&=-\tfrac{2}{3}(a^2-r^2) \\ \mathrm{T}&=\ \ 0 \end{aligned}$$

Thus throughout the spheroid, the principal stress-axes are parallel and perpendicular to the polar axis; also

$$\Delta=(a^2-r^2)$$

Hence the central stress-difference is a^2. If the solid be incompressible it was found to be $8a^2$. Hence infinite compressibility largely relieves the stress-difference due to ellipticity of figure. Next take the case when the harmonic is of the 4th order.

Then at the equator we have by (50)

$$\Delta=\tfrac{4\cdot 7}{6}r^2(a^2-r^2)=\tfrac{14}{3}r^2(a^2-r^2)$$

The maximum is reached, when $r^2=\frac{1}{2}a^2$ or $r=\cdot707a$, and is equal to $\frac{7}{6}=1\cdot167$. Comparing this with the Table V. (a) § 7, we see that infinite compressibility makes very little difference, for ·707 differs little from ·714 and 1·167 from 1·118.

It will now be shown that whatever may be the compressibility of the solid, we get identically the same solution in the case when the harmonic is of an infinitely high order. This is the problem of the harmonic mountains and valleys corrugating a mean level surface, which was considered in § 6. The same notation will be adopted here.

Both i and a are infinite, x becomes $a-\xi$, and $ia^2/(i-1)-r^2=a^2-r^2=2a\xi$. If the substitutions here suggested be made in (47) and (48), it will be found that the terms with ω as a factor are multiplied by ai (two infinites), whilst none of the terms with k as a factor involve more than one infinite. Hence the latter terms are negligeable compared with the former.

Again i being infinite, $K=2i^2\omega$. Thus if i and a be infinite (47) and (48) reduce to

$$\mathrm{P}=\frac{1}{2i^2}(i2a\xi)\frac{d^2W}{d\xi^2},\ \mathrm{R}=\frac{1}{2i^2}(i2a\xi)\frac{d^2W}{dz^2},\ \mathrm{T}=-\frac{1}{2i^2}(i2a\xi)\frac{d^2W}{d\xi dz}$$

but $a/i=b$, therefore

$$\mathrm{P}=b\xi\frac{d^2W}{d\xi^2},\ \mathrm{R}=b\xi\frac{d^2W}{dz^2},\ \mathrm{T}=-b\xi\frac{d^2W}{d\xi dz}$$

In (36) we found the effective potential, which produced the same state of stress as the weight of the mountains whose equation was $-\xi=h\cos z/b$; the result was

$$W=-gwh\,\epsilon^{-\xi/b}\cos\frac{z}{b}$$

Hence in the present case

$$\mathrm{P}=-gwh\frac{\xi}{b}\epsilon^{-\xi/b}\cos\frac{z}{b},\ \mathrm{R}=gwh\frac{\xi}{b}\epsilon^{-\xi/b}\cos\frac{z}{b},\ \mathrm{T}=gwh\frac{\xi}{b}\epsilon^{-\xi/b}\sin\frac{z}{b}$$

These results are identical with equations (38), which gave the stresses when the elastic solid was incompressible.

It may be concluded from this that, except for the case of the 2nd harmonic inequality, compressibility makes very little difference, and for the higher harmonics it makes no difference at all.

II.

SUMMARY AND DISCUSSION.

The existence of dry land proves that the earth's surface is not a figure of equilibrium appropriate for the diurnal rotation.

Hence the interior of the earth must be in a state of stress, and as the land does not sink in, nor the sea-bed rise up, the materials of which the earth is made must be strong enough to bear this stress.

We are thus led to inquire how the stresses are distributed in the earth's mass, and what are their magnitudes. These points cannot be discussed without an hypothesis as to the interior constitution of the earth.

In this paper I have solved a problem of the kind indicated for the case of a homogeneous incompressible elastic sphere, and have applied the results to the case of the earth.

It may of course be urged that the earth is not such as this treatment postulates.

The view which was formerly generally held was that the earth consists of a solid crust floating on a molten nucleus. It has also been lately maintained by Dr. AUGUST RITTER in a series of interesting papers that the interior of the earth is gaseous.[*] A third opinion, contended for by Sir WILLIAM THOMSON, and of which I am myself an adherent, is that the earth is throughout a solid of great rigidity; he explains the flow of lava from volcanoes either by the existence of liquid vesicles in the interior, or by the melting of solid matter, existing at high temperature and pressure, at points where diminution of pressure occurs.

There is another consideration, which is consistent with Sir WILLIAM THOMSON'S view, and which was pointed out to me by Professor STOKES. It may be that underneath each continent there is a region of deficient density; then underneath this region there would be no excess of pressure.

For the present investigation it is to some extent a matter of indifference as to which of these views is correct, for if it is only the crust of the earth which possesses rigidity, or if Professor STOKES'S suggestion of the regions of deficient density be correct, then the stresses in the crust or in the parts near the surface must be greater than those here computed—enormously greater if the crust be thin,[†] or if the region of deficient density be of no great thickness.

With regard to the property of incompressibility which is here attributed to the elastic sphere, it appears from §10 that even if we suppose the elastic solid to be *very* highly compressible, yet the results with regard to the internal stresses are almost the same as though it were incompressible. I think the hypothesis of great incompressibility is likely to be much nearer to the truth than is that of great compressibility. I shall therefore adhere to the supposition of infinite incompressibility, bearing in mind that even great compressibility would not much affect most of the results.

I take then a homogeneous incompressible elastic sphere, and suppose it to have the power of gravitation and to be superficially corrugated. In consequence of mathematical difficulties the problem is here only solved for the particular class of surface inequalities called zonal harmonics, the nature of which will be explained below.

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[*] 'Anwendung der mechanischen Wärmetheorie auf kosmologische Probleme.' CARL RÜMPLER, Hannover, 1879. This is a reprint of six papers in WIEDEMANN'S Annalen.

Dr. RITTER contends that the temperature in the interior of the planet is above the critical temperature and that of dissociation for all the constituents, so that they can only exist as gas. Data are wanting with regard to the mechanical properties of matter at, say 10,000° Fahr., and a pressure of many tons to the square inch. Is it not possible that such "gas" may have the density of mercury and the rigidity and tenacity of granite? Although such a conjectural "gaseous" solid might possess high rigidity, it almost certainly would have great compressibility: but it is proved in § 10 that the compressibility will make exceedingly little difference in the result of the present investigation excepting in the case of the 2nd harmonic inequality.

[†] The evaluation of the stresses in a crust, with fluid beneath, would be tedious, but not more difficult than the present investigation. I may perhaps undertake this at some future time.

Before discussing the state of stress produced by these inequalities, it will be convenient to explain the proper mode of estimating the strength of an elastic solid under stress.

At any point in the interior of a stressed elastic solid there are three lines mutually at right angles, which are called the principal stress-axes. Inside the solid at the point in question imagine a small plane (say a square centimeter or inch) drawn perpendicular to one of the stress-axes; such a small plane will be called an inter-face.* The matter on one side of the ideal inter-face might be removed without disturbing the equilibrium of the elastic solid, provided some proper force be applied to the inter-face; in other words, the matter on one side of an inter-face exerts a force on the matter on the other side. Now a stress-axis has the property that this force is parallel to the stress-axis to which the inter-face is perpendicular. Thus along a stress-axis the internal force is either purely a traction or purely a pressure. Treating pressures as negative tractions, we may say that at any point of a stressed elastic solid, there are three mutually perpendicular directions along which the stresses are purely tractional. The traction which must be applied to an inter-face of a square centimeter in area, in order to maintain equilibrium when the matter on one side of the inter-face is removed, is called a principal stress, and is of course to be measured by grammes weight per square centimeter.

If the three stresses be equal and negative, the matter at the point in question is simply squeezed by hydrostatic pressure, and it is not likely that in a homogeneous solid any simple hydrostatic pressure, *absolutely* equal in all directions, would ever rupture the solid. The effect of the equality of the three stresses when they are positive and tractional is obscure, but at least physicists do not in general suppose that this is the cause of rupture when a solid breaks.

If the three principal stresses be unequal, one must of course be greatest and one least, and there is reason to suppose that tendency of the solid to rupture is to be measured by the difference between these principal stresses.

In one very simple case we know that this is so, for if we imagine a square bar, of which the section is a square centimeter, to be submitted to simple longitudinal tension, then two of the principal stresses are zero (namely, the stresses perpendicular to the faces of the rod), and the third is equal to the longitudinal traction. The traction under which the rod breaks is a measure of its strength, and this is equal to the difference of principal stresses.

If at the same time the rod were subjected to great hydrostatic pressure, the breaking load would be very little, if at all affected; now the hydrostatic pressure subtracts the same quantity from all three principal stresses, but leaves the difference between the greatest and least principal stresses the same as before.

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* This term is due to Professor JAMES THOMSON.

Difference of principal stresses may also be produced by crushing.

In this paper I call the difference between the greatest and least principal stresses the "stress-difference," and I say that, if calculation shows that the weight of a certain inequality on the surface of the earth will produce such and such stress-difference at such and such a place, then the matter at that place must be at least as strong as matter which will break when an equal stress-difference is produced by traction or crushing.

I shall usually estimate stress-difference by metric tonnes (a million grammes) per square centimeter, or by British tons per square inch.

In Table VII., § 9, are given the experimentally determined values of the breaking stress-difference for various substances. The table is divided into two parts, in the former of which the stress-difference was produced by tension, and in the latter by crushing. It is not necessary here to advert to the difference in meaning of the numbers given in the first column and those given in the two latter columns in the first half of the table.

The cases of wood and cast brass are the only ones where a comparison is possible between the two breaking stress-differences, as differently produced. It will be seen that the material is weaker for crushing than for tension. For the reasons given in that section, I am inclined to think that these tables rate the strength of the materials somewhat too highly for the purposes of this investigation. I conceive that the results derived from crushing are more appropriate for the present purpose than those derived from tension; and fortunately the results for various kinds of rocks seem to have been principally derived from crushing stresses.

This table will serve as a means of comparison with the numerical results derived below, so that we shall see, for example, whether or not at 500 miles from the surface the materials of the earth are as strong as granite.

We may now pass to the mathematical investigation. It appears therefrom that the distribution of stress-difference is quite independent of the absolute heights and depths of the inequalities. Although the questions of distribution and magnitude of the stresses are thus independent, it will in general be convenient to discuss them more or less simultaneously.

The problem has only been solved for the class of superficial inequalities called zonal harmonics, and their nature will now be explained.

A zonal harmonic consists of a series of undulations corrugating the surface in parallels of latitude with reference to some equator on the globe; the number of the undulations is estimated by the order of the harmonic. The harmonic of the second order is the most fundamental kind, and consists of a single undulation forming an elevation round the equator, and a pair of depressions at the poles of that equator; it may also be defined as an elliptic spheroid of revolution, and the absolute magnitude is measured by the ellipticity of the spheroid.

If the order of the harmonic be high, say 30 or 40, we have a regular series of mountain chains and intervening valleys running round the sphere in parallels of latitude.

For the sake of convenience I shall always speak as though the equator were a region of elevation, but the only effect of changing elevations into depressions, and *vice versâ*, is to diametrically reverse the directions of all the stresses.

The harmonics of the orders 2, 6, 10, &c., have depressions at the poles of the sphere; those of orders 4, 8, 12, &c., have elevations at the pole.

The harmonic of the fourth order consists of an equatorial continent and a pair of circular polar continents, with an intervening depression. That of the sixth order consists of an equatorial continent and a pair of annular continents in latitudes (about) 60° on one and the other side of the equator. The 8th harmonic brings down these new annular continents to about latitude 45°, and adds a pair of polar continents; and so on.

By a continuation of this process the transition to the mountain chains and valleys is obvious.

In § 5 the case of the 2nd harmonic is considered. As above explained the sphere is deformed into a spheroid of revolution. The investigation also applies to the case of a rotating spheroid, such as the earth, with either more or less oblateness than is appropriate for the figure of equilibrium.

The lines throughout a meridional section of the spheroid along which the stress-difference is constant are shown in Plate 19, fig. 1, and the numbers written on the curves give the relative magnitude of the stress-difference.

It is remarkable that the stress-difference is the same all over the surface. In the polar regions the stress-difference diminishes as we descend into the spheroid and then increases again; in the equatorial regions it always increases as we descend. The maximum value is at the centre, and there the stress-difference is eight times as great as at the surface.

If the elastic solid be highly compressible the stress-differences are not nearly so great as on the hypothesis of incompressibility. In all the other cases considered in this paper compressibility makes practically no difference in the results.

On evaluating the stress-difference, on the hypothesis of incompressibility, arising from given ellipticity in a spheroid of the size and density of the earth, it appears that if the excess or defect of ellipticity above or below the equilibrium value (namely $\frac{1}{232}$ for the homogeneous earth) were $\frac{1}{1000}$, then the stress-difference at the centre would be 8 tons per square inch, and accordingly, if the sphere were made of material as strong as brass (see Table VII.), it would be just on the point of rupture. Again if the homogeneous earth, with ellipticity $\frac{1}{232}$, were to stop rotating, the central stress-difference would be 33 tons per square inch, and it would rupture if made of any material excepting the finest steel.

A rough calculation* will show that if the planet Mars has ellipticity $\frac{1}{80}$ (about twice the ellipticity on the hypothesis of homogeneity) the central stress-difference must be 6 tons per square inch. It was formerly supposed that the ellipticity of the planet was even greater than $\frac{1}{80}$, and even if the latest telescopic evidence had not been adverse to such a conclusion, we should feel bound to regard such supposed ellipticity with the greatest suspicion, in the face of the result just stated.

The state of internal stress of an elastic sphere under tide-generating forces is identical with that caused by ellipticity of figure.† Hence the investigation of § 5 gives the distribution of stress-difference caused in the earth by the moon's attraction. In Plate 19, fig. 1, the point called "the pole" is the point where the moon is in the zenith.

Computation shows that the stress-difference at the surface, due to the lunar tide-generating forces, is 16 grammes per square centimeter, and at the centre eight times as much. These stresses are considerable, although very small compared with those due to terrestrial inequalities, as will appear below.

In § 6 the stresses produced by harmonic inequalities of high orders are considered. This is in effect the case of a series of parallel mountains and valleys, corrugating a mean level surface with an infinite series of parallel ridges and furrows. In this case compressibility makes absolutely no difference in the result, as shown in § 10.

It is found that the stress-difference depends only on the depth below the mean surface, and is independent of the position of the point considered with regard to ridge and furrow; the direction of the stresses does however depend on this latter consideration.

In Plate 19, fig. 2, is shown the law by which the stress-difference increases and then diminishes as we go below the surface. The vertical ordinates of the curve indicate the relative magnitude of the stress-difference, and the horizontal ones the depth below the surface. The depth OL on the figure is equal to the distance between contiguous ridges, and the figure shows that the stress-difference is greatest at a depth equal to $\frac{1}{6 \cdot 28}$ of OL.

The greatest stress-difference depends merely on the height and density of the mountains, and the depth at which it is reached merely on the distance from ridge to ridge.

Numerical calculation shows that if we suppose a series of mountains, whose crests are 4000 meters or about 13,000 feet above the intermediate valley-bottoms, formed of rock of specific gravity 2·8, then the maximum stress-difference is 2·6 tons per square inch (about the tenacity of cast tin); also if the mountain chains are 314 miles apart the maximum stress-difference is reached at 50 miles below the mean surface.

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[*] The data for the calculation are: Ratio of terrestrial radius to Martian radius 1·878. Ratio of Martian mass to terrestrial mass ·1020. Whence ratio of Martian gravity to terrestrial gravity ·3596. Central stress-difference, due to ellipticity e, $996e$ tons per square inch. "Homogeneous" ellipticity of Mars $\frac{1}{166}$; and $\frac{996}{166}$ equal to 6.

[†] This is subject to certain qualifications noticed in § 5.

It may be necessary to warn the geologist that this investigation is approximate in a certain sense, for the results do not give the state of stress actually within the mountain prominences or near the surface in the valley-bottoms. The solution will however be very nearly accurate at some five or six miles below the valley-bottoms. The solution shows that the stress-difference is *nil* at the mean surface, but it is obvious that both the mountain masses and the valley-bottoms are in some state of stress.

The mathematician will easily see that this imperfection arises, because the problem really treated is that of an infinite elastic plane, subjected to simple harmonic tractions and pressures.

To find the state of stress actually within the mountain masses would probably be difficult.

The maximum stress-difference just found for the mountains and valleys obviously cannot be so great as that at the base of a vertical column of this rock, which has a section of a square inch and is 4000 meters high. The weight of such a column is 7·1 tons, and therefore the stress-difference at the base would be 7·1 tons per square inch. The maximum stress-difference computed above is 2·6, which is about three-eighths of 7·1 tons per square inch. Thus the support of the contiguous masses of rock, in the case just considered, serves as a relief to the rock to the extent of about five-eighths of the greatest possible stress-difference. This computation also gives a rough estimate of the stress-differences which must exist if the crust of the earth be thin. It is shown below that there is reason to suppose that the height from the crest to the bottom of the depression in such large undulations as those formed by Africa and America is about 6000 meters. The weight of a similar column 6000 meters high is nearly 11 tons.

In § 7 I take the cases of the even zonal harmonics from the 2nd to the 12th, but for all except the 2nd harmonic only the equatorial region of the sphere is considered.

Plate 19, fig. 3, shows an exaggerated outline of the equatorial portion of the inequalities; it only extends far enough to show half of the most southerly depression, even for the 12th harmonic. It did not seem worth while to trace the surfaces of equal stress-difference throughout the spheroid, but the laborious computations are carried far enough to show that these surfaces must be approximately parallel to the surface of the mean sphere. It is accordingly sufficient to find the law for the variation of stress-difference immediately underneath the equatorial belt of elevation. It requires comparatively little computation to obtain the results numerically, and the results of the computation are exhibited graphically in Plate 20, fig. 4.

Table V. (*b*), § 7, gives the maximum stress-differences, resulting from these several inequalities, computed under conditions adequately noted in the table itself. It will be convenient to postpone the discussion of the results.

In § 8 I build up out of these six harmonics an isolated equatorial continent. The nature of the elevation is exhibited in Plate 20, fig. 5, in the curve marked "representation;" no notice need be now taken of the dotted curve. This curve exhibits a belt of elevation of about 15° of latitude in semi-breadth, and the rest of the spheroid is approximately spherical. This kind of elevation requires the 2nd as one of its harmonic constituents, and this harmonic means ellipticity of the whole globe. Now it may perhaps be fairly contended that on the earth we have no such continent as would require a perceptible 2nd harmonic constituent. I therefore give in Plate 20, fig. 5, a second curve which represents an equatorial belt of elevation counterbalanced by a pair of polar continents in such a manner that there is no second harmonic constituent.

I have not attempted to trace the curves of equal stress-difference arising from these two kinds of elevation, but I believe that they will consist of a series of much elongated ovals, whose longer sides are approximately parallel with the surface of the globe, drawn about the maximum point in the interior of the sphere at the equator. The surfaces of equal stress-difference in the solid figure will thus be a number of flattened tubular surfaces one within the other.

At the equator however the law of variation of stress-difference is easy to evaluate, and Plate 20, fig. 6, shows the results graphically, the vertical ordinates representing stress-difference and the horizontal the depths below the surface. The upper curve in Plate 20, fig. 6, corresponds with the "representation curve" of Plate 20, fig. 5, and the lower curve with the case where there is no 2nd harmonic constituent.

The central stress-difference, which may be observed in the upper curve, results entirely from the presence of the 2nd harmonic constituent in the corresponding equatorial belt of elevation.

The maximum stress-differences in these two cases occur at about 660 and 590 miles from the surface respectively.

We now come to perhaps the most difficult question with regard to the whole subject—namely, how to apply these results most justly to the case of the earth.

The question to a great extent turns on the magnitude and extent of the superficial inequalities in the earth. As the investigation deals with the larger inequalities, it will be proper to suppose the more accentuated features of ridges, peaks, and holes to be smoothed out.

The stresses caused in the earth by deficiency of matter over the sea beds are the same as though the seas were replaced by a layer of rock, having everywhere a thickness of about $\frac{1\cdot02}{2\cdot75}$ or nearly $\frac{4}{11}$ of the actual depths of sea.

The surface being partially smoothed and dried in this manner, we require to find an ellipsoid of revolution which shall intersect the corrugations in such a manner that the total volume above it shall be equal to the total volume below it.

Such a spheroid may be assumed to be the figure of equilibrium appropriate to the earth's diurnal rotation; if it departs from the equilibrium form by even a little, then we shall much underestimate the stress in the earth's interior by supposing it to be a form of equilibrium.

Professor Bruns has introduced the term "geoid" to express any one of the "level" surfaces in the neighbourhood of the earth's surface, and he endeavours to form an estimate of the departure of the continental masses and sea-bottoms from some mean geoid.* From the geodesic point of view the conception is valuable, but such an estimate is scarcely what we require in the present case. The mean geoid itself will necessarily partake of the contortions of the solid earth's surface, even apart from disturbances caused by local inequalities of density, and thus it cannot be a figure of equilibrium.

Thus, even if we were to suppose that the solid earth were everywhere coincident with a geoid—which is far from being the case—a state of stress would still be produced in the interior of the earth.

An example of this sort of consideration is afforded by the geodesic results arrived at by Colonel Clarke, R.E.,† who finds that the ellipsoid which best satisfies geodesic measurement, has three unequal axes, and that one equatorial semi-axis is 1524 feet longer than the other. Now such an ellipsoid as this, although not exactly one of Bruns' geoids, must be more nearly so than any spheroid of revolution; and yet this inequality (if really existent, and Colonel Clarke's own words do not express any very great confidence) must produce stress in the earth. Colonel Clarke's results show an ellipticity of the equator equal to $\frac{1}{13731}$, and this in the homogeneous elastic earth will be about equivalent to ellipticity $\frac{1}{27000}$; such ellipticity would produce a central stress-difference of $\frac{7698}{27000}$ or nearly one-third of a British ton per square inch.

From this discussion it may, I think, be fairly concluded that if we assume the sea-level as being the figure of equilibrium and estimate the departures therefrom, we shall be well within the mark.

The average height of the continents is about 350 meters (1150 feet), and the average depth of the great oceans is in round numbers 5000 meters (16,000 feet); but the latter datum is open to much uncertainty.‡ When the sea is solidified into rock the 5000 meters of depth is reduced to 3200 meters below the actual sea-level. Thus the average effective depression of sea-bed is about nine times as great as the average height of the land. I shall take it as exactly nine times as great, and put the depth as 3150 meters; but it is of course to be admitted that perhaps eight and perhaps ten might be more correct factors.

In the analytical investigation of this paper the outlines of the vertical section of the continents and depressions are always sweeping curves of the harmonic type, and the magnitude of the elevations and depressions are estimated by the greatest heights and depths, measured from a mean surface which equally divides the two.

We have already supposed the outlines of continents and sea-beds to have been smoothed down into sweeping curves, which we may take as being, roughly speaking, of the harmonic type. The smoothing will have left the averages unaffected.

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* 'Die Figur der Erde.' Von Dr. H. Bruns. Berlin: Stankiewicz, 1878.

† Phil. Mag., Aug., 1878.

‡ In a previous paper, "Geological Changes, &c.," Phil. Trans., Vol. 167, Part I., p. 295, I have endeavoured to discuss this subject, and references to a few authorities will be found there.

The averages are not however estimated from a mean spheroidal surface, but from one which is far distant from the mean.

The questions now to be determined are as follows:—What is the proper greatest height and depression, estimated from a mean spheroid, which will bring out the above averages estimated from present sea-level, and what is the position of the mean spheroid with reference to the sea-level.

From the solution of the problem considered in the note below,* it appears that, if

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* Conceive a series of straight harmonic undulations corrugating a mean horizontal surface, and suppose them to be flooded with water. This will represent fairly well the undulations on the dried earth, and the water-level will represent the sea-level.

Suppose that the average heights and depths of the parts above and below water are known, and that it is required to find the position of the mean horizontal surface with reference to the water-level, and the height of the undulations measured from that mean surface.

Take an origin of coordinates in the water-level, the axis of x in the water-level and perpendicular to the undulations, and the axis of y measured upwards.

Let

$$y=h(\cos x-\cos a)$$

be the equation to the undulations.

The average height of the dry parts is clearly $\frac{1}{2a}\int_{-a}^{+a} y\,dx$ or $\frac{h}{a}(\sin a-a\cos a)$. Similarly the average depth below water is $\frac{h}{\pi-a}[\sin(\pi-a)-(\pi-a)\cos(\pi-a)]$ or $\frac{h}{\pi-a}\sin a+(\pi-a)\cos a$

If the latter average be p times as great as the former

$$ph\cos a\left(\frac{1}{a}\tan a-1\right)=h\cos a\left(\frac{1}{\pi-a}\tan a+1\right)$$

This is an equation for determining a.

Now I find that $a=34^\circ\ 30'$ gives $p=8{\cdot}983$, which corresponds very nearly with $p=9$ of the text above.

This value of a corresponds with an average equal to $\cdot 1165h$ for the height above water, and $1{\cdot}0469h$ for the depth below water. Now if we put

$$1{\cdot}0469h=3150 \text{ meters}$$

$$\text{which gives } \cdot 1165h=350 \text{ meters very nearly,}$$

we have $h=3009$ meters.

The depth below water-level of the mean level is $h\cos 34^\circ\ 30'$ or 2480 meters.

The greatest height of the dry part above the water-level is $3009-2480$ or 429 meters, and the greatest depth of the submerged part below water-level is $3009+2480$ or 5489 meters.

[After the proof-sheets of this paper had been corrected, Professor Stokes pointed out to me that, according to Rigaud (Cam. Phil. Soc., vol. 6), the area of land is about four-fifteenths of the whole area of the earth's surface. Now, in the ideal undulations we are here considering the area above water is about one-tenth of the whole area; hence in this respect the analogy is not satisfactory between these undulations and the terrestrial continents. If I have not considerably over-estimated the average depth of the sea (and I do not think that I have done so), the discrepancy must arise from the fact that actual continents and sea-beds do not present in section curves which conform to the harmonic type; there must also be a difference between corrugated spherical and plane surfaces.

The geological denudation of the land must, to some extent, render our continents flat-topped.—*Added May 4th*, 1882.]

the continents and sea-beds have sections which are harmonic curves, then if we take,—

The mean level bisecting elevations and depressions as 2480 meters (8150 feet) below the sea-level, and the greatest elevation and depression from that mean level as 3009 meters (9840 feet), it results that the *average* height of the land above sea-level is 350 meters and the *average* depression of dried sea-bed is 3150 meters.

It thus appears that 3000 meters would be a proper greatest elevation and depression to assume for the harmonic analysis of this paper, if the earth were homogeneous. But as the density of superficial rocks is only a half of the mean density of the earth, I shall take 1500 meters as the greatest elevation and depression from the mean equilibrium spheroid of revolution.

It is proper here to note that the height of the undulations of elevation and depression in the zonal harmonic inequalities is considerably greater towards the poles than it is about the equator; it might therefore be maintained that by making 1500 meters the equatorial height, we are taking too high an estimate. But the state of stress caused in the sphere at any point depends very much more on the height of the inequality in the neighbourhood of a superficial point immediately over the point considered, than it does on the inequalities in remote parts of the sphere.

Now in all the inequalities, except the 2nd harmonic, I have considered the state of stress in the equatorial region, and it will therefore I think be proper to adhere to the 1500 meters for the greatest height and depression.

We have next to consider, what order of harmonic inequalities is most nearly analogous to the great terrestrial continents and oceans. The most obvious case to take is that of the two Americas and Africa with Europe. The average longitude of the Americas is between 60° and 80° W., and the average longitude of Africa is about 25° E., hence there is a difference of longitude of about a right-angle between the two masses. These two great continents would be more nearly represented by an harmonic of the sectorial class,[*] rather than by a zonal harmonic, nevertheless I think the solution for the zonal harmonic will be adequate for the present purpose.

Now it has been explained above that the harmonic of the fourth order represents an equatorial continent and a pair of polar continents. In the case of the 4th harmonic therefore there is a right angle of a great circle between contiguous continents. We may conclude from this that the large terrestrial inequalities are about equivalent to the harmonic of the fourth order.

Table V. (*b*), § 7, gives the maximum stress-differences under the centre of the equatorial elevation of the several zonal harmonics, the height of each being 1500 meters.

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[*] The sectorial harmonic of the fourth order $\sin^4\theta \cos 4\phi$ would well represent these two great continents. It would fairly represent China and Australia; but would annihilate the Himalayan plateau, and place another great continent in mid-Pacific. It is not at all difficult to find the stress-difference under the centre of a sectorial inequality, but to find it generally involves the solution of a cubic equation.

The point at which this maximum is reached is given in each case, and Plate 20, fig. 4, illustrates graphically the law of variation of stress-difference.

The second harmonic cannot be said to represent a continent, and the table shows that in each of the other cases the maximum stress-difference is very nearly 4 tons per square inch. The depths of the maximum point are of course very different in each case.

We have concluded above that Africa and America are about equivalent to an harmonic of the fourth order, hence it may be concluded that the stress-difference under those continents is at a maximum at more than 1100 miles from the earth's surface, and there amounts to about 4 tons per square inch. A comparison with Table VII. shows that marble would break under this stress, but that *strong* granite would stand.

The case of the isolated continent investigated in § 8 appeared likely to prove the most interesting one, for the purpose of application to the case of the earth. But unfortunately I have found it difficult to arrive at a satisfactory conclusion as to the proper height to attribute to the continent.

The average height of the American continent is about 1100 feet above the sea, and the average depth of the Pacific Ocean about 15,000 feet. If the water of the Pacific be congealed into rock, it will have an effective depth of 10,000 feet. The greatest height of the American continent above the bed of the dried Pacific when smoothed down must be fully 12,000 feet or 3700 meters. The height of the great central Asian plateau above the average bed of the southern ocean (after drying) must be considerably more than this.

Now in the application to the homogeneous planet the heights are to be halved to allow for the smaller density of surface rock.

I therefore take 2000 meters as the height of the top of the equatorial table-land above the remaining approximately spherical portion of the sphere.

The investigation of § 8 then shows that the equatorial table-land will give rise to a stress-difference of 4·1 tons per square inch at a depth of 660 miles; and that the equatorial table-land counterbalanced by the pair of polar continents (the second harmonic constituent being absent) gives a stress difference of about 3·8 tons per square inch at a depth of 590 miles.

This estimate of stress-difference agrees in amount, with singular exactness, with that just found from the case of the 4th zonal harmonic, but the maximum is reached 400 or 500 miles nearer to the earth's surface.

I think there can be no doubt but that there *are* terrestrial inequalities of much greater breadth than that of my isolated continent; thus this investigation for the isolated continent will give a position for the maximum stress-difference too near the surface to correspond with the largest continents. On the other hand, I do not feel at all sure that I have not considerably underestimated the height of such a comparatively narrow plateau.

In the present paper it has been impossible to take any notice of the stresses produced by the most fundamental inequality on the earth's surface, because it depends essentially on heterogeneity of density.

It is well known that the earth may be divided into two hemispheres, one of which consists almost entirely of land, and the other of sea. If the south of England be taken as the pole of a hemisphere, it will be found that almost the whole of the land, excepting Australia, lies in that hemisphere, whilst the antipodal hemisphere consists almost entirely of sea. This proves that the centre of gravity of the earth's mass is more remote from England, than the centre of figure of the solid globe.

A deformation of this kind is expressed by a surface harmonic of the first order, for such an harmonic is equivalent to a small displacement of the sphere as a whole, without true deformation. Now if we consider the surface forces produced by such a deformation in a homogeneous sphere, we find of course that there is an unbalanced resultant force acting on the whole sphere in the direction diametrically opposed to that of the equivalent displacement of the whole sphere.

The fact that in the homogeneous sphere such an unbalanced force exists shows that in this case the problem is meaningless; it is in fact merely equivalent to a mischoice in the origin for the coordinates. But in the case of the earth such an inequality does exist, and the force referred to must of course be counterbalanced somehow. The balance can only be maintained by inequalities of density, which are necessarily unknown. The problem therefore apparently eludes mathematical treatment.

It is certain that so wide-spreading an inequality, even if not great in amount, must produce great stress within the globe. And just as the 2nd harmonic produces a more even distribution of stress than the 4th, so it is likely that the first would produce a more even distribution than the 2nd.

It is difficult to avoid the conclusion that the whole of the solid portion of the earth is in a sensible state of stress.

I would not however lay very much emphasis on this point, because we are in such complete ignorance as to the manner in which the equilibrium of the solid part of the earth is maintained.

From this discussion it appears that if the earth be solid throughout, then at a thousand miles from the surface the material must be as strong as granite. If it be fluid or gaseous inside, and the crust a thousand miles thick that crust must be stronger than granite, and if only two or three hundred miles in thickness much stronger than granite. This conclusion is obviously strongly confirmatory of Sir WILLIAM THOMSON'S view that the earth is solid throughout.

V. *Experimental Determination of the Velocity of White and of Coloured Light.*

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Received May 17,—Read May 19, 1881.

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PART I.

ABSOLUTE VELOCITY OF WHITE LIGHT.

Introductory.

IT is remarkable that, although the importance of an accurate knowledge of the velocity of light has been very generally appreciated, no attempt has hitherto been made in this country to measure that velocity by experiment. Our own experiments date from many years back, but we have been prevented by various interruptions to our work from giving a result which could lay claim to the greatest accuracy. In 1878 we made at Pitlochry, in Perthshire, between 600 and 700 observations, but the toothed wheel which was made for us not having the number of teeth in it which we had ordered, we were not able to eliminate perfectly certain unknown quantities occurring in the formulæ, and we felt that it would be better to wait until we could give a result in which we had perfect confidence. At the same time we resolved so to alter our apparatus that we should not have to depend upon the mean of a very large number of experiments to give us a good result, but that each observation should give us an accurate measurement, free from all doubt. This has now been accomplished by the experiments conducted in 1880–81 between Kelly House, Wemyss Bay, and the hills behind Innellan, across the mouth of the River Clyde.

The chief importance of a determination of the velocity of light is that it gives us the means, considered by many to be the best means, of determining the solar parallax, by combining the result with the constant of aberration determined by astronomers. The investigation has also acquired a further interest from the speculations of the late Professor CLERK MAXWELL, according to which the propagation of light is an electro-magnetic phenomenon, and its velocity should be the same as that of the propagation of an electro-magnetic displacement.

Our researches have, however, led us to results of great importance in another direction. We find reasons for believing that the different colours of which white light is composed do not travel with the same velocity, but that the more refrangible rays travel more rapidly through a vacuum, and that this difference is by no means very small, so that we may expect its presence to be determined by independent tests. The influence of this conclusion upon our views about the constitution of the ether is considerable, and will doubtless lead to some further knowledge of its properties.

The general theory of the method employed in the present research resembles that of M. FIZEAU. In his experiments, and in those of M. CORNU, a beam of light is sent through between the teeth of a toothed wheel to a distance of some miles, whence it is reflected back again by means of certain optical arrangements, which bring it back to the same point. If the wheel be rotating at a suitable rate, then, by the time that the light has gone to the distant station and back, the position of the space between two teeth is now occupied by a tooth and the light cannot pass. But if the velocity of the toothed wheel be doubled that position will be occupied by the next space and the light is able to pass. If the velocity be increased threefold, fourfold, &c., we have alternately eclipses and full brightness.

If m be the number of teeth in the wheel, it must, in the time that light has gone to the distant station and back again, have completed $\frac{1}{2m}$ of a revolution in order to bring a tooth into the position previously occupied by a space at the commencement of that time. If the wheel be found to be making N revolutions a second at the time of the first eclipse, then $\frac{1}{2m\mathrm{N}}$ is the fraction of a second taken by light to perform the double journey; and if the distance between the toothed wheel and the distant reflector be D then the velocity of light is

$$\mathrm{V}=4m\mathrm{ND}.$$

In the method which we devised, instead of having only one distant reflector, we have two, nearly in the same line, but one of them being at a greater distance than the other and a little to one side of it. Let us call the most distant reflector A and the other one B. The light reflected from A is eclipsed with a slower revolution of the toothed wheel than that from B; because the number of revolutions required is N, and we have

$$N=\frac{V}{4mD}.$$

But D_A (the distance to A) is greater than D_B (the distance to B); hence N_A (the speed of revolution producing the first eclipse with A) is less than N_B (the speed of revolution producing the first eclipse of B).

After the light from A has been eclipsed it begins to increase in brightness, while that from B is still diminishing. In the method of the present research we determine the speed of revolution when the two lights appear to be of equal brightness.

When we proceed to the second, third, &c., eclipses the difference in speed required to produce an eclipse in A and in B increases, and it may happen that at a certain speed the light from A reaches a maximum at the time when that from B is at a minimum, or *vice versâ*.

The superiority of this method over that of M. FIZEAU seemed to be that instead of having to determine the instant at which a light disappears we have only to determine the instant at which two lights seem to be of equal brightness. Every one who has been engaged in photometry is aware that the former is an operation of great difficulty and doubt, whereas the latter is one of very great delicacy.

The particular way in which we deduce the velocity of light from an observation of the equality of these two lights will now be explained. It must be especially noticed that we never have occasion to use the lights when near an eclipse, at which time, as CORNU has shown, irregularities are introduced into the formulæ.

We found that a great simplification was introduced in the formulæ by observing the 12th and 13th equalities (the ratio D_B to D_A being that of 12 to 13); so that two observations, one at each of these equalities, sufficed to give us a value of the velocity of light. We were also able to utilise some pairs of observations at the 13th and 14th equalities.

This method enabled us to use the electric light, which was in many ways the most convenient to us; for we are not dependent upon the absolute brightness of the light (which of course is liable to variations) but only on the proportionate brightness of the two reflected stars; and to prevent any error arising from any variation in this proportion (owing to fog, &c.), we never used a pair of observations which were separated in time by more than a few minutes.

Mathematical theory of our method.

If k be the width of a tooth, and $1-k$ be the width of the space between two teeth, and if E be the brightness of the star of light seen by reflection from the distant collimator when the toothed wheel is not in position, then the brightness of that star when the toothed wheel is in position and revolving slowly, but fast enough to cause the persistence of visual impressions to prevent us from distinguishing the separate teeth, is $E(1-k)$.

When the speed is increased the intensity depends upon the time taken by light to go to the distant reflector and to return. As the speed of the toothed wheel is gradually increased the star of light diminishes and increases in regular phases.

If the teeth be wider than the spaces the maximum light is less than $\frac{1}{2}E$, and the light is eclipsed not only at certain critical speeds, but also during a change of speed of rotation, which may be considerable if k be much greater than $\frac{1}{2}$.

If the spaces be wider than the teeth the maximum light is greater than $\frac{1}{2}E$, and the light is at no speed completely eclipsed, the minimum light being considerable if k be much less than $\frac{1}{2}$.

If the teeth of the wheel be not perfectly uniform, and if some teeth are wider than others, this will alter the law according to which the intensity varies with the speed of rotation. But, if the inequalities be not very great indeed, it is only in the neighbourhood of the maxima and minima that its influence is shown. The general effect is to increase the intensity of light at the minimum and to diminish it at the maximum.

If there be no very great irregularities in the width of the teeth, then (1) the *diminution* of intensity of the light with a definite increase of speed in the odd phases is perfectly constant, provided that in the interval of time taken by light to perform the double journey, the advancing part of each tooth of the wheel passes beyond the advancing part of the r^{th} tooth in front of it, and does not reach the following part of the $\overline{r+1}|^{\text{th}}$ tooth nor its following part reach the advancing part of the r^{th} tooth in front, where r is any whole number from zero upwards.*

Similarly, if there be no very great irregularities in the width of the teeth, then (2) the *increase* of intensity of the light with a definite increase of speed in the even phases is perfectly constant, provided that in the interval of time taken by light to perform the double journey the advancing part of each tooth in the wheel passes beyond the following part of the r^{th} tooth in front of it, and does not reach the advancing part of that tooth. This is in the condition where the width of a tooth is less than that of a space. If the width of a tooth be greater than that of a space

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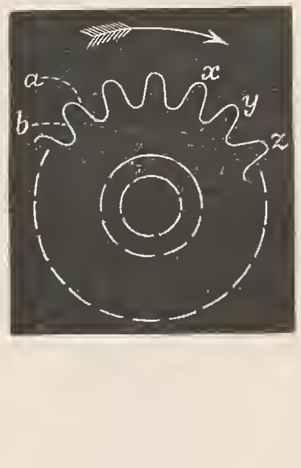
* We mean, by this notation, that if in the figure the arrow represents the direction of rotation of the toothed wheel a is the advancing and f the following part of that tooth, x, y, and z are the third, fourth, and fifth teeth in front of it.



the condition is that in the same interval of time the following part of each tooth should pass beyond the advancing part of the r^{th} tooth beyond it, but should not reach the following part of the $\overline{r+1}|^{\text{th}}$ tooth.

It is seen then that with a wheel which is carefully cut the rate of change of intensity of the light with change of speed of the toothed wheel is quite constant at that part of a phase which is about half way between the maximum and minimum brightness.

Our method of experimenting has reference to this part of a phase alone.

Let us call the period,—

From original brightness to the first extinction, the first phase;
From first extinction to next full brightness, the second phase;
From second full brightness to second extinction, the third phase;

and so on.

Let N be the number of revolutions per second made by the wheel when the first central eclipse is attained.

Let m be the number of teeth in the wheel.

Let D be the distance from the toothed wheel to the distant reflector.

Let I be the intensity of the star of light when the number of revolutions a second made by the toothed wheel is n.

Let V be the velocity of light.

Let τ be the time taken by light to perform the double journey, $\tau=\frac{2\text{D}}{\text{V}}$.

1. In the case of a wheel with teeth of the same width as the spaces (*i.e.*, $\kappa=\frac{1}{2}$), $\frac{d\text{I}}{dn}$ is constant, then

$$\frac{d\text{I}}{dn}=a;\ \text{I}=an+\text{C}$$

But if we be considering the first phase, then when $n=0$, $\text{I}=\frac{1}{2}\text{E}$; and when $n=\text{N}$, $\text{I}=0$.

Therefore

$$\text{C}=\tfrac{1}{2}\text{E},\ a=-\tfrac{1}{2}\frac{\text{E}}{\text{N}},$$

and

$$\text{I}=-\tfrac{1}{2}\frac{\text{E}}{\text{N}}n+\tfrac{1}{2}\text{E}$$

$$\frac{d\text{I}}{dn}=-\tfrac{1}{2}\frac{\text{E}}{\text{N}}$$

2. In the case of a wheel with any width of teeth, the value of $\frac{d\text{I}}{dn}$, in the middle of an odd phase, is the same as in the last example, but the maximum intensity is $\text{E}(1-\kappa)$. Suppose that it is the p^{th} phase which we are considering, p being odd; then

$$\frac{dI}{dn}=-\tfrac{1}{2}\frac{E}{N}$$

$$I=-\tfrac{1}{2}\frac{E}{N}(n+b)$$

but when

$$n=(p-1)N,\ I=E(1-\kappa)$$

whence

$$b=a-N\{2(1-\kappa)+p-1\}$$

$$I=\frac{E}{2}\left\{2(1-\kappa)+p-1-\frac{n}{N}\right\}$$

3. In the case of a wheel with any width of teeth, the value of $\frac{dI}{dn}$, in the middle of an even phase, is the same as in the two previous examples in magnitude, but of opposite sign, and the maximum intensity is $E(1-\kappa)$. Suppose that it is the p^{th} phase which we are considering, p being even; then

$$\frac{dI}{dn}=+\tfrac{1}{2}\frac{E}{N}$$

$$I=\tfrac{1}{2}\frac{E}{N}(n+c)$$

but when

$$n=pN,\ I=E(1-\kappa)$$

whence

$$c=N\{2(1-\kappa)-p\}$$

and

$$I=\frac{E}{2}\left\{2(1-\kappa)-p+\frac{n}{N}\right\}$$

If we have two distant reflectors, A and B, nearly in the same line, but at different distances D_A and D_B, and having consequently different speeds N_A and N_B, and different intrinsic brightnesses E_A and E_B. Then, if D_A be greater than D_B, N_A is less than N_B, and we shall have equalities of brightness of A and B at different speeds. The first equality is in the first phase of B and the second of A. The second equality is in the second phase of B and the third of A, and so on. The r^{th} equality is in the r^{th} phase of B and the $\overline{r+1}|^{\text{th}}$ of A, provided that $r(N_B-N_A)$ is less than $2N_A$. We need not consider any other case.

4. Let us consider the case of the r^{th} equality when r is even. Here B is increasing, A is decreasing. We have then $p_A=r+1$, and $p_B=r$,

$$I_A=I_B\,;\ \text{or}\frac{E_A}{2}\left\{2(1-\kappa)+(r+1)-1-\frac{n}{N_A}\right\}=\frac{E_B}{2}\left\{2(1-\kappa')-r+\frac{n}{N_B}\right\}^{*}$$

---

* We are taking account of the possibility of κ being different for A and B. But in practice this is never the case.

If $E_B = \rho E_A$ this becomes

$$2(1-\kappa)+r-\frac{n}{N_A}=\rho\left\{2(1-\kappa')-r+\frac{n}{N_B}\right\}$$

If ρ, κ and κ' were known we could determine the values of N_A and N_B from this equation when the velocity n giving equality of brightness had been determined, and when the distances D_A and D_B were known $\left(\text{for } \frac{N_A}{N_B}=\frac{D_B}{D_A}\right)$. But these quantities, ρ, κ, and κ', are not known.

5. Let us now consider the $\overline{r+1}|^{\text{th}}$ equality, when r has the same value as before. Here B is decreasing, A is increasing. We have $p_A = r+2$, $p_B = r+1$.

$$I'_A = I'_B\;;\ \text{or } \frac{E'_A}{2}\left\{2(1-\kappa)-(r+2)+\frac{n'}{N_A}\right\}=\frac{E'_B}{2}\left\{2(1-\kappa')+(r+1)-1-\frac{n'}{N_B}\right\}$$

Since E'_B still $=\rho E'_A$, this becomes

$$2(1-\kappa)-(r+2)+\frac{n'}{N_A}=\rho\left\{2(1-\kappa')+r-\frac{n'}{N_B}\right\}$$

6. From these two measurements, viz., of n and n', we can determine N_A and N_B in special circumstances. Let $\frac{D_A}{D_B}=g$. Then we have two equations

$$2(1-\kappa)+r-\frac{n}{N_A}=\rho\left\{2(1-\kappa')-r+\frac{n}{gN_A}\right\}$$

$$2(1-\kappa)-(r+2)+\frac{n'}{N_A}=\rho\left\{2(1-\kappa')+r-\frac{n'}{gN_A}\right\}$$

Subtracting, we have

$$2(r+1+\rho r)=\frac{1}{N_A}\left(\frac{\rho}{g}+1\right)(n+n')=\frac{g+\rho}{N_B}(n+n')$$

If now the distances D_A and D_B have been carefully chosen so as to make $g=\frac{r+1}{r}$, this equation becomes

$$2r(g+\rho)=\frac{g+\rho}{N_B}(n+n')$$

and

$$N_B=\frac{n+n'}{2r}$$

From this we can deduce the velocity of light. For

$$\tau_B=\frac{2D_B}{V}$$

and also

$$\tau_B = \frac{1}{2mN_B}$$

whence

$$V = 4mN_B . D_B$$
$$= \frac{2m(n+n')D_B}{r}$$

7. If, however, $\frac{r+1}{r}$ be not quite exactly equal to g, let

$$\frac{r+1}{r} = g+\delta$$

our equation becomes

$$2r(g+\rho+\delta) = \frac{g+\rho}{N_B}(n+n')$$

and

$$N_B = \frac{n+n'}{2r} . \frac{g+\rho}{g+\rho+\delta}$$
$$= \frac{n+n'}{2r}\left(1 - \frac{\delta}{g+\rho} + \frac{\delta^2}{(g+\rho)^2} - \ldots\right)$$

and

$$V = \frac{2m(n+n')D_B}{r}\left(1 - \frac{\delta}{g+\rho} + \frac{\delta^2}{(g+\rho)^2} - \ldots\right)$$

By employing the first term alone, in this last factor, we obtain an approximate value of N_B which enables us to calculate the value of ρ on the supposition that $\kappa = \kappa' = \frac{1}{2}$ (which is always very closely arranged so in practice). Thus we find the value of the small correction involved in the second term. The third term can always be neglected in our experiments.

8. To find the approximate value of ρ under these circumstances, we have now the two first equations in § 6, reduced to

$$1+r-\frac{gn}{N} = \rho\left(1-r+\frac{n}{N_B}\right)$$

and

$$1-(r+2)+\frac{gn'}{N_B} = \rho\left(1+r-\frac{n'}{N_B}\right)$$

Adding these we have

$$\frac{g}{N_B}(n'-n) = \rho\left\{2 - \frac{1}{N_B}(n'-n)\right\}$$

and

$$\rho = \frac{g(n'-n)}{2N_B-(n'-n)}$$

also

$$g+\rho = \frac{2gN_B}{2N_B-(n'-n)}$$

The smallness of the correction due to the term $\frac{\delta}{g+\rho}$ in our experiments can be seen by examining the details of the observations.

In our experiments $g=\frac{13}{12}$ almost exactly. Most of our measurements were made with the 12th equality. Some however were made with the 13th equality. We will now examine the formula for this case.

9. Let us consider the case of the $\overline{r+2}|^{\text{th}}$ equality, r having the same value as before. Here B is increasing, A is decreasing. We have $p_A=r+3$, $p\ =r+2$.

$$I''_A=I''_B \text{ or } \frac{E''_A}{2}\left\{2(1-\kappa)+(r+3)-1-\frac{n''}{N_A}\right\}=\frac{E''_B}{2}\left\{2(1-\kappa')-(r+2)+\frac{n''}{N_B}\right\}$$

Since E''_B still $=\rho E''_A$, this becomes

$$2(1-\kappa)+r+2-\frac{n''}{N_A}=\rho\left\{2(1-\kappa')-(r+2)+\frac{n''}{gN_A}\right\}$$

But at the $\overline{r-1}|^{\text{th}}$ equality

$$2(1-\kappa)-(r+2)+\frac{n'}{N_A}=\rho\left\{2(1-\kappa')+r-\frac{n'}{gN_A}\right\}$$

subtracting we have

$$2(r+2)+2\rho(r+1)=\frac{1}{N_A}\left(1+\frac{\rho}{g}\right)(n'+n'')=\frac{g+\rho}{N_B}(n'+n'')$$

If now D_A and D_B be so related that $g=\frac{r+2}{r+1}$, this equation becomes

$$2(r+1)(g+\rho)=\frac{g+\rho}{N_B}(n'+n'')$$

$$N_B=\frac{n'+n''}{2(r+1)} \text{ and } V=\frac{2m(n'+n'')D_B}{r+1}$$

If however $\frac{r+2}{r+1}=g+\delta'$ the true value of N_B is

$$N_B=\frac{n'+n''}{2(r+1)}\left(1-\frac{\delta'}{g+\rho}+\frac{\delta'^2}{(g+\rho)^2}-\ \dots\right).$$

To approximate to the value of $g+\rho$ on the assumption that $\kappa=\kappa'=\frac{1}{2}$ we notice that the above equations reduce to

$$1+r+2-\frac{gn''}{N_B}=\rho\left\{1-(r+2)+\frac{n''}{N_B}\right\}$$

$$1-(r+2)+\frac{gn'}{N_B}=\rho\left\{1+r-\frac{n'}{N_B}\right\}$$

adding these we have

$$2-\frac{g}{N_B}(n''-n')=\rho\left(\frac{n''-n'}{N_B}\right)$$

and

$$\rho=\frac{2N_B-g(n''-n')}{n''-n'}$$

also

$$g+\rho=\frac{2N_B}{n''-n'}$$

Thus

$$V=\frac{2m(n'+n'')D_B}{r+1}\left\{1-\frac{\delta'(n''-n')}{2N_B}+\ldots\right\}$$

when the 13th and 14th equalities are used.

Description of the apparatus.

The optical arrangements devised by M. Fizeau were admirably suited to the purpose, and were adopted, with modifications, by M. Cornu.

The telescope of emission, which is also the observing telescope, is pointed towards the distant stations. At its focus is placed the revolving toothed wheel; between that and the eye-piece is a diagonally-inclined piece of unsilvered glass.

The reflector designed by Fizeau for the distant station may be called a reflecting collimator; it consists of a telescope pointing towards the observing telescope, but, instead of having an eyepiece, it has in its principal focus a silver reflector. The advantages of this arrangement are its extreme simplicity and the facility of directing it—great accuracy of adjustment being unnecessary.

The rays of light coming from the sun, or any source of light,* are concentrated by means of a lens to throw an image of the source of light, after reflection at the diagonal mirror, upon the edge of the toothed wheel. If a tooth be not in the way these rays spread out and fill the object-glass, whence they proceed to the distant reflecting collimator, where they arrive all nearly parallel in direction. There they are caused by the object-glass to throw upon the focal mirror a luminous image of the object-glass of the observing telescope. The rays are then reflected to the object-glass, whence they proceed to the observing telescope and produce at its focus an image of the object-glass of the reflecting collimator. If no tooth be in the way, the rays proceed onwards through the eye-piece to the eye of the observer, who sees an illuminated image of the distant object-glass, which from its small size looks like a star of light. If now the toothed wheel revolves very slowly he sees the star eclipsed at intervals by the teeth; but if it be revolving so that at least ten teeth pass in a second, then, owing to the persistence of visual impressions he sees the star as a continuous light upon a brightish field produced by the illuminated rotating teeth. If the speed be further increased the brightness of the star diminishes owing to the light which passed between two teeth in leaving the observing station being partially stopped on its return by the advance of a tooth towards that space. So with increasing speed he sees the star disappear, then re-appear, attain its full brightness, diminish, disappear, reappear, &c., passing through similar phases with perfect regularity.

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* In the experiments of 1880–81 we always used the electric light.

One thing more is required either in the method of M. **Fizeau** or our own, and that is to have the means of determining at any instant the velocity of rotation of the toothed wheel. **Cornu** was the first to attempt to do this with accuracy: he connected the mechanism of the toothed wheel electrically with a chronograph so as to make a mark every 100 revolutions of the toothed wheel. A clock at the same time marked seconds, and by means of a vibrating spring tenths of a second were marked; while a fourth marker was under the control of the observer, who signalled the instant when he wished the velocity to be determined. **Cornu** did not attempt to maintain a uniform speed of revolution in the toothed wheel, but was able by means of the chronographic record to tell the velocity and rate of change of velocity at definite times, and hence, by interpolation, the exact velocity at any instant.

The plan of the present research was arranged in 1872, but it was not until 1875 that the apparatus was made. Since then the apparatus has been partially modified in order to overcome the optical and mechanical difficulties which arose in the course of the work. The most important pieces were—(1) the telescope, (2) the reflectors, (3) the revolving toothed wheel, (4) the clock, (5) the chronograph, (6) the dynamo-electric machine, and (7) the lamp.

1. *The telescope.*—This consists of a 5-inch achromatic object-glass of good quality, with a focal length of 7 feet. A **Bohnenberger's** eye-piece is employed, consisting of an erecting eyepiece with a piece of plain glass in front of the field lens and inclined to the axis of the telescope at an angle of 45°. A lateral hole in the tube of the eye-piece allows the light from a lamp, &c., at the side, to be reflected by this diagonal mirror along the axis of the telescope, and thence to the distant reflectors. The adapter, which connects the eye-piece with the body of the telescope, is a tube whose lower half is cut away so as to allow the revolving mechanism to be placed below in such a way as to bring the upper part of the revolving toothed wheel into the axis of the telescope and exactly in its focus. The light from a lamp or from the sun can be concentrated by a lens so as to throw its image upon the top of the toothed wheel. By looking along the axis of the telescope, from the object-glass, an observer can notice whether any of the light falls upon the inner sides of the telescope, and in this manner

he is able to adjust the direction of the beam of light so as to be central, in which case it must go directly to the distant reflectors whose images lie close together, not $\frac{1}{100}$th of an inch apart, on the teeth of the wheel.

Various optical difficulties presented themselves, and the arrangements were altered at various times. These chiefly consisted in illumination of the field of view, and were successively removed. Four improvements specially deserve notice:—

1. A general and intense illumination of the whole field was soon traced to a reflection from the centre of the object-glass. To obviate this a circle of black velvet about one inch in diameter was fastened to the centre of the object-glass on the inside. This was quite successful.

2. In using powerful lights an intense blaze was reflected from the toothed wheel, which made delicate observations impossible. We first tried smoking the toothed wheel, but not only did it still reflect a good deal of light but the regularity of form of the teeth was impaired; finally we used a highly-polished toothed wheel slightly bevelled, and by tilting the revolving mechanism the light was reflected to the upper parts of the interior of the adapter, which being blackened absorbed the light. This arrangement succeeded admirably, but some experiments were also made with the smoked wheel.

3. The field still had some general illumination owing to reflection from grains of dust, or scratches, or other imperfections in the plane-glass diagonal reflector. We objected to the glass reflector also for another reason: at an angle of 45° a *double layer* of glass reflects only 0·16 of the incident light; on the return of the light from the distant reflector 0·84 of this quantity passes through to the eye of the observer. The whole light obtained from this arrangement is then $0{\cdot}16 \times 0{\cdot}84 = 0{\cdot}136$; but if we could have an arrangement to reflect 0·5 of the light, so that on its return we should see $0{\cdot}5 \times 0{\cdot}5 = 0{\cdot}25$ of the light, we should nearly double the intensity. We accomplished this by substituting for the diagonal glass reflector a silvered reflector with an oval hole whose projection on a plane perpendicular to the axis of the telescope is a circle, and whose dimensions are such as to allow one-half of the light to pass through. The diagonal reflector is three inches from the toothed wheel. This arrangement certainly doubled the light, and owing to the darkness of the field its superiority over the glass reflector was enormous.

4. To prevent the slight illumination still remaining on the toothed wheel from causing inconvenience to the observer, a strip of metal with a series of holes of varying size was sometimes placed in the secondary focus of the eye-piece. It could be easily removed and replaced, so that when everything was prepared without its aid this diaphragm was inserted, using so small a hole as to show only the distant reflectors and two teeth of the wheel.

The toothed wheel.[*]—In contriving the revolving toothed wheel and the mechanism which gives it motion, considerable forethought is necessary. It must revolve at a very great speed, and must be capable of going at least for some minutes so as to avoid the necessity of continually winding it up. Our apparatus was constructed by Messrs. E. Dent and Co. It is mounted upon three screws, which rest in three holes countersunk in a plate of iron, which latter has an oblong hole in it to admit the passage of the catgut supporting the weight. This base plate of iron is, as before stated, generally slightly inclined by resting on a wedge, so that the light striking the toothed wheel is reflected upwards. Each wheel works into a pinion so as to multiply the velocity tenfold. These wheels and pinions are on five separate arbors; so that the multiplication is altogether 10,000-fold. On the first arbor the drum, 3 inches in diameter, is fixed. To the second a circular disc with a milled edge is attached. This enables the observer to increase or diminish the velocity of the wheel by a touch of the hand. It also enables him to reduce the velocity before putting on the brake, a sudden application of which might injure the mechanism. On the third arbor there is a cam which makes contact between two platinum points supported by two springs once in a revolution (*i.e.*, 100 revolutions of *the* toothed wheel), by means of which a current of electricity is transmitted to the chronographs. On the fourth arbor there is a brake consisting of a light disc of metal caught between two springs. To release this brake a handle or key behind the mechanism is turned through a right angle. This separates the springs and the brake ceases to act. The fifth arbor supports *the* toothed wheel, which is $1\frac{1}{2}$ inch in diameter, and contains 400 teeth cut to a depth of $\frac{1}{16}$th of an inch. The wheel is bevelled so as to reflect the light away from the observer's eye. Teeth of different shapes were tried, but the best was found to be that of saw teeth (*i.e.*, with pointed teeth and pointed spaces). The width of the teeth could then be varied by raising or lowering the mechanism by means of the foot screws. In the same way the toothed wheel could be brought exactly into the focus of the telescope. Wheels with varying numbers of teeth were also tried, and that one with 400 teeth was chosen as giving the best results.

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[*] The term "*the* toothed wheel" applies in this description to that toothed wheel in the mechanism between the teeth of which the light is made to pass.

The reflectors.—The two reflecting collimators are of identical construction. Those which we found to be most suitable were a pair constructed by Messrs. Troughton and Simms. They are supplied with achromatic object-glasses of 3 inches diameter and 3 feet focus. At the other end of the tube a cap is screwed on, and to the centre of this cap a circular silver mirror is attached by three screws which admit of adjustment. This mirror is ground to a spherical form, the radius of the sphere being 3 feet, and the centre of the sphere of which it is a part being the centre of the object glass. Other collimators were tried, in which the object-glasses were replaced by 9-inch silvered glass reflectors of parabolic form. This pair was constructed by a maker who makes a speciality of such reflectors for telescopes; but the whole workmanship was so disgracefully bad in every part, every conceivable fault being found in it, and every known device for patching up and concealing bad work having been resorted to, that in the form supplied to us they were absolutely useless. They were discarded altogether when we found that the atmospheric conditions were seldom favourable enough to admit of our using a larger aperture than 3 inches. We have always used the 3-inch collimators in our observations. Three adjustments must be made: (1), for focus; (2), for centering; (3), for direction.

(1.) To adjust the focus, the cap with the mirror attached is removed and another one put in its place. This one is similar to it in every way except that the mirror is replaced by a piece of ground-glass whose surface has the same position with respect to the cap that the mirror has in the other one. The image of a star or the sun can be thrown upon the ground-glass and focussed. The number of turns and parts of a turn of the screw of the cap are then counted, and the cap with mirror is now screwed on to the same position. A final adjustment can be made with the collimators in position, altering the focus until an observer at the telescope sees the reflected light most distinctly.

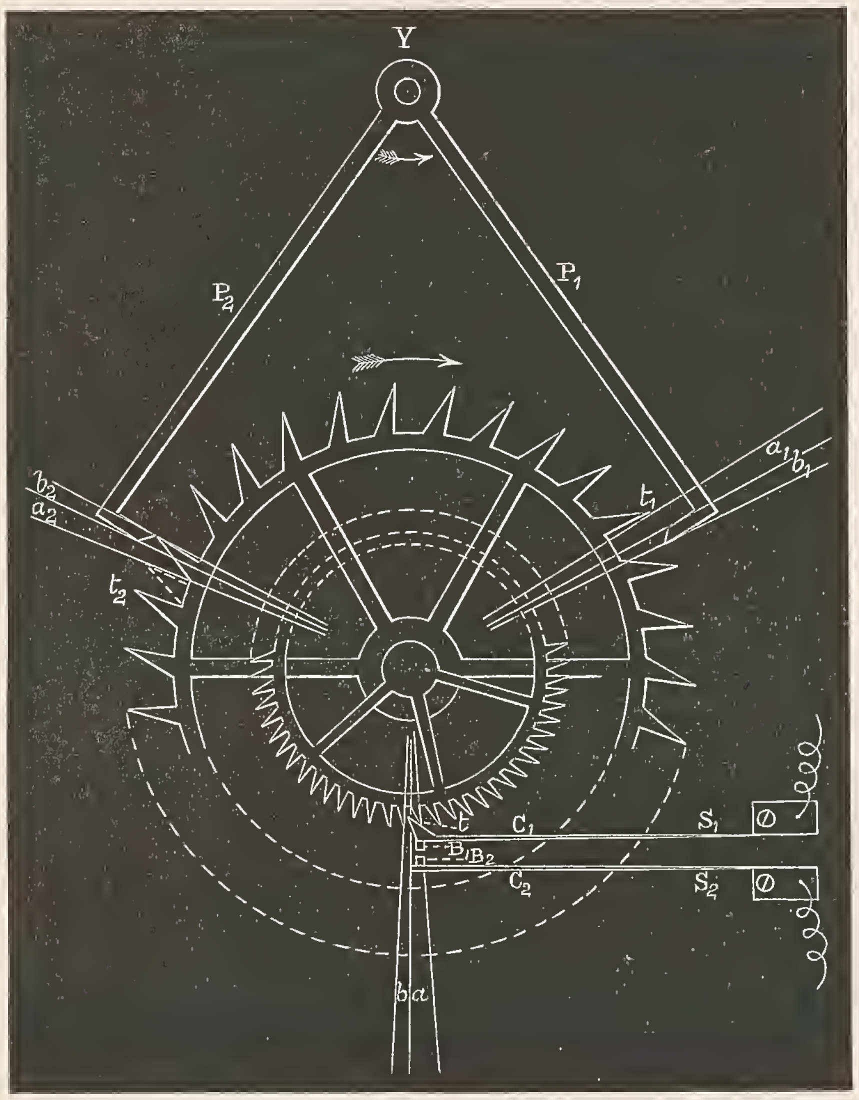
(2.) The adjustment which we call centering is accomplished by the three screws which attach the mirror to the cap, and causes the centre of the sphere of which the mirror forms a part to be at the centre of the object-glass. To test this point a tube 1 foot long is put on the collimator projecting in front of the object-glass. At the end of this tube a small ring is supported in the centre by three strips of metal. On looking through this ring the observer ought to see an image of his eye, and the screws are adjusted until this is the case.

(3.) To direct the collimators so that their axes should point to the observing telescope it was found most convenient to look through the object-glass at the mirror. If the head be so far as possible prevented from covering the object-glass an image of distant objects is seen in the mirror, and the direction is changed until the light coming from the observing telescope is seen in the centre of the mirror.

The clock.—The clock was constructed by Messrs. E. Dent and Co. specially for this research. It is driven by a weight attached to an endless chain passing over the drum. On the arbor of the drum, and connected with it by a toothed wheel and ratchet, is a wheel of 100 teeth working into a pinion of 10 teeth, which is on the same arbor as the scape wheel. This arbor bears a hand which marks seconds, and the arbour of the large wheel bears a hand which marks the minutes from 1 to 10. Any additional wheelwork was unnecessary for our purpose and would only tend to introduce errors. There is a dead-beat escapement, and the pendulum rod is made of varnished wood. The bob weighs 6 lbs., and the driving weight only 4 ozs., or one-twenty-fourth of the weight of the bob. The clock is best wound up by unhooking the weight from the endless chain after having hooked on another higher up. It goes without attention for two hours.

The arbor of the scape wheel bears another similar one, with 120 teeth, for making electrical contact once a second, by which means a mark shall be made on the chronograph. The system of electrical contacts adopted by us seemed to be free from the defects of those which tend to disturb the pendulum, because the springs whose contact sends the electric current are moved during the drop of the scape wheel from one pallet to the other. In the figure (fig. 1) a tooth has just passed from the impulse face of the pallet P_2. t_1 now falls through the angle a_1 on to the dead face of the pallet P_1. *During this interval* the tooth t of the smaller* wheel mounted on the same axis as the scape wheel moves the contact spring C_1 S_1 against the contact spring C_2 S_2, and the current passes. (The point of C_1 is agate, and the points of contact are platinum.) It will be seen that when the tooth t_1 goes on to give impulse to the pallet P_1 through the angle b_1 the tooth t has allowed the springs to separate and it passes through its angle b undisturbed. The motion of the tooth t_2 can be easily traced from the drawing. Careful workmanship was required to carry out this plan, as the angles a, a_1, a_2 ought to be made smaller than in the drawing. The result has been very successful.

Fig. 1.



Nevertheless, although this arrangement seems admirably adapted for not interfering with the regularity of the pendulum's vibration, we considered that it was not perfectly satisfactory for our purpose. For the length of a second marked by it on the chronograph would depend upon the exact form and size of the teeth in the auxiliary wheel. Consequently, we extemporised a contact free from this defect. A hole of the same size as the arbor of the crutch was bored in a piece of cork, and a slit was made in the cork to allow it to be fixed upon that arbor. Through the cork a thick copper wire was passed stretching horizontally for 3 inches, where it was bent downwards at a right angle. This bent arm was half an inch long, was amalgamated, and dipped, at each oscillation of the pendulum, into a cup of mercury connected with the electric circuit. The stout copper wire was also connected with that circuit by means of a very fine copper wire. This arrangement was very perfect.

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* The wheel is here drawn smaller for clearness of description. It is really of the same size.

The chronograph.—In all the experiments previous to 1880 we used one of the ingenious portable chronographs constructed by M. HYPP, of Neuchatel. In this instrument a strip of paper is run off from a drum, and upon it the signals from the clock and wheel are inscribed by two siphon pens. Uniformity of motion is given by the mechanism of the chronograph giving motion to a spur wheel against which the end of a spring presses. This spring permits one, two, &c., teeth of the wheel to pass in the course of a single vibration, the number allowed to pass being under control of the experimenter. A large number of observations were made with this apparatus. But we came to the decided opinion that while it is admirably adapted for observatory work where a greater accuracy than $\frac{1}{100}$th second is not required, yet in order to get the most perfect possible results a different class of instrument must be employed. Consequently we designed a new chronograph, which was constructed for us by Messrs. ELLIOTT Brothers. The principle of this apparatus is that we depend for uniformity of motion on the inertia of the apparatus. It was our object to get rid of all clockwork, and by making use of a fly-wheel, which has no work to do, to get rid of a host of irregularities which affect (it may be in a very slight degree) all other chronographs.

The base of the instrument consists of two strong triangular castings bound together in a horizontal position by three brass pillars. Between these triangles a fly-wheel, 12 inches in diameter, rotates on a vertical axis. The lower end of this axis is a hard steel socket which rests upon the point of a strong screw working in the centre of the lower triangle and firmly fixed there by means of a nut.

The lower triangle is supported upon three feet adjustable for levelling by screws. To the upper triangle two vertical brass pillars, 16 inches long, are fixed. They are connected at the top by a cross piece of iron, in the middle of which a thick pin works vertically with a screw motion, and it can be fixed by a nut.

The lower end of this pin terminates in a point, supporting the hard steel socket at the upper part of the axis of the registering cylinder. This cylinder is of brass. It is 12 inches long and 4 inches diameter. Its axis projects an inch at each end. The upper end, as before said, rests against the point of the upper screw. The lower end is, during an observation, firmly fixed to the axis of the fly-wheel by means of a solid brass tube closely fitting these two axes, with six screw nails to ensure rigidity. Round the upper end of the axis a silk thread is wound which, passing over a pulley mounted on friction wheels, carries a small weight. The axis of the cylinder is made vertical by the levelling screws, a spirit-level being placed on the top of the cylinder and observed in different positions of the fly-wheel. Thus we have a rigid mass rotating about a vertical axis with no cause for irregular motion except the acceleration produced by the weight and a slight friction, which combine to produce an accelerating or retarding force which may be regarded as constant. Two cylinders, exactly similar, were employed, and these could be interchanged. The brass cylinders were covered with a thin layer of smoke from a paraffin lamp.

The marks made by the currents of electricity from the clock and wheel respectively, were produced by two aluminium points attached to two springs, which were attracted by the poles of two electro-magnets in the circuits of the clock and wheel respectively. Thus, while the pens are in contact with the cylinder if there be no currents passing, and the fly-wheel be rotated, two circles are traced upon the cylinder. The electro-magnets are mounted on a stand which slides up and down one of the vertical brass pillars by means of a rack and pinion. Thus the assistant can, by turning the pinion, convert the two circular marks into spirals. We judged that it was much better to make an assistant perform this work than to use up the energy of the fly-wheel in the same way, as is often done in chronographs. Moreover, he turns the pinion only while an observation is being recorded, and not in every case when the fly-wheel is rotating, thus effecting a great saving in the space on the cylinder. The electro-magnets are not fixed rigidly to this support which moves up and down on the vertical column. They are attached to it by a vertical axis about which they have a small motion. Thus we are able to keep the pens in contact with the cylinder, or not, as we please. Ordinarily the pens are kept away from the cylinder by a light spring. By sending a current of electricity through a third electro-magnet, the pens are brought into contact with the cylinder. This current is sent by the observer by means of a contact maker, a few seconds before making an observation. When the assistant sees this he moves the pinion so as to cause the pens to describe a spiral, and the observer, at the instant of an observation, breaks and makes contact rapidly. This leaves a break on the smoked cylinder for about one-eighth of a second, the beginning of the break indicating the time of the observation.

When it is desired to read off the records of observations the cylinder is first taken off, and the collar which connected it rigidly to the fly-wheel removed. A ring, to which an arm carrying a vernier is attached, is now screwed to the centre of the upper triangle, and forms a bearing in which the axis of the fly-wheel can work. This part of the axis is conical, having the smallest diameter above. On this conical pivot a conical collar, which carries a 5-inch divided circle, is jammed. The cylinder is replaced with the lower end of its axis resting on the pointed upper end of the axis of the fly-wheel. It is prevented from rotating independently of the fly-wheel by an upright piece of metal and an upright spring, which are attached to the divided circle, and which catch between them one of four radial arms at the base of the cylinder.

It has been stated that one of the vertical pillars carries the marking pens or styles. The other pillar bears an arm to which is attached a microscope with cross wires in its focus. This microscope can be raised or lowered on the pillar which supports it. On looking through this microscope the marks made by one of the pens can be seen and brought, by rotating the fly-wheel, into super-position on the cross wires. The position of the cylinder is then read off by the vernier. The circle is divided into 500 equal parts, and by aid of the vernier we can read to one-twentieth part of one of these divisions, *i.e.*, to $\frac{1}{10000}$th of a revolution. We found it best in practice to rotate the apparatus at the rate of about one revolution a second. Hence one division of the vernier corresponds to about $\frac{1}{10000}$th of a second.

The perfect working of the chronograph depends upon a number of conditions which must all be fulfilled simultaneously. They are dependent (1) upon the manufacture of the instrument, (2) upon the mechanical adjustments, and (3) upon the electrical adjustments. The two last alone can be improved by the observer. The principal objects to be sought after in the manufacture of the instrument are absence of friction, rigidity, and the reduction of the moment of the momentum of the fly-wheel and cylinder together about their axis to zero.* The mechanical adjustments are as follows :—The vertical position of the axis of the instrument must be carefully tested by a level. The points of support must be oiled. The screws connecting the cylinder to the fly-wheel must be quite tight. The pressure of the upper point of support upon the axis must be very nicely adjusted. This point of support must be kept rigidly in position by the nut. The electrical adjustments are as follows :—The batteries and magnets must be in good working order. The pens must move freely and over a suitable range, and they must press with a suitable pressure upon the smoked cylinder.

The dynamo-electric machine.—The electric current to work our electric lamp was obtained from a small sized SIEMENS' dynamo-electric machine requiring three horse-power and rotating at the rate of 1,400 turns a minute. The axis of the machine is attached directly to a turbine of that kind known by the name of a vortex turbine. This was fed by a supply of water with a head of 300 feet coming from a distance of about a mile, and led to the house by a 3-inch water pipe. The current was led by means of stout wires from the generator to the lamp, a distance of about 100 feet. No further account of this machine is necessary, as our practice only, and not our results, could be affected by any imperfections.

The source of light.—We employed for our source of light, in the experiments of 1880–81, a SIEMENS' electric lamp. A condensing lens was used to throw an image of the incandescent carbon, after reflection by the diagonal reflector, upon the toothed wheel. Since with this lamp motion is given to only one of the carbon points it was necessary to mount the condensing lens upon a framework which should admit of its having an up and down movement. For the same reason the diagonal reflector in the eye-piece of the telescope was turned round about the axis of the telescope at each observation so as to send a maximum quantity of light along the axis to the distant reflectors. In some of our later experiments we used a bisulphide of carbon bottle-prism to send particular colours to the distant reflector. This prism was laid upon a ledge on the support of the condensing lens and immediately below it. With a little care a very pure spectrum could thus be formed.

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* The maker did not quite fully carry out our ideas on the two last of these points. The rigidity of the apparatus would have been increased by having three vertical pillars in place of two.

Establishment of the apparatus at Kelly, Wemyss Bay.

The room in Kelly House (the property of Dr. YOUNG) which was used for the observatory was the billiard room, facing the front of the house (the west). It is the room immediately to the north of the entrance hall or lobby. A pane of glass was removed from the window for observation. On the window-sill a brass plate has been fixed, with an inscription, indicating the exact relative position of the toothed wheel. The telescope was mounted upon two strong wooden supports. The toothed wheel mechanism rested upon two very solid beams (between which the cord for the weights could pass). These beams rested at one end upon a standard fixed to the wall of the room, and at the other end upon a strong box which rested on the billiard table. This end was also supported by a standard; and the floor was strengthened by thick wooden props below. The horizontal beams also acted as a support for the framework bearing the condensing lens and prism. The electric lamp was on a separate table. At one end of the beams and at the observer's right hand mercury cups were placed, with metallic contact pieces, by means of which the batteries could be put out of action when not required. LECLANCHÉ cells were used for the chronograph connexions. Twelve cells were used in all. The clock was attached to the wall of the room facing the observer. The levelling screws of the chronograph rested upon three metal plates on a piece of wood on the billiard table.

We have inserted a brass plate in the stone outside the window, with an arrow pointing towards the position occupied by the toothed wheel, the distance from the point being 106 inches.

Establishment of the reflecting collimators.—The reflecting collimators are placed on the hills behind the village of Innellan, separated from the observing telescope by the Firth of Clyde. The nearest one (B) is on the summit of the hill called *the Tom.** It rests upon two iron Y's which are imbedded in the solid rock. The reflector was 8 inches further from Kelly (*i.e.*, more to the west) than the most westerly of these two Y's, whose positions will always be marked by the holes. The collimator was covered by a wooden box with a hole in the east side. This box was attached to four iron rods placed in holes in the rock.

The more distant reflector (A) is in the face of the hill to the west of the Tom, nearly in the same line as a line drawn from Kelly to the top of the Tom, but two feet to the north of this line. It was impossible to gain a solid foundation, but the face of the hill was partially excavated, and two holes were "jumped" in a massive stone, in which two Y's were fixed to support the reflecting collimator. The reflector A was $8\frac{3}{4}$ inches further from Kelly than the most westerly of these two holes.

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* Kelly House and *the Tom* are both shown on the Ordnance Survey Maps.

The reflector A was frequently shifted in direction (not in absolute position) by the unsatisfactory condition of the soil. This was especially the case during the severe frost of the winter, and on the occasion of the subsequent thaw. It was covered by a wooden box, open to the east.

The reflector A was not interfered with by any inquisitive persons. But the box covering B was once broken open at the west end, and the centreing screws abstracted, doubtless with the aim of obtaining a view of Kelly House with what was supposed to be a powerful telescope.

We made some sketches, and took some photographs of the apparatus, from which the accompanying plates have been prepared.

[*Note.*—It has been decided that the descriptions of the apparatus are sufficiently clear, and that it is unnecessary to supplement them by the plates to which allusion is here made.]

The general arrangement of our apparatus gave us considerable satisfaction. We could have wished, however, to have had a chronograph of the same kind, but more convenient in use in some ways, with greater rigidity, and with a smaller moment of momentum about the axis. The mechanism of the toothed wheel might perhaps be constructed so as to give a greater velocity than we were able to obtain. If a better climate were experienced considerable advantage would be gained, and the telescope and reflectors might then be increased in size, and the distances lengthened.*

†[*Measurement of the distances between the toothed wheel and the reflecting collimators.*

These measurements were made with the ordnance survey 20-inch theodolite. The base-line was the distance between the ordnance survey centre-marks on Knock Hill and Innellan Hill. The necessary information was furnished us by Colonel Clarke. It seems unnecessary to give details of the triangulation, the manuscript being left in the charge of the Royal Society for reference. Calling Kelly House Station C, and the reflectors Stations A and B respectively, we obtained

$$CA=18{,}210{\cdot}6 \text{ feet}$$
$$CB=16{,}825{\cdot}3 \text{ feet.}]$$

These distances require several corrections.

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* We wish to record our indebtedness during the whole series of the observations which follow, to the able and skilful assistance rendered to us by Mr. D. Stewart, who superintended the working of the chronograph, and aided us in the work generally.

† Details of triangulation have been left out. The part in square brackets here is substituted.—G. F., December 17, 1881.

(1.) The distances of the reflectors and toothed wheel form the points used in the triangulation. These are

at A−9·18 feet
at B−0·97 foot
at C+8·83 feet

giving a correction to

CA=−0·35 foot

and to

CB=+7·86 feet.

(2.) The thickness of the object-glasses, giving to CA and to CB a correction of +0·1 feet to each.

These two corrections make CA=18,210·3 feet and CB=16,833·3 feet.

(3.) The reduction to a sphere whose radius is the distance of Kelly from the earth's centre instead of the sea level. The height of Kelly above the sea level is 100 feet. This correction is quite insensible.

(4.) The reduction of the circular measure of the angle subtended by CA or CB at the earth's centre to the chord of that arc. The angle is about 3′, and the reduction is quite insensible.

(5.) The station B is 414 feet higher than Kelly, and A is also in the same line. Owing to this cause there is a correction of +1·7 feet to CB and +1·9 feet to CA.

(6.) Lastly, the rays of light do not go in a straight line from C to A or B because they are bent into a curve by refraction. The curvature is so small that we may consider it as a part of a circle of very large radius. Supposing that the refraction amounted to half as much as the horizontal refraction of a heavenly body, *i.e.*, to 15′, then the angle subtended by this part of a circle would be 30′. But the difference between the circular measure of 30′ and the chord of the arc is insensible. Hence this correction is insensible.

The final result is that the distance CA=18,212·2 feet and CB=16,835·0 feet. Or D_A=CA=3·44,928 miles. D_B=CB=3·18,845 miles.

$$g=\frac{D_A}{D_B}=1·08181$$

$$\delta=\frac{13}{12}-\frac{D_A}{D_B}=+0·00152$$

$$\frac{r+1}{r}=\frac{13}{12}=1·08333$$

$$\delta'=\frac{14}{13}-\frac{D_A}{D_B}=-0·00484$$

$$\frac{r+2}{r+1}=\frac{14}{13}=1·07697$$

Translation of the chronograph records.

The most usual way of observing with the chronograph was to complete the electric circuits for the clock and the rotating mechanism during the hour or two that we might be at work. The circuit which when closed brings the pens into contact with the smoked cylinder was however kept open, the means of closing it being a contact key in the hand of the observer. The assistant winds up the chronograph (which can only go for about 20 seconds) and levels it. He also examines the pens to see that they are in working order. The observer winds up the mechanism. He notices that his contact maker is in good working order. He oils the pivot holes of the mechanism. He puts on the number of weights which experience tells him will give the required speed approximately. He asks "Are you ready?" When the assistant is ready the observer releases the brake on the mechanism. He may if he please count the number of eclipses of A or B before his final speed is obtained. He adjusts the width of the teeth by raising or lowering the mechanism by means of one of the levelling screws. When the speed which gives equality of lights is nearly obtained he notices and records whether A or B is increasing with increase of speed. He makes the equality of lights more exact by adding to or taking from the driving weights, or by pulling the catgut supporting the weights either up or down, or by touching the milled head on the arbor of the second wheel on the mechanism, or in such a way as he has decided upon, until he feels that he has complete power to produce equality of lights. He then says to the assistant "Begin." The assistant gives a definite speed (judged by the eye) to the fly-wheel and cylinder of about one revolution a second. After a few seconds the observer assumes that the cylinder is rotating uniformly. He makes contact. The pens touch the smoked cylinder. The assistant immediately begins to lower the support of the pens by means of the rack and pinion, so as to make the pens describe spirals close to each other, the pens meanwhile marking seconds of the clock, and hundreds of revolutions of the toothed wheel respectively. The observer now adjusts the speed so as to produce equality of brightness in the two stars. When he is quite certain that he has attained this he breaks contact for one-eighth of a second or thereabouts. This leaves a blank in the spiral traces, the commencement of which indicates the time at which the speed is required.

The 12th and 13th equalities were generally used, and sometimes the 14th. Successive observations were always made at successive equalities to allow of the complete elimination of errors alluded to in the theory of our method.

The cylinder is completely covered after four or five observations have been made.

After the spirals have been traced by the clock-pen and the wheel-pen upon the smoked cylinder of the chronograph, the divided circle and vernier are attached to the instrument, the former being divided into 500 divisions, and the latter reading to $\frac{1}{10000}$th of a revolution. The microscope with its cross wires is placed so that the irregularities marked by the pens on the spiral traces can be readily seen, and brought to coincide in succession with the vertical wire. This done, a reading is taken, and the cylinder is turned round until another mark coincides with the vertical wire. So the operation is continued, first with the wheel marks, and afterwards with the clock marks. In reading off the marks on each side of a break in the traces care must be taken to notice whether a mark (or it may be even two marks) have been omitted. Care must also be taken to notice the number of whole revolutions of the cylinder. But it is only in the case of the clock-pen that this is necessary; and even in this case the omission of this precaution could hardly lead us into error. In this way we may obtain a series of readings for the wheel and clock respectively such as the following:—

January 20, 1881, No. 6.					
Wheel.			Clock.		
Reading.	Differences.	At mean reading.	Reading.	Alternate differences.	Alternate second differences.
			515		
231					
			s_1 9,890	22,557	
	2,411	1,437			
			23,072	(22,419)	329
2,642					
			s_2 32,309	22,228	295
	2,404	5,046			
. .					
			45,300	22,124	
7,449					
			54,433		
Signal at 3,810			Signal at 24,108		

The signals are indicated by the commencement of the break in the pen traces. Notice that in the reading of the wheel marks the third one is a blank, indicating that one mark is omitted in the break. In consequence of this the corresponding "difference" is half of the difference between the two numbers 2,642 and 7,449. Notice also that in taking the clock differences we must subtract the *alternate* readings, partly because the clock cannot be perfectly "in beat," and partly because the electrical connexion is such as to make the length of the second, as registered, depend upon the direction of vibration of the pendulum.

The "difference," such as 2,411, in the column of the wheel record, is the number of divisions (or ten-thousandths of a revolution) passed over by the cylinder in the course of 100 revolutions of the toothed wheel.

The "alternate difference," such as 22,419 in the column of the clock record, is the number of divisions passed over by the cylinder in the course of two seconds of time.

If the cylinder rotated with perfect uniformity the quotient of these two numbers, viz., $\frac{22419}{2411}=9\cdot2986$, would be the number of hundreds of revolutions completed by the toothed wheel in two seconds of time, and if we multiply this number by 50, we get 464·93 revolutions per second as the speed of the toothed wheel.

But as a matter of fact the chronograph-cylinder is constantly accelerated by the small weight, and retarded by the friction of the instrument. The total effect is generally a retardation, gradually diminishing the velocity of rotation of the chronograph. In the short interval of time (usually about one-fourth or one-fifth of a second) between two wheel marks this retardation does not affect us. We can say, in the above example, that the cylinder is passing over 0·2411 revolution in 100 turns of the toothed wheel, at the instant when the pen is at the reading 1;437; and that it is passing over 0·2404 revolution in 100 turns of the toothed wheel when the pen is at the reading 5,046. A simple proportion tells us that when the pen is at the reading of the signal, *i.e.*, 3,810, the number of revolutions in 100 turns of the toothed wheel is $0{\cdot}2411-\frac{2373}{3609}\times 0{\cdot}0007=0{\cdot}2406$.

But in the case of the clock trace we use intervals of two seconds. Here the case is very different. From the readings of the chronograph we must determine what was the actual velocity of the cylinder at some particular reading of the chronograph, what was the retardation produced by the excess of friction, what is the law according to which the velocity of rotation of the cylinder varies with the reading upon the cylinder; and thence what was the velocity of its rotation at the time when the signal was indicated.

Let s be the reading at any time t, and v the velocity of rotation of the cylinder.

When $t=0$, let $s=s_0$, and let $v=u$.

Let $s=s_1$ and s_2 when the times are t_1 t_2 respectively.

Let f be the excess of friction, or the retardation.

Then

$$s_1=s_0+ut_1-\tfrac{1}{2}ft_1^2$$

$$s_2=s_0+ut_2-\tfrac{1}{2}ft_2^2$$

$$s_2-s_1=u(t_2-t_1)-\tfrac{1}{2}f(t_2^2-t_1^2)$$

$$\bar{v}=\frac{s_2-s_1}{t_2-t_1}=u-f\frac{(t_2+t_1)}{2}$$

Therefore $\bar{v}=$the velocity of rotation at the time $\frac{t_2+t_1}{2}$.

Now let $\bar{s}$ be the reading corresponding to the velocity of rotation $\bar{v}$, or the reading at that time $\frac{t_2+t_1}{2}$.

$$\bar{s}=s_0+u\frac{t_2+t_1}{2}-\tfrac{1}{2}f\left(\frac{t_2+t_1}{2}\right)^2$$

but

$$\frac{s_1+s_2}{2}=s_0+u\frac{t_2+t_1}{2}-\tfrac{1}{2}f\left(\frac{t_2^2+t_1^2}{2}\right)$$

Therefore

$$\bar{s}=\frac{s_1+s_2}{2}+\tfrac{1}{8}f(t_2^2+t_1^2-2t_1t_2)$$

$$=\frac{s_1+s_2}{2}+\tfrac{1}{2}f\left(\frac{t_2-t_1}{2}\right)^2$$

In our case $t_2-t_1=2$ seconds. Hence we have

$$\bar{s}=\frac{s_1+s_2}{2}+\tfrac{1}{2}f, \text{ corresponding to } \bar{v}=\frac{s_2-s_1}{2}.$$

To determine the value of f we notice above that

$$s_2-s_1=u(t_2-t_1)-\tfrac{1}{2}f(t_2^2-t_1^2)$$

so

$$s_3-s_2=u(t_3-t_2)-\tfrac{1}{2}f(t_3^2-t_2^2)$$

(where s_3 is the reading of the clock mark two seconds later than the mark indicated by s_2), but $t_2-t_1=t_3-t_2=2$ seconds. Therefore, subtracting, we have

$$\begin{aligned}(s_3-s_2)-(s_2-s_1)&=f(t_3-t_1)\\&=4f\end{aligned}$$

whence

$$f=\frac{(s_3-s_2)-(s_2-s_1)}{4}$$

It follows then that f is equal to the second alternate difference in the above example divided by 4.

This value of f, thus determined, is never perfectly constant; but we can interpolate so as to find its mean value in the interval between the reading $\frac{s_1+s_2}{2}+\frac{1}{2}f$ and the reading corresponding to the signal given by the observer.

[*N.B.*—When through imperfection of adjustment the values of f were very discordant, the observation was always rejected.]

To deduce the velocity v of rotation of the cylinder when the reading (corresponding to the signal) was s, we notice that

$$\begin{aligned}v^2&=\bar{v}^2-2f(s-\bar{s})\\&=\left(\frac{s_2-s_1}{2}\right)^2-2f\left(s-\frac{s_1+s_2}{2}-\tfrac{1}{2}f\right) \quad \text{. . . . . (A)}\end{aligned}$$

The first thing to be done in applying these formulæ is to determine the value of f which is to be adopted. The record given above shows us that, at the reading 23,072, $4f=329$; and at the reading 32,309, $4f=295$; and we obtain for the value of $4f$, at the signal reading 24,108, by simple proportion,

$$4f_1=329-\frac{1036}{9237}\times 34=325.$$

Taking this as a first approximation to the value of $4f$ which is to be adopted, we find

$$\begin{aligned} s_1 &= 9{,}890 \\ s_2 &= 32{,}309 \\ \hline s_1+s_2 &= 42{,}199 \end{aligned}$$

$\bar{s}=\frac{s_1+s_2}{2}+\frac{1}{2}f=21{,}100+41=21{,}141$ (assuming f_1 is sufficiently near to the correct value of f).

The value of $4f$ corresponding to this value of s is

$$4f_2=329+\frac{1931}{9237}\times 34=336.$$

The value for $4f$ which we must adopt is the mean of these two, $4f_1$ and $4f_2$.

$$4f=\tfrac{1}{2}(325+336)=331$$

[We might adopt this improved value for f to find a truer value for $\bar{s}$, and thence we might get a still nearer approximation to the correct value of f. In practice we find that no greater accuracy would be thus attained.]

Substituting now in equation (A) we have

$$\frac{s_2-s_1}{2}=11{,}210\,;\ s-\left\{\frac{s_1+s_2}{2}+\tfrac{1}{2}f\right\}=24{,}108-21{,}141$$

$$= 2{,}967$$

$$v_2=(11{,}210)^2-166\times 2967$$

and

$$v=11{,}188.$$

We have neglected all account of the position of the decimal point up to this stage. It is easy to see that the above value of v means that the cylinder of the chronograph is, at the instant of equality of the lights, rotating at a speed which if uniform would accomplish 1·1188 revolutions per second. At the same instant the cylinder rotated 0·2406 of a revolution in the time taken by the toothed wheel to complete 100 revolutions. Hence the speed of rotation of the toothed wheel is

$$n'=\frac{1{\cdot}1188}{0{\cdot}002406}=465{\cdot}00 \text{ revolutions per second.}$$

The value of v is calculated by the help of seven-figure logarithms.

Having now studied in detail the method of reduction of this particular observation, the following systematic form for the tabulation of results will easily be understood :—

January 20, 1881, No. 6.

Wheel.			Clock.		
Reading.	Differences.	At mean reading.	Reading.	Alternate differences.	Alternate second differences.
231			515		
	2,411	1,437	9,890	22,557	
2,642			23,072	(22,419)	329
..	2,404	5,046	32,309	22,228	295
7,449			45,300	22,124	
			54,433		
Signal at 3,810			Signal at 24,108		
$v'=0{\cdot}002406$			$\bar{s}=21{,}141$ $4f=\ \ 331$		
$w=\frac{v}{v'}=465{\cdot}00$			$\bar{v}=\ \ 1{\cdot}1210$ Correction for friction, &c. $=-\ \ {\cdot}0028$ $v=\ \ 1{\cdot}1182$		

Referring to the mathematical theory of our method it will be seen that in order to calculate for each observation the slight correction which must be applied, owing to $\frac{D_A}{D_B}$ being not exactly equal to $\frac{13}{12}$, it is necessary to make a first approximation to the velocity of light neglecting this small correction. Our formula for this first approximation, studying the 12th and 13th equality (which is the best pair for our purpose), is

$$V=\frac{2m(n+n')D_B}{r}$$

The mean value of all our determinations of

$n+n'$ is 880·02	log	$=2{\cdot}9444922$
$D_B=3{\cdot}18845$	,,	·5035796
$2m=800$	,,	2·9030900
		6·3511618
$r=12$	,,	1·0791812
$\left\{\begin{array}{l}V=187{,}060\\ \text{first approximation}\end{array}\right.$		5·2719806

Also from § 6 of the mathematical theory the first approximation to the value of $2N_B$ is $\frac{n+n'}{r}$

$\log(n+n')$	2·9444922
$\log r$	1·0791812
$2N_B=73,350$ first approximation	1·8653110
$g=1·08181$	·0341510
$2gN_B=79·3345$ first approximation	1·8994620

These are the values which are adopted in calculating the second term in the following reductions.

The rejection of observations.—We resolved never to reject a single observation simply because it differed largely from our average result. The only cases where we rejected an observation were (1) when there was no corresponding observation of the next equality taken at a very short interval of time, and (2) when the traces made by the clock pen on the chronograph showed great irregularities in the second differences, or when they showed that the friction on the axle was abnormally great.

With reference to No. (1) we may state that a quarter of an hour even might alter the value of ρ by fog, &c., and this would invalidate the result.

With reference to No. (2) it is right to say that we generally read off a large number of clock-pen marks corresponding to each observation. From these we could easily judge if the chronograph was working well.

December 21, 1880, No. 1.

Wheel.			Clock.		
Reading.	Differences.	At mean reading.	Reading.	Alternate differences.	Alternate second differences.
10,051			5,652		
	2,563	11,333	15,553	21,825	
12,614			27,477	21,714	254
. .	2,573	15,187	37,267	(21,571)	317
17,759			49,048	21,397	
			58,664		
Signal at 13,657			Signal at 33,915		
$v'=\cdot002569$			$\bar{s}=38,302$ $4f=310$		
$n=\frac{v}{v'}=421\cdot08$			$\bar{v}=1\cdot0786$ Correction for friction, &c. $=+\cdot0032$ $v=1\cdot0818$		

December 21, 1880, No. 2.

Wheel.			Clock.		
Reading.	Differences.	At mean reading.	Reading.	Alternate differences.	Alternate second differences.
5,790			5,370		
	2,316	6,948	17,091	21,385	
8,106			26,755	(21,198)	294
. .	2,297	10,403	38,289	21,091	185
12,700			47,846	21,013	
			59,302		
Signal at 8,679			Signal at 28,958		
$v'=\cdot002306$			$\bar{s}=27,726$ $4f=280$		
$n'=\frac{v}{v'}=459\cdot28$			$\bar{v}=1\cdot0599$ Correction for friction, &c. $=-\cdot0008$ $v=1\cdot0591$		

$$\text{From Nos. 1 and 2}\left\{\begin{array}{c} 2m\dfrac{(n+n')\mathrm{D_B}}{r}=187,132 \\ 1-\dfrac{\delta}{g+\rho}=0\cdot999323 \\ \text{Product}=\mathrm{V}=187,007 \end{array}\right\}\text{Correction for second term}=-125.$$

December 21, 1880, No. 5.					
Wheel.			Clock.		
Reading.	Differences.	At mean reading.	Reading.	Alternate differences.	Alternate second differences.
			7,953		
1,197			17,419	16,742	
	1,964	2,179	24,695		294
3,161			. .	(16,448)	
. .	1,972	5,133	41,143		320
7,104			50,225	16,128	
			57,271		
Signal at 3,482			Signal at 33,692		
$v'=\cdot001968$			$\bar{s}=32{,}958$ $4f=\quad 308$ $\bar{v}=\quad \cdot8224$ Correction for friction, &c. $=-\quad \cdot0006$ $v=\quad \cdot8218$		
$n=\frac{v}{v'}=417\cdot58$					

December 21, 1880, No. 6.					
Wheel.			Clock.		
Reading.	Differences.	At mean reading.	Reading.	Alternate differences.	Alternate second differences.
			5,952		
8,448			14,799	19,866	
	2,110	9,503	25,818	19,699	339
10,558			34,498	(19,527)	282
. .	2,083	12,641	45,345	19,417	
14,723			53,915		
Signal at 12,241			Signal at 32,524		
$v'=\cdot002086$			$\bar{s}=35{,}616$ $4f=\quad 285$ $\bar{v}=\quad \cdot97640$ Correction for friction, &c. $=+\quad \cdot00224$ $v=\quad \cdot97864$		
$n'=\frac{v}{v'}=469\cdot14$					

$$\text{From Nos. 5 and 6}\left\{\begin{array}{l} 2m\dfrac{(n+n')\mathrm{D_B}}{r}=188{,}484 \\ 1-\dfrac{\delta}{g+\rho}=0\cdot999583 \\ \text{Product }=\mathrm{V}=188{,}405 \end{array}\right\}\text{ Correction for second term }=-79.$$

December 21, 1880, No. 8.					
Wheel.			Clock.		
Reading.	Differences.	At mean reading.	Reading.	Alternate differences.	Alternate second differences.
7,785			7,927		
	1,843	8,707	17,479	17,425	
9,628			25,352	17,277	259
..	1,825	11,453	34,756	(17,166)	304
13,277			42,518	16,973	
			51,729		
Signal at 10,263			Signal at 30,460		
$v'=\cdot001833$			$\bar{s}=33,972$ $4f=292$		
$n=\frac{v}{v'}=469\cdot88$			$\bar{v}=\cdot85830$ Correction for friction, &c. $=+\cdot00298$ $v=\cdot86128$		

December 21, 1880, No. 9.					
Wheel.			Clock.		
Reading.	Differences.	At mean reading.	Reading.	Alternate differences.	Alternate second differences.
5,630			7,000		
	2,306	6,783	15,737	19,172	
7,936			26,172	(19,053)	244
..	2,295	10,231	34,790	18,928	277
12,525			45,100	18,776	
			53,566		
Signal at 8,924			Signal at 29,096		
$v'=\cdot002299$			$\bar{s}=25,295$ $4f=248$		
$n=\frac{v}{v'}=413\cdot32$			$\bar{v}=\cdot95270$ Correction for friction, &c. $=-\cdot00247$ $v=\cdot95023$		

$$\text{From Nos. 8 and 9}\left\{\begin{aligned} 2m\frac{(n+n')D_B}{r}&=187,736\\ 1-\frac{\delta}{g+\rho}&=\cdot999678\\ \text{Product}=V&=187,676\end{aligned}\right\}\text{Correction for second term}=-60.$$

December 21, 1880, No. 10.					
Wheel.			Clock.		
Reading.	Differences.	At mean reading.	Reading.	Alternate differences.	Alternate second differences.
3,542 5,795 .. 10,310	2,253 2,258	4,669 8,053	9,604 19,223 30,755 40,272 51,644 61,034	21,151 21,049 (20,889) 20,762	262 287
Signal at 6,918			Signal at 37,186		
$v'=\cdot002256$			$\bar{s}=41,236$ $4f=$ 285		
$n'=\frac{v}{v'}=464\cdot24$			$\bar{v}=$ 1·0445 Correction for friction, &c. $=+$ ·0028 $v=$ 1·0473		

From Nos. 9 and 10 $\left\{\begin{array}{r} 2m\frac{(n+n')\mathrm{D}_{\mathrm{B}}}{r}=186,537 \\ 1=\frac{\partial}{g+\rho}=\cdot999570 \\ \text{Product}=\mathrm{V}=186,457 \end{array}\right\}$ Correction for second term $=-80$.

December 21, 1880, No. 11.					
Wheel.			Clock.		
Reading.	Differences.	At mean reading.	Reading.	Alternate differences.	Alternate second differences.
1,897 4,721 .. 10,373	2,824 2,826	3,309 7,547	2,422 13,039 25,757 36,229 48,770 59,164	23,235 (23,190) 23,013 22,935	322 255
Signal at 5,690			Signal at 25,918		
$v'=\cdot002825$			$\bar{s}=24,675$ $4f=$ 325		
$n=\frac{v}{v'}=410\cdot12$			$\bar{v}=$ 1·1595 Correction for friction, &c. $=-$ ·0009 $v=$ 1·1586		

From Nos. 10 and 11 $\left\{\begin{array}{r} 2m\frac{(n+n')\mathrm{D}_{\mathrm{B}}}{r}=185,857 \\ 1-\frac{\partial}{g+\rho}=\cdot999632 \\ \text{Product}=\mathrm{V}=185,788 \end{array}\right\}$ Correction for second term $=-79$

January 20, 1881, No. 3.

Wheel.			Clock.		
Reading.	Differences.	At mean reading.	Reading.	Alternate differences.	Alternate second differences.
9,610			6,415		
	3,056	11,138	21,264	25,390	
12,666			31,805	(25,192)	310
..	3,051	15,717	46,456	25,080	304
18,767			56,885	24,888	
			71,344		
Signal at 13,914			Signal at 34,168		
$v'=\cdot003053$			$\bar{s}=33{,}899$ $4f=\ \ 309$		
$n=\frac{v}{v'}=412\cdot53$			$\bar{v}=\ \ 1\cdot2596$ Correction for friction, &c. = − ·0002 $v=\ \ 1\cdot2594$		

January 20, 1881, No. 4.

Wheel.			Clock.		
Reading.	Differences.	At mean reading.	Reading.	Alternate differences.	Alternate second differences.
2,214			7,554		
	2,746	3,587	22,621	25,768	
4,960			33,322	(25 586)	299
..	2,755	7,715	48,207	25,469	230
10,469			58,791	25,356	
			73,563		
Signal at 5,291			Signal at 35,490		
$v'=\cdot002750$			$\bar{s}=35{,}450$ $4f=\ \ 289$		
$n'=\frac{v}{v'}=465\cdot18$			$\bar{v}=\ \ 1\cdot2793$ Correction for friction, &c. 0·0 $v=\ \ 1\cdot2793$		

$$\text{From Nos. 3 and 4}\left\{\begin{array}{r} 2m\frac{(n+n')D_B}{r}=186{,}569\\ 1-\frac{\delta}{g+\rho}=\cdot999603\\ \text{Product}=V=186{,}495\end{array}\right\}\text{Correction for second term}=-74.$$

January 20, 1881, No. 5.

Wheel.			Clock.		
Reading.	Differences.	At mean reading.	Reading.	Alternate differences.	Alternate second differences.
2,926			8,863		
	2,761	4,307	22,476	23,264	
5,687			32,127	(23,074)	369
. .	2,778	8,465	45,550	22,895	408
11,243			55,022	22,666	
			68,216		
Signal at 6,119			Signal at 36,377		
$v'=\cdot002768$			$\bar{s}=34{,}060$ $4f=\ \ 378$		
$n=\frac{v}{v'}=415\cdot15$			$\bar{v}=\ \ 1\cdot1537$ Correction for friction, &c. = $-\ \ \cdot0046$ $v=\ \ 1\cdot1491$		

January 20, 1881, No. 6.

Wheel.			Clock.		
Reading.	Differences.	At mean reading.	Reading.	Alternate differences.	Alternate second differences.
231			515		
	2,411	1,437	9,890	22,557	
2,642			23,072	(22,419)	329
. .	2,404	5,046	32,309	22,228	295
7,449			45,300	22,124	
			54,433		
Signal at 3,810			Signal at 24,108		
$v'=\cdot002406$			$\bar{s}=21{,}141$ $4f=\ \ 331$		
$n'=\frac{v}{v'}=465\cdot00$			$\bar{v}=\ \ 1\cdot1210$ Correction for friction, &c. = $-\ \ \cdot0022$ $v=\ \ 1\cdot1188$		

$$\text{From Nos. 5 and 6}\left\{\begin{array}{r}2m\dfrac{(n+n')\mathrm{D_B}}{r}=187{,}088\\ 1-\dfrac{\delta}{g+\rho}=\cdot999550\\ \text{Product}=\mathrm{V}=187{,}003\end{array}\right\}\text{Correction for second term}=-85.$$

December 21, 1880, No. 2					
Wheel.			Clock.		
Reading.	Differences.	At mean reading.	Reading.	Alternate differences.	Alternate second differences.
5,790			5,370		
	2,316	6,948	17,091	21,385	
8,106			26,755	(21,198)	294
..	2,297	10,403	38,289	21,091	185
12,700			47,846	21,013	
			59,302		
Signal at 8,679			Signal at 28,958		
$v'=\cdot002306$			$\bar{s}=27,726$ $4f=280$		
$n'=\frac{v}{v'}=459\cdot28$			$\bar{v}=1\cdot0599$ Correction for friction, &c. $=-\cdot0008$ $v=1\cdot0591$		

December 21, 1880, No. 3.					
Wheel.			Clock.		
Reading.	Differences.	At mean reading.	Reading.	Alternate differences.	Alternate second differences.
			9,432		
8,430			21,958	23,000	
	2,336	9,598	32,432		345
10,766			..	(22,646)	
..	2,317	13,083	55,078		203
15,400			67,330	22,443	
			77,521		
Signal at 12,051			Signal at 42,348		
$v'=\cdot002323$			$\bar{s}=43,790$ $4f=282$		
$n''=\frac{v}{v'}=487\cdot89$			$\bar{v}=1\cdot1323$ Correction for friction, &c. $=+\cdot0011$ $v=1\cdot1334$		

$$\text{From Nos. 2 and 3}\left\{\begin{array}{r} 2m\dfrac{(n'+n'')D_B}{r+1}=185,846 \\ 1-\dfrac{\delta'}{g+\rho}=1\cdot00185 \\ \text{Product }=V=186,190 \end{array}\right\}\text{Correction for second term }=+344.$$

January 20, 1881, No. 6.

Wheel.			Clock.		
Reading.	Differences.	At mean reading.	Reading.	Alternate differences.	Alternate second differences.
231			515		
	2,411	1,437	9,890	22,557	
2,642			23,072	(22,419)	329
. .	2,404	5,046	32,309	22,228	295
7,449			45,300	22,124	
			54,433		
Signal at 2,406			Signal at 24,108		
$v'=\cdot 002406$			$\bar{s}=21,141$ $4f=\quad 331$		
$n'=\frac{v}{v'}=465\cdot 00$			$\bar{v}=\quad 1\cdot 1210$ Correction for friction, &c. $=-\quad \cdot 0022$ $v=\quad 1\cdot 1188$		

January 20, 1881, No. 7.

Wheel.			Clock.		
Reading.	Differences.	At mean reading.	Reading.	Alternate differences.	Alternate second differences.
6,229			4,722		
	2,381	7,420	18,357	23,272	
8,610			27,994	(23,022)	358
. .	2,358	10,968	41,379	22,914	306
13,325			50,908	22,716	
			64,095		
Signal at 9,144			Signal at 29,346		
$v'=\cdot 002370$			$\bar{s}=29,912$ $4f=\quad 352$		
$n''=\frac{v}{v'}=485\cdot 87$			$\bar{v}=\quad 1\cdot 1511$ Correction for friction, &c. $=+\quad \cdot 0004$ $v=\quad 1\cdot 1515$		

$$\text{From Nos. 6 and 7}\left\{\begin{array}{r} 2m\,\dfrac{(n'+n'')\mathrm{D}_{\mathrm{B}}}{r+1}=186{,}572 \\ 1-\dfrac{\delta'}{g+\rho}=1\cdot 00138 \\ \text{Product }=\mathrm{V}=186{,}830 \end{array}\right\}\text{Correction for second term }=+258.$$

January 20, 1881, No. 8.					
Wheel.			Clock.		
Reading.	Differences.	At mean reading.	Reading.	Alternate differences.	Alternate second differences.
2,823			139		
	2,891	4,269	16,186	27,330	
5,714			27,469	(27,140)	264
..	2,907	8,621	43,326	27,066	276
11,528			54,535	26,864	
			70,190		
Signal at 8,014			Signal at 28,248		
$v'=\cdot002905$			$\bar{s}=29{,}789$ $4f=\ \ 266$		
$n'=\frac{v}{v'}=467\cdot37$			$\bar{v}=\ \ 1\cdot3570$ Correction for friction, &c. $=+\ \ \cdot0007$ $v=\ \ 1\cdot3577$		

$$\text{From Nos. 7 and 8}\left\{\begin{aligned} 2m\frac{(n'+n'')\mathrm{D}_{\mathrm{B}}}{r+1}&=187{,}037\\ 1-\frac{\delta'}{g+\rho}&=1\cdot00122\\ \text{Product}=\mathrm{V}&=187{,}266\end{aligned}\right\}\text{Correction for second term }=+229$$

January 21, 1881, No. 2.					
Wheel.			Clock.		
Reading.	Differences.	At mean reading.	Reading.	Alternate differences.	Alternate second differences.
4,207			4,058		
	2,598	5,506	17,526	24,761	
6,805			28,819		415
..			..	(24,346)	
	2,590	10,690	53,165		394
..			65,962	23,952	
14,574			77,117		
Signal at 9,000			Signal at 39,234		
$v'=\cdot002593$			$\bar{s}=41{,}043$ $4f=\ \ 410$		
$n'=\frac{v}{v'}=470\cdot04$			$\bar{v}=\ \ 1\cdot2173$ Correction for friction, &c. $=+\ \ \cdot0015$ $v=\ \ 1\cdot2188$		

January 21, 1881, No. 3.					
Wheel.			Clock.		
Reading.	Differences.	At mean reading.	Reading.	Alternate differences.	Alternate second differences.
7,503			5,879		
	2,446	8,726	16,947	23,799	
9,949			29,678	(23,672)	460
..	2,412	12,361	40,619	23,339	462
14,773			53,017	23,210	
			63,829		
Signal at 12,071			Signal at 32,305		
$v'=\cdot002415$			$\bar{s}=28,841$ $4f=\ \ 460$		
$n''=\frac{v}{v'}=487\cdot58$			$\bar{v}=\ \ 1\cdot1836$ Correction for friction, &c. $=-\ \ \cdot0061$ $v=\ \ 1\cdot1775$		

From Nos. 2 and 3 $\left\{\begin{array}{r} 2m\dfrac{(n'+n'')\mathrm{D}_{\mathrm{B}}}{r+1}=187,897 \\ 1-\dfrac{\delta'}{g+\rho}=1\cdot00116 \\ \text{Product}=\mathrm{V}=188,110 \end{array}\right\}$ Correction for second term $=+213$

January 21, 1881, No. 4.					
Wheel.			Clock.		
Reading.	Differences.	At mean reading.	Reading.	Alternate differences.	Alternate second differences.
7,376			4,725		
	2,517	8,635	15,768	23,609	
9,893			28,334	(23,449)	293
..	2,483	12,376	39,217	23,316	245
14,859			51,650	23,204	
			62,421		
Signal at 11,605			Signal at 31,786		
$v'=\cdot002940$			$\bar{s}=27,529$ $4f=\ \ 288$		
$n'=\frac{v}{v'}=469\cdot85$			$\bar{v}=\ \ 1\cdot1725$ Correction for friction, &c. $=-\ \ \cdot0026$ $v=\ \ 1\cdot1699$		

From Nos. 3 and 4 $\left\{\begin{array}{r} 2m\dfrac{(n'+n'')\mathrm{D}_{\mathrm{B}}}{r+1}=187,860 \\ 1-\dfrac{\delta'}{g+\rho}=1\cdot00117 \\ \text{Product}=\mathrm{V}=188,079 \end{array}\right\}$ Correction for second term $=+219$

The following is a summary of these results :—

12th and 13th equalities.

1880 December 21, Nos.	1 and 2	V=187,707	miles per second.
,, ,,	5 ,, 6	188,405	,,
,, ,,	8 ,, 9	187,676	,,
,, ,,	9 ,, 10	186,457	,,
,, ,,	10 ,, 11	185,788	,,
1881 January 20, Nos.	3 ,, 4	186,495	,,
,, ,,	5 ,, 6	187,003	,,
Mean for 12th and 13th equalities :		V=187,076	,,

13th and 14th equalities.

1880 December 21, Nos.	2 and 3	V=186,190	miles per second.
1881 January 20, Nos.	6 ,, 7	186,830	,,
,, ,,	7 ,, 8	187,266	,,
,, January 21, Nos.	2 ,, 3	188,110	,,
,, ,,	3 ,, 4	188,079	,,
Mean for 13th and 14th equalities :		V=187,295	,,
General mean of both sets . .		V=187,167	,,

Multiplying this by the mean refractive index of air (=1·00029) we obtain the value for the velocity in vacuo, viz.: 187,221 miles per second.

This must be corrected for the rate of our clock.

One second of our clock is equal to 0·999723 of a mean solar second.

Dividing the value found for V by this quantity, we obtain the final value for the velocity of the white light from an electric lamp in vacuo, viz. :—

V=187,273 miles per second (log=5·2724757)
=301,382 kiloms. per second (log=5·4791167)
Using Struve's constant of aberration 20″·445.
The resulting parallax of the sun is =8″·77.
Distance of the sun =93,223,000 miles.

The value obtained by Cornu,[*] using the method of Fizeau, was 300,400 kiloms. per second. He nearly always used the Drummond (or lime) light. A few experiments were made with a petroleum lamp.

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[*] "Annales de l'Observatoire de Paris" (Mémoires, tome xiii.), 1876.

The value obtained by MICHELSON,[*] using a modification of the method of FOUCAULT, was 299,940 kiloms. per second. He always used the light of the sun when near the horizon (in the early morning or late afternoon), except in a single set of observations where he used the electric light, and which he considered unsatisfactory. Thus we have three series of very carefully conducted experiments to determine the velocity of light, each one differing essentially from the others in their method of research; the results are all very close to each other, and we believe that in the sequel we shall be able to show reasons for the outstanding differences. Grouping the three sets in order we have:—

	Usual source of light.	Method.	Results for V.
			Kiloms.
MICHELSON[*] . . .	The sun near horizon .	Deflection by mirror	299,940
CORNU[†]	Lime light	Toothed wheel and eclipses .	300,400
YOUNG and FORBES .	Electric light . . .	Toothed wheel and equalities .	301,382

CORNU—MICHELSON	460	kiloms.	per second.
YOUNG and FORBES—MICHELSON.	1,442	,,	,,
YOUNG and FORBES—CORNU . .	982	,,	,,

After we had completed the observations which have now been reduced, we found reason for believing that the velocity of light depends upon its colour, and further examination of the question confirmed us in this opinion. It seemed useless then to continue to measure the velocity of a light whose colour changes considerably and quite sufficiently to give us values for the velocity of light varying much more than any errors of observation could make them vary. We then devoted our attention to an examination of the question involved in the second part of our research, viz.: whether the velocity of light depends upon its colour, and if so to what degree.

PART II.

RELATIVE VELOCITY OF LIGHTS OF DIFFERENT COLOURS.

Does the velocity of light depend upon its colour?

Before describing the observations which furnish an answer to this question, we will briefly recapitulate the general arrangements in our method of working, so that those who wish it may study this part of our research independently of the rest.

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[*] "Astronomical Papers prepared for the use of the American Ephemeris and Nautical Almanac." Vol. i., part iii., 1880.

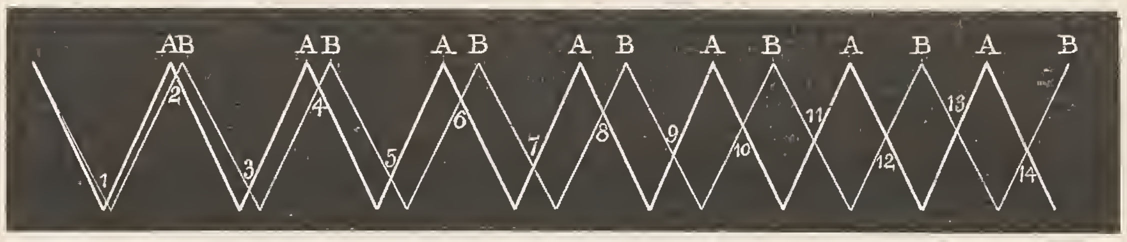
[†] "Annales de l'Observatoire de Paris" (Mémoires, tome xiii.), 1876.

Our method is very like that of M. Fizeau. We have an observing telescope, in the focus of which, and in a plane perpendicular to the axis of which, we have a toothed wheel revolving with very great velocity. The light from the sun or from an electric or other lamp is condensed by a lens, and reflected by a diagonal reflector in the eye-piece, so as to throw an image of the incandescent carbons upon the toothed wheel. The light passing between the teeth goes to a distant reflector and returns by reflection to the object-glass of the observing telescope, by which the rays are brought to a focus in the plane of the toothed wheel at that exact part of the focal plane whence the rays had emerged which were capable of striking the distant reflector. This point in the focal plane is of course that point at which the observer sees the distant reflector (through a small aperture in the diagonal reflector), or at which he sees a star of light when the lamp is in action.

If the double distance to the distant reflector and back were about six miles, we know, roughly speaking, light takes about $\frac{1}{31000}$ second to traverse the double distance. Now suppose the wheel to rotate very slowly. We see alternately a tooth of the wheel, and a star of light shining in the interval between two teeth. If the speed be increased so that more than 10 teeth pass in a second, the persistence of visual impressions causes us to be unable to distinguish these alternate phases. We see a star of light continuously upon a partially illuminated field. If in $\frac{1}{31000}$ second a tooth passes into the position previously occupied by a space, then the light which passed away through a space to the distant reflector is on its return stopped by a tooth and we see nothing but the tooth, while when a space between two teeth is at that part of the field of view where the star should appear no star of light is seen, because $\frac{1}{31000}$ second ago a tooth occupied that position, and no light could get through to go to the distant reflector. If there be 400 teeth in the wheel this speed of revolution is $\frac{800}{31000}$ of a second to one revolution, or 38·75 revolutions a second. At this speed of revolution no star of light would be seen, but if the speed be doubled the light passing out through a space to the distant reflector can on its return pass through the next space to the eye of the observer, and the light is seen with its full intensity. If the speed 38·75 revolutions a second be increased threefold, fivefold, &c., or any odd number of times, we have an eclipse of the star. If that speed be increased twofold, fourfold, &c., or any even number of times, we have full brightness.

Fizeau, and Cornu after him, measured the speed required to produce an eclipse, and thence they deduced a value for the velocity of light. Our method, however, differs distinctly from theirs in this way: that in place of having a single reflector in the distance, we have two at different distances from the observing telescope, but nearly in the same line with it, so that the observer in looking through the telescope sees two stars side by side separated from each other by a distance of about 25″ of arc. While the toothed wheel is being rotated with gradually increasing speed, the star coming from the more distant reflector (which we call A) is eclipsed before that one coming from the nearer reflector (which we call B). As the speed increases A grows brighter, while B is still diminishing in brightness, and at a certain speed they are of equal brightness. In the same way in the different phases, as the speed of the wheel is increased more and more, we have a succession of equalities in the two stars. The distance of A from the observer is to that of B in the ratio of about 13 to 12. Hence it follows that the sixth maximum of B coincides with the seventh minimum of A. In the following figure abscissæ represent speeds of revolution of the toothed wheel, and ordinates intensities of the stars. The intersection of the two lines indicates equality of brightness and shows the speed required to produce it.

Fig. 2.



The lines in the above figure which indicate the brightness are subject to certain alterations dependant upon the necessary imperfections in the optical and mechanical parts of the apparatus. It has been shown in the previous part of this memoir that our method is in general unaffected by these alterations. Most of our observations have been made at speeds corresponding to the 12th, 13th, and 14th equalities, and it will be unnecessary in the diagrams which represent further development of the theory to delineate other parts of the diagram.

Distinctive colours observed in the return light.

Having made these preliminary remarks, we will now proceed to trace the steps by means of which the relation between colour and velocity has been suspected, and the quantity of the effect has been approximately determined.

In the course of our observations made with sunlight at Pitlochry in 1878, and in those made with the electric light at Kelly in 1880–81, we were frequently annoyed by the presence of colour in the stars, one of them appearing reddish and the other bluish. This made it very difficult to appreciate the exact speed which might be said to produce equality in the lights; for, as is well known, it is very difficult, if not impossible, to judge accurately of the equality of two lights of different colours. We considered that these colours arose from a want of accurate adjustment of the distant reflectors. These consist each of a telescope tube with an achromatic object-glass at one end, but with no eye-piece, and having a silver mirror at the focus of the object-glass. An image of the object-glass of the observing telescope is thrown by the rays from the source of light upon the silver mirror, whence the light is reflected back to the observing telescope. Now the quantity of light which is reflected back *into* the observing telescope depends largely upon the accuracy of focus of the reflecting collimator. If, then, the object-glass be not accurately achromatised the one reflector may be focussed accurately for blue rays and the other for red rays. Thus we should have one star intrinsically redder than the other. In consequence of these considerations, we used, whenever we noticed a difference in colour, to mention the fact, in order that this cause of inaccuracy in the observation might be taken account of. We did not take particular notice which star was red and which blue, though we sometimes noted the fact. We had no idea that any information might be gained by always noticing which star was red and which one blue.

On different days the distinctness and steadiness of the stars varied enormously. The days when the stars were steady and distinct were the days on which we got the best observations, and felt most certainty about the exact speeds which produced equality in the two stars, except that it was often on these days that the difference in colour troubled us most.

On the 11th February, 1881, we were making the regular observations for determining the speed of revolution of the toothed wheel required to give the 12th, 13th, and 14th equalities, corresponding to speeds of about 410, 450, and 490 revolutions a second. These speeds were obtained by using three, four, or five weights to drive the mechanism. The observations in the observing book are numbered from 1 upwards. The following observations, 1 to 5, were taken between 9.30 A.M. and 9.50 A.M. The remarks are extracted from the observing book, and were written at the time, an entry being made after each observation.

"*February* 11, 1881.—Splendid morning. A and B very bright and steady. If anything, A is greater than B.

1.—Three weights. B increasing with increase of speed. B reddish, A bluish (?).
2.—Four weights. A increasing ,, ,, ,, A ,, B ,,
3.—Five weights. B increasing ,, ,, ,, B ,, A ,,
4.—Four weights. A increasing ,, ,, ,, A ,, B ,,
5.—Three weights. B increasing ,, ,, ,, B ,, A ,,
(Not a very good observation.)"

The mark (?) expressing doubt about the colour in the first observation was inserted after the second observation had been made, and was so inserted because they seemed to be antagonistic.

When the observations numbered 6 and 7 respectively were being taken at 10.55 A.M., the following remark is entered in the observing book: "The same phenomenon as above." This refers to the colours.

A number of trials were then made at different speeds (in which no use was made of the chronograph) to examine still further this remarkable phenomenon, and the following statement is made in the observing book:—"Always the light which is increasing with respect to the other, with increase of velocity [of the toothed wheel], appears red; and the other one blue." The words in square brackets are not in the original, and are inserted here to make the statement clearer.

These observations clearly proved to us that the colour which we had often observed was not always due to the adjustment of the distant reflectors. For here sometimes the one and sometimes the other was the red one. At each successive equality (*e.g.*, the 11th and 12th, the 12th and 13th, &c.) the colours of A and B are reversed.

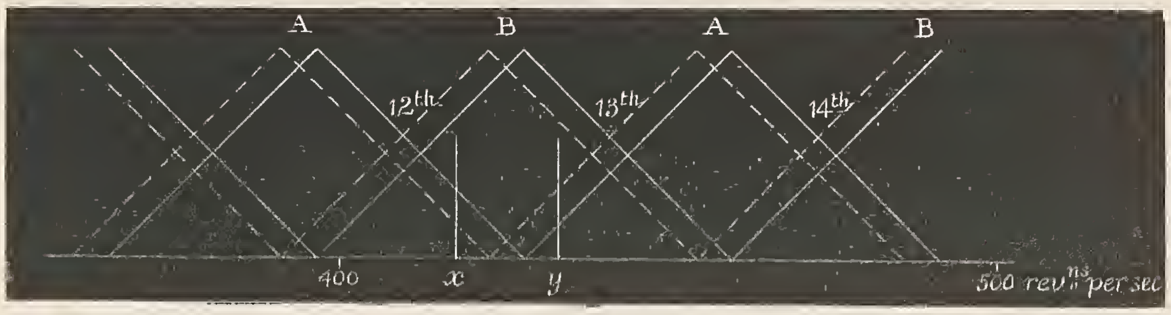
Since February 11 there certainly have been many days when the colour-differences were not perceptible. It may perhaps have been because the stars were not steady or were flickering or indistinct. On these occasions the atmospheric refraction disturbs the course of the rays, so that the teeth of the wheel being extremely minute, a ray of light which, if there were no irregular atmospheric refraction, would not reach the reflector, does so under these circumstances. In such a case the stars do not alter their intensities, with change of speed of the toothed wheel, so regularly as they do when the atmosphere is not unequally heated and disturbed.

The general result however was established by the observations on February 11, 1881, but it is not a common observation.

Explanation of the colours perceived in the return light.

The simplest explanation which can be given of these phenomena, and the only explanation which seems to be capable of standing all kinds of tests, is that *the different colours travel with different velocities, the more refrangible rays, or those with shortest wave-length, travelling quickest.* If this were the case we should be forced to alter our diagram indicating the intensities of A and B. We should have as many curves of intensity for each of the two stars of light as there are colours in the light we are employing. Let us examine only two of these colours (red and blue for example). If the red light travel slower than the blue a smaller velocity is required to produce an eclipse with red light and with blue. For this reason the curve representing intensity in terms of speed of rotation for red light will have its maxima and minima lagging gradually more and more behind those for the blue light. We are in general dealing only with the speeds of rotation which produce the 12th, 13th, and 14th equalities; and during that small variation in speed the lines for red and for blue light may, for purposes of illustration, be drawn sensibly parallel. The curves for the two stars A and B would then be shown approximately by the following diagram, in which dotted lines represent red light, and full lines blue light. Here we notice that at x the light of A is diminishing with increase of speed, and the abscissa corresponding to blue light is greater than that corresponding to red light. Hence, *when the intensity is diminishing with increase of speed the star should have a blue tinge.* But at y the light of A is increasing with increase of speed, and the abscissa corresponding to red light is greater than that corresponding to blue light. Hence, *when the* *intensity is increasing with increase of speed the star should have a red tinge.* Observation confirms these statements. Hence the observations can be explained on the assumption that blue light travels quicker than red light.

Fig. 3.



The analytical expression of this result is quite simple. §§ 2 and 3 of the mathematical theory give the following values for the intensities of red and violet light (indicated by the suffixes R and V):

1. Light increasing with increase of speed
$$\begin{cases} I_R = \frac{E}{2}\left\{2(1-\kappa)-p+\frac{n}{N_R}\right\} \\ I_V = \frac{E}{2}\left\{2(1-\kappa)-p+\frac{n}{N_V}\right\} \end{cases}$$

But if the velocity for violet is greater than for red light, N_V is greater than N_R, and hence I_R is greater than I_V in the even phases, and the return light will appear tinged with red.

2. Light diminishing with increase of speed
$$\begin{cases} I_R = \frac{E}{2}\left\{2(1-\kappa)+p-1-\frac{n}{N_R}\right\} \\ I_V = \frac{E}{2}\left\{2(1-\kappa)+p-1-\frac{n}{N_V}\right\} \end{cases}$$

Here, on the other hand, if N_V be greater than N_R, we have I_V greater than I_R, and the return light will be tinged with blue, in the odd phases.

First measurements of the difference in velocity of red and white light.

While we were quite prepared to examine every possible source of error in these new and unexpected conclusions, we considered it to be of first importance to attempt to get, even in a rough manner, some actual measurements of the difference in velocity of red and blue light, on the assumption that such a difference is the explanation of our results. From the above figure it will be seen that the speed of rotation necessary to give equality of lights must always be greater for blue than for red light. It is also clear that the difference in speed of rotation for red and for blue light bears the same relation to the absolute speed of rotation for either of those colours as the difference in velocity between rays of red and blue light bears to the absolute velocity of that colour.* The same thing will hold, though to a less extent, on comparing red and white light. Our only means at hand on February 11 was to determine the speed which produced an equality (1) in the ordinary way with the white light of the electric lamp, and (2) with the eye screened by a piece of ruby red glass.

The observations made with this object on February 11 are named in the observing book No. 13 (white), No. 13 (red), No. 14 (white), and No. 14 (red).

They were not very satisfactory, for the differences found between the velocities of red and white light were small. The observations No. 13 were at the speed producing the 13th equality. The observations No. 14 were at the speed producing the 14th equality. The speeds of rotation finally deduced from the chronograph records were as follows :—†

		Difference.
Observation No. 13 (red) speed of rotation	$456^{r}\cdot 84$	$4^{r}\cdot 14$
,, No. 13 (white) ,, ,, ...	$460^{r}\cdot 98$	
,, No. 14 (red) speed of rotation	$494^{r}\cdot 85$	$1^{r}\cdot 57$
,, No. 14 (white) ,, ,, ...	$496^{r}\cdot 42$	

$$\frac{\text{Difference of velocity (red and white)}}{\text{Absolute velocity (white)}}=\begin{cases}\cdot 90 \text{ per cent. from No. 13}\\ \cdot 32 \quad ,, \quad ,, \text{ No. 14}\end{cases}$$

These differences are small; but on the whole they indicate a greater speed for white than for red light. But these differences might be suspected to be due to irregularities in the action of the chronograph. The general result seemed to be that we must obtain a greater difference in speed by choosing two colours of light, differing considerably in wave-length, and that we might with advantage discard the chronograph as an absolute measurer of speeds, and adopt some more delicate means of measuring minute differences of speed.

Great difficulty was found in obtaining a blue medium which would sufficiently stop out the red rays. The ordinary blue glass, coloured with cobalt, allows large quantities of red light to pass. We tried eight or nine solutions, which we put into glass cells with parallel sides and tested with a prism. We found that a nitrate of copper solution gave the least quantity of red.

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* § 4 of the mathematical theory gives us the value of n for the r^{th} equality for red light, viz.,

$$n_{\text{R}}=\frac{\{2(1-\kappa)(1-\rho)+r(1+\rho)\}\text{N}_{\text{A}}\text{N}_{\text{B}}}{\text{N}_{\text{B}}+\rho\text{N}_{\text{A}}}=\kappa\,.\,\frac{\text{N}_{\text{A}}\text{N}_{\text{B}}}{\text{N}_{\text{B}}+\rho\text{N}_{\text{A}}}$$

If now $\sigma=\frac{\text{velocity of violet light}}{\text{velocity of red light}}$ we notice that for violet light N_{A} and N_{B} must be multiplied by σ.

Thus we have

$$n_{\text{V}}=\kappa.\sigma.\frac{\text{N}_{\text{A}}\text{N}_{\text{B}}}{\text{N}_{\text{B}}+\rho\text{N}_{\text{A}}}=\sigma.n_{\text{R}}$$

$$\frac{n_{\text{V}}}{n_{\text{R}}}=\sigma$$

also $\frac{n_{\text{V}}-n_{\text{R}}}{n_{\text{R}}}=\frac{\text{difference of velocities for red and violet}}{\text{velocity of red light}}$ Q. E. D.

† The observations and reductions are in the hands of the Royal Society.

First differential observations for red and blue light.

After February 11 we never had a clear day or night to continue our research until the 21st of the month. We then made observations in the following manner:—A thick piece of indiarubber tubing was attached to the top of the pulley which supports the weights driving the toothed wheel. At its upper end it was attached to a string, which, passing over a fixed pulley, was held by the observer. The observer adjusted the driving weights so as to be a little in excess of what was required to produce equality of the lights. He then fixed the string tightly, and as the weights descended, the indiarubber was stretched, and diminished the effective driving weight. In this way a beautifully gradual diminution of velocity was obtained, accompanied by as beautifully gradual an increase and decrease in the brightness of the two stars respectively. Our plan was to place the blue solution between the lamp and the diagonal reflector. When equality of lights was attained the observer said "Stop" to the assistant, who then commenced to count seconds on the clock. At the same time the blue solution was replaced by a piece of ruby glass. When equality of red lights was attained the observer again said "Stop." The interval of time was then noted in the observing book. Our intention was to measure by means of the chronograph, at our leisure, the diminution in velocity produced by the action of the indiarubber during a given number of seconds.

Alongside of each observation we entered in the observing book the weights we used. But in the observations of February 21 these were nearly always the same, and it was always the 12th equality which was observed, being a speed of about 400 revolutions a second. It will nevertheless be well here to tabulate the names of the weights employed and their absolute weights, as reference is frequently made in the observing book to them. We usually had one large lead weight to begin with, and this was the one used when one lead weight is spoken of. It weighed 56 lbs. We had two other lead weights which might be added, each weighing 12 lbs. 2 oz. We had five iron weights, each weighing 10 lbs. 7 oz. These were generally supported by an additional iron hook weighing 1 lb. 11 oz. Smaller weights, two of which were called *a* and *b*, each weighing about 1 lb., and others weighing a few ounces, were used as well. The length of the indiarubber tubing which we generally, if not always, used was 10½ inches when unstretched. It lengthens 3 inches with the addition of 1 lb. weight.

With one lead weight (56 lbs.) the fall of the weight when driving the mechanism is 1 inch in 7 seconds.

With three lead weights, one iron weight and hook (92 lbs. 7 oz.), the fall is 1 inch in 5·3 seconds.

[*N.B.*—It must be remembered that the effective weight is half of the actual weight owing to the action of the pulley.]

On February 21 eight observations were made in the manner described above. A little practice was required to get accustomed to the method of observation, but after a few trials the results were wonderfully accordant. The following extract from the observing book contains all the observations made upon that day :—

"*February* 21.—No. 1. At 5.30 P.M. Two lead weights and *a*. Blue to red—interval =28 seconds."

The following were between 7 and 8 P.M. :—

"No. 2. Two lead weights and *a* + hook. Interval =9½ seconds.
"No. 3. Weights as before . . . . Interval =20 seconds.
"No. 4. „ „ . . . . Interval =24 seconds.
"No. 5. „ „ +*b*. Interval =45 seconds.
"No. 6. „ „ . . . . Interval =20½ seconds. Very good.
"No. 7. „ „ . . . . Interval =20 seconds.
"No. 8. „ „ . . . . Interval =21 seconds."

The following remark is added :—

"During the whole of to-day—while trying different methods to detect the difference in velocity of different coloured rays—I have at every step been struck by the enormous difference that exists. It is easy to get a velocity for which A is greater for blue light while B is greater for red light. Sometimes, when A and B are equal for blue, A or B is almost invisible for red, and the other at [near ?] its maximum."

On the evening of February 21 chronograph measurements were made to determine the loss of speed after the indiarubber had been in action for some definite interval of time. The interval adopted was about 18 seconds. The observations and reductions are in the hands of the Royal Society. The results of four such determinations are as follows :—

	No. 1.	No. 2.	No. 3.	No. 4.
Revolutions per second—				
At beginning of observation . .	487^{r}·22	514^{r}·77	502^{r}·33	502^{r}·33
At end „ „ . .	480^{r}·28	499^{r}·64	491^{r}·45	496^{r}·60
Diminution of speed in 18 seconds . .	6^{r}·94	13^{r}·13	10^{r}·88	4^{r}·73
„ „ 1 second . .	0^{r}·38	0^{r}·73	0^{r}·60	0^{r}·26

Average . . . . =0·49 revolution per second.

Although these measurements vary a little amongst each other, they give us sufficiently well a rough knowledge of the rate at which the indiarubber reduces the speed.

The average interval of time between the equality of red and blue lights was 23·5 seconds. This multiplied by 0·49 gives us a difference of 11·5 revolutions a second, or about 2·82 per cent. of the speed producing equality of white light (410 revolutions per second).

It appears then from these experiments that the difference in velocity of red and blue lights is about 2·82 per cent. of the total velocity of light.

The eight separate observations give the following values: 3·36, 1·14, 2·40, 2·88, 5·40, 2·46, 2·40, 2·52.

On February 23 a red solution was substituted for the ruby red glass, as it allowed fewer of the more refrangible rays to pass. In these experiments the chronograph was used directly; the velocity of rotation of the toothed wheel being reduced as before by means of the indiarubber tubing.

The observation book contains the following entry :—

"Lights very bright. A greater than B decidedly. Observations perfectly satisfactory. Interval between first and second signals [those for blue and red lights] about 20 seconds. Whole set [four observations] completed in 10 minutes. Atmosphere very clear all the time.

"No. 1. A increasing with increase of speed. Blue light. (When blue lights were equal, on changing to red light, A was far too bright.)

"No. 2. A increasing with increase of speed. Red light.
"No. 3. B ,, ,, ,, ,, Blue light.
"No. 4. B ,, ,, ,, ,, Red light."

The chronograph was used this day in a different manner to that usually adopted. Contrary to our expectation, the result was not satisfactory, and we do not think that the speeds obtained can be relied upon. We can only say therefore to-day that the general effect was the same as before. The defect we allude to is probably due to a small mistake in counting the number of pen marks in a certain space upon the chronograph cylinder. The readings of the chronograph and the reductions are, however, in the hands of the Royal Society.

Exceptional observation.

On February 24, 10 P.M., the following entry was made :—"Tried two speeds. (1) A increasing with increase of speed, (2) B increasing. Used red and blue lights. There seemed to be a decided tendency for *red* to require a *greater* velocity than *blue*, to produce equality. This is contrary to all our previous experience. A was decidedly brighter than B, but both were tolerably steady and bright."

A few minutes later the following observation was made :—

"With white light.

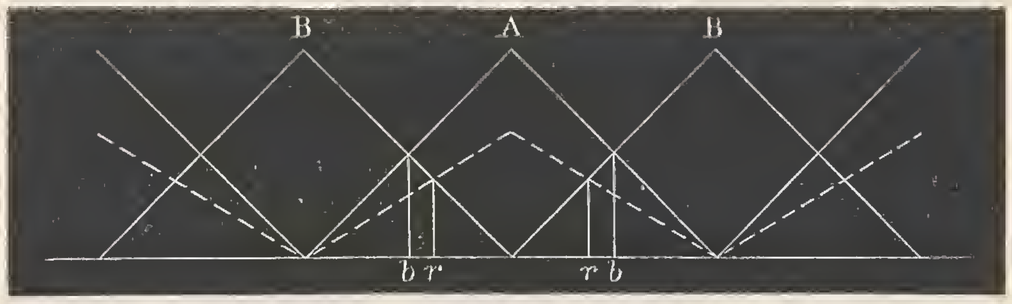
"A blue, B red. Increase of speed increases brightness of A. [This is contrary to previous experience.] This was with speed about 440.

"A blue, B red. Increase of speed increases brightness of B. [This is in accordance with previous experience.] This was with speed about 400.

"It seems then that to-night A is intrinsically bluer than B, owing probably to adjustment of focus. But will this explain all?"

Let us see now what would be the effect of A being intrinsically bluer than B. The following figure represents such a case on the supposition that blue and red light travel with the same velocity, and that A is deficient in red light. Full lines represent blue light, and dotted lines red. These coincide for B.

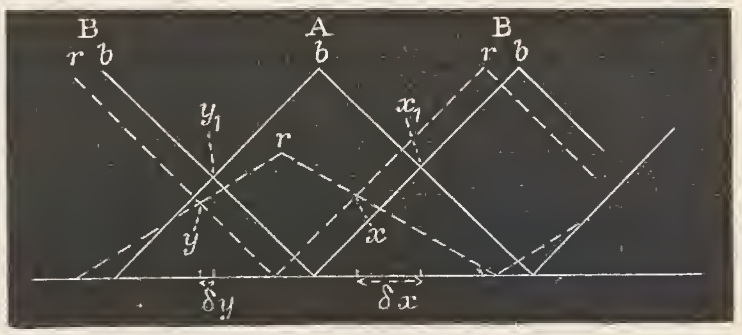
Fig. 4.



The result is clear that at different successive phases the blue and red rays alternately require a greater speed of rotation to produce equality of lights. But this does not in the least represent the results of our observations.

The following diagram illustrates the case where A is deficient in red light, on the assumption that blue rays travel quickest.

Fig. 5.



The maxima of A and B with different speeds are indicated by the letters A, B. The red is shown to reach its maximum before the blue. The red in A is shown to have a smaller maximum than the red in B. The intersections of the full and dotted lines represent the speeds producing equality of the blue and red rays respectively. Here δy is almost nil, whereas δx is too great. Hence, if we observe only at one speed, we may get far too great or far too small a difference between the velocities of red and blue light.

The mean of δy and δx is (nearly) that due to the difference in velocity of red and blue rays.

[*N.B.*—Our observations on February 23 were taken at the 12th and at the 13th equalities, so that this cause of uncertainty is eliminated.]

On February 25, 1881, 4 P.M., B was greater than A. A was bluish compared with B when the toothed wheel was removed. Three observations were made, but they show no decided difference between δx and δy. The indiarubber was used as before.

No. 1. 12th equality of lights . . . . . . Blue-red=2·5 seconds.
No. 2. 13th „ „ . . . . . . „ „ =4·0 „
No. 3. (at 5·30 P.M.) 12th equality of light . „ „ =6·0 „

Now the loss of speed caused by the indiarubber in one second is 0·49 revolution per second. The 12th equality is at a speed of about 420 revolutions a second; and the 13th equality about 460 revolutions a second. From these we get the difference between the velocity for red and for blue light, in percentages of the whole, as follows:—

From No. 1 we have 0·29 per cent.
„ No. 2 „ 0·43 „
„ No. 3 „ 0·70 „

On February 27, between noon and 1 P.M., measurements were made in the usual way with the indiarubber, which was a most convenient method of working at different speeds:—

No. 1. 12th equality . . . . Blue-red =11 seconds.
No. 2. 13th „ . . . . „ „ =11½ „

(In this observation the indiarubber did not seem to act well after the blue equality).

The observations were continued at 4 P.M. at various speeds and slower than those we had previously experimented upon:—

No. 3. 9th equality . . . . Blue-red = 9·0 seconds.
No. 4. 10th „ . . . . „ „ =15·5 „
No. 5. 9th „ . . . . „ „ =16·5 „
No. 6. 10th „ . . . . „ „ =11·0 „
No. 7. 10th „ . . . . „ „ =14·0 „
No. 8. 11th „ . . . . „ „ = 9·5 „

The following remarks are appended:—

"Observations from 3 to 8, lights splendidly steady and equal. The difference in velocity for red and blue most striking. A and B are precisely the same colour. Both A and B looked like clearly defined circles with no diffraction phenomena. This is very uncommon, and the observations are therefore valuable."

On the same day some more chronographic measurements were made to determine the reduction in speed per second of time produced by the indiarubber, at the speeds of the 10th and 11th equalities. The reductions are in the hands of the Royal Society, and they gave us—

10th equality . .	0·34	revolutions per second	diminution	in 1 second.
11th ,, . .	0·29	,,	,,	,,
9th ,, . .	0·31	,,	,,	,,

Reducing the first and second observations as before, allowing for a diminution of speed of 0·49 revolution a second, per second—

$$\text{From No. 1 we have } \frac{\cdot 49 \times 11 \times 100}{420} = 1\cdot 28 \text{ per cent.}$$

$$\text{,, No. 2 ,, } \frac{\cdot 49 \times 11\cdot 5 \times 100}{460} = 1\cdot 22 \text{ ,,}$$

Chronograph records were afterwards taken at about the speeds of the 9th, 10th, and 11th equalities, and the speeds read off at intervals of about 8 or 10 seconds. Thus we obtained the percentage loss of speed in one second. Reducing the observations of this day we have—

No. 3.	9th equality;	$\frac{\text{difference of velocity (blue and red)}}{\text{total velocity}}$	=1·71 per cent.
No. 4.	10th ,,	,, ,,	=1·55 ,,
No. 5.	9th ,,	,, ,,	=3·14 ,,
No 6.	10th ,,	,, ,,	=1·10 ,,
No. 7.	10th ,,	,, ,,	=1·40 ,,
No. 8.	11th ,,	,, ,,	=0·68 ,,

Mean of the day's observations=1·51

March 1, 1881.—In the interval between February 27 and March 1 we removed the wedge under the toothed wheel and brought the mechanism into an upright position. Seeing that the solutions which we had hitherto used to produce our colours allowed colours of very different refrangibilities to pass, we thought it would be satisfactory to try the effect of pure prismatic colours. To do this we mounted a bisulphide of carbon bottle-prism in front of the condenser and succeeded in throwing a pure spectrum upon the toothed wheel. The distance traversed by the rays from the prism to the toothed wheel was 24½ inches. The electric lamp was mounted upon rollers so that we could easily move it to change the colours. A motion of only about ½-inch of the spectrum changed the colour of the stars from blue to red. This was a very convenient and pleasant method of working. The indiarubber was used as before. The following remarks are extracted from the observing book:—

"Reduced velocity by stretched indiarubber. Used prism for colours. Deflection of spectrum in inches given below [this was measured approximately by our assistant].

" No. 1.	Green to orange. . .	$\frac{1}{2}$-inch,	12·5 seconds,	12th equality,	good
" No. 2.	Green to reddish-orange	$\frac{5}{8}$-inch,	15 ,,	13th ,,	good
" No. 3.	Blue to orange . . .	$\frac{7}{16}$-inch,	9 ,,	14th ,,	fair
" No. 4.	Deep blue to blood-red	$\frac{5}{8}$-inch,	9 ,,	14th ,,	very good.

" This was a most successful mode of observation. Though B was much greater than A the equality was well determined each time."

In reducing these observations it is clearly impossible to take into account the exact refrangibilities or wave-lengths of the colours named. The general change was from a slightly greenish-blue to a red tinged with orange. Reducing as before, we have—

No. 1. $\frac{\text{Difference in velocity (blue and red)}}{\text{total velocity}} = 1{\cdot}35$ per cent.
No. 2. ,, ,, $= 1{\cdot}56$,,
No. 3. ,, ,, $= 0{\cdot}90$,,
No. 4. ,, ,, $= 0{\cdot}90$,,

March 8, 1 P.M.—It might, perhaps, be thought that we had now thoroughly tested and confirmed our first conclusions by the variety of our tests. But we were anxious to leave no room for doubt and to vary the tests in every possible way, and we thought that some advantage might be gained by changing entirely our method of altering the speed, so as gradually to increase it, the reverse of what we had done with the indiarubber. Accordingly we attached an iron crucible to the weights and led a thick indiarubber tube from it up to a funnel with a stop-cock, in reach of the observer. We filled this with mercury, and having put on such weights as were barely sufficient to produce equality, we turned on red light (using the prism method) and opened the stop-cock. So soon as equality of red lights was attained, we said " Stop " to the assistant, who then commenced to count seconds on the clock. Meantime he had instantaneously changed the colour of the light to blue. When the blue equality arrived we again said " Stop." He gave us the interval in seconds, and the deflection of the spectrum in fractions of an inch, which we entered in the observing book. Chronograph tests were afterwards made to measure the increase in speed produced in a second by the flow of mercury. This method seemed to us to be hardly of so great delicacy as the indiarubber method, but we felt much interest in seeing whether so entirely a different method could give results approximating to those obtained in the previous researches.

The following remarks are extracted from the observing book :—

" *March* 8, 1 P.M.—No. 1. Blue-red = 16 seconds. Exact time uncertain to a few seconds. General effect positive " [*i.e.*, the blue equality required a higher speed].

"After blue equality turned again to red, and B was much brighter" [than A], "B increasing with increase of speed. Spectrum deflection about $\frac{5}{8}$ inch." [This was the 12th equality.]

"No. 2. Ditto. Same in every way. Blue-red $=9$ seconds. $\frac{5}{8}$ inch deflection.

"No. 3. Same arrangements. Blue-red $=5$ seconds nearly. Deflection $=\frac{3}{4}$ inch.

"Want of oil in (1), (2), and (3) made velocity irregular. We tried one more observation, but the motion of the wheel seemed quite irregular, and though it gave the same value (16 seconds) as No. 1, we could not trust it, because on turning on red light again A was brightest. We now oiled the mechanism with great effect in steadying the motion and diminishing noise. But the lights had begun to flicker and we got no good observations."

At 4 P.M. the lights improved.

"4 P.M.—Oiled axles.

"No. 4. 12th equality. Blue-red $=8$ seconds. Deflection $=\frac{9}{16}$ inch; but it did not look so great a change of colour as usual.

"No. 5. Same arrangements. Blue-red $=10{\cdot}5$ seconds. Deflection $=\frac{11}{16}$ inch. (The change in colour looked greater.) After equality of blue, tried red again, B was far brightest.

"No. 6. Same arrangements. Blue-red $=6$ seconds. Deflection $=\frac{11}{16}$ inch.

"No. 7. 11th equality. Blue-red $=7$ seconds. Deflection $=\frac{5}{8}$ inch.

"No. 8. Ditto. Blue-red $=6{\cdot}5$ seconds. Deflection $=\frac{5}{8}$ inch."

"8.45 P.M.—No. 9. Same conditions as No. 1" [12th equality]. "Blue-red $=16$ seconds. Deflection $=\frac{5}{8}$ inch.

"No. 10. Same conditions. Blue-red $=23$ seconds. Deflection $=\frac{5}{8}$ inch. Speed varied very slowly.

"Oiled the mechanism.

"No. 11. Blue-red $=10$ seconds. Deflection $=\frac{1}{2}$ inch [12th equality].

"No. 12. Blue-red $=9{\cdot}5$ seconds. Deflection $=\frac{5}{8}$ inch [12th equality].

"No. 13. Blue-red $=9$ seconds. Deflection $=\frac{3}{4}$ inch [11th equality].

"No. 14. Blue-red $=7$ seconds (good). Deflection $=\frac{3}{4}$ inch [11th equality]."

Reducing these results with the aid of the chronograph tests for the mercury method, we obtain the following 14 percentages for the values for

$$\frac{\text{difference of velocity of blue and red light}}{\text{mean velocity of light}} :—$$

(1) 3·20
(2) 1·80
(3) 1·00
(4) 1·60
(5) 2·10
(6) 1·20
(7) 1·40
(8) 1·30
(9) 3·20
(10) 4·60
(11) 2·00
(12) 1·90
(13) 1·80
(14) 1·40

The mean of these is 2·03 per cent., giving a result which confirms in a satisfactory manner the results arrived at by the other methods.

April 26 and 27.—After an absence of a month from Kelly we again tested the colour effect on April 26 and 27. We used wheels with 250 teeth and with 400 teeth, bright and smoked, upright and inclined. We ascertained the fact that blue light required the greater velocity of rotation to produce equality; but we did not make any actual measurements. We were unable to detect the successive phases of colour which were observed on February 11.

By collecting results of all the observations on the effect of colour on velocity we shall be able to judge of the general effect:—

February 11.—The star which was increasing with increase of speed was red; the star which was waning with increase of speed was blue.

The percentage difference of red and white light, two observations, 0·90 and 0·32.

February 21.—Ruby-red glass and blue copper solution. Speed diminished gradually by indiarubber. Eight observations—

(1) 3·36 (3) 2·40 (5) 5·40 (7) 2·40
(2) 1·14 (4) 2·88 (6) 2·46 (8) 2·52

Mean percentage difference $=2·82$.

February 23.—Red solution and blue solution. Direct measurements of the change of velocity. Two observations at different speeds—

(1) 1·96 (2) 0·36

Mean percentage difference $=1·16$.

February 24.—The effect seemed to be of the opposite kind to that hitherto obtained.

February 25.—Red and blue solutions. Indiarubber method. Three observations—

(1) 0·29 (2) 0·43 (3) 0·70

Mean percentage difference $=0·47$.

February 27.—Red and blue solutions. Indiarubber method. Eight observations—

(1) 1·28 (3) 1·71 (5) 3·14 (7) 1·40
(2) 1·22 (4) 1·55 (6) 1·10 (8) 0·68

Mean percentage difference $=1·51$.

March 1.—Pure spectrum thrown by bisulphide of carbon prism upon the toothed wheel. Blue and red parts of the spectrum used in succession. Indiarubber method. Four observations—

(1) 1·35 (2) 1·56 (3) 0·90 (4) 0·90

Mean percentage difference $=1·17$.

March 8.—Spectrum used for colour. Mercury used to increase speed gradually. Fourteen observations—

(1) 3·20
(2) 1·80
(3) 1·00
(4) 1·60
(5) 2·10
(6) 1·20
(7) 1·40
(8) 1·30
(9) 3·20
(10) 4·60
(11) 2·00
(12) 1·90
(13) 1·80
(14) 1·40

Mean percentage =2·03.

TABULAR statement of percentage differences in the velocity of red and of blue light.

February 21.	February 23.	February 24.	February 25.	February 27.	March 1.	March 8.	April 26.	April 27.
+3·36	Positive effect.	— . . .	+0·29	+1·28	+1·35	+3·20	Positive effect.	Positive effect.
1·14			0·43	1·22	1·56	1·80		
2·40			0·70	1·71	0·90	1·00		
2·88				1·55	0·90	1·60		
5·40				3·14		2·10		
2·46				1·10		1·20		
2·40				1·40		1·40		
2·52				0·68		1·30		
						3·20		
						4·60		
						2·00		
						1·90		
						1·80		
						1·40		
Mean +2·82	+ . . .	— . . .	+0·47	+1·51	+1·17	+2·03	+ . . .	+ . . .

General mean of 37 observations =1·8.

We cannot account for the apparently negative effect obtained on February 24. But when we consider that it was only for a short time that the observations were attempted, and that the appearance of a negative effect was extremely feeble (almost doubtful), and that in so delicate an observation the state of our health might affect the sensitiveness of our appreciation of colour effects, and that on other days the positive effect was most marked and indubitable, as recorded in numerous passages in our observing book, and quoted above,—when we consider all these things we cannot place this single dubious night's evidence in opposition to the overwhelming testimony of so many positive observations. It was too much to expect that we should be able in the first instance to measure, by the differential methods described above, the difference produced by a given difference of wave-length with very great precision. But there seems little room for doubt that the general mean of these 37 observations cannot be far from the truth, and we may affirm that when the wave-length changes from about λ=50 tenth-metres to about λ=60 tenth-metres, the velocity changes about 1·8 per cent., or in any case somewhat over 1 per cent.

Let us now consider some of the effects which must follow from these results.

Refraction and dispersion.—There is nothing in the undulatory theory of light, independent of our views as to the kind of force acting in the ether, which is opposed to the view that red and blue lights travel with different velocities in vacuo. We know that the ratios of the velocities of red or blue lights, in a refracting medium and in vacuo, are equal to their refractive indices. But the theory of refraction or dispersion tells us nothing about the ratio of the velocities of red and blue lights either in vacuo or in refracting media.

Interference and diffraction.—So with the phenomena of interference and diffraction, which give us a measure of the wave-length of the different colours. If we could measure the period of vibration of the different colours, this would show us the difference in velocity in the different colours. But our knowledge of the period of vibration is dependent on the velocity of light, and these phenomena give us no information upon the phenomena we are discussing.

Experiments of CORNU *and* MICHELSON.—Why did not these experimenters notice the effect of colour? First, consider the work of CORNU. We might expect that he should have seen colours in his star of light near the eclipses. But when we have a very feeble light changing its intensity it is very difficult to appreciate differences in colour, and he had no light for comparison as we had. He depended largely for the accuracy of his results on two facts: (1) that he took a mean of 546 pairs of observations, so reducing the probable error to one twenty-third part of that of a single pair of observations, and (2) that he used eclipses as high as the 21st eclipse, which reduces the personal error of the eye observation to one forty-first part. We may add that he had five phases (which he calls U, *u*, V, *v*, and *ν*) differing slightly, and he had considerable latitude in placing any single observation in one or other of these phases. We may also add that into each set of observations he introduced what he called "rectifications probables," which assisted to eliminate discordances.

If, however, we examine his non-rectified results, and classify them according to the source of light used, we shall, we believe, obtain confirmation of our views.

Neglecting the small correction depending upon the phase ($U+u$ or $V+v$) we have the following results:—

Source of light.	Number of observations.	Approximate velocity.
		Kiloms.
Petroleum	20	298,776
Sun near horizon . .	77	300,242
Lime light	449	300,290

Compare this with the absolute determinations given on p. 270.

	Usual source of light.	Velocity.
MICHELSON	Sun near horizon. .	299,940
CORNU	Lime light	300,400
YOUNG and FORBES .	Electric light . . .	301,382

In every case the more refrangible the mean colour of the source of light the greater is the velocity.

In MICHELSON'S observations the image of the slit was described as indistinct and covering a sensible space. From our results it would appear that the width of his spectrum between mean red and blue would be about 2 millims. But it would be a very impure spectrum, and it is only by employing absorptive media, or part of a pure spectrum, to give colour to the light used, that we should expect him to detect the difference.

Solar eclipses and occultations.—We might expect that the sun or a star on disappearing behind the moon would appear to be red, and on reappearing would first flash out blue. But light takes only 1½ second to come from the moon and the one-fiftieth or one-hundredth part of this is too small an interval of time for us to have any hopes in this direction.

Eclipses of Jupiter's satellites.—There must be a difference in time between the disappearance or reappearance of the blue or red parts of the light of a satellite, amounting to about half a minute. But the change is so gradual and the light when near eclipse so feeble that we fear this would be a very difficult determination.

Aberration.—A star exposed fully to the action of aberration must be drawn out into a spectrum parallel to the direction of the earth's motion. The length of the spectrum between the mean red and blue must be about 0″·36. It is possible that with a reflector of good definition this effect might be detected.

Temporary stars.—The mean light from the nearest star whose parallax is known takes 3½ years to reach the earth. The difference in time taken by the blue and red rays to reach us must be 13 days. The star T Corona flashed out suddenly in 1866. It was first seen on May 9, and there seems to be good evidence to show that it was not conspicuous a weak earlier. On May 9 it was of the second magnitude, and it diminished in brightness for some days at the rate of a magnitude a day. On the 12th May it was examined by Dr. HUGGINS with a spectroscope and showed bright lines in very different parts of the spectrum. This is somewhat contrary to what we should have expected. But we are not aware that the evidence for the identity of the temporary star and the small one known to be in that position is irresistible, nor do we know that the parallax of that small star has been very carefully studied, nor is the evidence as to the invisibility of the star at the beginning of the month conclusive.

Variable stars.—When a variable star brightens the first colour to reach us in increased intensity should be blue, and when it fades the last colour to cease shining in full splendour should be red. If we could determine the difference in time between the time of maximum for blue and for red rays, the present research would enable us to determine approximately the distance of the star from us.

Variable stars are certainly known to change colour; but the only reference to a systematic law in this connexion which we have ever come upon is in WEBB'S 'Celestial Objects for Common Telescopes,' where (Edition II. (1866), p. 208) the following remarkable passage occurs:—

"HIND thinks several variable stars increase blue, are yellow after maximum, and flash red in decreasing."

This is in exact accordance with the results of the present research.

We have applied to Mr. HIND for details of his observations. He informs us that the remark quoted by WEBB was made in a letter to the "Times," some twenty-six years since, on the occasion of notifying a newly discovered variable. He does not, however, feel justified in now advising us to place reliance on this as a law.

It appears, then, that our conclusions as to the relation between colour and velocity must for the present rest upon the merit of our observations. We have ourselves no hesitation in saying that the effects were so striking and so decided on the greatest number of occasions, that we ourselves have no doubt as to the general conclusion. But we admit that much is still left to be done in the way of absolute determinations. There is little doubt that a further investigation into the matter would help to a further knowledge of the properties of the ether (if there be such a substance). This end would be specially aided by an exact determination of the law of dependence of velocity upon wave-length. We believe that, with certain improvements in our apparatus, in a better climate, this determination might be completed.*

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* The details of our observations on the dependence of velocity on colour are deposited with the Royal Society.

VI. *Action of Free Molecules on Radiant Heat, and its Conversion thereby into Sound.*

By John Tyndall, *F.R.S.*

Received January 23, 1882.*

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§. 1. *Introduction.*

The experimental researches of Rumford and Leslie raised the subject of Radiant Heat to an extraordinary pitch of interest and importance. Both of these philosophers occupied themselves with what may be called superficial emission and absorption.

Melloni is to be regarded as the founder of our knowledge of the transmission of radiant heat through solids and liquids. Save in a passing inference, to be noticed immediately, Melloni left untouched the gaseous form of matter; thinking, probably, that gases and vapours, though their diathermancy could hardly be supposed theoretically perfect, came in this respect so near perfection as to be placed beyond the grasp of laboratory experiment. It was doubtless the general prevalence of this conviction which caused this field of inquiry to lie fallow for so many years after the discovery of the thermo-electric pile.

By an experimental arrangement characteristic of the genius of the man, though, it may be, not quite equal to the requirements of the problem, Melloni proved that the law of inverse squares held good for radiant heat *in air*, and from this he inferred the absence of all sensible absorption, by air, within the distance embraced by his experiments*. MELLONI extended to radiation his conclusion regarding absorption. "On ne connait," he writes, "aucun fait qui démontre directement le pouvoir émissif des fluides élastiques purs et transparents."† Such was MELLONI'S relation to the subject now before us.

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* The substance of this paper was delivered orally as the Bakerian Lecture on November 24, 1881.

In 1855 Dr. FRANZ of Berlin published a paper "On the Diathermancy of certain gases and coloured liquids."‡ He found that air contained in tubes 452 and 900 millimeters long, absorbed 3·54 of the radiation from an ARGAND lamp, and he concluded that all transparent gases would behave like air. I have given reasons for holding that Dr. FRANZ in these experiments did not touch the question in hand.§ In the arrangement which he describes, the absorption by air was quite insensible. But 60 per cent. of the radiation from his powerful source was lodged in the glass ends of his tubes; these, as secondary sources, radiated directly and indirectly against his pile, and it was their chilling by the cold air that slightly lowered his deflection and produced the supposed absorption.

It is not improbable that other attempts were made to bring gaseous matter under the dominion of experiment; but none to my knowledge are recorded.

§. 2. *Partial Summary of previous work.*

My researches on magne-crystallic action carried with them the incessant use of conceptions and reasonings touching molecular constitution and arrangement. At an early period of these studies it occurred to me that heat, both in its radiant and in its ordinary thermometric form, might be turned to good account as an explorer of molecular condition. The first fruit of this idea was a paper "On Molecular Influences,"‖ in which it was shown that wood possesses three axes of calorific conduction coincident with the axes of elasticity discovered by SAVART. Experiments on certain crystals recorded in this paper suggested a possible connexion between diathermancy and conductivity, and in 1853 I worked at this question. The substances then submitted to experiment were rock crystal, amethyst, topaz, beryl, rock-salt, smoky quartz, fluor spar, tourmaline, Iceland spar, dichroite, arragonite, heavy spar, flint, and glass of various kinds. These minerals were employed in the shape of cubes carefully cut and polished, the transmission through each of them, in different directions, both of radiant and conducted heat being determined.

A desire for completeness, not then attained, caused me to postpone, and finally to forego the publication of the results of this inquiry. It, however, kept alive reflections on the influence of molecular constitution on the phenomena of radiation and absorption. Encountered continually by the thought that in liquids and solids the pure molecular action was, or might be, hampered by cohesion, the desire to bring, if possible, free molecules under the dominion of experiment beset me more and more.

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* "Pour un intervalle de cinq à six mètres, l'air n'exerce aucune absorption sensible pour le rayonnement des corps chauds," 'La Thermochrose,' p. 136.

† Annales de Chimie et de Physique, vol. xxii., p. 494.

‡ POGGENDORFF'S Annalen, vol. xciv., p. 337.

§ Philosophical Transactions, 1861, Vol. 151, p. 27, and elsewhere.

‖ Ibid., 1853, Vol. 143, p. 217.

At the beginning of 1859 I definitely attacked this problem, meeting at the outset difficulties and negations the reverse of encouraging. But after some weeks of labour, I found myself in secure possession of the result that gases and vapours exhibited, in relation to radiant heat, phenomena far more surprising than those observed by MELLONI in liquids and solids. On the 26th of May 1859 the subject was brought before the Royal Society,[*] and on the 10th of June I was able, by illuminating the dial of a galvanometer and casting its image upon a screen, to demonstrate in the Royal Institution not only the fact of absorption, but the astonishing differences of absorption which gases and vapours equally transparent to light manifested in regard to radiant heat.[†]

The following gases and vapours were then examined:—Air, oxygen, hydrogen, nitrogen, carbonic oxide, carbonic acid, nitrous oxide, coal gas, ammonia, olefiant gas, bisulphide of carbon, chloroform, benzol, iodide of ethyl, cyanide of ethyl, formate of ethyl, acetate of ethyl, propionate of ethyl, iodide of amyl, chloride of amyl, amyline, absolute alcohol, amylic alcohol, methylic alcohol, ethylic ether, ethylamylic ether, sulphuric ether, and some others. In the Philosophical Magazine for 1862 I have given samples of the results obtained with a few of these substances; and I will here confine myself to the remark that were the measurements there recorded multiplied a hundred-fold, they would fall far short of the number actually executed in 1859.

With a view of compelling the feeblest gases and vapours to show, if they possessed it, their capacity to absorb radiant heat, the "method of compensation" was invented.[‡] Without prejudice to the delicacy of the galvanometer, this method enabled me to bring into play quantities of heat far greater than those ever previously invoked, my object being so to exalt the total radiation that a minute fraction of that total should reveal itself to experiment. By this method not only were the feebler gases and vapours coerced, but the vastness of the diathermic range, if I may use the phrase, was established with a clearness and an evidence unattainable by any other means then existent.[§]

Notices of the investigation having appeared in many English and continental journals,[*] I was induced to defer the detailed publication of the experiments. The investigation itself had taught me the difficulties and dangers which beset it. These had reference both to the methods of experiment and to the purity of the substances employed. To secure the perfect constancy of the sources of heat, and the perfect steadiness of the galvanometer, when the flux of heat was powerful, involved a lengthened discipline. With neither gases nor vapours, moreover, was it easy to obtain uniform results. When generated in different ways, the action of the same gas would sometimes prove itself so discordant as to suggest to me the possible existence of novel allotropic conditions to account for such variations of behaviour. Two samples, moreover, of nominally the same liquid, would furnish vapours yielding results far too divergent to be tolerated. The drying apparatus also contributed its quota of disturbance. These anomalies were finally traced to the fact that an incredibly small amount of impurity derived from the stronger gases or vapours sufficed to disguise and falsify the action of the weaker ones. All this had to be learnt; and when learnt, I thought it desirable, for the sake of accuracy, not to publish the results which had been gained with so much labour, but to go once more, with improved appliances, over the same ground. This I did; though it involved the total abandonment of seven weeks' uninterrupted experimental work in 1859, of seven weeks' similar work in 1860, and of many fragmentary efforts. On the 10th of January, 1861, the memoir containing an account of the investigation was handed in to the Royal Society.[†]

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[*] Proceedings of Royal Society, vol. x., p. 37.

[†] Proceedings of Royal Institution, vol. iii., p. 155.

[‡] Philosophical Transactions, 1861, Vol. 151, pp. 6 and 7.

[§] With moderate total heats the method of compensation is extremely easy of application; but when the total radiation is very large, some discipline is required to keep the galvanometer needle steady in its most sensitive position. With due training, however, perfect mastery over this difficulty may be obtained.

The first point of importance established in 1859 and developed in the memoir just mentioned was that already referred to; namely, the fact of absorption, and large differences of absorption. The second point—destined, I think, to throw light on the deeper problems of molecular physics—was the proof that while elementary gases offered a scarcely sensible impediment to radiant heat, equally transparent compound gases exhibited, in many cases, an energy of absorption comparable to that of the most athermanous solids and liquids. Determining, for example, the action of a mechanical mixture of two elementary gases, it was proved that without altering either the quantity of matter, or its perfect transparency to light, the absorption of invisible heat might be increased many hundred-fold by the passage of the constituents of the mixture into a state of chemical combination.

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[*] Proceedings of the Royal Society, May 26, 1859; Proceedings of the Royal Institution, June 10, 1859; Bibliothèque Universelle, July, 1859; Cosmos, vol. 15, p. 321; Nuovo Cimento, vol. 10, p. 196; Comptes Rendus, 1859; and in other journals.

[†] Section 3 of the Bakerian Lecture for 1861 reveals some of the difficulties which beset the earlier stages of these inquiries. To secure strength of radiation and steadiness of the needle I passed from source to source, obtaining my temperatures in turn from water, oil, fusible metal, sheets of copper heated by regulated flames, and from other things. Approximate results were readily obtainable; but I aimed at a degree of accuracy which would render any material retractation afterwards unnecessary. Soundness of work I thought preferable to rapidity of publication.

A similar deportment may be detected in liquids and solids. The quantity of iodine vapour generated at ordinary temperatures is so small that its action on radiant heat is, as might be expected, insensible. But iodine itself, when liquefied by a powerful solvent, behaves as an almost perfectly transparent body to the obscure calorific rays, even when it is able to extinguish totally the light of the sun. Liquid bromine is also highly diathermanous. The same may be said of phosphorus. In MELLONI's table, Sicilian sulphur comes next to rocksalt in transmissive power. A concentrated solution of sulphur in bisulphide of carbon exerts no sensible action on radiant heat. By fusing together iodine and sulphur Professor DEWAR has produced a "ray filter" which separates with extreme sharpness the visible from the invisible rays. The remarkable diathermancy of certain specimens of vulcanite, brought to light in the experiments of Mr. GRAHAM BELL and Mr. PREECE, is probably due to the sulphur they contain. MELLONI showed that lampblack is to some extent diathermanous. But when a suitable source of heat is chosen, lampblack proves far more pervious to radiant heat than MELLONI found it to be. An opaque layer of this substance transmits 41 per cent. of the radiation from a hydrogen flame. Were the lampblack optically continuous, the transmission would, doubtless, be still greater. An opaque solution of iodine transmits 99 per cent. of the radiation from the same source, while a layer of pure water 0·07 of an inch in thickness, transmits only 2 per cent. of the radiation from a hydrogen flame. Such results indicate that a profound change in the relation of ponderable matter to the luminiferous ether accompanies the act of chemical combination.

One of my principal aims in the Bakerian Lecture of 1861 was to illustrate the hold which experiment had obtained of a subject previously considered intractable. The densities of the gases and vapours employed were therefore varied within wide limits. In the experimental tube first made use of, a full atmosphere of olefiant gas absorbed more than 80 per cent. of the entire radiation, and it was therefore evident that a small fraction of an atmosphere of such a gas would exert a measurable action. On trial, it was found possible to measure the absorption of $\frac{1}{10,000}$th of an atmosphere of olefiant gas. The action of this gas was determined at sixteen different densities, the absorption, as long as the density was very small, being accurately proportional to the quantity of gas present. Similar experiments were made, and similar results obtained with other gases. The action of sulphuric ether vapour upon radiant heat was proved to be still more powerful than that of olefiant gas. The vapour was first carried into the experimental tube by a current of dry air; and the pure vapour was afterwards examined at seventeen different densities. Bisulphide of carbon was tested at twenty different densities; amylene at ten; benzol at twenty; and so of the others.

Considering the views previously entertained regarding the diathermancy of gases and vapours I was naturally impressed with these results. Sceptical when I first observed them, I scrutinised them closely, until repeated scrutiny abolished every doubt. For my own instruction I illustrated the action of the stronger gases and vapours in a variety of ways. Turning, for example, once rapidly round a cock connecting the exhausted experimental tube with a holder containing a powerful gas, the needle would fly aside, owing to the stoppage of the heat by the infinitesimal amount of gas which entered the tube during the rotation. Discharging a powerful gas or vapour in free air, between the source of heat and the thermopile, a similar energetic action would be produced by the perfectly invisible agent.

I was schooled in such actions before the thought of testing the omnipresent vapour of our atmosphere occurred to me. When it did occur there was in my mind no *à priori* ground for supposing that its action would prove insensible; for why should I assume that $\frac{1}{100}$th of an atmosphere of aqueous vapour would prove neutral, after I had proved a small fraction of this fraction, on the part of other gases and vapours, to be active? There was no reason for such an assumption on my part—nothing to deter me from hopefully submitting the question to experiment. I accordingly tested the water vapour of the atmosphere in which I worked, and found its action on a first trial to be thirteen times that of the air in which it was diffused.

It is not uninstructive to compare this approach to the problem with that of a very distinguished man—the late Professor MAGNUS, of Berlin.* Subsequent to me, he subjected the aqueous vapour of our atmosphere to an experimental test; but he made the experiment under the assured conviction that his result would be negative. "It could," he says, "be foreseen with certainty that the small amount of aqueous vapour taken up by air at ordinary temperatures could exert no influence on the transmission." I think it must be obvious that if MAGNUS had gone through the discipline to which I had been subjected, he would not have used this language. His mistake however was a natural one. In fact during the earlier stages of the inquiry my mind was exactly in the condition of his mind—I also thinking, until practically instructed to the contrary, that the action of aqueous vapour at ordinary temperatures must be immeasurably small. It is well known that MAGNUS tested his foregone conclusion, and found it verified; while I, on the other hand, as above stated, justified mine.

The various gases which had been examined in the experimental tube with regard to their powers of absorption, were next tested as regards their powers of radiation. Columns of the heated gases were allowed to ascend in free air, and to radiate against the pile. In this simple way, the radiative power of "transparent elastic fluids" was for the first time established. The order in which the gases ranged themselves, in regard to radiation, was exactly the order of their absorptions. Here, as in other cases, I instructed myself by observing how gases might be made to play the part of solids. Permitting, for example, a film of one of the stronger gases to glide over a heated surface of polished silver, the radiation from that surface was augmented as it might have been by a coating of isinglass or lampblack.

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* In many respects my generous and helpful friend, but, in regard to this question, my steadfas antagonist for many years.

A surmise has been mentioned regarding new allotropic conditions, as occurring to me amid the perplexities of my earlier experiments. In one instance—that of electrolytic oxygen—the surmise proved correct. My first experiments indicated that the modicum of ozone which went forward with the oxygen, exerted four times the absorption of the gas in which it was diffused.[*] Subsequently, by changing the apparatus, and taking pains to augment the quantity of ozone, the multiple rose successively from 4 to 20, 35, 47, 85, ascending finally to 136.[†] The behaviour of ozone was thus proved to be similar to that of molecules composed of heterogeneous atoms. Hence the conclusion, drawn at the time, that the molecule of ozone was formed of oxygen atoms so grouped as to render their action upon radiant heat virtually that of a compound body. This, it is needless to say, is the constitution now assigned to ozone.

With the view of including corrosive gases and vapours among the number of those examined, and for other reasons, the brass experimental tube was displaced by a tube of glass of the same diameter and nearly 3 feet long. The source of heat was also changed from a LESLIE's cube, containing boiling water, to a plate of copper against which a sheet of flame was permitted to play. Extraordinary precautions were found necessary to insure perfect steadiness on the part of the flame. With this arrangement the practical inability of the elementary gases to absorb radiant heat was further illustrated and confirmed. Chlorine gas and bromine vapour, for example, were proved to be highly diathermanous.

At the pressure of an atmosphere it was found that the diathermic range of colourless gases extended from 1 to about 1000. The portion of gas first entering the experimental tube, having the whole heat to act upon, produced, as might be expected, the greatest effect, the increment of absorption, after a certain quantity of gas or vapour had entered, being infinitesimal.[‡] It was therefore interesting to

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[*] Philosophical Transactions, Vol. 151, p. 8.

[†] Ibid., Vol. 152, pp. 84, 85.

[‡] This is well illustrated by an experiment on sulphuric ether vapour recorded in the Bakerian Lecture for 1861:—

Pressure.	Absorption.
1 inch	214
2 ,,	282
3 ,,	315
4 ,,	330
5 ,,	330

The absorption of air being taken as unity, that of sulphuric ether vapour at 1 inch mercury pressure is here shown to be 214. When, however, vapour corresponding to a pressure of 4 inches was already in the experimental tube, the addition of another inch did not sensibly augment the absorption.

compare together the various gases at very small pressures. When the pressure was that of 1 inch of mercury the diathermic range was greatly increased, the absorption by olefiant gas being then at least 6000 times the absorption by atmospheric air.

With the changed apparatus the action of the aqueous vapour of our atmosphere was again taken up, and proved to be, not 13 times, as I had at first supposed, but, on fairly humid days, at least 60 times that of the air in which it was diffused. When, moreover, dried air was caused to pass over moistened glass and then carried into the experimental tube, the absorption was still greater.

A power has been claimed for mist or haze which has been denied to aqueous vapour; but in these experiments concentrated luminous beams, which would have infallibly brought into view the least trace of suspended matter, revealed no mist or dimness of any kind. It is, moreover, demonstrable that an amount of turbidity rendered strikingly evident by a luminous beam, exerts only a fractional part of the action of the pure aqueous vapour. When well dried air was led, not through water or over wet glass, but over bibulous paper, taken apparently dry from the drawers of the laboratory, the amount of vapour carried forward from the pores of the paper produced 72 times the absorption of the air which carried it. After five repetitions of the experiment, wherein the same air was carried over the same paper, a quantity of vapour was still sent forward capable of exerting 47 times the absorption of the air in which it was diffused.

Here the possible action of odours upon radiant heat naturally suggested itself. Many perfumes were accordingly subjected to examination, the odorous substance being in each case carried into the experimental tube by a current of dry air. Thus tested pachouli exerted 30 times, cassia 109 times, while aniseed exerted 372 times the absorption exerted by the air in which it was diffused.

A novel method of exhibiting the absorption and radiation of gaseous bodies, the germ of which had been previously discovered,* was illustrated and developed by the new apparatus. Suppose the experimental tube exhausted and the needle, under the joint action of the two sources, to be at 0°. On admitting a strong vapour the usual deflection would occur. Suppose it to be 50 galvanometric degrees. Let dry air be now introduced until the experimental tube is filled. Although fresh matter is thus thrown athwart the rays of heat, the needle behaves as if the matter within the experimental tube had wholly disappeared. It sinks to zero, and not only so, but passes say to 50° on the other side.

After the first moments of perplexity succeeding the observation of this effect, its cause became clear. On entering the experimental tube the air, having its *vis viva* destroyed, was heated dynamically. Incompetent to radiate itself, it imparted its warmth to the vapour, and this powerful radiator poured the heat thus received against the pile. This heat sufficed not only to neutralise the deflection of 50° due to

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[*] Philosophical Transactions, Vol. 151, p. 32.

absorption, and indicating cold, but to carry the needle up to 50° on the side of heat. So likewise, when the experimental tube was filled with mixed air and vapour, the needle being at 0°, a stroke of the pump, though opening a freer passage for the rays from the source, caused a deflection indicative not of heat, but of cold. Here, the vapour within the tube being chilled by the dilatation of the air, the pile radiated its uncompensated warmth into the vapour and produced the observed deflection.

Such observations suggested a new means of demonstrating the absorption and radiation of heat by gases and vapours. Abandoning all external sources of heat, and permitting the various gases already examined to enter the experimental tube at a common velocity, they became self heated and radiated against the pile. Their radiation, thus determined, corresponded exactly with the results obtained when heated columns of these gases were permitted to rise freely in the atmosphere.

Both the radiation and absorption of vapours were determined in the same manner. The external source of heat was abandoned, and a measured quantity of every vapour was introduced into the experimental tube. Through an orifice of fixed dimensions dry air was then permitted to enter the tube, where the destruction of its *vis viva* raised its temperature. The heated air warmed the vapour, which in its turn radiated the heat imparted to it against the pile. The deflection of the galvanometer declared the strength of this radiation. Absorption was determined by permitting the mixed air and vapour to dilate by a measured quantity, the pile being here the warm body, and the chilled vapour the absorbent. The order in which the vapours stood as regards absorption was here exactly the order of their radiation, while both absorption and radiation, thus determined, agreed with the results obtained by sending the rays from an external source of heat through the pure vapours in the experimental tube.

What has been called "vapour-hesion," whereby liquid films are produced, has been supposed to play a dominant part in my experiments. But it can hardly be imagined that an irregular action of this kind could produce results of such precision and consistency as those here recorded. Such results are, in my opinion, only compatible with the conclusion that the veritable radiators and absorbers are the molecules of the vapours. Apart from all experiment, the notion that vapours must act in this way comes commended to us by the proved, and conceded, deportment of gases. It would be unreasonable to admit that a compound gaseous molecule is active, and at the same time to affirm that a compound vaporous molecule is inert.

This hypothesis of liquid films formed on the interior surface of the experimental tube, and on the plates of rocksalt, becomes, I think, more embarrassed as we proceed. It depends on the unproved assumption that liquids possess powers of absorption which are denied to their vapours. To water and brine, for instance, MAGNUS largely concedes such powers, but not to aqueous vapour. That the state of aggregation exerts an influence is not denied, but that it is here the dominant factor is open to doubt. To admit this would be to concede that the seat of absorption is the molecule as a whole, to the practical exclusion of the constituent atoms of the molecule. For if the atoms exert any influence, the mere passage from the liquid to the vaporous condition, which separates the molecules from each other, but leaves them individually intact, cannot destroy their powers of absorption.

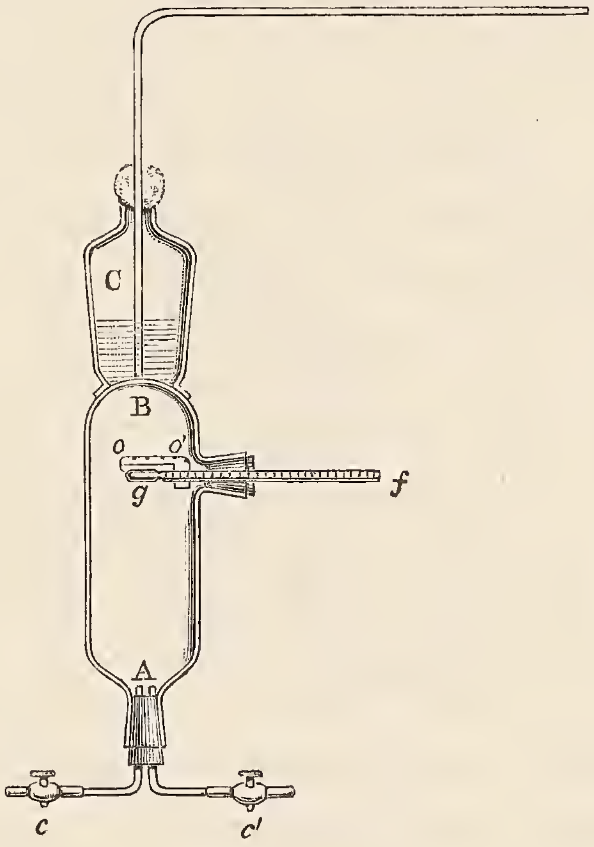
At an early stage of these researches the parallelism of liquid and vaporous absorption forced itself upon my attention. Thus, my experiments on bisulphide of carbon vapour were connected with the deportment of liquid bisulphide, as set forth in Melloni's table. The vapours, moreover, of chloride of sulphur and chloride of phosphorus, whose liquids stand in Melloni's table next to bisulphide of carbon, were afterwards proved by me to possess a diathermancy corresponding to that of their liquids. After various references to this subject in preceding memoirs, a portion of the Bakerian Lecture for 1864 was devoted to its examination. Liquid layers enclosed between plates of transparent rocksalt were tested in regard to their diathermancy, and for the sake of control and verification they were employed in five different thicknesses. The vapours of these liquids were examined in quantities proportional to the quantity of liquid, the same quality of heat being applied both to liquids and vapours. By these experiments, it seemed to me placed beyond a doubt, that the liberation of the molecule from the liquid condition did not destroy its absorbent power, the order of absorption being proved to be precisely the same for liquids and their vapours. Ten different substances were shown at the time here referred to to follow this rule. The list has since been extended, and I am not acquainted with a single real exception to the rule. Any reasoning, therefore, which ascribes a potent absorption to perfectly impalpable films, condensed on the surface of my apparatus, and which denies that absorption to the free molecules within the experimental tube, is in my opinion untenable.

The relation between liquids and their vapours here indicated is very thorough. It extends beyond the field of experiment which we have hitherto had in view. I have, for example, published some researches on the action of rays of high refrangibility on gaseous matter, and have shown in a great variety of cases, that the molecules are shaken asunder by such rays. The actinic clouds, as I have called them, produced by this decomposition, reveal vividly the track of the beam by which they are generated, and render it easy to observe the distance to which the action penetrates. In the case of nitrite of amyl, for example, the power of decomposition is soon exhausted, the actinic cloud ceasing abruptly at a point about 18 inches from the place where the beam enters the vapour. An experimental tube 3 feet long, has therefore one half of its vapour shielded by the other half, and on reversing the tube, the shielded half comes instantly down as an actinic cloud. In the case of iodide of allyl vapour, on the other hand, the beam may pass through a charged experimental tube 5 feet long, fill it with an actinic cloud, and still effect decomposition in another tube placed beyond it. What is true of these vapours is true equally of their liquids; for while a layer of the liquid nitrite $\frac{1}{8}$th of an inch thick prevents, when placed in the track of the beam, the decomposition of its vapour, a layer of the liquid iodide, of quadruple thickness, does not arrest the decomposition. The power, and the lack of power, to penetrate considerable depths is shared alike by the liquids and their vapours. Other, and still more subtle and penetrating illustrations of parallelism between liquid and vaporous absorption are mentioned in the Bakerian Lecture for 1864.*

§. 3. *Researches of* MAGNUS.

Prompted by the experiment of GROVE, illustrating the chilling action of hydrogen, MAGNUS, in 1860, began an investigation on the power of gases to *conduct* heat.† His apparatus, traced in outline from his own plate, is shown in fig. 1, where A B is the recipient for the gases, and C a flask containing water kept boiling by a current of steam. The bottom of C, which formed the top of A B, was the source of heat. A thermometer $g\,f$, shielded by a cork or metal screen $o\,o$ from the radiation of the source, was intended to receive and measure the heat transmitted by conduction. The recipient A B was mounted in a space surrounded by water of a constant temperature. The heating

Fig. 1.



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* Carbonic acid is one of the feeblest of the compound gases, as regards the radiation from solid bodies; but, for the radiation from a carbonic oxide flame, it transcends all other gases in absorbent power. The action of aqueous vapour is also enhanced when it acts upon the rays emitted by a hydrogen flame. The enhancement extends to water. Curious reversals of diathermic position, when heat from different sources is employed, are moreover shown to occur simultaneously with liquids and vapours.

† A Preliminary Note of this inquiry is published in the "Bericht" of the Berlin Academy for July 30th, 1860. No measurements are given, but certain results are announced. The experiments were first published in POGGENDORFF'S Annalen for April, 1861.

of the thermometer, when A B was exhausted,* was compared with its heating, when A B was filled with various gases, and in every case but one the heating through the gas was found less than the heating through the vacuum. The exception was hydrogen, which carried more heat to the thermometer than was transmitted by the vacuum. The conclusion drawn by its author from this experiment was that hydrogen conducted heat like a metal.

One remark only in this Note has any reference to the diathermancy of gases, but it is a significant one. MAGNUS had no doubt as to the power of every one of his gases to conduct heat. There could, he supposed, be only a difference of degree between them and hydrogen. Whence, then, the lowering of the thermometer? He answers thus:—"From this it is not to be inferred that the gases do not conduct heat, but merely that in their case conduction is so feeble as to be neutralised by adiathermancy." These are the only words in the note which have any reference to radiation.

In his next inquiry, MAGNUS dealt directly with the subject of diathermancy, a Preliminary Note of the investigation being published in the Monatsbericht for February 7th, 1861. This note, like its predecessor, consisted of general and descriptive statements, no actual measurements being given. The completed memoir was first published in POGGENDORFF'S Annalen for April, 1861. For the purposes of this new inquiry the apparatus used in the experiments on gaseous conduction was modified as shown in fig. 2. To the recipient A B a second one G F was attached, both being connected by the tubulure shown in the figure. The recipient G F rested upon the plate of an air-pump, on which also stood the thermopile *p*, with one of its faces turned towards the source. From the pile, through the air-pump plate, wires passed to the galvanometer. With this apparatus, the absorption by atmospheric air and by oxygen was found to be 11·12, and by hydrogen 14·1 per cent. of the total radiation. The alleged conductivity of hydrogen did not therefore manifest itself in these experiments.

Let us analyse these results. In the first experiments, the distance of the thermometer from the source of heat was 35 millimeters. The action on the thermometer through a vacuum being represented by 100, the action through air and through oxygen of this depth was found to be 82. The loss of 18 per cent. in air and in oxygen was alleged to be due to the adiathermancy of these media, to which percentage, if we wish to ascertain the *total* absorption by air, we should have to add such heat as reached the thermometer by conduction.

Turning now to the modified apparatus, which is evidently drawn to scale, the gas here traversed by the radiant heat was about 275 millimeters in depth, while the stratum traversed in the first experiments was, as stated, only 35 millimeters. Yet in these first experiments an absorption of 18 per cent., while in the later ones an absorption of only 11·21 per cent. is assigned to air. In other words, when the depth of the aerial stratum was augmented more than seven-fold, the absorption, instead of increasing, fell to less than two-thirds of that of the shallower stratum. It is pretty obvious that an influence different from pure absorption came here into play. That influence was convection.

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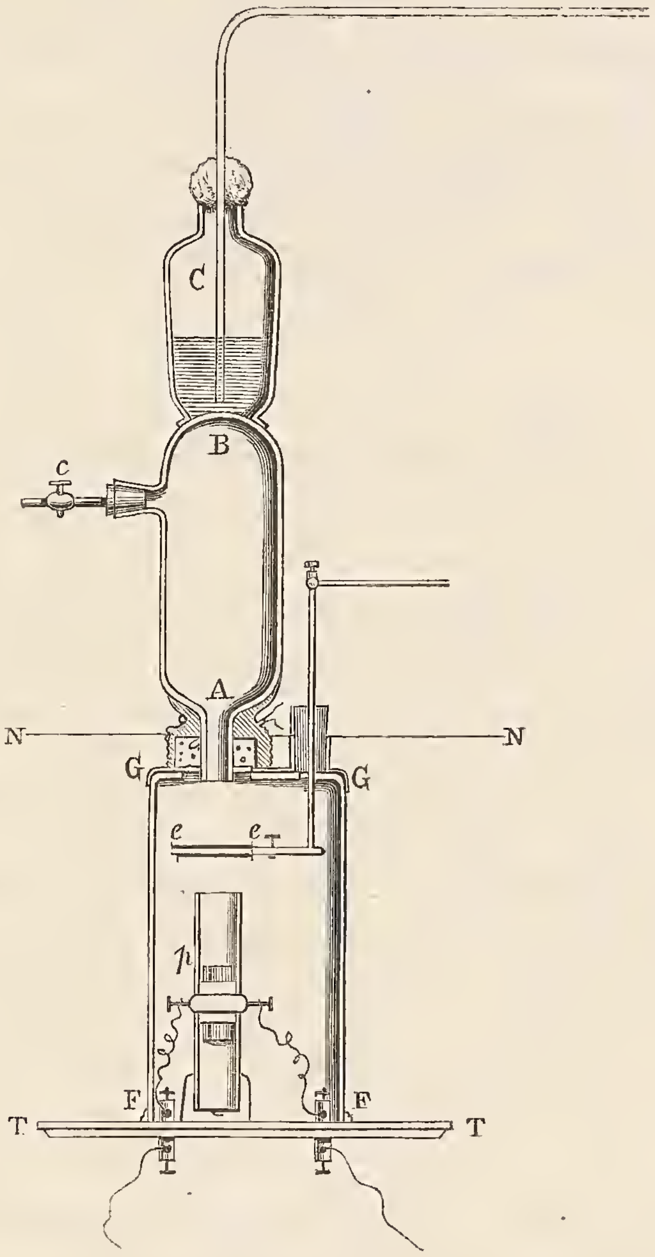
* This vacuum temperature, at least in so far as it exceeded that of the sides of the recipient, was obviously derived from the screen.

Fig. 2.



Anxious to probe this matter to the bottom, and to abolish, or account for, the differences between my friend and myself, I wrote to him proposing an exchange of apparatus—that he should send his to London, and I mine to Berlin. I afterwards had a fac-simile of his apparatus constructed in London, and satisfied myself by actual trial that it was really hampered with the defects I had ascribed to it. By means of the striæ of incense smoke and of chloride of ammonium, the fact of convection in air was rendered plainly visible to the eye, while the behaviour of hydrogen, under like circumstances, revealed the cause of its transporting more heat than the vacuum in the first experiments, and less heat, not only than the vacuum, but than air or oxygen, in the second experiments. In the one case, the thermometer, being close to the source, came within the range of the convection currents of this mobile gas, the heat being transported to it by these currents. In the other case, a considerable distance intervened between the source and the pile, which was further effectually protected by the narrow tubulure. Through it the currents could not pass, but they nevertheless existed in the recipient A B, lowering the temperature of the source without heating the thermoscopic instrument.

The experimental resources of MAGNUS were great, and he here applied them; but the defects of his method were radical and irremovable. These defects reached their culmination in the subsequent researches of Professor BUFF,* who by pursuing substantially the same method, arrived at the result that a stratum of air 2½ inches thick absorbed 60 per cent. of the radiation from a source of 100° C.† BUFF also found olefiant gas to be more diathermanous than air, whereas at atmospheric pressures, it is many hundred, and at a pressure of $\frac{1}{30}$th of an atmosphere, many thousand times, more opaque to heat.

This is the point at which aqueous vapour enters into the experiments of MAGNUS. When dry and humid air were compared together in his apparatus, no difference between them was observed. But, apart from all disturbance, it would require an instrumental arrangement far more delicate and powerful than that here employed, to bring into view the action of a stratum of mixed air and aqueous vapour 11 inches deep, and having a temperature of only 15° C. Disturbances, however, were not absent. In the first place, the convection currents which enabled dry air to reduce the radiation by 11·12 per cent., were more than sufficient to mask the action of the vapour. Secondly, dry and humid air were brought in succession into direct contact with the face of the thermopile. The pile was therefore affected by any difference of temperature between it and the air, and it could scarcely be supposed that these temperatures were always alike. It was also affected by the condensation and evaporation which occurred when humid air and dry air were brought successively into contact with its lampblack-coated face. To "vapour-hesion" MAGNUS subsequently ascribed very large effects. Here we have the conditions specially suited to the development of the action, and yet no reference is made to it. Either then the disturbance was overlooked, or the apparatus was not sufficiently delicate to reveal it. To these two sources of disturbance—the lowering of the temperature of the source by convection, and the warming and chilling of the pile by contact, condensation, and evaporation—is to be added another, due to the warming which must have occurred when the dynamically heated air came into direct contact with the thermopile—an action which, in my apparatus, proved sufficient to whirl the needle of the galvanometer more than once through an entire circle.

MAGNUS next experimented with glass tubes 1 meter long and closed at the ends with plates of glass. His source of heat was a strong gas flame aided, as in the experiments of Dr. FRANZ, by a parabolic mirror. Two tubes were employed, the one blackened within and the other unblackened. With the blackened tube an absorption of 2·44 per cent. was found for air, and an absorption of 3·75 per cent. for hydrogen. In the unblackened tube the absorption by air was 14·75 per cent., and by hydrogen 16·27 per cent. of the total radiation.

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* Phil. Mag., 5th ser., vol. 4, p. 401. For my reply see Proc. Roy. Soc., vol. 30, p. 10.

† Ibid., 1877, vol. 4, p. 424.

I went over this ground with the utmost care, using invisible as well as visible heat. But, substituting plates of pellucid rocksalt for the plates of glass, I failed to realise the effect obtained by MAGNUS. He ascribed the difference between the results obtained with his blackened and his unblackened tube to a change of quality in the heat, produced by reflection from the interior surface of the latter. With plates of rocksalt, however, though the reflection abides, the change of quality does not occur. My position, therefore, in regard to these experiments, is similar to what it was in regard to those of Dr. FRANZ. The results obtained with air, oxygen, and hydrogen, were, I hold, due to the chilling of the heated glass ends of the tube by the cold gases, and the consequent lowering of the secondary radiation.

It was shown by MAGNUS himself, and is moreover obvious at first sight, that the unblackened tube sent a far greater amount of heat to the glass plate adjacent to the thermopile than the blackened one. That plate being more heated by the source, was more chilled by the air when it entered. The greater cooling power of hydrogen accounts, moreover, for the advance of the supposed absorption from 14·75 to 16·27 per cent. With carbonic acid MAGNUS detected a difference which had escaped Dr. FRANZ. Instead of making the action of this gas equal to that of air, he found in the blackened tube an absorption of 8·19 per cent., and in the unblackened tube an absorption of 21·92 per cent. exerted by carbonic acid. Here true absorption mixed itself with the effect of mere chilling, while, with still more powerful gases, the effect of chilling retreated by comparison more and more.

Such were the experiments which determined, in the first instance, the attitude of this distinguished man towards that portion of my work which related to the action of the air and vapour of our atmosphere on radiant heat. In the defence of his position he brought to bear all the resources of consummate skill and large experience. His position, however, was by no means a wholly defensive one. He dwelt repeatedly and emphatically on the dangers—and they are real—to which the method pursued by me was exposed. I had closed my experimental tube with plates of transparent rocksalt and he urged against me the hygroscopic character of this substance. Placing rocksalt beside a vessel of water under a glass shade, he found that it could be rendered dripping wet.* Hence his argument, that, instead of measuring the action of vapour, I had really measured the action of brine. This, however, I could not admit. I was aware of the danger and had avoided it. In many hundred instances the rocksalt plates had been detached from my experimental tube while filled with the very air which had produced the observed absorption, and found to be as dry as polished plate glass. For a week at a time I have charged my experimental tube alternately with dried and undried air, removing every evening the plates of salt while the humid air filled the tube. Their dryness and polish were found unimpaired.* I have frequently flooded the experimental tube with light, and watched narrowly whether any dimness showed itself on the salt, or on the interior surface, when the humid air entered. There was nothing of the kind. I finally abandoned the plates of salt altogether, and obtained in a tube open at both ends substantially the same effects as those obtained when the tube was closed with plates of rocksalt.

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* It has been shown by Professor DEWAR that the exposure of a dry plate of rocksalt for 5 minutes to saturated air sensibly augments the weight of the salt as determined by a delicate balance.

In 1862 MAGNUS came to London. He had been previously working at the points of difference between us, and had strengthened his first conviction. The action of the air he had found to be considerable, and the action of aqueous vapour practically nil. I also had been working, but with an entirely different result. It was hoped by both of us that our differences would be settled during this visit. With my closed experimental tube I showed him the neutrality of dry air and the activity of humid air; and while the latter was in the tube I detached the rocksalt plates and placed them in his hand. He closely inspected them, passed his dry handkerchief over them, and frankly and emphatically pronounced them perfectly dry. I then executed in his presence the experiments with the open tube, and reproduced the results which I had previously published. I subjected the method of compensation to a severe test and showed him how exact it could be made. He frankly confessed his inability to find any flaw in my experiments, and save in one particular made no attempt to reconcile our differences. He accounted for the neutrality of dry air observed by me by pointing to my thermopile, between which and the experimental tube a space of air intervened. He argued, and justly argued, that though the calorific rays were permitted to enter the tube from a vacuum, if the air intervening between tube and pile could produce the effect which he ascribed to it, the heat would be robbed of its absorbable rays before the dry air entered the tube, the subsequent neutrality of dry air being a matter of course. The logic was good, but its basis I knew to be more than doubtful; and I therefore asked him whether a layer $\frac{1}{20}$th of an inch thick between pile and tube would produce any sensible effect. His reply was an emphatic negative. In subsequent experiments, therefore, the conical reflector was removed from my pile, and placed within the experimental tube, its narrow end being caused to abut against the plate of rocksalt. The face of the pile was then brought within less than $\frac{1}{20}$th of an inch of the rocksalt plate; and in this way my former measurements, which had declared the pure air of our atmosphere to be a practical vacuum to radiant heat, were verified to the letter.

The well-earned fame of MAGNUS as an experimenter, and his personal friendliness to myself, rendered it specially incumbent on me to deal respectfully with every one of his suggestions. He once intimated to me that the absorption which I had supposed due to aqueous vapour, might be really due to the smoke and dust suspended in London air. To meet this I carried air myself from the Isle of Wight, had it carried from Epsom Downs and other places, and found the aqueous vapour diffused in such air to be from 60 to 70 times more energetic than the air itself. London air, moreover, was freed from its suspended matter and tested when dry: it proved neutral. The self same air was then rendered humid: its absorbent power was restored. Then, with dry air as a carrier, I introduced smoke into the experimental tube, until it far exceeded in density that suspended in the London air when MAGNUS drew my attention to it. The quantity of heat intercepted by this smoke proved to be only a fraction of that absorbed by the perfectly invisible aqueous vapour.

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* This mastery over the apparatus was not attained without training. Any lapse of care soon declared itself by the condition of the plates of salt.

On his return to Berlin he resumed his labours. He had been especially impressed by the experiments with the open tube, and to this point he directed his chief attention. "The result of this experiment," he writes, "was so surprising and so little in accord with what I had found by other methods, that on reaching home I determined to repeat the experiment." He did so, with this result:—"I have," he says, "repeated the blowing in of dry air and moist air many hundred times; but in no single case was the deflection such as to indicate a greater absorption by moist air."[*] Humid air in his experiments produced the deflection of heat; dry air the deflection of cold—a result diametrically opposed to mine. In London he had seen that my deflections were as large as I had affirmed them to be, but he had not criticised them with a view of ascertaining whether they were or were not in the right direction. In these new experiments, however, he had, he thought, hit upon their origin. The moving air had reached the face of the thermopile, producing, when humid, heat by condensation, and when dry, cold by evaporation.

I read the account of these experiments with some concern; for it was thereby made plain to me that MAGNUS had by no means realised the anxious care that I had bestowed upon my work. The testimony of an independent observer would, I thought, set the matter right. My apparatus, carefully adjusted, was accordingly handed over to Dr. FRANKLAND, who minutely tested every point involved in, or arising out of, the objection of MAGNUS. He verified all my results. His opinion as to the accuracy of the method of compensation is worth recording. "In conclusion," he writes, "I cannot but express my surprise and admiration at the precision and sharpness of the indications of your apparatus. Without having actually worked with it I should not have thought it possible to obtain these qualities in so high a degree in determinations of such extreme delicacy."[†] To this may be added the subsequent testimony of Professor

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[*] POGGENDORFF'S Annalen, 1863, vol. 118, p. 580; Phil. Mag., 1863, vol. 26, p. 25.

[†] The total heat here employed amounted to 86·2° of a quadrant. This exceedingly large deflection was neutralised by the radiation from the compensating cube. But so accurately were the two sources balanced, and so constant was the radiation on both sides, that the determinations were made with ease, and without sensible disturbance or fluctuation.

WILD, now of the University of St. Petersburg, who went carefully over the same ground. "In all my experiments," he says, "conducted according to TYNDALL'S method, which included more than a hundred distinct observations, I have never obtained deflections of the galvanometer needle in contradiction to the statements of Professor TYNDALL."*

In an extremely able paper, a translation of which is published in the Philosophical Magazine for October, 1866, the Petersburg philosopher compares the methods pursued by MAGNUS and myself respectively. Insufficient sensitiveness and the disturbance due to convection currents, caused him, he says, to abandon the method of MAGNUS. "Although," he continues, "this method of investigating absorption may, in the hands of so experienced and expert an experimenter as Professor MAGNUS, be an appropriate one for determining absolute values with great certainty, I feel bound, from my own experience, to give a decided preference to TYNDALL'S method, not only on account of the greater facility with which it furnishes qualitative [quantitative] results, but also in consequence of its greater delicacy. It is principally in consequence of this greater delicacy that, notwithstanding the negative results furnished by MAGNUS'S method, I maintain that the greater absorptive power of moist air, as compared with dry, has been fully established by the experiments made according to TYNDALL'S method; and I am of opinion that meteorologists may without hesitation accept this new fact in their endeavours to explain phenomena which hitherto have remained more or less enigmatical."

In 1866 MAGNUS varied his method of experiment, seeking to solve the question of absorption by observations on radiation. "I have," he says, "made a few determinations of the radiation of dry and moist air, and of some other gases and vapours. Up to the present time," he continues, "the capacity of these bodies to transmit heat has alone been determined."† He then describes his arrangement:—"The gases and vapours were passed through a brass tube of 15 millimeters internal diameter, which was placed horizontally and heated by gas flames. One end of the tube was bent upwards so that the heated air ascended vertically, while at a distance of 400 millimeters from the vertical current, was placed the thermopile." When dry air was sent through this tube, the deflection produced was three divisions of a scale; when air which had passed through water at a temperature of 15° C. was sent through the tube, the deflection rose to five divisions; when the water was warmed to 60° or 80° FAHR., the deflection was 20 divisions; and when the water boiled, the deflection was 100 divisions. In this last experiment, however, a mist appeared, so that, as urged at the time, the radiation could not be said to have been purely from vapour. In the other cases no mist was visible, but it was nevertheless concluded that the 20-division deflection was due to the formation of mist at the boundary of the ascending current.

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* Phil. Mag., 4th ser., vol. 32, p. 252.

† This is an inadvertence. Exhaustive experiments on the radiation of gases and vapours had been made and published many years previously.

I should be disposed to claim these experiments as telling in my favour. The first of them, in my opinion, dealt with the radiation not from dry air but from the adjacent aqueous vapour which had been warmed by the dry air. That the deflection in the second experiment was small is not surprising. The radiation which could reach the pile from a jet of air only 15 millimeters in diameter, and containing such moisture as could be taken up at 15° C., must have been extremely small under any circumstances. But in the present case, even this small radiation was diminished by the passage of the heat through 400 millimeters of undried air. I should demur to the explanation of the third experiment, and question the warrant to imagine a mist which could not be seen. Even the fourth experiment, where mist was visible, yielded, I doubt not, a mixed result; part of the effect, and probably the smallest part, being due to the mist, and part of it to the vapour.

With regard to the radiation from hot aqueous vapour, the following experiment is typical of some hundreds which I have had occasion to make. A burner consisting of two rings provided with numerous small apertures was placed within a square tin chimney. At some height above the burner the chimney was perforated so as to enable the radiation from a heated gaseous column within the chimney to reach a distant thermopile. The side of the chimney facing the pile was so protected by screens that the radiation from the chimney itself was nil. Connecting the burner with a bottle of compressed hydrogen the gas was ignited. A column of hot vapour rose from the burner and passed the aperture in the chimney, through which it sent its rays to the pile. Mere tips of flame were first employed; the column of vapour rising from them sufficed nevertheless to produce a permanent deflection of

40°

A slight augmentation of the flame sent the needle up to

60°

A still further augmentation sent it up to

75°

This last deflection was equivalent to more than 400 of the degrees in the neighbourhood of zero.

The radiating column was here considerably above the flame. To examine the condition of the column a concentrated luminous beam was directed upon it. There was no precipitation. On the contrary, the suspended matter in the air of the chimney was much less than that of the surrounding air. Instead of a white mist, we had the blackness due to the destruction of the floating matter by the hydrogen flame.

On quenching the flame the needle returned accurately to zero.

In his objections Magnus, for the most part, dealt with true causes, but he erred as to their scope of action. I never denied the existence of the dangers which he emphasized. The hygroscopic character of rocksalt, for example, to which he recurred so often, cannot be questioned. It has a strong attraction for moisture, especially when cold. On this point my experience has been large, and I applied it in the execution of my experiments. These, as I have so frequently stated, were conducted with plates of salt as dry as polished glass or rock crystal. Thus, while conceding the fact that rocksalt is hygroscopic, I demur to its application.

A similar remark applies to the last solution offered by MAGNUS of the differences between us. In 1867, he showed that vapours were condensed by surface attraction to a greater extent than had previously been supposed. Blowing vapour-laden air into a metal tube, he found that heat was generated. He inferred, and rightly inferred, that this heat was produced by the condensation which occurred on the interior surface. This condensation he found to depend on the condition of the surface, being greater when it was tarnished or coated than when it was polished. He saturated air with moisture at a temperature of 16° C., and then raised both it and his pile to a temperature of 38°. When such air was blown against the dry face of the pile heat was generated. Condensation therefore occurred upon a surface 22° C. higher than the dew point of the vapour. Against this I have nothing to urge. But the fact by no means justifies the inference drawn from it, which was, that the vapours in my experimental tube were converted by "vapour-hesion" into liquid layers of high opacity to radiant heat. These layers, acting upon the calorific rays impinging on the interior surface of my tube, produced the absorption which I had erroneously ascribed to the vapours. More than this, the liquid layers were supposed to be broken up into discontinuous patches which not only absorbed the heat but scattered it. "Vapour-hesion," it may be added, was found to vary with the liquid which produced the vapour, being particularly strong in the case of alcohol.

MAGNUS brought this generalization to the test of experiment, but failed to verify it. He urged humid air against a dry mirror, from which radiant heat was reflected; but unless he wetted the mirror visibly no effect was produced on the reflected beam. Still he held that reflection, oft repeated, rendered sensible an action which eluded a single reflection. My position here is clear. I do not doubt surface attraction, or deny the existence of impalpable films. No experiment was ever made on the reflection of light or radiant heat in which such films did not intervene, but they had as little effect upon my results as they had upon those of DE LA PROVOSTAYE and DESAINS,* and of other refined experimenters. As early as 1859, I was made aware of the danger which might arise from condensation. Warned by the action of chlorine on my brass experimental tube, I coated it inside with lampblack, and re-tested with it all my vapours. The result removed from my mind the suspicion that surface condensation had anything to do with the observed absorption. Many similar experiments with blackened tubes were subsequently made by me for my own safety and instruction. There was no substantial difference between the results obtained with such tubes, and those obtained with polished tubes in which internal reflection came into play.

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* Considering the energy of water as a radiator, exceeding according to LESLIE that of lampblack itself, the film of this liquid which must have covered the plates of silver in the experiments of the two French philosophers ought, if MAGNUS be correct, to have sensibly raised the emission. Calling the emission from lampblack 100, that from polished silver plus the film was only 2·1.

Such are the general features and phases of a discussion which, though dealing only with a small item of my work, has consumed a considerable amount of time. Other able experimenters have entered this field, the latest of whom, MM. LECHER and PERNTER, have published a long and learned memoir in WIEDEMANN'S Annalen, which has been translated in the Philosophical Magazine for January, 1881. My experiments with gases they corroborate, but not those with vapours. Regarding the action of aqueous vapour they are especially emphatic, their conclusion being: "that moist air does not perceptibly absorb the heat rays from a source of 100° C." In fact they found moist air a little more transparent than a vacuum. "No imaginable source of error," they affirm, "has here been left out of account." The arrangement for filling with moist air was varied, the air allowed to stand for a long time over the water in the gas-holder, and this moist air then passed through several wash-bottles into the experimental space, but with the same *negative* result." In common with MAGNUS, MM. LECHER and PERNTER ascribe my results to the condensation of liquid films on the rocksalt plates, and on the polished inner surface of my tube.*

§. 4. *Experiments resumed: Verifications.*

With a view to my own instruction, and to the removal of uncertainty from other minds, these researches on radiant heat were resumed in November, 1880. A brass experimental tube 4 feet long, $2\frac{3}{8}$th inches in diameter, and polished within, was first employed.† Interposed between it and the source was a "front chamber" through which, when exhausted, the rays passed into the experimental tube. A plate of transparent rocksalt separated the tube from the chamber; while a second plate of salt closed the distant end of the experimental tube. The source of heat was at first a LESLIE'S cube containing water at 100° C., to one of the faces of which the end of the front chamber was carefully soldered. The chamber also passed air-tight through a copper cell in which a continuous circulation of cold water was kept up. The heat which might otherwise have reached the experimental tube by conduction from the source was thus cut off. One face of a thermopile, provided with a reflecting cone, received the rays which passed through the experimental tube. The other face, also provided with a cone, received the rays from a "compensating cube," used, as formerly, to neutralise the radiation from the source, and to bring the needle of the galvanometer to zero when the experimental tube was exhausted. On the entrance then of any absorbent gas or vapour the equilibrium was destroyed, the needle moved from zero, and from the observed permanent deflection the absorption was calculated. Other qualities of heat, and other experimental tubes than that here described, were afterwards introduced into the inquiry.

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* However I may otherwise differ from MM. LECHER and PERNTER, I agree with their opening remark, that few other questions of experimental physics present difficulties so great as the one here under consideration. Nor do I see reason to differ from their closing words, that "the extraordinary difficulty of investigations of this sort would be richly repaid by the attainment of quantitative results; whilst the corresponding optical investigations (immeasurably easier) will always remain more of a qualitative nature." It is the difficulty here signalised that has caused so many distinguished investigators to go astray in this field of inquiry. I may state here that on the receipt of their paper I wrote to MM. LECHER and PERNTER, but my communication was returned from Vienna through the dead letter office.

† The plate answering to this description will be found in the Philosophical Transactions for 1861.

I here give the measurements executed in 1880 with the vapours of nine different liquids, in experimental tubes of the dimensions above given.

Table I.

		Vapours.		Liquids.
	Pressures.	Leslie's cube vacuum.	Leslie's cube free.	Red hot spiral.
Bisulphide of Carbon	·48 in. mer.	4·4	5·0	7·6
Chloroform	·36 ,,	12·8	12·9	28·8
Benzol	·32 ,,	14·8	15·0	44·5
Iodide of Ethyl	·36 ,,	18·4	19·3	47·0
Iodide of Methyl	·46 ,,	25·0	26·2	59·0
Amylene	·26 ,,	26·1	27·2	65·0
Sulphuric Ether	·28 ,,	35·0	35·6	71·0
Acetic Ether	·29 ,,	43·3	43·7	77·5
Formic Ether	·36 ,,	43·3	44·0	78·0

The "pressures" in this table are chosen with a view to the comparison of liquids and vapours. They express quantities of vapour which are proportional to the quantities of matter in the respective liquids at a common thickness. The two next columns contain the absorptions per 100, of the heat from two Leslie's cubes, the one with a vacuum in front of it, the other placed in free air, and well protected from air-currents. The close agreement of the two columns proves the "front chamber" to be a superfluity. It also illustrates the coincidence to be attained in these measurements when they are carefully made. In the last column I have placed the absorptions exerted by liquid layers of the respective substances at a common thickness of one millimeter. The source of heat here was an incandescent platinum spiral. The order of absorption of liquids and vapours is the same.

This order is, as might be expected, undisturbed, when we apply heat of the same quality to liquids and vapours respectively. This is shown by the following table:—

TABLE II.

	Polished tube.		Blackened tube.
	Spiral (dull).	Spiral (bright).	Spiral (dull).
Bisulphide of Carbon	3·0	2·4	3·0
Chloroform	7·0	4·6	5·0
Benzol	9·4	7·5	10·0
Iodide of Ethyl	13·6	8·6	12·3
Iodide of Methyl	15·6	10·7	13·8
Amylene	20·6	14·3	17·5
Sulphuric Ether	27·0	21·0	23·8
Acetic Ether	33·0	23·0	29·0
Formic Ether	33·0	23·0	30·0

The vapour pressures here are the same as those employed in Table I. The order of absorption is the same in both tables; but its amount is diminished. This was to be expected from the difference in quality of the heat. We are dealing with transparent vapours—in other words, vapours pervious to the luminous rays; and the greater the proportion of these rays in the calorific beam, the less will be the absorption. This is well illustrated by the second column in the table, which shows the fall of the absorption when the spiral is raised from a dull red to almost a white heat. The polished tube was used in both these cases.

The third column of figures in Table II. shows the results obtained when the experimental tube was coated within with lampblack. The absorptions are in the same order, and almost of the same amount as those of the first column. The case is representative and might be multiplied to any extent. It is incompatible with the notion that my results were due to films collected on the polished interior surface of my experimental tube.

Placing the substances recently experimented on in the order of their absorption, and also in the order which they exhibited in 1864, we have the following two columns:—

1880.	1864.
Bisulphide of Carbon	Bisulphide of Carbon
Chloroform	Chloroform
Benzol	Iodide of Methyl
Iodide of Ethyl	Iodide of Ethyl
Iodide of Methyl	Benzol
Amylene	Amylene
Sulphuric Ether	Sulphuric Ether
Acetic Ether	Acetic Ether
Formic Ether	Formic Ether

In 1864, therefore, the iodide of methyl proved itself more diathermanous than the iodide of ethyl, while both of them were more diathermanous than benzol. In 1880, the case was precisely the reverse. Suspecting that the discrepancy might be due to impurity, I requested my friend Professor DEWAR to subject the liquids to a further process of purification. Tested afterwards, they produced the following deflections :—

	A	B	C
Bisulphide of Carbon	5·0°	5·0°	4·0°
Chloroform	17·0	15·0	6·0
Iodide of Methyl	39·4	33·0	8·0
Iodide of Ethyl	33·0	35·0	12·5
Amylene	42·0	41·0	16·0
Sulphuric Ether	44·3	43·5	18·2
Acetic Ether	46·2	45·5	22·0
Formic Ether	47·5	46·9	22·2

Under A and B respectively are the deflections produced by the liquids prior to and after purification. The iodide of methyl falls from 39·4° to 33°, while the iodide of ethyl rises from 33° to 35°. The relative positions occupied by the liquids in 1864 are thus restored. Benzol, however, remained permanently lower than before. Under C are the deflections produced by the vapours of the purified liquids. Here also the positions of the two iodides are reversed, vaporous absorption following the order of liquid absorption. I have frequently encountered cases of this character. The simultaneous change of diathermic position of liquid and vapour indicates that the foreign ingredient, whatever it was, possessed approximately the same volatility as the substance which it vitiated.

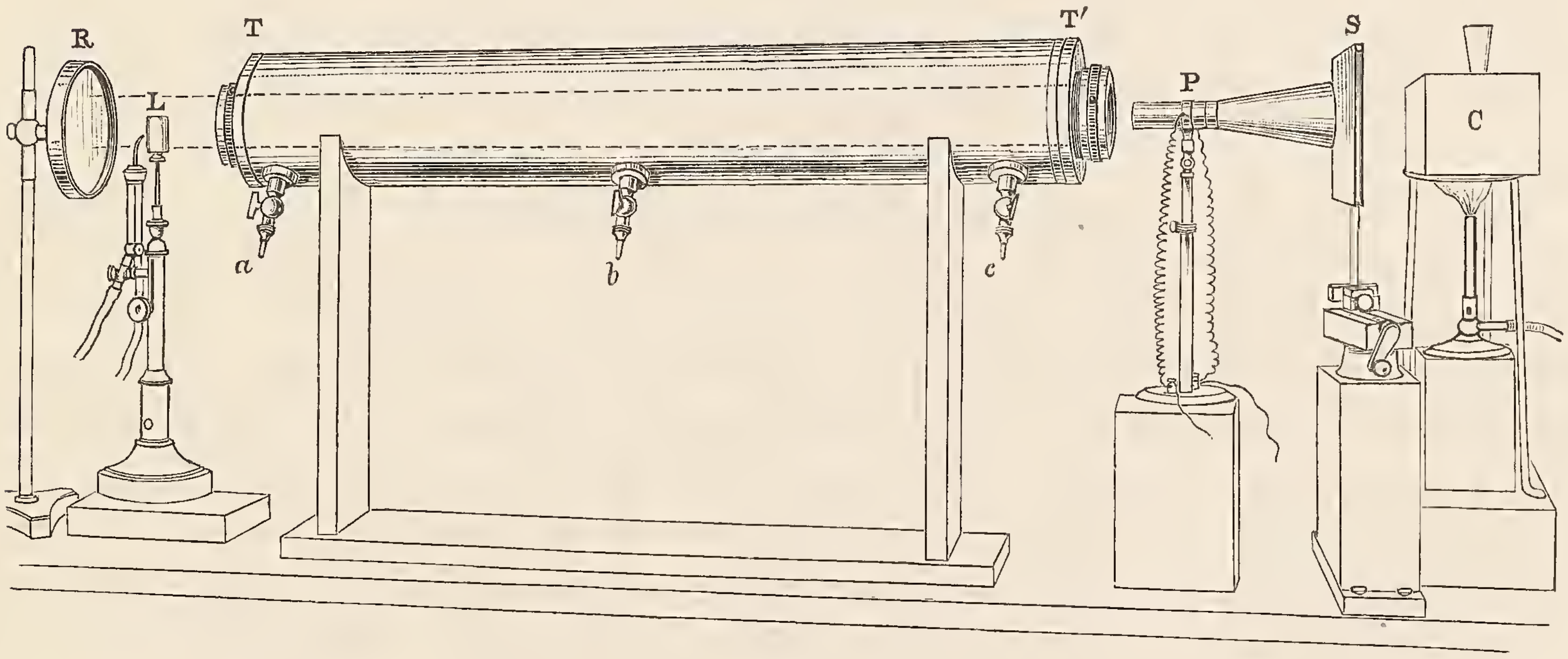
§. 5. *New Experiments. Hypothesis of Internal Films.*

a. The Experimental Tube.

I wish now to come to closer quarters with the argument urged by MAGNUS and repeated by MM. LECHER and PERNTER, namely that my results were due to "vapourhesion"—that is to say to liquid films condensed on my experimental tube and on my plates of rocksalt. The two investigators last named express unqualified surprise that I should have neglected the simple precaution of experimenting with blackened tubes. But this precaution was by no means neglected by me. I have repeatedly fortified myself by experiments of this character, with the result recorded in Table I. But I went further. A smooth coating of lampblack, however powerful as an absorber, might be held competent to reflect a certain portion of the incident heat. Hence my desire to get entirely rid of reflection, by avoiding all contact with the interior surface of the experimental tube.

In fig. 3 an apparatus is sketched by means of which this has been accomplished. T T′ is a stout tube of brass, 36 inches long and of 6 inches internal diameter. Projecting from its otherwise closed ends are screws 1 inch each in depth, surrounding circular apertures $3\frac{1}{8}$ inches in diameter. By means of screw caps these apertures may be closed air-tight by transparent plates of rocksalt, the plates being protected from crushing by washers of indiarubber. *a* is a cock leading to an air-pump and a barometer tube, *b* is a cock to which flasks can be attached, while *c* is a cock connected with a purifying apparatus (not shown). This consists of a U tube filled with fragments of clean glass moistened with sulphuric acid, and of a second tube filled with fragments of Carrara marble wetted with caustic potash. A plug of cotton wool intercepts the floating matter of the air. The source of heat L is a cylinder of carefully prepared lime, against which a flame of coal gas and oxygen impinges. The incandescent lime faces the concave mirror R, which receives the rays and sends them back in a parallel beam through the tube T T′. P is the thermopile from which wires proceed to a distant galvanometer, the reflected dial of which is observed through a telescope. S is an adjusting screen. At C is what, in my former researches, I have called the "compensating cube," used to neutralise the radiation from the source L, and to bring the galvanometer needle to zero. The chief object of this arrangement is to enable the experimenter to send a calorific beam along the axis of the tube T T′, which shall never touch its internal surface. The great width of the tube, aided when necessary by diaphragms, renders this easy of accomplishment.

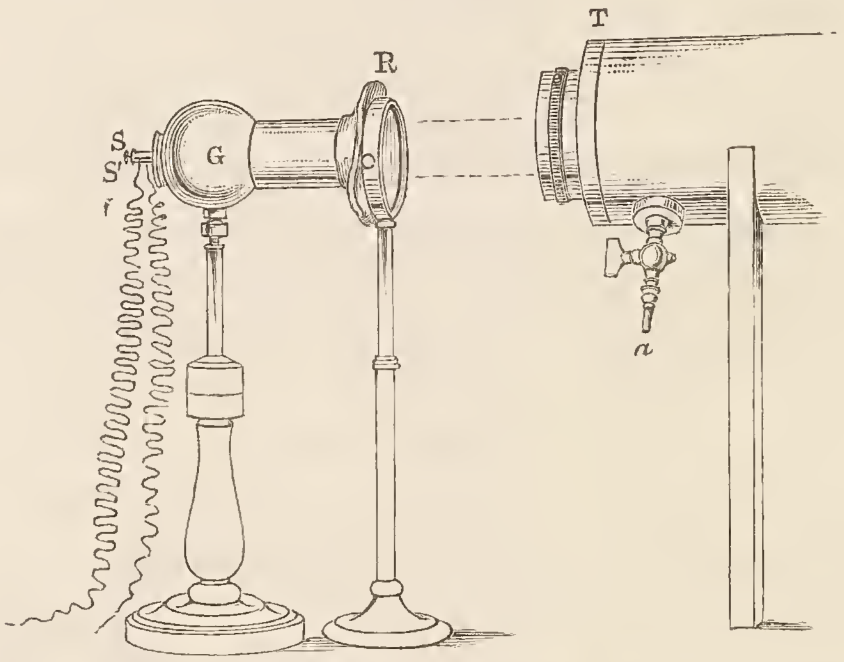
Fig. 3.



The mode of operation is readily understood. Suppose the heat impinging on the pile to produce a galvanometric deflection of 50°. This, the "total heat," is neutralised by the radiation from the compensating cube. To render the compensation accurate, the double screen S is shifted by an extremely fine screw motion. Even when the total heat is very large it is possible, by means of this screen, to neutralise accurately the radiation of the source and bring the needle to zero. Suppose it to be there when the tube T T′ is exhausted; on permitting a gas or vapour capable of absorbing radiant heat to enter T T′, the pre-existing equilibrium is destroyed; the needle moves promptly away from zero, and from the observed deflection the absorption may be calculated.

Instead of the concave reflector, a rocksalt lens of great purity is sometimes used to render the beam parallel. When the lens is used, the incandescent portion of the cylinder of lime is caused to face the tube T T′, between which and the lime the lens is introduced. Instead of the incandescent lime, a spiral of platinum wire heated to redness by a voltaic current is frequently employed as the source of heat. The spiral is placed in the centre of a brass globe G, fig. 4, and is connected with the screws S S′ fixed in the back of the globe, from which wires pass to the battery. R shows the position of the rocksalt lens, while one end of the experimental tube is shown at T, The greatest care is here necessary to protect the spiral from any agitation of the air. Such care, however, it is always in the experimenter's power to bestow. To secure constancy in the radiation, the battery was charged three times a day, the strength of the current being regulated by a rheostat, and checked by a tangent galvanometer.

Fig. 4.



An experimental tube of the description shown in fig. 3 was first employed by me about twelve years ago, and with it I then verified all the experiments I had previously made on the absorption of radiant heat by gases and vapours. The experiments were not published in any scientific journal, but they are thus referred to at page 394 of my volume of collected memoirs, entitled "Contributions to Molecular Physics in the domain of Radiant Heat":—"The two ends of an experimental tube 38 inches long and 6 inches in diameter were each provided with an aperture 2·6 inches in diameter. These apertures were closed with plates of rocksalt. The source of heat was a platinum spiral, well defended from air-currents, and heated to redness by an electric current. In front of the spiral was a rocksalt lens, which sent a slightly convergent beam through the tube, Behind the most distant plate was formed a sharply defined image of the spiral, its size being such *that it was wholly embraced by the plate of salt.* Here then was a beam of heat passing through an experimental tube without coming into contact either with the surface of the tube itself, or with any coating or lining of that surface. With this apparatus all my old experiments on vapours have been frequently repeated. There is no substantial difference between the results thus obtained, and those obtained with an experimental tube, where nineteen-twentieths of the heat which reached the pile, was reflected heat."

The tube referred to in this extract was of rough brass, tarnished within. Hence, when air entered it, after exhaustion, the dynamic heating of the tube and the partial condensation of the vapour when the air was moist, produced an amount of radiation from its internal surface which, though small, was a source of some disturbance. In my present experiments, therefore, another tube of the dimensions above given is employed, and to reduce to a minimum any radiation from its internal surface, it is coated within with silver, deposited electrolytically and highly polished. Experiments with this tube show that it does not in any way disturb the true radiation from the source.

In Table I. are recorded five series of measurements executed with a brass tube polished within. But as the liquids, though reported pure by the manufacturing chemist, vary slightly from time to time, I thought it advisable to adhere to the same samples in experiments wherein a tube with reflecting interior is to be compared with one permitting of no reflection. The following five series of new measurements were therefore executed with the tube first mentioned.

TABLE III.

BRASS tube with internal reflection.

	Pressures.	A Spiral (dark).	B Spiral (bright).	C Lime (free).	D Lime (mirror).	E Lime (lens).
Bisulphide of Carbon	·48	5·0	3·1	3·4	3·1	2·5
Chloroform	·36	7·9	5·3	5·6	4·8	5·0
Benzol	·32	9·6	8·6	7·9	6·9	6·3
Iodide of Methyl	·46	12·1	8·8	10·2	8·1	7·5
Iodide of Ethyl	·36	15·0	11·9	12·5	11·3	10·6
Amylene	·26	21·9	15·6	16·0	14·4	12·5
Sulphuric Ether	·28	30·9	23·1	22·7	19·8	18·1
Acetic Ether	·29	36·9	27·3	29·6	24·1	..
Formic Ether	·36	..	..	29·6	25·0	23·8

Column A in this table contains the absorptions of the respective vapours, in hundredths of the total radiation, when the source was a platinum spiral just under incandescence, unaided by either lens or mirror. Under B are the absorptions when the source was the same spiral heated to bright redness. The same vibrating atoms are preserved, but in A they vibrate on the whole more slowly than in B; and as the atomic periods of the transparent vapours synchronise best with the slower oscillations, the absorptions in A are considerably higher than those in B. Under C are the absorptions when the source was a moderate limelight, produced by coal gas and oxygen. Under D are the absorptions when the source was the limelight, with its heat gathered up and sent through the experimental tube by a concave mirror, while under E are the absorptions of the heat of the incandescent spiral aided by a rocksalt lens. The absorptions in D and E are somewhat less than those in C, because the path of the rays was diminished, through the reduction of internal reflection by the mirror and lens.

With the foregoing results, obtained with the brass experimental tube polished within, where the greater portion of the heat reaching the pile had undergone reflection, are now to be compared those obtained with the silvered tube, where internal reflection was wholly avoided.

Table IV.

Wide silvered tube with no reflection.

	L A Spiral (dark).	L B Spiral (bright).	M C Lime (bright).	M D Lime (moderate).	L E Lime (dull red).
Bisulphide of Carbon	4·0	1·8	1·5	1·5	3·3
Chloroform	5·6	3·0	3·5	4·3	6·0
Benzol	7·0	5·1	4·4	6·5	8·0
Iodide of Methyl	7·5	5·9	5·0	6·5	9·2
Iodide of Ethyl	11·3	7·6	6·8	8·5	11·2
Amylene	15·0	11·8	8·8	12·0	17·3
Sulphuric Ether.	21·2	17·1	12·5	15·0	25·0
Acetic Ether	26·0	20·0	16·3	24·0	32·0
Formic Ether	27·0	21·0	17·0	25·0	34·0

The vapour pressures here were those given in Table III. Column A contains the absorptions measured when the source of heat was a spiral under incandescence, aided by a rocksalt lens. B contains the absorptions measured with the same arrangement, the spiral being raised to bright redness. As usual the heat of lowest refrangibility is most absorbed. C contains the absorptions of a tolerably intense limelight aided by the silvered mirror. D the absorptions of the same light with its intensity reduced, while E contains the absorptions of the rays from a lime cylinder heated to dull redness by a steady flame of hydrogen, burning in air, and aided by a rocksalt lens.

Here, then, where no trace of reflected heat is sent to the pile, we have substantially the same results as those obtained when by far the greater portion of the heat reaching the pile had undergone reflection. The agreement shows that the films, the action of which had been posited, but never proved, are, in regard to this action, what I always knew them to be, imaginary.

b. The Plates of Rocksalt.

Having thus, I trust effectually, disposed of the hypothesis which ascribed my results to liquid layers covering the interior surface of the experimental tube, I proceed to the examination of the plates of rocksalt which have also been credited with a liquid deposit. This hypothesis is to a great extent disposed of in a paper of mine published in the Philosophical Transactions for 1864; which, like other papers, has been overlooked by writers on this question. An apparatus is there described which enabled me to operate upon gaseous strata of different thicknesses, the thickness, in each case, being measured with great accuracy by aid of a vernier. Let two polished plates of rocksalt be supposed in close contact with each other, and let them be gradually separated by a suitable rack-and-pinion movement. From the first moment of their separation let the space between them be copiously supplied with vapour. Liquid films, if they form at all, will be deposited when the distance between the plates of salt is but small. As far as the films are concerned, they will be as influential when the plates are $\frac{1}{20}$th of an inch apart as when they are 2 inches apart. Hence, if the hypothesis of my opponents be correct, the absorption ought to declare itself with the former amount of separation as clearly as with the latter. But if, as I allege, the absorption be the act of the vapour molecules, then the deepening of the vapour stratum will be accompanied by an increase of the absorption. The following experiment bearing directly on this subject was executed in 1864. My plates of salt were first fixed at a distance of $\frac{1}{20}$th of an inch asunder, and the space between them was copiously supplied with air saturated with sulphuric ether vapour. The distance between the plates of salt was then augmented by steps, as shown in the following table, and at each step the absorption was determined. Here are the results :—

Thickness of vapour stratum.	Absorption.
0·05 inch	2·1 per cent.
0·1 „	4·6 „
0·2 „	8·7 „
0·4 „	14·3 „
0·8 „	21·0 „
1·5 „	34·6 „
2·0 „	35·1 „

We thus see that when the depth of the vapour stratum augments from $\frac{1}{20}$th of an inch to 2 inches, the absorption augments from 2 per cent. to 35 per cent. of the total radiation.

It is only with sulphuric ether that an absorption of 2 per cent. by the thinnest stratum could be obtained. With most other vapours the action of a layer $\frac{1}{20}$th of an inch in thickness proved insensible. We may thus begin with a layer which yields an absorption immeasurably small, gradually augment the thickness of the absorbing layer until it becomes a column 38 inches long, and find, throughout, the increase of absorption running hand in hand with the increase of the length of the absorbent layer. This result is utterly irreconcilable with the hypothesis that liquid layers on the plates of salt played any important part in my experiments.

With regard to "vapour-hesion," I have to make the following further remarks. Let a thin plate of polished rocksalt be placed upon, or against, the face of the thermopile, with its lampblack removed, so as to expose a clean metallic surface. Let an open glass tube end about a quarter of an inch above the plate of salt; and through this tube let a current of mixed air and vapour be gently urged downwards against the plate. No fine spray, which might readily arise from bubbling through a liquid, is to be permitted to mingle with the vapour. My vapours, I may say, have been usually formed, without bubbling, in large flasks, each containing a portion of a volatile liquid, the vapour of which was permitted to diffuse in the air of the flask. FLETCHER's foot-bellows were employed to urge the vapours forward. The needle being at zero, or thereabouts, on causing the mixed current to impinge upon the rocksalt, the needle immediately swings aside, the deflection varying from 20° to 80° and more, according to the delicacy of the galvanometer and the quality of the vapour.

From such experiments, which prove heat to be liberated when vapour comes into contact with the salt, the condensation of the vapour has been inferred. I accept the inference.

This, however, does not imply the acceptance of the inference from this inference, that the condensed films exert the action ascribed to them on radiant heat. To test whether they do so or not I had a circular plate of rocksalt mounted on its edge, and so placed that a beam of heat passed through it normally to a distant thermopile. The source of heat was an incandescent spiral, aided by a concave mirror; and the total radiation, after passing through the salt and impinging on the pile, produced a deflection of 51°. This radiation was accurately neutralised by a compensating cube, the needle under the operation of the two opposing forces pointing to 0°. By an arrangement which every experimenter can imagine for himself, sheets of air, laden with various vapours, could be poured in succession over the plate of salt. If, under these circumstances, absorbent films were formed, the equilibrium would be destroyed and the needle would move from 0°. My slit, which was the flattened base of an open tin cone, was placed across the upper part of the plate of rocksalt, so that the vapours, which were all heavier than air, should fall over the plate downwards. Here are the results obtained when the following vapours were permitted to form films upon the plate:—

Bisulphide of Carbon	A barely sensible action.
Chloroform	No action.
Benzol	No action.
Iodide of Methyl	No action.
Iodide of Ethyl	No action.
Amylene	No action.
Sulphuric Ether	A barely sensible action.
Acetic Ether	No action.
Formic Ether	No action.
Alcohol	No action.

The minute deflections produced by bisulphide of carbon and sulphuric ether had nothing to do with liquid films, so that the words "no action" might have been written against these substances as against the others. While therefore recognising the fact of condensation, these simple experiments prove how incorrect it is to credit the condensed films with the effects which have been ascribed to them. According to MAGNUS, alcohol exhibits a force of vapour-hesion particularly strong: but we here see that even this vapour produces no sensible effect. The vapours have been tested with other sources of heat, with the same result.

The deportment of dry and humid air is, as usual, very instructive. When the thin plate of rocksalt resting on the naked face of the pile, was exposed to the common air of the laboratory, it, of course, contracted a film corresponding to the humidity of the air. The sweeping away of this film by dry air produced a very small deflection indicating cold. When, on the contrary, humid air was urged against the salt, the deflection indicating heat was prompt and large. We have now to test the action of the liquid film thus formed upon radiant heat. The circular plate of rocksalt before referred to was mounted, with the incandescent spiral on one side and the pile on the other. The slit was so arranged that dry and humid air could be sent in succession over the surface of the plate. As before, the radiation from the spiral was neutralised by the compensating cube, the needle pointing to zero when the two sources were in equilibrium. On sweeping dry air over the plate of salt, so as to remove the film contracted from the laboratory air, there was no motion of the needle. On pouring humid air from the slit over the salt, there was no motion of the needle. To moisten the air it was urged from bottom to top through a tall jar filled with wet bibulous paper. The same apparatus had furnished humid air which produced a swing of 80° when urged against the thin plate of rocksalt resting on the face of the pile. As a barrier to radiant heat it was nevertheless powerless. The condensation may even be considerably enhanced without producing any sensible effect. Through a glass tube I urged my breath against the plate of salt so as to produce the colours of thin plates, without any sensible effect upon the galvanometer.

Having thus clearly shown what films, even in an exaggerated form, cannot accomplish, I hope, in testing the action of humid air, that the use of the silvered tube, closed with polished plates of rocksalt, on which no trace of visible moisture is deposited, will be conceded to me. This tube involves the use of a source of heat of small dimensions. The lime light, though fulfilling this condition, is not suitable, because of the high refrangibility of its heat. The incandescent platinum spiral would be better; still, the radiation from this source is but feebly absorbed by the aqueous vapour taken up by air at ordinary temperatures. The oxyhydrogen flame fulfils the required conditions best.* It has the advantage of high temperature and low refrangibility, while the fact that its heat, as coming from aqueous vapour, is absorbed with special energy, by aqueous vapour, is also a strong recommendation. In the Bakerian Lecture for 1864, I have illustrated this point. It was then shown that when a platinum spiral, rendered incandescent by a voltaic current, had 5·8 per cent. of its heat absorbed by un-dried air, a hydrogen flame had from 17 to 20 per cent. of its heat absorbed. The mere plunging of a platinum spiral into the flame caused the absorption to fall from 17 per cent. to 8·6 per cent. Hence my reason for choosing a hydrogen flame in the present instance.

Dry air and humid air being caused to occupy the experimental tube in succession, both of them were compared with the radiation through the tube when very perfectly exhausted by a BIANCHI's pump. The following results were obtained.

	Deflection.	Absorption per 100.
Vacuum	0·0°	0·0
Dry air.	0·0°	0·0
Humid air.	8·0°	11·7
Vacuum	0·0°	0·0
Dry air.	0·0°	0·0
Humid air.	7·6°	11·2

The deflection through a vacuum, otherwise the total heat, was 47° which, according to my calibration table, is equivalent to 68 units.

A fresh supply of hydrogen was here introduced into the gas holder. The total heat being 46° or 65 units, the following results were obtained.

	Deflection.	Absorption per 100.
Dry air	0·0°	0·0
Humid air	6·2°	9·5
Dry air	0·0°	0·0
Humid air	6·7°	10·3

Mean of the four determinations with humid air 10·7.

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* I was careful to assure myself that, unless it amounted to the visible wetting of the plates of salt, there was no sensible stoppage of the rays from the oxyhydrogen flame. This quite agrees with the result obtained by MAGNUS himself, in the experiment with the concave mirror already referred to. Unless he visibly wetted the mirror he failed to impair the energy of the reflected beam.

A column of humid air 38 inches long absorbs, according to these experiments, 10·7 per cent. of the radiation from a hydrogen flame. I have been criticised for estimating the absorption of the earth's rays within 10 feet of the earth's surface at 10 per cent. This estimate I consider a moderate one, and the foregoing experiments prove it to be so.

It would be an error to suppose that determinations like these are easily made. They require the most scrupulous care for their successful accomplishment. The hydrogen issued from the gas holder through an orifice of fixed dimensions in a stream of the utmost possible constancy. It was then led into a SUGG's regulator, whence it issued under an absolutely constant pressure. The flame issued from a circular brass burner with an aperture $\frac{3}{16}$ths of an inch in diameter. It was carefully surrounded by a hoarding, the space within the hoarding being packed with horsehair. Every precaution, in fact, was taken to avoid the agitation of the air around the flame. Proper care was also taken to secure the pile against disturbance by air currents. The air being first purified, by passing it through caustic potash and sulphuric acid, was rendered humid by carrying it over wet bibulous paper contained in a suitable tube. It required some minutes to enter, and it was therefore necessary, by prior patient observance of the needle, to make sure that during this interval no change occurred in the radiation save that effected by the humid air itself. The humid air was removed from the experimental tube, not by exhaustion which always causes precipitation, but by gently forcing, by means of a compressing pump, dry air through the tube. When this was done, the needle, in all the experiments above recorded, returned within a small fraction of a degree to zero.*

With the rough, wide experimental tube to which reference has already been made, I, ten years ago, found the absorption of a column of humid air 38 inches long, to be 8 per cent. of the total radiation from a flame of hydrogen.

§. 6. *Conservation of Molecular Action.*

If the absorption of radiant heat be the act of the constituent atoms of compound molecules, its amount depending solely on the number of molecules encountered by the calorific waves, then whatever may be the changes of density which gases and vapours undergo, so long as the number of molecules remains the same, the absorption ought to continue constant. Such constancy, should it be proved to exist, I name the "conservation of molecular action." The experiments now to be described deal with this question.

Besides the silvered experimental tube already described as 38 inches long, I had another constructed of the same diameter, and with similar terminal apertures. Its length was 10·8 inches. The one tube was, therefore, 3·5 times the length of the other. The shorter tube was constructed with the view of proving that the absorptions I had recorded in previous memoirs were exerted by the vapour molecules, and not by liquid layers deposited on my plates of salt. The hypothesis of such layers being, however, completely disposed of, we can carry the experiments a step further. Assuming that the absorption does not change with change of density, so long as the quantity remains constant, it would follow that 1 mercury inch of vapour in the long tube ought to quench as much heat as 3·5 inches in the short one. The same conclusion ought, of course, to hold good when we compare 2 inches of vapour in the long tube with 7 inches in the short one.

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* The temperature of the laboratory air during the foregoing experiments was 60° FAHR.

The experiments have been made, and with the following results :—

SULPHURIC ether vapour.

SHORT experimental tube.

Pressure.	Deflection.	Absorption per 100.
3·5 inches . . .	24°	30
7 ,, . .	30°	37·5
Total heat .	50°	

Again :—

Pressure.	Deflection.	Absorption per 100.
3·5 inches . . .	24°	30
7 ,, . . .	31·5°	39·4
Total heat .	50°	

Taking the means of these two experiments we have the absorptions—

For 3·5 inches	30 per cent.
For 7 ,, ...	38·5 ,,

Such is the result obtained with the short tube : we now turn to the long one.

LONG experimental tube.

Pressure.	Deflection.	Absorption per cent.
1 inch	23°	30·3
Total heat .	49°	
2 inches . . .	31°	38·8
Total heat .	50°	

These results are almost identical with those obtained with the short tube and greater pressures.

The source of heat in this case was a dull lime light. In subsequent experiments the light was brightened. Here is the result.

LONG experimental tube.

Pressure.	Absorption.
1 inch	22·3 per cent.
2 inches	29·5 „

SHORT experimental tube.

Pressure.	Absorption.
3·5 inches	22·5 per cent.
7·0 „	30·0 „

The agreement here is almost as close as that established by the first experiment. With a still brighter source of heat the absorptions were :—

LONG tube.

1 inch	18·4 per cent.
2 inches	25·7 „

SHORT tube.

3·5 inches	18·8 per cent.
7·0 „	25·6 „

When, therefore, the density of the vapour varies inversely as the length of it traversed by the calorific rays, the absorption remains constant.

The hydride of amyl lends itself conveniently to experiments of this character. With it, and with the lime light as source, the following measurements have been made :—

HYDRIDE of amyl.

LONG experimental tube.

Pressure.	Deflection.	Absorption.
1 inch	10°	12·8
2 inches . . .	15°	19·2
Total heat	49·4°	

SHORT experimental tube.

Pressure.	Deflection.	Absorption.
3·5 inches . . .	10°	12·2
7·0 „ . . .	15°	18·3
Total heat	50·5°	

The agreement here is close enough to illustrate the law; the greatest difference being under 1 per cent.

At this point the following entry appears in my note book:—"It might have been wise on my part to rest content with the comparison of the long and short tubes with the lime light as source of heat. But for the sake of completeness I wished to introduce the incandescent spiral. The fulfilment of this wish—that is to say, the successful performance of a single experiment—has cost me more than a week's work. Once however committed to the task, I could not leave it incomplete.

"The discrepancy between the two tubes was in no case great, hardly ever exceeding 2 per cent. But the difference was uniformly in favour of the long tube and small density. Diaphragms were employed, the position of the short tube was shifted, and it was finally placed so that the pile should occupy the same position in relation to its adjacent end as it did in relation to the adjacent end of the long tube. The discrepancies then disappeared, the absorptions in the two tubes proving practically identical.

"Many leaves of paper were covered with observations during the week, but it is useless to take up time and space in copying them here. One representative observation will suffice."

SULPHURIC ether.

Source of heat—bright red spiral with rocksalt lens.

LONG experimental tube.

Pressure.	Deflection.	Absorption.
1 inch	20·0°	23·5
2 inches . . .	27·3°	32·1
Total heat	51·0°	

SHORT experimental tube.

Pressure.	Deflection.	Absorption.
3·5 inches . . .	17·8°	23·4
7·0 ,, . . .	24·8°	32·6
Total heat	49·0°	

The agreement between the two tubes is as perfect as could be desired.

It is easy to record these experiments, but it is not so easy to make them. On every portion of the apparatus—the source, the tube, the thermopile, and the galvanometer—extraordinary care must be bestowed to make the experiments strictly comparable. The results were checked by taking the total heat after every experiment—a precaution which insured the detection of any variation on the part of the source. When the platinum spiral was used, the battery of ten GROVE's cells, employed to render it incandescent, had to be very carefully prepared, freshly amalgamated zinc plates being used in each fresh battery. The requisite deflection of a tangent compass being produced, it was kept constant by means of a rheochord throughout the day. When the battery showed signs of rapid falling it was always renewed. It would be tedious to dwell upon the precautions taken to protect the source and the pile from the least agitation of the air. Such precautions are essential, but their necessity and form must be learnt by each experimenter for himself.

§. 7. *Thermal Continuity of Liquids and Vapours.*

I have amply illustrated by experiments, recently made, the correspondence which subsists between vapour absorption and liquid absorption, when the quantities of matter traversed in the two cases by the calorific rays are proportional to each other. This correspondence, as I have already stated, was established eighteen years ago. And though the result goes to the very core of the discussion which my researches have aroused, though in relation to that discussion they had, in my estimation, a weight and import greater than those of any other experiments published by me, they seem never for a moment to have attracted the attention of those who have taken part in the discussion. Here is a result, published in 1864, which illustrates the point now under consideration:—

		Absorption per 100.	
		Vapour.	Liquid.
Bisulphide of Carbon	0·48	4·3	8·4
Chloroform	0·36	6·6	25·0
Iodide of Methyl	0·46	10·2	46·5
Iodide of Ethyl	0·36	15·0	50·7
Benzol	0·32	16·8	55·7
Amylene	0·26	19·0	65·2
Sulphuric Ether	0·28	21·5	73·5
Acetic Ether	0·29	22·2	74·0
Formic Ether	0·36	22·5	76·3
Alcohol	0·50	22·7	78·6

The magnitude of the absorption in the liquids is far greater than in the vapours; because the quantity of absorbent matter is far greater in the former than in the latter; but the order of absorption is the same.

When the vapours are doubled in quantity the absorptions are considerably increased. When trebled they are still further augmented; in other words, they approach more and more in magnitude to the absorptions of the liquids; but the harmony as regards order is never disturbed. What then would occur if the vapours were so increased as to render the quantities of matter in the two states, not proportional, but *equal* to one another? This is the question with which I now propose to deal. At the time when the results above recorded were obtained, I thought it probable that if a circular liquid layer of a given diameter could be vaporized in a tube of the same diameter, the absorption would remain unchanged. In other words, I thought that the liberation of the molecules from liquid cohesion would neither augment nor diminish their action upon radiant heat. Since 1864 this problem has been often in my mind. The wide silvered tube, I am happy to say, has rendered the solution of the problem possible.

It is only highly volatile liquids that lend themselves to this experiment, because from them alone can vapours be derived of sufficient density to produce liquid layers of practicable thickness. On the 22nd of last October the experiment was first attempted. The source of heat was the lime light, the rays of which were received by a concave mirror silvered in front, and sent in a nearly parallel beam through the experimental tube. At the end nearest the source the tube was provided with a diaphragm having a circular orifice 1 inch in diameter. At the other end was a diaphragm with an orifice ½ an inch in diameter. Beyond this was placed the thermopile furnished, not with its reflecting cone, but with a tube of brass (shown in fig. 3) 2 inches long, and blackened within. In this arrangement, the heat which reached the pile did not even approach the interior cylindrical surface. The total heat employed produced a deflection of 60 galvanometric degrees, which, when the tube was exhausted by a powerful BIANCHI's air-pump, was accurately neutralised by a compensating cube. Liquid sulphuric ether was then placed in a large flask provided with a sound stopcock, the object being to expose a considerable evaporating surface. The flask was plunged in water, with the view of keeping the liquid and its vapour at an approximately constant temperature. The air being carefully removed from the flask, it was attached to the experimental tube, and a quantity of vapour was allowed to enter sufficient to render a column 38 inches long equivalent to a liquid layer 1 millimeter in thickness. Two concurrent experiments made the deflection produced by the vapour

41°

Without altering the quality of the heat, the absorption exercised by a liquid layer of sulphuric ether 1 millimeter thick was next determined. The rocksalt cell with which the experiment was made is described in detail in the Bakerian Lecture for 1864 (Phil. Trans., Vol. 154, p. 328.) The annexed figure (fig. 5) will give a sufficiently clear notion of its construction and disposition. Between two stout plates of brass, *c c*, and its fellow, two rocksalt plates of extreme purity are firmly clasped by suitable screws, due care being taken to protect the plates from a crushing pressure. The two brass plates referred to are perforated by circular apertures as shown in the figure. The two plates of salt are not allowed to come into contact but are separated from each other by a carefully worked brass plate 1 millimeter thick, perforated like *c c*. A portion of this middle plate is cut away, opening a passage into the interior of the cell. Through this passage the cell is filled with liquid by means of the funnel *f*. The cell is placed on a platform P P riveted on to the double brass screen S S. The source was placed at A, and the thermopile with its blackened tube at B. It is not necessary to figure them.

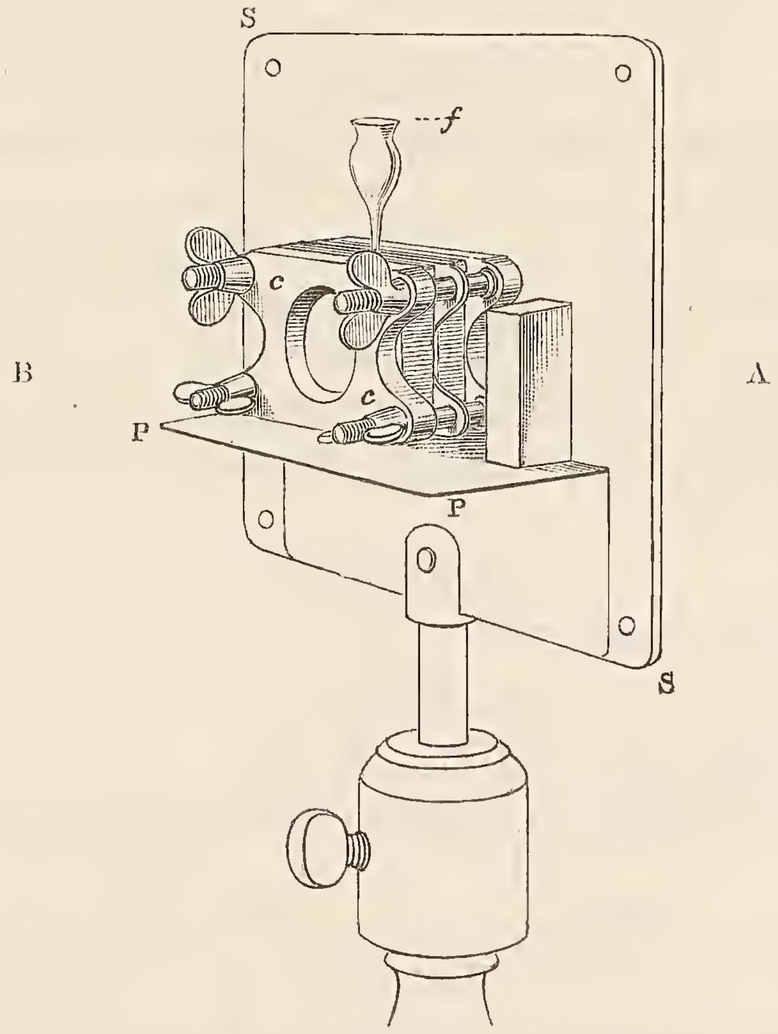
The needle being brought accurately to zero by the compensating cube when the cell was empty, liquid sulphuric ether was poured in. The consequent deflection was

42°

Here vapour absorption and liquid absorption were so nearly alike as to stimulate me to further efforts. The entry in my note book on the 22nd ends with the remark: "I purpose repeating this experiment on Monday, with the view of rendering the result secure."

On Monday, accordingly, the experiments were resumed, but they proved by no means so easy as I had hoped to find them. For five days I worked at the subject without coming to any satisfactory conclusion. The source, the mirror, the cell, the experimental tube, and the thermopile, were all in turn objects of scrutiny; but there still remained a difference between the action of the liquid and that of its vapour sufficient to throw doubt on the assertion of their identity.

Fig. 5.



At the conclusion of many trials and precautions I found the absorption of the vapour still distinctly in excess of that of the liquid. I had reduced the spherical aberration to a minimum, by confining the reflection to a small central area of the silvered mirror. Still the image of the incandescent lime, formed at the end T′ of the experimental tube, was large enough to encroach a little on the annular space surrounding the aperture closed by the plate of rocksalt. Diaphragms of polished metal had also been used to lessen the amount of heat falling upon the pile. I figured to myself the heat impinging on the annular space and diaphragm, reverberated back to the end T of the experimental tube, reflected from the annular space and diaphragm at that end, and thus in part sent back to the pile. Such heat, instead of passing once through the vapour, would pass through it three times, and if it formed a sensible part of the total heat, might make vapour absorption appear greater than liquid absorption. I had the tube dismounted, and the annular spaces and diaphragms carefully coated with lampblack. Remounting the tube and measuring once more the vapour absorption, it was found to be

32·4 per cent.

This was the mean of five concurrent series of observations, in which every care was taken to ensure exactitude. Lest the total heat should vary, during the execution of a series, it was taken at the conclusion of every individual experiment.

The absorption of sulphuric ether, acting as a liquid, was next determined. The mean of three series of experiments, two of which yielded identical results, and the third of which differred only 0·7 per cent. from the others, made the absorption of the liquid ether to be

32·9 per cent.

which, I need not say, is surprisingly close to the vapour absorption, differing therefrom by only 0·5 per cent.

Informed by experiment that the heat from the incandescent platinum spiral was more powerfully absorbed than that of the lime light, I thought it worth while to inquire whether the liquid followed its vapour in absorptive energy, when the quality of the heat was changed. On Monday the 31st, accordingly, the rays from the spiral being rendered parallel by a rocksalt lens, the absorption of sulphuric ether was determined and found to be

66·7 per cent.,

while the absorption of the liquid was

67·2 per cent.

which again gives a difference of only 0·5 per cent.

On another occasion I found the absorption by sulphuric ether to be:

Vapour	71 per cent.
Liquid	70 ,,

On the 1st November I checked the result obtained with the lime light and mirror, by using the lime light and rocksalt lens. Here are the absorptions of the vapour and liquid respectively:—

Vapour	33·3 per cent.
Liquid	33·3 ,,

The absorptions are identical; while the result agrees closely with that obtained with the concave reflector.

When the requisite quantity of sulphuric ether vapour, viz. 7·2 mercury inches, was in the experimental tube, I tried whether the radiation from a LESLIE'S cube, coated with lampblack and filled with boiling water, could pass through the vapour. About 14 per cent. of the incident heat was transmitted. Had I been asked at the time whether a liquid layer of sulphuric ether 1 millimeter thick was pervious to the heat of the cube, I should have replied with some confidence in the negative. Hence, for the moment, I thought the experiment opposed to the law that vapour absorption and liquid absorption, when equal quantities of matter are compared, are the same. On actually testing a layer of the liquid ether 1 millimeter thick, the transmission of upwards of 6 per cent. of the incident heat was observed. So that in this case also we have harmony of deportment between liquid and vapour. The absorption of the vapour exceeds that of the liquid because the heat from the cube radiated freely against the interior silvered surface of the experimental tube, and by its reflection from that surface had its path through the vapour augmented in length. This augmentation naturally carried with it an increase of the absorption.

The next substance examined was hydride of amyl, the boiling point of which is 30° FAHR., or 5° lower than that of sulphuric ether. When a large evaporating surface is exposed, there is therefore no difficulty in obtaining, from this liquid, vapour of a pressure of 6·6 inches of mercury. This, in a tube 38 inches long, would, if squeezed to liquefaction, produce a layer 1 millimeter thick. Vapour absorption and liquid absorption being measured in succession, this is the behaviour of the hydride of amyl:—

Absorption by vapour	51 per cent.
Absorption by liquid	51 per cent.

the two absorptions being absolutely identical.*

Combining this section and the last, their joint results may be thus summed up. Beginning with a column of sulphuric ether vapour 38 inches long at 7·2 inches pressure, or with a column of hydride of amyl vapour 38 inches long and at 6·6 inches pressure, and gradually shortening the column without altering the quantity, the vapour would gradually augment in density and pass wholly, when reduced to a thickness of 1 millimeter, into the liquid state of aggregation. Suppose a beam of heat of constant value, after passing through the vapour, to impinge upon a thermopile and to produce a definite galvanometric deflection; this deflection would remain absolutely fixed during all the changes of density and aggregation which we have supposed the vapour to undergo. In other words—as regards the absorption of radiant heat, the vapour would pass, without breach of continuity, through all its stages of condensation into the liquid form of matter.

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* When the rocksalt cell was empty, reflection of course occurred at its two interior surfaces. A perfectly diathermanous liquid, with the refractive index of rocksalt, would annul this reflection. And though the liquids actually employed had a smaller refractive index than rocksalt, and though they were far from being perfectly diathermanous, their introduction into the cell must nevertheless have diminished the reflection and thus added to the transmitted heat. This addition, having been determined by calculation, was sensibly neutralised by the introduction of washers of thin note paper which slightly augmented the thickness of the liquid stratum traversed by the calorific rays.

A general law of molecular physics is, I apprehend, here illustrated.

§. 8. *Rhythmic Absorption of Radiant Heat by Gases and Vapours.*

Conclusive as the foregoing experimental argument must appear, as regards the action of free molecules upon radiant heat, I am nevertheless glad to supplement it by another of a totally different character. On the 29th of November, 1880, I had the pleasure of witnessing, in the laboratory of the Royal Institution, the remarkable experiments of Mr. GRAHAM BELL, wherein musical sounds were evoked by causing an intermittent beam of light to impinge upon thin discs of various kinds of matter. I was soon convinced that the effects were due to the rhythmic gain and loss of heat. Being occupied with experiments on gases and vapours at the time, I thought that they might be invoked to decide the nature of the action revealed by Mr. BELL. The result was mentally clear before the experiment was made. I pictured a highly absorbent vapour exposed to the shocks of the intermittent beam, suddenly expanding during the moment of exposure, and as suddenly contracting when the beam was intercepted. Pulses of an amplitude probably far greater than those obtainable with solids would, I thought, be thus produced; and these pulses, if caused to succeed each other with sufficient rapidity, would be sure to produce musical sounds.

This idea was tested and verified on the spot. The Journal of Telegraph Engineers for December 8th, 1880, contains the following record of what occurred. "When Professor BELL was good enough to show me his experiments, I happened to be myself experimenting on the action of vapours upon radiant heat. Old experiments had revealed, and new ones had confirmed the fact that, as regards the absorption of heat, there existed vast differences between vapours. This is well illustrated by the deportment of bisulphide of carbon and of sulphuric ether, one of which is highly transparent, and the other highly opaque to radiant heat. It occurred to me that, if the action were due to the absorption of heat, we might possibly extract musical sounds from sulphuric ether vapour, whereas bisulphide of carbon vapour being transparent to heat-rays they would for the most part go through this vapour unabsorbed, and produce no sonorous effect. I think Professor BELL will bear me witness as to the result. We placed a quantity of sulphuric ether vapour in a test tube, and allowed an intermittent beam of light to strike upon the vapour far above the liquid, and we heard distinctly a musical tone of a pitch corresponding to the rapidity of the flashes. We then took the bisulphide of carbon vapour, and tried it in a similar manner, but neither Professor BELL nor myself could hear any trace of a musical sound."*

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* Mr. BELL has given a perfectly accurate account of this occurrence in the Philosophical Magazine,

It was obvious, however, that the arrangement of Mr. BELL—a truly beautiful one—was not suited to bring out the maximum effect. He had employed a series of glass lenses to concentrate his beam, and these, however pure, would, in the case of transparent gases, absorb a large portion of the rays most influential in producing sound. This may be illustrated by comparing a rocksalt lens, in my collection, with a glass lens of the same focal length prepared in the workshop of M. DUBOSCQ. Transmitted through the former, the radiation from an incandescent platinum spiral produced a galvanometric deflection of 55°, possessing, according to the table of calibration, a value of more than 100. Transmitted through the latter, the deflection fell to 10°, or to less than $\frac{1}{10}$th of the radiation transmitted by the rocksalt. The $\frac{9}{10}$ths here shown to be intercepted by the transparent glass, consist of heat on which transparent gases and vapours would exert a specially absorbent power. Hence the desirability of maintaining this important factor, in the radiation employed to test the sonorous power of such substances.

It was with the view of preserving intact these powerful calorific rays that I employed in my experiments on calorescence* small concave mirrors silvered in front; and to these mirrors I now resorted. My more intense sources of heat comprised a SIEMENS' lamp connected with a dynamo machine; an ordinary electric lamp connected with a voltaic battery; and a lime light, produced sometimes by the combustion of oxygen and hydrogen, and sometimes by oxygen and coal gas. The lime light (which was used by me in 1859) is so handy, steady, and otherwise effective, that I have applied it almost exclusively throughout this part of the inquiry. Sources of heat, however, of much lower temperature than the lime light, have proved competent to evoke musical sounds. A candle flame, a red hot coal, a red hot poker, the same poker at the temperature of boiling water, and an incandescent platinum spiral, have all been proved effective, though of course far less so than the concentrated lime light.†

To produce the required intermittence I first employed a circle of sheet zinc 16 inches in diameter provided with radial slits. This was afterwards exchanged for a second disc of the same diameter, but furnished with circumferential teeth and interspaces. The disc was mounted vertically on a whirling table, and caused to rotate across the beam near the focus of the concave mirror. Immediately behind the disc was placed the flask containing the gas or vapour to be examined, while an india-

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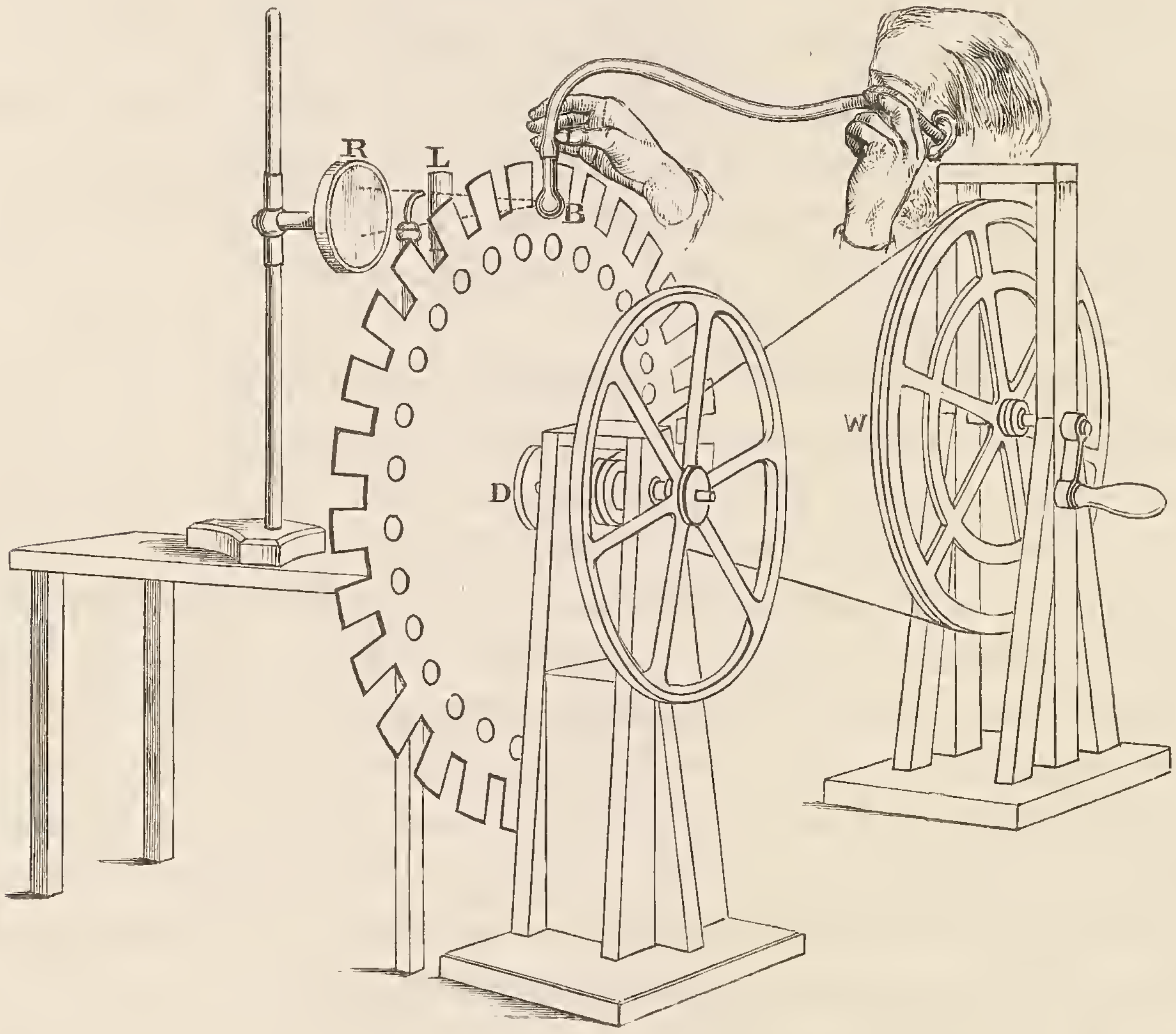
vol. xi., p. 519. With reference to what occurred on the 29th of November he writes thus:—"Professor TYNDALL at once expressed the opinion that the sounds were due to rapid changes of temperature in the body submitted to the action of the beam. Finding that no experiments had been made at that time to test the sonorous properties of different gases, he suggested filling one test-tube with the vapour of sulphuric ether (a good absorbent of heat), and another with the vapour of bisulphide of carbon (a poor absorbent); and he predicted that, if any sound was heard, it would be louder in the former case than in the latter. The experiment was immediately made; and the result verified the prediction."

* Philosopical Transactions, 1866, Vol. 156, p. 1.

† These earlier experiments will be found recorded in the Proceedings of the Royal Society, vol. 31, pp. 307, 478.

rubber tube, ending in a hollow cone of ivory or boxwood, connected the flask with the ear. With this arrangement, simple as it is, sounds of surprising intensity were obtained with all those gases and vapours which my previous experiments with the experimental tube and thermopile had proved to be powerful absorbers of radiant heat. The final arrangement was that shown in fig. 6.

Fig. 6.



The source of heat is the carefully worked and centred lime-cylinder L, heated by the oxy-hydrogen flame. The rays from this source are received by the concave mirror R, and converged upon the bulb B which contains the substance to be tested. The bulb is connected with the ear by a piece of india-rubber tubing, ending in a tapering tube of boxwood or ivory. The intermittence of the calorific beam is effected by the disc D of strong cardboard, 2 feet in diameter, and provided at the circumference with 29 teeth and corresponding interspaces.* The disc is caused to rotate by the wheel W, with which it is connected by a band. The positions of the sonorous bulb and of its ear-tube are shown in the figure. In the case of gases lighter than air the bulb B is turned upside down. With the heavier gases it is held erect. When vapours are tested, a small quantity of liquid is poured into the bulb, which is shaken so as to diffuse the vapour in the air above the liquid. The bulb is held so that the point of maximum concentration of the beam falls upon it.

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* Intermittence is sometimes produced by the series of equidistant circular apertures shown in the figure.

With this apparatus I have tested more than once the sounding power of ten gases and of about eighty vapours. As a sound producer chloride of methyl is supreme. It is however closely followed by aldehyde, olefiant gas and sulphuric ether, the two latter being very nearly equal to each other. The volatility of the liquid from which the vapour is derived is of course an important factor in the result. For, however high the inherent capacity of the molecule as an absorber may be, if the molecules be scanty in number the effect is small. Feeble vapours, on the other hand, may to some extent atone by quantity for the inherent weakness of their molecules. A few examples will suffice to show how the specific action of the molecules over-rides the effect of volatility. Bisulphide of carbon with a boiling point of 43° C. is less powerful than acetic ether with a boiling point of 74°. Tetrachloride of carbon boils at 77° but its sound by no means equals that of acetal which boils at 104°. Chloroform with a boiling point of 61° is less powerful as a sound producer than valeral with a boiling point of 100°, or even than valerianic ether with a boiling point of 144°. Cyanide of methyl boils at 82°, but produces less sound than acetate of propyl with a boiling point of 102°. In the experimental tube, these vapours follow, as absorbers, the order of their sounds. When tested in liquid layers they follow the same order. I have examined about a score of liquids with boiling points varying from 163° to 308°. At ordinary temperatures the vapours of these liquids were practically inaudible. The liquids being plunged in a bath of heated oil the vapours so produced emitted, for the most part, powerful sounds. The measured absorptions of a sufficient number of substances, in relation to their sounding power, shall be tabulated immediately.

The fact is worth a passing reference that the chlorides of the elements appear one and all to be feeble sound producers, because they are one and all feeble absorbers of radiant heat. Many years ago I had found them highly diathermanous, and accepted chloride of sodium as representative of the class. Silicium chloride, for example, though very volatile is weak as a sound producer. Tetrachloride of carbon, and terchloride of phosphorus are also volatile, but not strong. Bichloride of tin, chloride of arsenic, chloride of titanium, and chloride of sulphur, are all feeble sound producers. In these three cases the boiling points are high, but non-volatility is not the cause of the weakness, for when the vapours are raised, by heating their liquids, almost to the pressure of the atmosphere, they are still but feebly sonorous. Whatever then the condition may be which renders these substances pervious to radiant heat, it appears to be a condition common to them all.

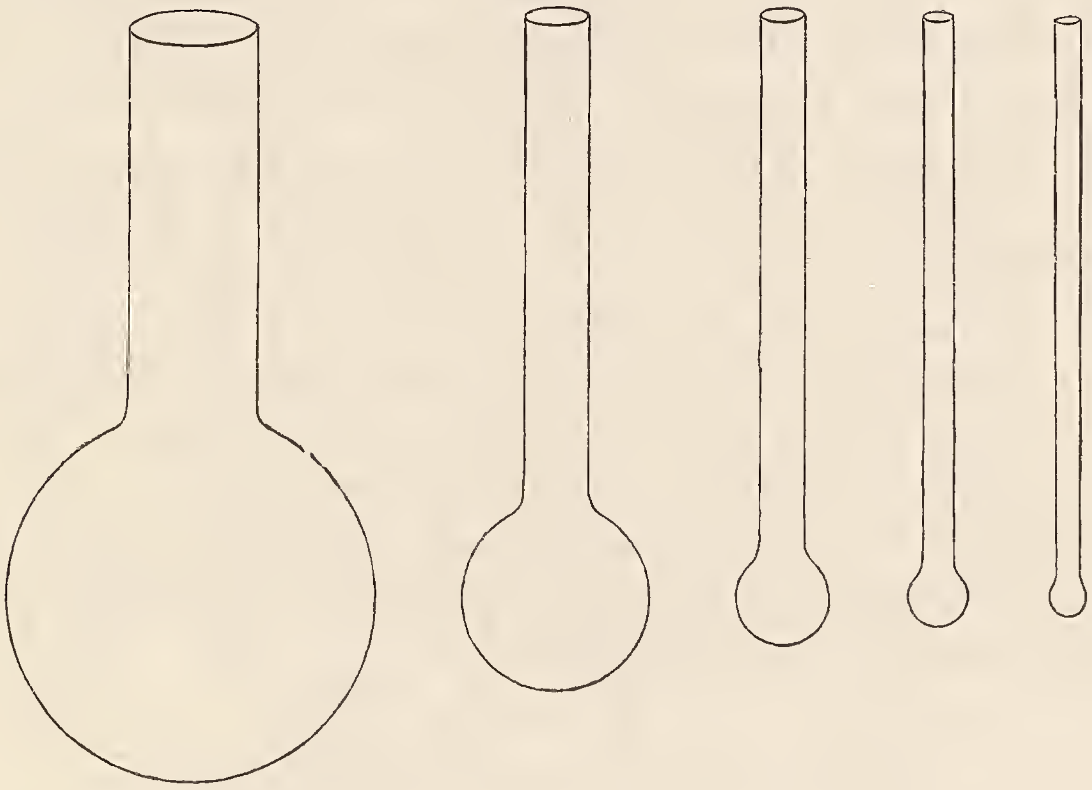
In experiments with the chlorides care must be taken to abolish all fumes. Bichloride of tin sounds loudly with fumes, but is weak without them. Simply heating the upper part of the flask frequently suffices to lower the sounds from loudness almost to silence.

The universality of its presence, and the discussions which it has provoked, rendered the action of water vapour especially interesting to me. I did not imagine at the outset that the modicum of vapour diffused in atmospheric air at ordinary temperatures could produce sonorous pulses of sensible intensity. In my first experiment, therefore, I warmed water in a flask nearly to its boiling point. I heated the flask above the water with the spirit lamp flame, thus dissipating every trace of haze, and then exposed the clear vapour to the intermittent beam. The experiment was a virtual question put to the vapour as to whether I was right or wrong in ascribing to it the power of absorbing radiant heat. The vapour answered by emitting a musical note which, when properly converged upon the tympanum, seemed as loud as the peal of an organ. When the temperature was lowered from 100° C. to 10° C., the sound did not vanish, as I expected it would. It remained not only distinct but strong. The flasks employed in these experiments were dried in a variety of ways, of which I have already given some account, and which will suggest themselves to every experimenter in this field. Taken open from the laboratory, and exposed to the intermittent beam, the flasks are always to some extent sonorous. Placed beside sulphuric acid underneath the receiver of an air-pump, and permitted to dry there, they are reduced to silence. The slightest invasion of humid air renders them again sonorous. Breathing for a moment into a dried and silent flask, a loud sounding power is immediately manifested.

Flasks without lips have been specially blown for these experiments, the india-rubber tubing being readily pushed over their necks. Large flasks are not the most suitable. To produce effective pulses sudden and intense expansions and contractions are required, and these are best obtained when the beam, at its place of maximum concentration, covers a large portion of the matter in the flask. Thin bulbs about a cubic inch in volume are both handy and effective; but the bulb may be reduced to $\frac{1}{50}$th, or even $\frac{1}{100}$th of a cubic inch, without rendering the sound insensible. A speck of water introduced into such tiny bulbs, when vaporised by heat, produces sounds which are not only sensible but loud. A series of bulbs which I have actually employed in my experiments are represented in their natural dimensions in fig. 7.

It cannot be necessary to state that the absorption which produces the pulses is direct and immediate, being the act of the gaseous molecules. The pulses are not due to the heating of the glass envelope and the communication of its heat to what it contains. For, were this the mode of heating, air would be as sonorous as olefiant gas. Nor are the pulses due to the sudden vaporization of a liquid layer which might be supposed to overspread the interior of the flask. When water at a low temperature is purposely caused to cover the interior surface, exposure to the beam produces sound of a certain intensity. When the flask is so heated in a spirit flame as to chase away every trace of the adherent liquid, the exposure of the pure vapour, then within the flask, to the beam, generates a sound far louder than that produced when the liquid film was there. Holding the bulb containing the hot vapour for a little time in the intermittent beam, its temperature falls; the quantity of vapour diminishes, and the sound sinks in intensity. On quitting the spirit flame, the bulb in some cases must have been near a red heat, but even at this temperature the vapour sounded loud.

Fig. 7.



It has, I think, been amply shown that when the molecules of a liquid are rendered free by vaporization they carry with them their absorbent power, liquids and vapours being pervious and impervious to the same quality of heat. Hence the inference, that prior transmission through a liquid of sufficient thickness ought so to sift a calorific beam as to render it powerless to act on the vapour of that liquid. Even with the loudest sounding vapours this proves to be the case, a layer of liquid $\frac{1}{8}$th of an inch thick being found generally sufficient to deprive the beam of its efficient rays, and the vapour of its sounding power.

In transparent liquids, the visible rays have free transmission, the destruction of sounding power by such liquids must, therefore, be due to the absorption of the invisible calorific rays. This induction, which hardly needs verification, is nevertheless capable thereof. Many years ago I pointed out the astonishing transparency of dissolved iodine to the invisible heat rays. Placed in the path of the intermittent beam, a layer of this substance, perfectly opaque to light, does not sensibly diminish the sound of transparent gases and vapours. To such substances, the iodine is exactly complementary, arresting the rays which they transmit, transmitting the rays which they absorb, and, therefore, not interfering with the sounding power.

That sounds may also be produced by the absorption of the visible rays is well exemplified by the deportment of iodine and bromine vapours, both of which yield with the lime light forcible sounds. Here the intervention of a transparent liquid, however adiathermanous it may be, produces no sensible effect upon the sound, the reason being that it permits the particular rays which act upon the coloured vapours to pass freely through it. A layer of dissolved iodine, on the other hand, deprives the beam of its power of evoking sounds from either iodine or bromine vapour.

The rotation which produces the maximum effect is soon ascertained by experiment. The sound is loudest when the pulses succeed each other in periods which invoke the resonance of the flask. I possess a hollow cone with a well-polished rocksalt base, by which this point is well illustrated. Filling this cone with chloride of methyl, while the base is turned towards the source of heat, the apex of the cone being connected by a tube with the ear, sounds of extraordinary intensity are produced. Abandoning the ear-tube, the sound can be heard at a distance. But to obtain this effect the speed of rotation must be definite and constant. The maximum sound once obtained, either the lowering or the heightening of the speed rapidly enfeebles it. It is difficult by hand-turning to keep the rate of rotation constant. Hence the desirability of a mechanical arrangement, which would ensure the proper rapidity and necessary uniformity. One or two motive powers have been tried, including a small steam-engine and an electro-magnetic engine, but the arrangement has not yet been brought to perfection.

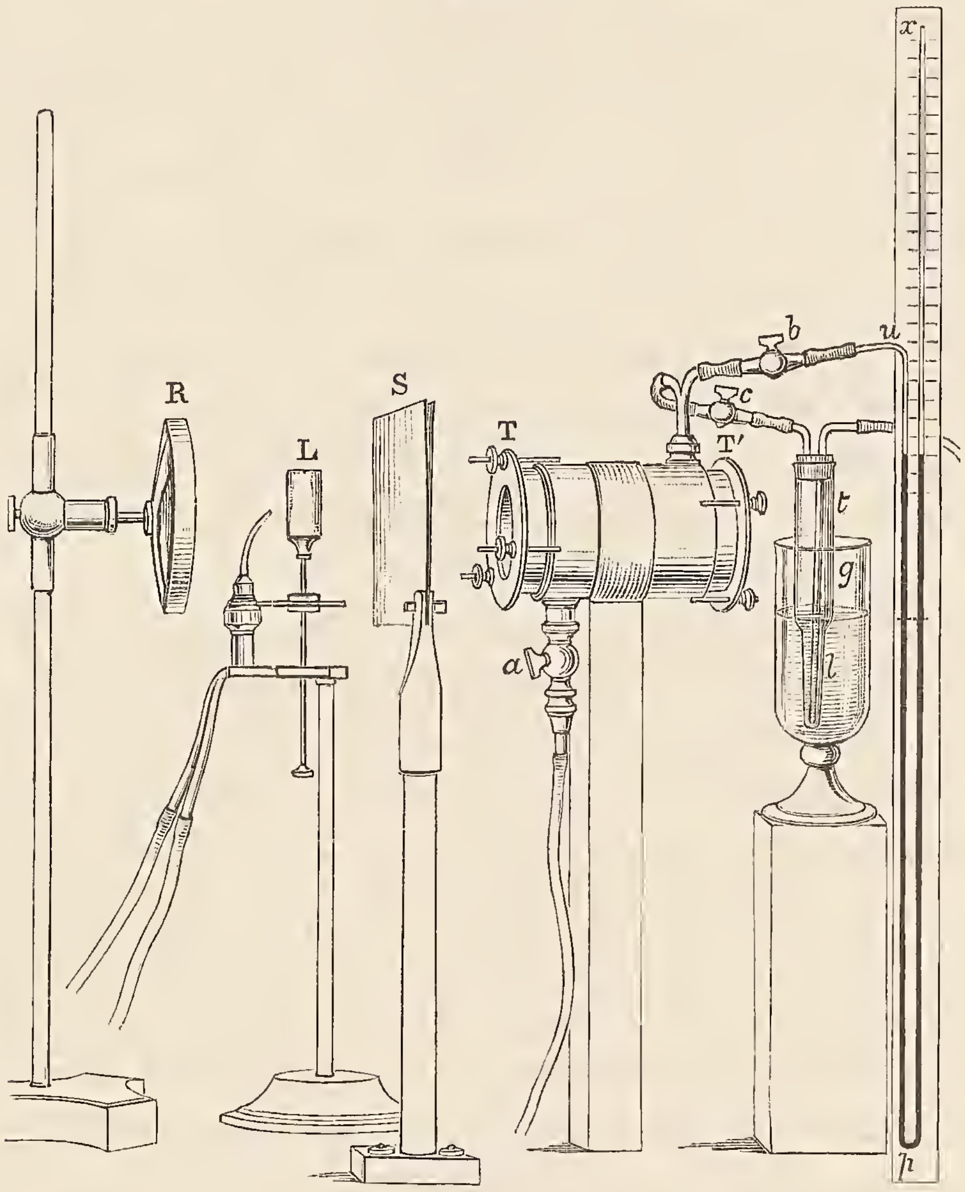
§. 9. *Manometric measurements.*

Some time before the visit of Mr. GRAHAM BELL in November, 1880, I had inserted into my old experimental brass cylinder a narrow tube of glass, which being bent at a right angle a few inches above the cylinder could hold an index of coloured liquid in its horizontal portion. I had long known that the absorption of radiant heat must be accompanied by the expansion of the absorbing body, but thought that such expansion would furnish only a rough measure of the absorption. With ordinary sources of heat I found the expansion small, even when sulphuric ether occupied the experimental tube; but when a pair of stout carbons, rendered incandescent by a SIEMENS' machine, were employed as a source, the liquid index was driven forcibly out of the narrow glass tube.

The experimental tube, however, was but a rude manometer, and I therefore sketched and described to my assistant at the time, with a view to its construction, a handier instrument. The apparatus was to consist of a short tube with rocksalt ends, capable of being exhausted and filled with any required gas or vapour. Through this tube it was proposed to send a concentrated calorific beam, whose action on an absorbent gas or vapour should be declared by the depression of a liquid column in one leg of a U tube, and its elevation in the other. Two rocksalt plates were to be employed with the view of allowing the beam free escape from the tube after it had done its work upon the gas or vapour. The warming of the apparatus by the reverberation of the heat would be thus avoided. The point aimed at was to effect the expansion of the gaseous body purely by radiant heat, and undisturbed as far as possible by heat derived from the envelope.*

A number of manometric tubes of different lengths and materials were constructed on this principle, some being of glass and some of metal. The instrument with which the measurements now to be recorded were executed is represented in fig. 8. T T′ is a glass tube 4 inches long and 3 inches in diameter. It is provided with brass flanges at the ends which reduce the diameter to 2·5 inches. Against these flanges, transparent plates of rocksalt are fixed air-tight. The tightness of the tube was secured, sometimes by india-rubber washers properly greased, and sometimes by cement.† A stop-cock *a*

Fig. 8.



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* Professor Röntgen was, I believe, the first to turn the expansion of gases to account in demonstrating the absorption of radiant heat. The very day, moreover, on which I made my communication to the Society of Telegraph Engineers, viz. the 8th of December, 1880, he forwarded to a scientific journal the announcement of his having obtained sounds from coal-gas and ammonia (see Wiedemann's Annalen, Jan. 1881). His subsequent experiments with aqueous vapour, &c., agree with mine.

† The absorption of the vapours by india-rubber—which was in some cases extraordinary—caused the washers to be abandoned.

near one end of T T′ was connected with a barometer tube and an air-pump. A T-piece at the other end was connected on the one side with a purifying apparatus (not shown), consisting of two U tubes, one containing fragments of Carrara marble wetted with caustic potash, the other containing fragments of glass wetted with sulphuric acid. Before entering these U tubes the air was freed from suspended matter by a plug of cotton wool. On the other side the T-piece was connected with a quill tube of glass bent into the shape of a U, in the two legs of which a coloured liquid stood at the same level. The liquid column when standing at the same level in both arms of the U was 350 millimeters high in each, while the free leg of the U rose to a height of about 500 millimeters above the surface of the liquid (shortened in the figure). The source of heat was the lime cylinder L, rendered incandescent by a flame of coal gas and oxygen. The rays from the lime cylinder were received by a concave mirror R silvered in front, and sent by it in a convergent beam through the manometer tube. The focus of the beam was within the tube and near its most distant end. The gas and oxygen were supplied from gasholders specially constructed for these and similar experiments; long and futile experience of gas from the public mains, or compressed in iron bottles, having shown independent gasholders which could be kept at an unalterable pressure to be essential.

The experiments were conducted thus:—A test-tube *t*, plunged in water, held by the glass *g*, contained the liquid whose vapour was to be examined. Through a cork which stopped the test-tube passed a narrow tube of glass, ending in a small orifice near the bottom of the test-tube, and at a considerable depth below the surface of the liquid. To augment this depth, and to economise the liquid, the lower part of the test-tube was drawn out to half the diameter of its upper part. A second narrow tube passed also air-tight through the cork, and ended immediately beneath it. Both tubes were bent at a right angle above the cork. The manometric tube being exhausted, air freed from its carbonic acid, its moisture, and its suspended matter, was allowed to bubble through the liquid in the test-tube, and to pass thence into the manometric tube. To spare the oxygen in the gasholder it was cut off during the interval between two consecutive experiments, the coal gas being kept continually alight. When the manometric tube was filled, which was always accomplished through an orifice of fixed dimensions, the oxygen was turned on, and the cylinder was allowed to remain for one minute under the action of the intensified flame. During this time a double silver screen S intercepted the radiation. At the end of a minute this screen, which moved on a hinge, was withdrawn, the beam then passing through the mixed air and vapour. The liquid standing in the adjacent leg of the narrow U-tube was immediately depressed, that in the opposite leg being equally elevated. The rise of this latter column above its starting point, marked zero on a millimeter scale, was accurately measured. Double this rise gave the difference of level in the two legs of the U, and this "water pressure" expressed the augmentation of elastic force by the absorption of radiant heat.

Here follow a certain number of the measurements which have been thus made. They do not comprise the whole of the substances examined.

Table V.

Vapours.

Increase of elastic force by Radiant Heat.

Name of Liquid.	Boiling point.	Mean Water pressure.	Character of sound.
1. Sulphuric Ether	35°	300 millims.	Very strong
2. Hydride of Amyl	30°	279 ,,	,,
3. Acetone	58°	267 ,,	,,
4. Bromide of Ethyl	39°	264 ,,	,,
5. Formic Ether	55°	261 ,,	,,
6. Acetic Ether	74°	248 ,,	,,
7. Acetal	104°	237 ,,	,,
8. Chloride of Allyl	46°	235 ,,	,,
9. Iodide of Methyl	45°	233 ,,	,,
10. Dichloride of Ethidene	57°	217 ,,	Strong
11. Nitrate of Ethyl	86°	208 ,,	,,
12. Nitrite of Amyl	99°	205 ,,	,,
13. Chloride of Butyl	69°	185 ,,	,,
14. Butyric Ether	121°	183 ,,	,,
15. Formic Acid	99°	180 ,,	,,
16. Valeral	100°	172 ,,	,,
17. Valerianic Ether	144°	168 ,,	,,
18. Acetate of Propyl	102°	166 ,,	,,
19. Methylic Alcohol	—	162 ,,	,,
20. Iodide of Ethyl	72°	148 ,,	Moderate
21. Bromide of Butyl	92°	134 ,,	,,
22. Dutch Liquid	85°	127 ,,	,,
23. Acetate of Butyl	114°	120 ,,	,,
24. Benzol	81°	117 ,,	,,
25. Carbonic Ether	126°	108 ,,	,,
26. Chloride of Amyl	102°	105 ,,	,,
27. Chloropicrin	112°	94 ,,	,,
28. Iodide of Allyl	101°	92 ,,	,,
29. Chloroform	61°	89 ,,	,,
30. Iodide of Butyl	121°	88 ,,	,,
31. Allylic Alcohol	97°	84 ,,	,,
32. Bisulphide of Carbon	43°	81 ,,	,,
33. Bromide of Amyl	119°	78 ,,	,,
34. Cyanide of Ethyl	98°	77 ,,	,,
35. Butyl Alcohol	110°	72 ,,	Weak
36. Nitrate of Amyl	147°	67 ,,	,,
37. Oxalate of Ethyl	—	66 ,,	,,
38. Cyanide of Methyl	82°	64 ,,	,,
39. Tetrachloride of Carbon	77°	58 ,,	,,
40. Bromoform	150°	44 ,,	,,
41. Xylol	140°	44 ,,	,,
42. Amylic Alcohol	130°	42 ,,	,,

TABLE V (continued).

Name of Liquid.	Vapours Boiling point.	Mean Water pressure.	Character of sound.
43. Iodide of Amyl	146°	42 millims.	Weak
44. Terebine	160°	39 „	„
45. Cymole	175°	38 „	„
46. Butyric Acid	163°	Very small absorptions and very feeble sounds at ordinary temperatures. Sounds in most cases very strong when liquid is heated to its boiling point.	
47. Butyrate of Amyl	176°		
48. Caprylic Alcohol	180°		
49. Valerianic Acid	175°		
50. Pure Aniline	184°		
51. Œnanthic Ether	188°		
52. Valerianate of Amyl	196°		
53. Salicylous Acid	196°		
54. Caproic Acid	205°		
55. Nitro Benzol	205°		
56. Kreosote	210°		
57. Menthol	213°		
58. Chinoline	238°		
59. Eugenol	247°		
60. Nicotine	250°		
61. Monobromnapthaline	277°		
62. Sebasic Ether	308°		

TABLE VI.

GASES.

Name of gas.	Water pressure.
Chloride of Methyl	350 millims.
Aldehyde	325 „
Olefiant Gas	315 „
Sulphuric Ether	300 „
Nitrous Oxide	198 „
Marsh Gas	164 „
Carbonic Acid	144 „
Carbonic Oxide	116 „
Oxygen	5 „
Hydrogen	5 „
Nitrogen	5 „
Dry air	5 „
Humid air at 50° C.	130 „

Sulphuric ether is here inserted with the view of connecting this Table with the last. Of all gaseous bodies hitherto examined chloride of methyl is the most energetic absorber and the most powerful sound producer. After it comes aldehyde, with a boiling point of 21° C. The figure 5 attached to the elementary gases, and to dry air, expresses, not absorption of radiant heat, but expansion, due to contact with the slightly warmed apparatus. The nitrous oxide employed was derived from an iron bottle in which it was preserved for medical purposes. In some of my experiments marsh gas showed itself a better absorber than nitrous oxide. This, for instance, was the case in experiments made in the spring of 1880 with the manometer. The sample of marsh gas wherewith the foregoing result was obtained was very carefully prepared in our chemical laboratory.

The temperature of 50° C. in the case of humid air was obtained in a wooden shed erected in our laboratory. The shed is traversed by two tubes of sheet iron 4 inches in diameter, which carry the heated air and products of combustion from two large ring-burners. It is 8′ 6″ long, 4′ 3″ wide, and 7′ feet high. The temperature of the air within it can be readily raised to 60° C. In the experiment recorded in Table VI. the air was taken from the outside laboratory through a tube passing through the wooden wall of the shed. It was caused to bubble through water contained in a large flask which had been permitted to remain for some time in the warm shed. The mixed air and vapour entered the manometer tube at a temperature some degrees lower than that of the tube itself. Closely examined, all parts of this tube were bright and dry when the vapour-laden air was within it. On permitting the beam from the lime light (produced by coal-gas and oxygen) to pass through the mixture, a prompt rise of 65 millimeters was the consequence. Cutting the beam off the column rapidly returned to zero. The double of 65, or 130 millimeters, gives the difference of level in the two legs of the U-tube.

I have done my best to render these determinations correct. They have been repeated both by myself and my assistant* a great number of times. The first measurements were made in the early part of last year, and were made known in the Royal Institution on the 8th of April, 1881. Difficulties were encountered in obtaining a powerful, and at the same time constant, source of heat. The mixture of coal-gas and oxygen issuing from independent holders was finally resorted to. The sounds are classified into "very strong," "strong," &c., but it is, of course, impossible to say where one class ends and another begins. They shade gradually into each other. But if the middle members of any class be compared with the corresponding members of another class, the difference of sounding power will appear.

§. 10. *Application of Results to Meteorology.*

If it be at length conceded that aqueous vapour exerts upon radiant heat the action which I so long ago ascribed to it, I think the knowledge of this action will prove of importance to the scientific meteorologist. Meteorology, as connected with heat, seems to me to abound in facts which it has hitherto been incompetent to explain. This, for example, I hold to be the case as regards the celebrated observations of PATRICK WILSON of Glasgow, made a century ago. WILSON brought strongly into light the great differences which sometimes exist between the temperature of the earth's surface, and of the air at a small elevation above the surface. His letter to Dr. MASKELYNE on this subject is published in the Philosophical Transactions for 1780 under the title:—"*An account of a most extraordinary degree of cold at Glasgow, together with some new experiments and observations on the comparative temperature of hoar frost and the air near it.*" On the afternoon of the 13th of January, 1780, the cold was intense, a thermometer at the high window of the observatory pointing, at 7 P.M., to 0° FAHR. At 8 P.M. WILSON and Dr. IRVINE laid two thermometers upon the snow, and hung up two others in the air 2 feet above the snow. Here follow the temperatures observed on the evening of the 13th and on the morning of the 14th of January:—

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* Who has aided me in this investigation with his usual zeal and intelligence.

Time of observation.	Snow temperature.	Air temperature.
8½ P.M.	−12°	−0°
9 ,,	−14°	−2°
10 ,,	−14°	−4°
11 ,,	−17°	−6°
11½ ,,	−18°	−6°
½ A.M.	−20°	−8°
1 ,,	−23°	−7°

The sign − signifies that the temperatures were all below zero FAHRENHEIT. These temperatures amply justifiy WILSON's statement that the cold was "extraordinary." Coexistent moreover with the general cold we have a difference of 16° between the temperature of the surface and that of the air 2 feet above it. Had the air thermometer been hung 10 feet, instead of 2 feet, above the surface the difference would have been still greater. The thermometer, moreover, must have been chilled, not only by its immersion in cold air, but also by its own radiation against the intensely cold snow. The chilling of the superficial snow was purely an effect of radiation. Beneath the surface its temperature reached +14°. WILSON filled a bread-basket with this warm snow at 2½ A.M. on the 14th. Within half an hour it had fallen 24°, and in two hours 32°.

I venture to predict that if WILSON's experiment be repeated during the cold of a Canadian winter the same result will be obtained; and it seems to me that until the action of water vapour upon radiant heat had been discovered no explanation of the phenomenon could have been given. It was accepted but not accounted for. On the night of WILSON's observations "a light air was felt coming from the east." With such an "air" and such a temperature the quantity of water vapour in the atmosphere must have been infinitesimal. Dry air being a practical vacuum to the rays of heat, were the vapour-screen entirely removed, the earth would find itself exchanging temperatures with celestial space, and the superficial chill would be commensurate. In WILSON'S case, though the vapour was not abolished, it was so far diminished as to produce the observed refrigeration. Meteorologists, I am informed, sometimes say that laboratory experiments, however well performed, have but little application to their field of observation.* I, on the other hand, submit that such experiments are necessary to rescue their science from empiricism. What could WELLS have done with dew had he not been preceded by LESLIE and RUMFORD? His whole theory is an application of results obtained in the laboratory.†

What I have stated regarding WILSON applies also to SIX, who concluded from his experiments "that the greatest differences at night in point of temperature, between bodies on the surface of the earth and the atmosphere near it, are those which take place in very cold weather." This is quoted from WELLS,‡ who, in his Essay on Dew, recurs more than once to the subject. He signalises, but does not explain, "the greater difference which takes place in very cold weather, if it be calm and clear, between the temperatures of the air and of bodies on the earth at night, than in equally calm and clear weather in summer."§ A considerable number of observations bearing upon this point are scattered through the Essay. The radiant power of the air being practically *nil*, it retains for a considerable time the warmth imparted to it during the day, while when it is dry, the rays from the surface of the earth pass unimpeded through it. Hence the relative refrigeration of the surface.‖

In regard to the action of water vapour MAGNUS considered experiment superfluous, as the phenomenon of dew sufficed to prove me wrong. If the vapour possessed the power which I ascribed to it, he contended that dew could not be formed. It is not difficult to dispose of this objection. The formation of dew and superficial refrigeration are connected, not by coincidence but by opposition. I would venture to predict that where the one is great the other, in general, will be small. "Very little dew," says

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* Mr. HILL, the Meteorological Reporter for the North-Western Provinces of India, writes thus:—"There is even, on the part of some, an evident reluctance to accept the decision of laboratory experiments on the question of atmospheric absorption as final, however ingenious, varied, and consistent with one another the experiments may be."—Proc. Roy. Soc., vol. 33, p. 216.

† "Its complete theory," says WELLS, "could not possibly, in my opinion, have been attained, before the discoveries on heat were made, which are contained in the works of Mr. LESLIE and Count RUMFORD."—Essays, p. 191.

‡ Essays, p. 176. WELLS thus generously refers to the labours of WILSON. "Indeed, several of my experiments upon dew were only imitations of some which had been previously made on hoar frost, by that ingenious and worthy man."—Essays, p. 151.

§ Ibid., p. 188.

‖ It ought to be stated that, contrary to SIX and to WELLS, Mr. GLAISHER has found that "the differences between the temperature of the air and of bodies on the earth at night, in equally clear and calm weather, were the same at every period of the year." (Phil. Trans., 1847, p. 126.) He moreover records differences considerably in excess of those observed by WILSON and by WELLS. Keeping the action of aqueous vapour in view, the elaborate paper of Mr. GLAISHER might repay further discussion.

Wells, "appeared on the two nights of the greatest cold I have ever observed on the surface of the earth, relatively to the temperature of the air, both of them having occurred after a long tract of dry weather."* This evidence is specially valuable in view of the fact that Wells knew nothing of the action of water vapour on radiant heat. On another occasion he observed a difference of 9½ degrees between the temperature of the air and that of wool placed upon the earth, without any deposition of dew whatever upon the chilled wool.† He supplements these observations by one equally important in the opposite direction. "On the night," he writes, "which afforded the most copious dew ever observed by me, the cold possessed by the grass beyond that of the air was for the most part only 3° and 4°."‡ The smallness of the refrigeration in this instance, and the copiousness of the dew, I refer to one and the same cause; namely, the abundance of vapour. Heavy dew implies this abundance; abundant vapour, if not too local, implies checked radiation, and checked radiation tends to abolish the difference of temperature between air and soil.

Wells had a theory of his own to account for the association of moderate refrigeration and heavy dew. The heat rendered free by the condensation of the vapour to liquid "prevented" the cold. He tried to determine the effect of condensation by the following experiment.§ To 10 grains of wool he added 21 grains of water, this being the quantity of dew deposited on wool in one of his observations. He placed the moistened wool in a saucer on a feather bed in a room, and determined the chill produced by its evaporation. After eight hours, while the wool still retained 2½ grains of moisture, its temperature was 4° lower than that of a dry saucer placed near it on the same feather bed. When the process is reversed, condensation instead of evaporation coming into play, the foregoing amount of heat, Wells contended, would be liberated on the grass, and thus prevent inordinate refrigeration.

In thus reasoning Wells went to the limit of the knowledge of his time, and the explanation here given is a philosophical one. But I do not think it a sufficient explanation. The grass is exposed to the open atmosphere, and the heat developed by every successive film of moisture condensed upon its blades, is instantly wasted by radiation. Those who are accustomed to work with the thermopile know how rapidly the associated galvanometer needle falls from a high deflection to zero, when the heat incident upon the pile is suddenly cut off. A similar rapidity of waste would assuredly occur during the slow formation of dew. The heat of condensation could not for this reason be housed in the manner supposed by WELLS.* The true explanation I hold to be that already indicated—the checking of radiation by vapour, the abundance of which was indicated by the copious deposition of dew.

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* Essays, p. 186.

† Ibid., p. 183. Wells sometimes found wetted wool to lose weight, while dry wool gained no weight though lowered many degrees below the temperature of the air, p. 184.

‡ Ibid., p. 169. From a remark occurring at page 135, it may be inferred that the night here referred to was that common to the 29th and 30th of July, 1813. On the two occasions first mentioned, when there was but little dew, the grass was in one instance 12°, and in the other 14° colder than the air.

§ Essays, p. 187.

If the experiments of WILSON could be made in an atmosphere still colder than that in which he worked,—on a large plain, for instance, and in a country remarkable for the dryness of its air—WELLS considered that a difference of at least 30° would be observed on serene nights between the air and a downy substance placed on the earth. And as SIX had found the air temperature at an elevation of 220 feet to be 10° higher than at 7 feet, these 10° being added to the 30° would make the surface at least 40° colder than the air at the height of 200 or 300 feet. With all this I agree. I would go even further, and reiterate here a statement made by me nineteen years ago, that the withdrawal of the aqueous vapour of our atmosphere, for a single calm night, would kill every plant in England capable of being killed by a freezing temperature.

PICTET, I believe, was the first to notice that the temperature of the air near the earth's surface on serene nights diminished as the surface was approached, the sequence of the day temperatures being thus inverted. To account for the chilling of the air say at 10 feet above the earth's surface beyond that at 100 feet above the surface, WELLS invoked the radiant power of the air itself. It is chilled, he thought, by its own emission against the cold earth underneath. WELLS takes great pains to prove that the air possesses this power; and if not the air, the floating matter of the air will, he contends, exert the necessary radiation. Difficulties of this nature not unfrequently crop up in works on meteorology, but they disappear in presence of the fact, that mixed with the air is a gaseous constituent, small in quantity, but capable of producing the effects needing explanation.

As an example of such difficulties, I have already referred to Sir JOHN LESLIE's paper "On certain impressions of cold transmitted from the Higher Atmosphere."†

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* WELLS himself observed in grass a fall of temperature of 7° in twenty minutes. This gives us some notion of the rapidity with which a radiant so powerful as water would dispose of its heat.—Essays, p. 157.

At the instance of my friend Mr. FRANCIS GALTON, and with the kind sanction of the Meteorological Council, the following instructive observations, showing the temperatures recorded by two thermometers, the one placed on cotton-wool resting on the earth, and the other hung at a height of four feet in the air, were recently made by Mr. WHIPPLE at Kew:—

Time.	Air.	Wool.
4.20 P.M.	34·8°	33·2°
„ 25 „	32·5	27·6
„ 30 „	32·4	25·7
„ 35 „	32·4	23·4
„ 40 „	32·2	21·7
„ 45 „	32·2	20·7

The rapidity of radiation is well shown by these observations, an exposure of twenty-five minutes sufficing to establish a difference of 11·5°.

† Transactions Roy. Soc. of Edinburgh, vol. viii., p. 483.

He there describes the Æthrioscope, an instrument used to measure these impressions. "The sensibility," he says, "of the instrument is very striking, for the liquor incessantly falls and rises in the stem with every passing cloud. Under a fine blue sky, it will sometimes indicate a cold of 50 millesimal degrees; yet on the other days, when the air seems equally bright, the effect is only 30°. The causes of these variations are not quite ascertained." He might have said, not at all ascertained. The causes, I submit, are the variations of the quantity of transparent aqueous vapour in the atmosphere, which, without affecting the visual brightness of the air, is competent to arrest radiation from the earth. Precisely of the same character is the difficulty noticed by Lieutenant HENNESSEY in his paper on "Actinometrical Observations in India." Like LESLIE he speaks of variations, the causes of which are not ascertained. "Again," he says, "there is a change of intensity from day to day apparently not due to alterations in the sun's declination, so that the average daily curve (about noon) is higher or lower without any visible reason."* The reason here is that applicable in LESLIE's case; namely, the variations of the invisible atmospheric vapour.

In 1866 my friend Professor SORET of Geneva favoured me with a letter from which the following is an extract:—"In two comparative experiments, made within a few days at Geneva and Bologna, the most powerful radiation was obtained at Geneva, although at Bologna the heavens were visibly *purer*. The result appears to me to support your views regarding the aqueous vapour of the air; for the tension of aqueous vapour at Bologna was 10·7 while at Geneva it was only 6·33."

Cautiously abstaining from drawing a general conclusion from a single fact, M. SORET, in 1868, made some further experiments on solar radiation. The intensity was measured by first allowing the rays to fall directly on the thermometer of the actinometer, and then by allowing them, prior to meeting the thermometer, to pass through 5 centimeters of water. Calling the first temperature T and the second t the ratio $\frac{t}{T}$ will be obviously greatest when the absorption by the water is least. And as we know that water exerts its chief absorbent power on the ultra-red rays of the spectrum, the variations in this ratio observed at different atmospheric thicknesses will enable us to infer the nature of the heat "arrested" by the atmosphere.

M. SORET found the ratio to be greater in the middle of the day than when the sun is near the horizon. At 12.30, for example, on the 9th of March the ratio was 0·594, while at 5.10, on the same day, it was only 0·409. A smaller fraction of the total heat was absorbed by the water near mid-day than near sunset. At mid-day therefore the solar heat was more thoroughly sifted of its calorific rays, and more transmissible by water, than it was when the atmospheric thickness was much greater. It would seem difficult to reconcile this result with the notion that aqueous vapour is the absorbing constituent of the atmosphere.

A year subsequently MM. DESAINS and BRANLEY found, both at Paris and at

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* Proc. Roy. Soc., vol. xix., p. 228.

Lucerne, that the sun's heat was always more transmissible through water and alum in the morning than at mid-day. I have too much confidence in the able experimenters here named to think any of them wrong. How then is the discrepancy between them to be accounted for? I think in the following way. What is called the glow of the Alps varies greatly with the quantity of suspended matter in the air. When pronounced, it shows that the more refrangible constituents have been in great part removed from the sun's rays. The proportion of the less refrangible rays in the total radiation is augmented in this way, the relative transmissibility of the heat being diminished. It was, I would suggest, heat that had its character impressed upon it in this way by scattering, and not by absorption, that yielded the result obtained by M. SORET.

Whatever may be the value of this explanation, one result of great interest to me was established by the two French experimenters. Simultaneous observations were made by them on the summit of the Rigi and at Lucerne, the vertical distance between both stations being 4,756 feet. Within this stratum 17·1 per cent. of the solar rays was absorbed.

Experiments were made at the same time at both stations on the perviousness of water to the solar rays. If, as I contend, a vapour and its liquid absorb the same rays, the withdrawal of 17 per cent. of the radiation by aqueous vapour must render the residual heat more transmissible by water. This is precisely what the French experimenters found it to be. "Through a glass trough 0·08 of a meter in length, and full of water, the rays on the Rigi passed in the proportion of 685, and at Lucerne in the proportion of 730, per 1,000 of the incident heat."

MAGNUS was so convinced of the impotency of aqueous vapour to arrest radiant heat, that in reference to various meteorological phenomena, where the action I had ascribed to it offered a satisfactory explanation of the facts, he put in its place mist or haze, the existence of which he assumed, even when neither mist nor haze was visible. There are various passages in the Essay on Dew which it would be difficult to reconcile with this assumption; for they show that even visible atmospheric turbidity has by no means the influence which MAGNUS ascribed to it.

Thus on the 7th of January, 1814, WELLS observed "a little after sunset" a refrigeration of 8°, at a time when some parts of the sky were covered with clouds, and the lower atmosphere a little obscure.[*] On another evening, "when the atmosphere was neither very clear nor very still" a difference of 14½° was observed between the temperatures of air and swan down. WELLS also observed a refrigeration of 5° when the sky was thickly covered with high clouds. A very definite observation in regard to haze was made on the 21st of January, 1814. The air at this time was "a good deal hazy."[†] Notwithstanding this, the temperature of swansdown placed on snow was 13½° lower than that of the air 4 feet above it. Thus, if other circumstances be favourable, that is to say, if the air be dry, even a visible haze does not prevent powerful refrigeration. I close these references to mist and haze by mentioning a most striking observation made by WELLS on the 1st January, 1814. "I found," he says, "during a dense fog, while the weather was very calm, a thermometer lying on grass thickly covered with hoar frost, 9° lower than another suspended in the air 4 feet above the former."[*] Here, as before, low temperature implies scanty vapour, the absence of which enabled the grass to pour its heat even through the interstices of a dense fog.[†]

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[*] Essays, p. 174.

[†] Ibid., p. 176.

I could draw still further on this admirable Essay in illustration of the thesis which I have so long defended. As a repertory of valuable facts and penetrative arguments it probably stands unrivalled in the literature of meteorology. One point remains which cannot be passed over. It has reference to the part played by clouds in arresting and returning the radiation from the earth. "No direct experiments," says WELLS, "can be made to ascertain the manner in which clouds prevent, or occasion to be small, the appearance of a cold at night, upon the surface of the earth, greater than that of the atmosphere; but it may, I think, be firmly (fairly ?) concluded from what has been said in the preceding article, that they produce this effect, almost entirely, by radiating heat to the earth, in return for that which they intercept in its progress from the earth towards the heavens."[‡] WELLS had the strongest analogies to adduce in favour of this view. He placed boards and sheets of paper above his thermometers, thus screening them from the clear sky; and in that beautiful passage where he speaks of "the pride of self knowledge," and refers to the simple devices which experience had taught gardeners to apply for the safety of their plants, he mentions the protection which even a thin cambric handkerchief can afford to thermometers over which it is spread. He was irresistibly led to conclude that clouds acted in the same fashion, and that when they occupied the firmament, they sent back to the earth the heat incident upon them, exactly as the board, and the paper, and the cambric, sent it back in experiments made close to the surface of the earth.

But in the enunciation of this hypothesis his knowledge and penetration as an observer came, as usual, into play. He is careful to distinguish between high clouds and low clouds. "Dense clouds," he says, "near the earth," must possess the heat of the lower atmosphere, and will therefore send to the earth as much, or nearly as much, heat as they receive from it by radiation. But similarly dense clouds, if very high, though they equally intercept the communication of the earth with the sky, yet being from this elevated situation colder than the earth, will radiate to it less heat than they receive from it, and may consequently admit of bodies on its surface becoming several degrees colder than the air."[§]

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[*] Ibid., p. 158.

[†] Mr. GLAISHER moreover has found differences of from 10° to 12° between grass and air "at times when the sky has been free from clouds but not bright, haze and vapour being prevalent." (Phil. Trans., 1847, p. 145.)

[‡] Essays, p. 205.

[§] Ibid., p. 206. "If," says WELLS, in another place, "the clouds were high and the weather calm, I

MAGNUS urged this point against me, and I may be permitted to say that I always considered it one of his strongest points; my holding of this opinion being however dependent on the views which I entertained, and which were opposed to those of MAGNUS, regarding the relation of liquid to vapour. If, as I believe, the absorbent power is not enhanced by condensation—if in this respect water behaves like hydride of amyl and sulphuric ether—then I do not think that such a process of reverberation, between earth and clouds, as that assumed by WELLS is possible. The aqueous vapour in a very few thousand feet of air, of average humidity, would, if condensed, form a layer of water 0·5 of an inch in thickness, and through such a layer, or even through a thinner layer, the earth's radiation could not pass. If the earth's radiation reach the clouds it must be by a process similar to that of handing buckets from man to man in the case of a fire. The heat must be taken up and re-radiated, we know not how many times, before the clouds are reached. I do not, however, think this mechanism of discharge necessary. Low clouds will not form above exposed thermometers, in weather previously serene, unless some change has occurred in the atmosphere; and change may occur where no cloud reveals it. It may extend, and in most cases probably does extend, from the low clouds to the earth. I think it in the highest degree probable that in most, if not in all the cases cited by WELLS, of thermometers rising when clouds were formed overhead, the precipitation was due to the intrusion of humid air, the humidity extending invisibly from the clouds downwards. To this, I believe, rather than to any immediate exchange of temperature with the clouds, the rapid and considerable changes of temperature referred to by him at pp. 156 and 157 of the Essay are to be ascribed. Future observations will, doubtless, bring this view to an experimental test.

I here recur with renewed pleasure, to a paper published by General STRACHEY in the Philosophical Magazine for July, 1866. It was probably intended as a reply to the strictures of MAGNUS; and to me it appears cogent in the highest degree. General STRACHEY calculated the fall of temperature from $6^h\ 40^m$ P.M., Madras time, to $5^h\ 40^m$ next morning, for a certain number of days, selected as sufficiently clear. He also calculated the mean vapour tension during the nights, and tabulated the results according to the quantity of vapour for the years 1841, 1842, 1843 and 1844. In such observations, as pointed out by STRACHEY, discrepancies are to be expected, but the general result is unmistakable, that the fall of temperature by radiation is greatest when the air is driest, and least when the air is most humid. A series of observations made at Madras between the 4th and the 25th of March, 1850, are particularly suitable for the illustration of this law of action. During the period referred to "the sky remained remarkably clear, while great variations of the quantity of vapour took place." Here are the results as tabulated by General STRACHEY:—

have sometimes seen on grass, though the sky was entirely hidden, no very inconsiderable quantity of dew."—Ibid., p. 128.

Tension of vapour.	·888.	·849.	·805.	·749.	·708.	·659.	·605.	·554.	·435.
Fall of temperature from 6h 40m P.M. to 5h 40m A.M.	6°·0	7°·1	8°·3	8°·5	10°·3	12°·6	12°·1	13°·1	16°·5

These results, if correct, and I am not aware that they have ever been questioned, show in the most impressive manner the influence of the aqueous vapour of our atmosphere on our planet's radiation. As the vapour diminishes, the door opens, which permits the escape of the earth's heat. The halving of the vapour tension nearly trebles the refrigeration of the thermometer.

Equally clear is the evidence given by General STRACHEY as to the action of aqueous vapour upon the radiation of the sun. Here are the results :—

Tension of vapour.	·824.	·737.	·670.	·576.	·511.	·394.
Rise of temperature from 5h 40m A.M. to 1h 40m P.M.	12°·4	15°·1	19°·3	22°·2	24°·3	27°·0

This table is the exact complement of the last. *There* the *fall* of temperature was powerfully promoted by the withdrawal of the vapour. *Here* the *rise* of temperature is powerfully promoted by the same cause.*

But the most impressive illustration of the action of aqueous vapour is now to be referred to. In 1865 I subjected to examination the radiation from the electric light produced by a battery of 50 of GROVE's cells, and found, by prismatic analysis, the invisible calorific radiation to be 7·7 times the visible. The determination was afterwards made by the method of filtration, whereby the one class of rays was detached with great sharpness from the other, and both of them rendered measurable. By this method the invisible radiation was found to be 8 times the visible. A close agreement was therefore established between the results of the two methods. Computed from the diagram of MÜLLER the invisible radiation of the sun is twice the visible. This smaller ratio might, of course, be referred to the original quality of the solar emission, the ratio holding good up to the surface of the sun. But having placed, as I thought beyond doubt, the action of aqueous vapour on radiant-heat, and believing the action of the vapour to be substantially the same as that of water, I reasoned and experimented as follows in 1865:—"The sun's rays, before reaching our earth, have to pass through the atmosphere, where they encounter the atmospheric vapour which exercises a powerful absorption on the invisible calorific rays. From this, apart from other considerations, it would follow that the ratio of the invisible to the visible radiation in the case of the sun must be less than in the case of the electric light. Experiment, we see, justifies this conclusion. If we cause the beam from the electric lamp to pass through a layer of water of suitable thickness, we place its radiation in approximately the same condition as that of the sun; and on decomposing the beam, after it has been thus sifted, we obtain a distribution of heat closely resembling that observed in the solar spectrum."

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* Mr. HILL, Meteorological Reporter for the North-West Provinces of India, in a paper recently presented to the Royal Society describes an attempt to determine the "Constituent of the Atmosphere which absorbs Radiant Heat." He uses for this purpose the careful observations made by Messrs. J. B. N. HENNESSEY and W. H. COLE, at Mussoree and Dehra respectively. From the absence of symmetry in the quantities of heat received by the actinometer on both sides of noon at Mussoree, and from the existence of this symmetry at Dehra, he infers the periodic lifting and lowering of the absorbing constituent above and below the higher station. He finds the variation of the absorption coefficient to follow the variations of vapour tension. From this and from a similar result obtained by a second method of calculation, he draws the conclusion "that there can be very little error in agreeing with Dr. TYNDALL that the absorptive power of dry air is sensibly nothing, and that the total absorptive power of the atmosphere is due to the water vapour it contains." A most interesting discourse on Solar Heat, by M. VIOLLE, of Grenoble, will be found in the Revue Scientifique for 1878, p. 944. I guard myself against saying that the diathermancy of dry air is *perfect*.

If therefore, we could get above the vapour-screen which swathes the earth, the "powerful absorption" referred to in the paragraph just quoted would disappear, the ratio of the invisible to the visible solar rays being augmented correspondingly. That such would be the case I have long taken for granted, but I hardly hoped for a corroboration so impressive as that furnished by the recent observations of Professor LANGLEY, in the Sierra Nevada Mountains of California. Professor LANGLEY is known to have highly distinguished himself by researches on radiant heat, with instruments of his own invention—he writes to me thus from Mount Whitney, California, Sept. 10, 1881:—

"I received your letter just as I was starting on the expedition to this point of which I wrote. I much regretted that I had not time to provide myself with your mercury pyrheliometer, so I have been obliged to use the old form, with its many disadvantages.

"Our route here has led us through the dryest parts of this continent, and across rainless deserts to this mountain, where the air is perhaps drier than at any other equal altitude ever used for scientific investigation. I write from an altitude of 12,000 feet, while the 'Peak' rises nearly 3,000 more above me. I have been successful in bringing up, and using here, the rather complex and delicate apparatus for investigating the absorption of the atmosphere on homogeneous rays, throughout the visible and invisible spectrum.

"You may be interested in knowing that the result indicates a great difference in the *distribution* of the solar energy here from that to which we are accustomed in regions of ordinary humidity; and that while the evidence of the effect of water vapour on the more refrangible rays is feeble, there is, on the other hand, a systematic effect due to its absence which shows by contrast its power on the red and ultra-red in a striking light.

"These experiments also indicate an enormous extension of the ultra-red spectrum beyond the point to which it has been followed below; and being made on a scale different from that of the laboratory—on one indeed as grand as nature can furnish—and by means wholly independent of those usually applied to the research, must, I think, when published, put an end to every doubt as to the accuracy of the statements so long since made by you, as to the absorbent power of this agent over the greater part of the spectrum, and as to its predominant importance in modifying to us the solar energy.

"I am, with much regard,
"Very truly yours,
"S. P. Langley."

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TRANSACTIONS
OF THE
ROYAL SOCIETY
OF
LONDON.

FOR THE YEAR MDCCCLXXXII.

VOL. 173.—PART II.

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MDCCCLXXXII.

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By F. M. Balfour, *LL.D., F.R.S., Fellow of Trinity College, Cambridge, and* W. N. Parker, *Assistant in the Biological Laboratory of the Normal School of Science, South Kensington.*

Received November 24,—Read December 8, 1881.

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INTRODUCTION.

THE following paper is the outcome of the very valuable gift of a series of embryos and larvæ of *Lepidosteus* by Professor ALEX. AGASSIZ, to whom we take this opportunity of expressing our most sincere thanks. The skull of these embryos and larvæ has been studied by Professor PARKER, and forms the subject of a memoir already presented to the Royal Society.

Considering that *Lepidosteus* is one of the most interesting of existing Ganoids, and that it is very closely related to species of Ganoids which flourished during the Triassic period, we naturally felt keenly anxious to make the most of the opportunity of working at its development offered to us by Professor AGASSIZ' gift. Professor AGASSIZ, moreover, most kindly furnished us with four examples of the adult Fish, which have enabled us to make this paper a study of the adult anatomy as well as of the development.

The first part of our paper is devoted to the segmentation, formation of the germinal layers, and general development of the embryo and larva. The next part consists of a series of sections on the organs, in which both their structure in the adult and their development are dealt with. This part is not, however, in any sense a monograph, and where already known, the anatomy is described with the greatest possible brevity. In this part of the paper considerable space is devoted to a comparison of the organs of *Lepidosteus* with those of other Fishes, and to a statement of the conclusions which follow from such comparison.

The last part of the paper deals with the systematic position of *Lepidosteus* and of the Ganoids generally.

GENERAL DEVELOPMENT.

The spawning of *Lepidosteus* takes place in the neighbourhood of New York about May 20th. AGASSIZ (No. 1)* gives an account of the process from Mr. S. W. GARMAN'S notes, which we venture to quote in full.

"Black Lake is well stocked with Bill-fish. When they appear, they are said to come in countless numbers. This is only for a few days in the spring, in the spawning season, between the 15th of May and the 8th of June. During the balance of the season they are seldom seen. They remain in the deeper parts of the lake, away from the shore, and, probably, are more or less nocturnal in habits. Out of season, an occasional one is caught on a hook baited with a Minnow. Commencing with the 20th of April, until the 14th of May we were unable to find the Fish, or to find persons who had seen them during this time. Then a fisherman reported having seen one rise to the surface. Later, others were seen. On the afternoon of the 18th, a few were found on the *points*, depositing the spawn. The temperature at the time was 68° to 69°

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* The numbers refer to the list of memoirs of the anatomy and development given at the end of this memoir (p. 433).

on the shoals, while out in the lake the mercury stood at 62° to 63°. The *points* on which the eggs were laid were of naked granite, which had been broken by the frost and heat into angular blocks of 3 to 8 inches in diameter. The blocks were tumbled upon each other like loose heaps of brickbats, and upon and between them the eggs were dropped. The *points* are the extremities of small capes that make out into the lake. The eggs were laid in water varying in depth from 2 to 14 inches. At the time of approaching the shoals, the Fish might be seen to rise quite often to the surface to take air. This they did by thrusting the bill out of the water as far as the corners of the mouth, which was then opened widely and closed with a snap. After taking the air, they seemed more able to remain at the surface. Out in the lake they are very timid, but once buried upon the shoals they become quite reckless as to what is going on about them. A few moments after being driven off, one or more of the males would return as if scouting. If frightened, he would retire for some time; then another scout would appear. If all promised well, the females, with the attendant males, would come back. Each female was accompanied by from one to four males. Most often, a male rested against each side, with their bills reaching up toward the back of her head. Closely crowded together, the little party would pass back and forth over the rocky bed they had selected, sometimes passing the same spot half-a-dozen times without dropping an egg, then suddenly would indulge in an orgasm; and, lashing and plashing the water in all directions with their convulsive movements, would scatter at the same instant the eggs and the sperm. This ended, another season of moving slowly back and forth was observed, to be in turn followed by another of excitement. The eggs were excessively sticky. To whatever they happened to touch, they stuck, and so tenaciously that it was next to impossible to release them without tearing away a portion of their envelopes. It is doubtful whether the eggs would hatch if removed. As far as could be seen at the time, upon or under the rocks to which the eggs were fastened there was an utter absence of anything that might serve as food for the young Fishes.

"Other Fishes, Bull-heads, &c., are said to follow the Bill-fish to eat the spawn. It may be so. It was not verified. Certainly the points under observations were unmolested. During the afternoon of the 18th of May a few eggs were scattered on several of the beds. On the 19th there were more. With the spear and the snare, several dozens of both sexes of the Fish were taken. Taking one out did not seem greatly to startle the others. They returned very soon. The males are much smaller than the average size of the females; and, judging from those taken, would seem to have as adults greater uniformity in size. The largest taken was a female, of 4 feet 1½ inch in length. Others of 2 feet 6 inches contained ripe ova. With the 19th of May all disappeared, and for a time—the weather being meanwhile cold and stormy—there were no signs of their continued existence to be met with. Nearly two weeks later, on the 31st of May, as stated by Mr. Henry J. Perry, they again came up, not in small detachments on scattered points as before, but in multitudes, on every shoal at all according with their ideas of spawning beds. They remained but two days. During the summer it happens now and then that one is seen to come up for his mouthful of air; beyond this there will be nothing to suggest the ravenous masses hidden by the darkness of the waters."

Egg membranes.—The ova of *Lepidosteus* are spherical bodies of about 3 millims. in diameter. They have a double investment consisting of (1) an outer covering formed of elongated, highly refractive bodies, somewhat pyriform at their outer ends (Plate 21, fig. 17, *f.e.*), which are probably metamorphosed follicular cells,* and (2) of an inner membrane, divided into two zones, viz.: an outer and thicker zone, which is radially striated, and constitutes the *zona radiata* (*z.r.*), and an inner and narrow homogeneous zone (*z.r′.*).

Segmentation.—We have observed several stages in the segmentation, which show that it is complete, but that it approaches the meroblastic type more nearly than in the case of any other known holoblastic ovum.

Our earliest stage showed a vertical furrow at the upper or animal pole, extending through about one-fifth of the circumference (Plate 21, fig. 1), and in a slightly later stage we found a second similar furrow at right angles to the first (Plate 21, fig. 2). We have not been fortunate enough to observe the next phases of the segmentation, but on the second day after impregnation (Plate 21, fig. 3), the animal pole is completely divided into small segments, which form a disc, homologous to the blastoderm of meroblastic ova; while the vegetative pole, which subsequently forms a large yolk-sac, is divided by a few vertical furrows, four of which nearly meet at the pole opposite the blastoderm (Plate 21, fig. 4). The majority of the vertical furrows extend only a short way from the edge of the small spheres, and are partially intercepted by imperfect equatorial furrows.

Development of the embryo.—We have not been able to work out the stages immediately following the segmentation, owing to want of material; and in the next stage satisfactorily observed, on the third day after impregnation, the body of the embryo is distinctly differentiated. The lower pole of the ovum is then formed of a mass in which no traces of the previous segments or segmentation furrows could any longer be detected.

Some of the dates of the specimens sent to us appear to have been transposed; so that our statements as to ages must only be taken as *approximately* correct.

Third day after impregnation.—In this stage the embryo is about 3·5 millims. in length, and has a somewhat dumb-bell shaped outline (Plate 21, fig. 5). It consists of (1) an outer area (*p.z.*) with some resemblance to the area pellucida of the Avian embryo, forming the parietal part of the body; and (2) a central portion consisting of the vertebral and medullary plates and the axial portions of the embryo. In hardened specimens the peripheral part forms a shallow depression surrounding the central part of the embryo.

---

* We have examined the structure of the ovarian ova in order to throw light on the nature of these peculiar pyriform bodies. Unfortunately, the ovaries of our adult examples of *Lepidosteus* were so badly preserved, that we could not ascertain anything on this subject. The ripe ova in the ovary have an investment of pyriform bodies similar to those of the just laid ova.

With reference to the structure of the ovarian ova we may state that the germinal vesicles are provided with numerous nucleoli arranged in close proximity with the membrane of the vesicle.

The central part constitutes a somewhat prominent ridge, the axial part of it being the medullary plate. Along the anterior half of this part a dark line could be observed in all our specimens, which we at first imagined to be caused by a shallow groove. We have, however, failed to find in our sections a groove in this situation except in a single instance (Plate 22, fig. 20, *x*), and are inclined to attribute the appearance above-mentioned to the presence of somewhat irregular ridges of the outer layer of the epiblast, which have probably been artificially produced in the process of hardening.

The anterior end of the central part is slightly dilated to form the brain (*b.*); and there is present a pair of lateral swellings near the anterior end of the brain which we believe to be the commencing optic vesicles. We could not trace any other clear indications of the differentiation of the brain into distinct lobes.

At the hinder end of the central part of the embryo a very distinct dilatation may also be observed, which is probably homologous with the tail swelling of Teleostei. Its structure is more particularly dealt with in the description of our sections of this stage.

After the removal of the egg-membranes described above we find that there remains a delicate membrane closely attached to the epiblast. This membrane can be isolated in distinct portions, and appears to be too definite to be regarded as an artificial product.

We have been able to prepare several more or less complete series of sections of embryos of this stage (Plate 22, figs. 18–22). These sections present as a whole a most striking resemblance to those of Teleostean embryos at a corresponding stage of development.

Three germinal layers are already fully established. The epiblast (*ep.*) is formed of the same parts as in Teleostei, viz. :—of an outer epidermic and an inner nervous or mucous stratum. In the parietal region of the embryo these strata are each formed of a single row of cells only. The cells of both strata are somewhat flattened, but those of the epidermic stratum are decidedly the more flattened of the two.

Along the axial line there is placed, as we have stated above, the medullary plate. The epidermic stratum passes over this plate without undergoing any change of character, and the plate is *entirely constituted of the nervous stratum of the epidermis.*

The medullary plate has, roughly speaking, the form of a solid keel, projecting inwards towards the yolk. There is no trace, at this stage at any rate, of a medullary groove; and as we shall afterwards show, the central canal of the cerebro-spinal cord is formed in the middle of the solid keel. The shape of this keel varies according to the region of the body. In the head (Plate 22, fig. 18, *m.c.*), it is very prominent, and forming, as it does, the major part of the axial tissue of the body, impresses its own shape on the other parts of the head and gives rise to a marked ridge on the surface of the head directed towards the yolk. In the trunk (Plate 22, figs. 19, 20) the keel is much less prominent, but still projects sufficiently to give a convex form to the surface of the body turned towards the yolk.

In the head, and also near the hind end of the trunk, the nervous layer of the epiblast continuous with the keel on each side is considerably thicker than the lateral parts of the layer. The thickening of the nervous layer in the head gives rise to what has been called by GÖTTE* "the special sense plate," owing to its being subsequently concerned in the formation of parts of the organs of special sense. We cannot agree with GÖTTE in regarding it as part of the brain.

In the keel itself two parts may be distinguished, viz.: a superficial part, best marked in the region of the brain, formed of more or less irregularly arranged polygonal cells, and a deeper part of horizontally placed flatter cells. The upper part is mainly concerned in the formation of the cranial nerves, and of the dorsal roots of the spinal nerves.

The mesoblast (*ms.*) in the trunk consists of a pair of independent plates which are continued forwards into the head, and in the prechordal region of the latter, unite below the medullary keel.

The mesoblastic plates of the trunk are imperfectly divided into vertebral and lateral regions. Neither longitudinal sections nor surface views show at this stage any trace of a division of the mesoblast into somites. The mesoblast cells are polygonal, and no indication is as yet present of a division into splanchnic and somatic layers.

The notochord (*nc.*) is well established, so that its origin could not be made out. It is, however, much more sharply separated from the mesoblastic plates than from the hypoblast, though the ventral and inner corners of the mesoblastic plates which run in underneath it on either side, are often imperfectly separated from it. It is formed of polygonal cells, of which between 40 and 50 may as a rule be seen in a single section. No sheath is present around it. It has the usual extension in front.

The hypoblast (*hy.*) has the form of a membrane, composed of a single row of oval cells, bounding the embryo on the side adjoining the yolk.

In the region of the caudal swelling the relations of the germinal layers undergo some changes. This region may, from the analogy of other Vertebrates, be assumed to constitute the lip of the blastopore. We find accordingly that the layers become more or less fused. In the anterior part of the tail swelling, the boundary between the notochord and hypoblast becomes indistinct. A short way behind this point (Plate 22, fig. 21), the notochord unites with the medullary keel, and a neurenteric cord, homologous with the neurenteric canal of other Ichthyopsida, is thus established. In the same region the boundary between the lateral plates of mesoblast and the notochord, and further back (Plate 22, fig. 22), that between the mesoblast and the medullary keel, becomes obliterated.

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* "Üb. d. Entwick. d. Central-Nerven Systems d. Teleostier," Archiv für mikr. Anat., vol. xv., 1878.

Fifth day after impregnation.—Between the stage last described and the next stage of which we have specimens, a considerable progress has been made. The embryo (Plate 21, figs. 6 and 7) has grown markedly in length and embraces more than half the circumference of the ovum. Its general appearance is, however, much the same as in the earlier stage, but in the cephalic region the medullary plate is divided by constrictions into three distinct lobes, constituting the regions of the fore-brain, the mid-brain, and the hind-brain. The fore-brain (Plate 21, fig. 6, *f.b.*) is considerably the largest of the three lobes, and a pair of lateral projections forming the optic vesicles are decidedly more conspicuous than in the previous stage. The mid-brain (*m.b.*) is the smallest of the three lobes, while the hind-brain (*h.b.*) is decidedly longer, and passes insensibly into the spinal cord behind.

The medullary keel, though retaining to a great extent the shape it had in the last stage, is no longer completely solid. Throughout the whole region of the brain and in the anterior part of the trunk (Plate 22, figs. 23, 24, 25) a slit-like lumen has become formed. We are inclined to hold that this is due to the appearance of a space between the cells, and not, as supposed by OELLACHER for Teleostei, to an actual absorption of cells, though we must admit that our sections are hardly sufficiently well preserved to be conclusive in settling this point. Various stages in its growth may be observed in different regions of the cerebro-spinal cord. When first formed, it is a very imperfectly defined cavity, and a few cells may be seen passing right across from one side of it to the other. It gradually becomes more definite, and its wall then acquires a regular outline.

The optic vesicles are now to be seen in section (Plate 22, fig. 23, *op.*) as flattish outgrowths of the wall of the fore-brain, into which the lumen of the third ventricle is prolonged for a short distance.

The brain has become to some extent separate from the superjacent epiblast, but the exact mode in which this is effected is not clear to us. In some sections it appears that the separation takes place in such a way that the nervous keel is only covered above by the epidermic layer of the epiblast, and that the nervous layer, subsequently interposed between the two, grows in from the two sides. Such a section is represented in Plate 22, fig. 24. Other sections again favour the view that in the isolation of the nervous keel, a superficial layer of it remains attached to the nervous layer of the epidermis at the two sides, and so, from the first, forms a continuous layer between the nervous keel and the epidermic layer of the epiblast (Plate 22, fig. 25). In the absence of a better series of sections we do not feel able to determine this point. The posterior part of the nervous keel retains the characters of the previous stage.

At the sides of the hind-brain very distinct commencements of the auditory vesicles are apparent. They form shallow pits (Plate 22, fig. 24, *au.*) of the thickened part of the nervous layer adjoining the brain in this region. Each pit is covered over by the epidermic layer above, which has no share in its formation.

In many parts of the lateral regions of the body the nervous layer of the epidermis is more than one cell deep.

The mesoblastic plates are now divided in the anterior part of the trunk into a somatic and a splanchnic layer (Plate 22, fig. 25, *so.*, *sp.*), though no distinct cavity is as yet present between these two layers. Their vertebral extremities are somewhat wedge-shaped in section, the base of the wedge being placed at the sides of the medullary keel. The wedge-shaped portions are formed of a superficial layer of palisade-like cells and an inner kernel of polygonal cells. The superficial layer on the dorsal side is continuous with the somatic mesoblast, while the remainder pertains to the splanchnic layer.

The diameter of the notochord has diminished, and the cells have assumed a flattened form, the protoplasm being confined to an axial region. In consequence of this, the peripheral layer appears clear in transverse sections. A delicate cuticular sheath is formed around it. This sheath is probably the commencement of the permanent sheath of later stages, but at this stage it cannot be distinguished in structure from a delicate cuticle which surrounds the greater part of the medullary cord.

The hypoblast has undergone no changes of importance.

The layers at the posterior end of the embryo retain the characters of the last stage.

Sixth day after impregnation.—At this stage (Plate 21, fig. 8) the embryo is considerably more advanced than at the last stage. The trunk has decidedly increased in length, and the head forms a relatively smaller portion of the whole. The regions of the brain are more distinct. The optic vesicles (*op.*) have grown outwards so as to nearly reach the edges of the area which forms the parietal part of the body. The fore-brain projects slightly in front, and the mid-brain is seen as a distinct rounded prominence. Behind the latter is placed the hind-brain, which passes insensibly into the spinal cord. On either side of the mid- and hind-brain a small region is slightly marked off from the rest of the parietal part, and on this are seen two more or less transversely directed streaks, which, by comparison with the Sturgeon,* we are inclined to regard as the two first visceral clefts (*br.c.*). We have, however, failed to make them out in sections, and owing to the insufficiency of our material, we have not even studied them in surface views as completely as we could have wished.

The body is now laterally compressed, and more decidedly raised from the yolk than in the previous stages. In the lateral regions of the trunk the two segmental or archinephric ducts (*sg.*) are visible in surface views: the front end of each is placed at the level of the hinder border of the head, and is marked by a flexure inwards towards the middle line. The remainder of each duct is straight, and extends backwards for about half the length of the embryo. The tail has much the same appearance as in the last stage.

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* SALENSKY, "Recherches s. le Développement du Sterlet." Archives de Biol., vol. ii., 1881, pl. xvii., fig. 27.

The vertebral regions of the mesoblastic plates are now segmented for the greater part of the length of the trunk, and the somites of which they are composed (Plate 23, fig. 30, *pr.*) are very conspicuous in surface views.

Our sections of this stage are not so complete as could be desired: they show, however, several points of interest.

The central canal of the nervous system is large, with well-defined walls, and in hardened specimens is filled with a coagulum. It extends nearly to the region of the tail.

The optic vesicles, which are so conspicuous in surface views, appear in section (Plate 22, fig. 26, *op.*) as knob-like outgrowths of the fore-brain, and very closely resemble the figures given by OELLACHER of these vesicles in Teleostei.*

From the analogy of the previous stage, we are inclined to think that they have a lumen continuous with that of the fore-brain. In our only section through them, however, they are solid, but this is probably due to the section merely passing through them to one side.

The auditory pits (Plate 22, fig. 27, *au.*) are now well marked, and have the form of somewhat elongated grooves, the walls of which are formed of a single layer of columnar cells belonging to the nervous layer of the epidermis, and extending inwards so as nearly to touch the brain.

In an earlier stage it was pointed out that the dorsal part of the medullary keel was different in its structure from the remainder, and that it was destined to give rise to the nerves. The process of differentiation is now to a great extent completed, and may best be seen in the auditory region (Plate 22, fig. 27, VIII.). In this region there was present during the last stage a great rhomboidal mass of cells at the dorsal region of the brain (Plate 22, fig. 24, VIII.). In the present stage, this, which is the rudiment of the seventh and auditory nerves, is seen growing down on each side from the roof of the hind-brain, between the brain and the auditory involution, and abutting against the wall of the latter.

Rudiments of the spinal nerves are also seen at intervals as projections from the dorsal angles of the spinal cord (Plate 23, fig. 29, *sp.n.*). They extend only for a short distance outwards, gradually tapering off to a point, and situated between the epiblast and the dorsal angles of the mesoblastic somites.

The process of formation of the cranial nerves and dorsal roots of the spinal nerves is, it will be seen, essentially the same as that already known in the case of Elasmobranchii, Aves, &c. The nerves arise as outgrowths of a special crest of cells, the *neural crest* of MARSHALL, which is placed along the dorsal angle of the cord. The peculiar position of the dorsal roots of the spinal nerves is also very similar to what has been met with in the early stages of these structures by MARSHALL in Birds,* and by one of us in Elasmobranchs.†

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* "Beiträge zur Entwick. d. Knochenfische," Zeit. f. wiss. Zool., vol. xxiii., 1873, taf. iii., fig. ix., 2.

In the parietal region a cavity has now appeared in part of the trunk between the splanchnic and somatic layers of the mesoblast (Plate 23, fig. 29, *b.c.*), the somatic layer (*so.*) consisting of a single row of columnar cells on the dorsal side, while the remainder of each somite is formed of the splanchnic layer (*sp.*). In many of the sections the somatic layer is separated by a considerable interval from the epiblast.

We have been able to some extent to follow the development of the segmental duct. The imperfect preservation of our specimens has, as in other instances, rendered the study of the point somewhat difficult, but we believe that the figure representing the development of the duct some way behind its front end (Plate 23, fig. 29) is an accurate representation of what may be seen in a good many of our sections.

It appears from these sections that the duct (Plate 23, fig. 29, *sg.*) is developed as a hollow ridge-like outgrowth of the somatic layer of mesoblast, directed towards the epiblast, in which it causes a slight bulging. The cavity of the ridge freely communicates with the body-cavity. The anterior part of this ridge appears to be formed first. Very soon, in fact, in an older embryo belonging to this stage, the greater part of the groove becomes segmented off as a duct lying between the epiblast and somatic mesoblast (Plate 23, fig. 28, *sg.*), while the front end still remains, as we believe, in communication with the body cavity by an anterior pore.

This mode of development corresponds in every particular with that observed in Teleostei by ROSENBERG and OELLACHER.

The structure of the notochord (*nc.*) at this stage is very similar to that observed by one of us in Elasmobranchii.‡ The cord is formed of transversely arranged flattened cells, the outer parts of which are vacuolated, while the inner parts are granular, and contain the nuclei. This structure gives rise to the appearance in transverse sections of an axial darker area and a peripheral lighter portion.

The hypoblast retains for the most part its earlier constitution, but underneath the notochord, in the trunk, it is somewhat thickened, and the cells at the two sides spread in to some extent under the thickened portion (Plate 23, fig. 29, *s.nc.*). This thickening, as is shown in transverse sections at the stage when the segmental duct becomes separated from the somatic mesoblast (Plate 23, fig. 28, *s.nc.*), is the commencement of the subnotochordal rod.

The tail end of the embryo still retains its earlier characters.

Seventh day after impregnation.—Our series of specimens of this stage is very imperfect, and we are only able to call attention to the development of a certain number of organs.

Our sections clearly establish the fact that the optic vesicles are now hollow processes of the fore-brain. Their outer ends are dilated, and are in contact with the external skin. The formation of the optic cup has not, however, commenced. The nervous layer of the skin adjoining the outer wall of the optic cup is very slightly thickened, constituting the earliest rudiment of the lens.

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* Journal of Anat. and Physiol., vol. xi., p. 491, plates xx. and xxi.

† 'Elasmobranch Fishes,' p. 156, plates x. and xiii.

‡ Ibid., p. 136, plate xi., fig. 10.

In one of our embryos of this day the developing auditory vesicle still has the form of a pit, but in the other it is a closed vesicle, already constricted off from the nervous layer of the epidermis.

With reference to the development of the excretory duct we cannot add much to what we have already stated in describing the last stage.

The duct is considerably dilated anteriorly (Plate 23, fig. 31, *sg.*); but our sections throw no light on the nature of the abdominal pore. The posterior part of the duct has still the form of a hollow ridge united with somatic mesoblast (Plate 23, fig. 32, *sg.*).

During this stage, the embryo becomes to a small extent folded off from the yolk-sac both in front and behind, and in the course of this process the anterior and posterior extremities of the alimentary tract become definitely established.

We have not got as clear a view of the process of formation of these two sections of the alimentary tract as we could desire, but our observations appear to show that the process is in many respects similar to that which takes place in the formation of the anterior part of the alimentary tract in Elasmobranchii.* One of us has shown that in Elasmobranchs the ventral wall of the throat is formed *not* by a process of folding in of the hypoblastic sheet as in Birds, but by a growth of the ventral face of the hypoblastic sheet on each side of and at some little distance from the middle line. Each growth is directed inwards, and the two eventually meet and unite, thus forming a complete ventral wall for the gut. Exactly the same process would seem to take place in *Lepidosteus*, and after the lumen of the gut is in this way established, a process of mesoblast on each side also makes its appearance, forming a mesoblastic investment on the ventral side of the alimentary tract. Some time after the alimentary tract has been thus formed, the epiblast becomes folded in, in exactly the same manner as in the Chick, the embryo becoming thereby partially constricted off from the yolk (Plate 23, figs. 33, 34).

The form of the lumen of the alimentary tract differs somewhat in front and behind. In front, the hypoblastic sheet remains perfectly flat during the formation of the throat, and thus the lumen of the latter has merely the form of a slit. The lumen of the posterior end of the alimentary tract is, however, narrower and deeper (Plate 23, figs. 33, 34, *al.*). Both in front and behind, the lateral parts of the hypoblastic sheet become separated from the true alimentary tract as soon as the lumen of the latter is established.

It is quite possible that at the extreme posterior end of the embryo a modification of the above process may take place, for in this region the hypoblast appears to us to have the form of a solid cord.

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* F. M. Balfour, 'Monagraph on the Development of Elasmobranch Fishes,' p. 87, plate ix., fig. 2.

We could detect no true neurenteric canal, although a more or less complete fusion of the germinal layers at the tail end of the embryo may still be traced.

During this stage the protoplasm of the notochordal cells, which in the last stage formed a kind of axial rod in the centre of the notochord, begins to spread outwards toward the sheath of the notochord.

Eighth day after impregnation.—The external form of the embryo (Plate 21, fig. 9) shows a great advance upon the stage last figured. Both head and body are much more compressed laterally and raised from the yolk, and the head end is folded off for some distance. The optic vesicles are much less prominent externally. A commencing opercular fold is distinctly seen. Our figure of this stage is not, however, so satisfactory as we could wish.

A thickening of the nervous layer of the external epiblast which will form the lens (Plate 23, fig. 35, *l.*) is more marked than in the last stage, and presses against the slightly concave exterior wall of the optic vesicle (*op.*). The latter has now a large cavity, and its stalk is considerably narrowed.

The auditory vesicles (Plate 23, fig. 36, *au.*) are closed, appearing as hollow sacs one on each side of the brain, and are no longer attached to the epiblast.

The anterior opening of the segmental duct can be plainly seen close behind the head. The lumen of the duct is considerably larger.

The two vertebral portions of the mesoblast are now separated by a considerable space from the epiblast on one side and from the notochord on the other, and the cells composing them have become considerably elongated from side to side (Plate 23, fig. 37, *ms.*).

In some sections the aorta can be seen (Plate 23, fig. 37, *ao.*) lying close under the sub-notochordal rod, between it and the hypoblast, and on either side of it a slightly larger cardinal vein (*cd.v.*).

The protoplasm of the notochord has now again retreated towards the centre, showing a clear space all round. This is most marked in the region of the trunk (Plate 23, fig. 37). The sub-notochordal rod (*s.nc.*) lies close under it.

A completely closed fore-gut, lined by thickened hypoblast, extends about as far back as the auditory sacs (Plate 23, figs. 35 and 36, *al.*). In the trunk the hypoblast, which will form the walls of the alimentary tract, is separated from the notochord by a considerable interval.

Ninth day after impregnation: External characters.—Very considerable changes have taken place in the external characters of the embryo. It is about 8 millims. in length, and has assumed a completely piscine form. The tail especially has grown in length, and is greatly flattened from side to side: it is wholly detached from the yolk, and bends round towards the head, usually with its left side in contact with the yolk. It is provided with well-developed dorsal and ventral fin-folds, which meet each other round the end of the tail, the tail fin so formed being nearly symmetrical. The head is not nearly so much folded off from the yolk as the tail. At its front end is placed a disc with numerous papillæ, of which we shall say more hereafter. This disc is somewhat bifid, and is marked in the centre by a deep depression.

Dorsal to it, on the top of the head, are two widely separated nasal pits. On the surface of the yolk, in front of the head, is to be seen the heart, just as in Sturgeon embryos. Immediately below the suctorial disc is a slit-like space, forming the mouth. It is bounded below by the two mandibular arches, which meet ventrally in the median line. A shallow but well marked depression on each side of the head indicates the posterior boundary of the mandibular arch. Behind this is placed the very conspicuous hyoid arch with its rudimentary opercular flap; and in the depression, partly covered over by the latter, may be seen a ridge, the external indication of the first branchial arch.

Eleventh day after impregnation: External characters.—The embryo (Plate 21, fig. 10) is now about 10 millims. in length, and in several features exhibits an advance upon the embryo of the previous stage.

The tail fin is now obviously not quite symmetrical, and the dorsal fin-fold is continued for nearly the whole length of the trunk. The suctorial disc (Plate 21, fig. 11, *s.d.*) is much more prominent, and the papillæ (about 30 in number) covering it are more conspicuous from the surface. It is not obviously composed of two symmetrical halves. The opercular flap is larger, and the branchial arches behind it (two of which may be made out without dissection) are more prominent.

The anterior pair of limbs is now visible in the form of two *longitudinal* folds projecting in a vertical direction from the surface of the yolk-sac at the sides of the body.

The stages subsequent to hatching have been investigated with reference to the external features and to the habits by Agassiz, and we shall enrich our own account by copious quotations from his memoir.

He states that the first batch were hatched on the eighth* day after being laid. "The young Fish possessed a gigantic yolk-bag, and the posterior part of the body presented nothing specially different from the general appearance of a Teleostean embryo, with the exception of the great size of the chorda. The anterior part, however, was most remarkable; and at first, on seeing the head of this young *Lepidosteus*, with its huge mouth-cavity extending nearly to the gill-opening, and surmounted by a hoof-shaped depression edged with a row of protuberances acting as suckers, I could not help comparing this remarkable structure, so utterly unlike anything in Fishes or Ganoids, to the Cyclostomes, with which it has a striking analogy. This organ is also used by *Lepidosteus* as a sucker, and the moment the young Fish is hatched he attaches himself to the sides of the disc, and there remains hanging immovable; so firmly attached, indeed, that it requires considerable commotion in the water to make him loose his hold. Aerating the water by pouring it from a height did not always produce sufficient disturbance to loosen the young Fishes. The eye, in this stage, is rather less advanced than in corresponding stages in bony Fishes; the brain is also comparatively smaller, the otolith ellipsoidal, placed obliquely in the rear above the gill-opening. . . . Usually the gill-cover is pressed closely against the sides of the body, but in breathing an opening is seen through which water is constantly passing, a strong current being made by the rapid movement of the pectorals, against the base of which the extremity of the gill-cover is closely pressed. The large yolk-bag is opaque, of a bluish-gray colour. The body of the young *Lepidosteus* is quite colourless and transparent. The embryonic fin is narrow, the dorsal part commencing above the posterior end of the yolk-bag; the tail is slightly rounded, the anal opening nearer the extremity of the tail than the bag. The intestine is narrow, and the embryonic fin extending from the vent to the yolk-bag is quite narrow. In a somewhat more advanced stage,—hatched a few hours earlier,—the upper edge of the yolk-bag is covered with black pigment cells, and minute black pigment cells appear on the surface of the alimentary canal. There are no traces of embryonic fin-rays either in this stage or the one preceding; the structure of the embryonic fin is as in bony Fishes—previous to the appearance of these embryonic fin-rays—finely granular. Seen in profile, the yolk-bag is ovoid; as seen from above, it is flattened, rectangular in front, with rounded corners, tapering to a rounded point towards the posterior extremity, with re-entering sides."

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* This statement of Agassiz does not correspond with the dates on the specimens sent to us—a fact no doubt due to the hatching not taking place at the same time for all the larvæ.

We have figured an embryo of 11 millims. in length, shortly after hatching (Plate 21, fig. 12), the most important characters of which are as follows:—The yolk-sac, which has now become much reduced, forms an appendage attached to the ventral surface of the body, and has a very elongated form as compared with its shape just before hatching. The mouth, as also noticed by AGASSIZ, has a very open form. It is (Plate 21, fig. 13, *m.*) more or less rhomboidal, and is bounded behind by the mandibular arch (*mn.*) and laterally by the superior maxillary processes (*s.mx.*). In front of the mouth is placed the suctorial disc (*s.d.*), the central papillæ of which are arranged in groups. The opercular fold (*h.op.*) is very large, covering the arches behind. A well-marked groove is present between the mandibular and opercular arches, but so far as we can make out it is not a remnant of the hyomandibular cleft.

The pectoral fins (Plate 21, fig. 12, *pc.f.*) are very prominent longitudinal ridges, which, owing to their being placed on the surface of the yolk-sac, project in a nearly vertical direction: a feature which is also found in many Teleostean embryos with large yolk-sacs.

No traces of the pelvic fins have yet become developed.

The positions of the permanent dorsal, anal, and caudal fins, as pointed out by AGASSIZ, are now indicated by a deposit of pigment in the embryonic fin.

In an embryo on the sixth day after hatching, of about 15 millims. in length, of which we have also given a figure (Plate 21, fig. 14), the following fresh features deserve special notice.

In the region of the head there is a considerable elongation of the pre-oral part, forming a short snout, at the end of which is placed the suctorial disc. At the sides of the snout are placed the nasal pits, which have become somewhat elongated anteriorly.

The mouth has lost its open rhomboidal shape, and has become greatly narrowed in an antero-posterior direction, so that its opening is reduced to a slit. The mandibles and maxillary processes are nearly parallel, though both of them are very much shorter than in the adult. The operculum is now a very large flap, and has extended so far backwards as to cover the insertion of the pectoral fin. The two opercular folds nearly meet ventrally.

The yolk-sac is still more reduced in size, one important consequence of which is that the pectoral fins (*pc.f.*) appear to spring out more or less horizontally from the sides of the body, and at the same time their primitive line of attachment to the body becomes transformed from a longitudinal to a more or less transverse one.

The first traces of the pelvic fins are now visible as slight longitudinal projections near the hinder end of the yolk-sac (*pl.f.*).

The pigmentation marking the regions of the permanent fins has become more pronounced, and it is to be specially noted that the ventral part of the caudal fin (the permanent caudal) is considerably more prominent than the dorsal fin opposite to it.

The next changes, as AGASSIZ points out, "are mainly in the lengthening of the snout; the increase in length both of the lower and upper jaw; the concentration of the sucker of the sucking disc; and the adoption of the general colouring of somewhat older Fish. The lobe of the pectoral has become specially prominent, and the outline of the fins is now indicated by a fine milky granulation. Seen from above, the gill-cover is seen to leave a large circular opening leading to the gill-arches, into which a current of water is constantly passing, by the lateral expansion and contraction of the gill-cover; the outer extremity of the gill-cover covers the base of the pectorals. In a somewhat older stage the snout has become more elongated, the sucker more concentrated, and the disproportionate size of the terminal sucking-disc is reduced; the head, when seen from above, becoming slightly elongated and pointed."

In a larva of about 18 days old and 21 millims. in length, of which we have not given a figure, the snout has grown greatly in length, carrying with it the nasal organs, the openings of which now appear to be divided into two parts. The suctorial disc is still a prominent structure at the end of the snout. The lower jaw has elongated correspondingly with the upper, so that the gape is very considerable, though still very much less than in the adult.

The opercular flaps overlap ventrally, the left being superficial. They still cover the bases of the pectoral fins. The latter are described by AGASSIZ as being "kept in constant rapid motion, so that the fleshy edge is invisible, and the vibration seems almost involuntary, producing a constant current round the opening leading into the cavity of the gills."

The pelvic fins are somewhat more prominent.

The yolk-sac, as pointed out by AGASSIZ, has now disappeared as an external appendage.

After the stage last described the young Fish rapidly approaches the adult form. To show the changes effected we have figured the head of a larva of about a month old and 23 millims. in length (Plate 21, fig. 15). The suctorial disc, though much reduced, is still prominent at the end of the snout. Eventually, as shown by AGASSIZ, it forms the fleshy globular termination of the upper jaw.

The most notable feature in which the larva now differs in its external form from the adult is in the presence of an externally heterocercal tail, caused by the persistence of the primitive caudal fin as an elongated filament projecting beyond the permanent caudal (Plate 28, fig. 68).

Delicate dermal fin-rays are now conspicuous in the peripheral parts of all the permanent fins. These rays closely resemble the horny fin-rays in the fins of embryo Elasmobranchs in their development and structure. They appear gradually to enlarge to form the permanent rays, and we have followed out some of the stages of their growth, which is in many respects interesting. Our observations are not, however, complete enough to publish, and we can only say here that their early development and structure proves their homology with the horny fibres or rays in fins of Elasmobranchii. The skin is still, however, entirely naked, and without a trace of its future armour of enamelled scales.

The tail of a much older larva, 11 centims. in length, in which the scales have begun to be formed, is shown in Plate 21, fig. 16.

We complete this section of our memoir by quoting the following passages from AGASSIZ as to the habits of the young fish at the stages last described :—

"In the stages intervening between plate iii, fig. 19, and plate iii, fig. 30, the young *Lepidosteus* frequently swim about, and become readily separated from their point of attachment. In the stage of plate iii, fig. 30, they remain often perfectly quiet close to the surface of the water; but, when disturbed, move very rapidly about through the water. . . . The young already have also the peculiar habit of the adult of coming to the surface to swallow air. When they go through the process under water of discharging air again they open their jaws wide, and spread their gill-covers, and swallow as if they were choking, making violent efforts, until a minute bubble of air has become liberated, when they remain quiet again. The resemblance to a Sturgeon in the general appearance of this stage of the young *Lepidosteus* is quite marked."

BRAIN.

I. *Anatomy.*

The brain of *Lepidosteus* has been figured by BUSCH (whose figure has been copied by MIKLUCHO-MACLAY, and apparently by HUXLEY), by OWEN, and by WILDER

(No. 15). The figure of the latter author, representing a longitudinal section through the brain, is the most satisfactory, the other figures being in many respects inaccurate; but even WILDER'S figure and description, though taken from the fresh object, appears to us in some respects inadequate. He offers, moreover, fresh interpretations of certain parts of the brain which we shall discuss in the sequel.

We have examined two brains which, though extremely soft, were, nevertheless, sufficiently well preserved to enable us to study the external form. We have, moreover, made a complete series of transverse sections through one of the brains, and our sections, though utterly valueless from a histological point of view, have thrown some light on the topographical anatomy of the brain.

Plate 25, figs. 47 A, B, and C, represent three views of the brain, viz.: from the side, from above, and from below. We will follow in our description the usual division of the brain into fore-brain, mid-brain, and hind-brain.

The fore-brain consists of an anterior portion forming the cerebrum, and a posterior portion constituting the thalamencephalon.

The cerebrum at first sight appears to be composed of (*a*) a pair of posterior and somewhat dorsal lobes, forming what have usually been regarded as the true cerebral hemispheres, but called by WILDER the prothalami, and (*b*) a pair of anterior and ventral lobes, usually regarded as the olfactory lobes, from which the olfactory nerves spring. Mainly from a comparison with our embryonic brains described in the sequel, we are inclined to think that the usual interpretations are not wholly correct, but that the true olfactory lobes are to be sought for in small enlargements (Plate 25, figs. 47 A, B, and C, *olf.*) at the front end of the brain * from which the olfactory nerves spring. The cerebrum proper would then consist of a pair of anterior and ventral lobes (*ce.*), and of a pair of posterior lobes (*ce'.*), both pairs uniting to form a basal portion behind.

The two pairs of lobes probably correspond with the two parts of the cerebrum of the Frog, the anterior of which, like that of *Lepidosteus*, was held to be the olfactory lobe, till GÖTTE'S researches showed that this view was not tenable.

The anterior lobes of the cerebrum have a conical form, tapering anteriorly, and are completely separated from each other. The posterior lobes, as is best shown in side views, have a semicircular form. Viewed from above they appear as rounded prominences, and their dorsal surface is marked by two conspicuous furrows (Plate 25, fig. 47 B, *ce'.*), which have been noticed by WILDER, and are similar to those present in many Teleostei. Their front ends overhang the base of the anterior cerebral lobes. The basal portion of the cerebrum is an undivided lobe, the anterior wall of which forms the lamina terminalis.

What we have above described as the posterior cerebral lobes have been described by WILDER as constituting the everted dorsal border of the basal portion of the cerebrum.

The portion of the cerebro-spinal canal within the cerebrum presents certain primitive characters, which are in some respects dissimilar to those of higher types, and have led WILDER to hold the posterior cerebral lobes, together with what we have called the basal portion of the cerebrum, to be structures peculiar to Fishes, for which he has proposed the name "prothalami."

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* The homologies of the olfactory lobes throughout the group of Fishes require further investigation.

In the basal portion of the cerebrum there is an unpaired slit-shaped ventricle, the outer walls of which are very thick. It is provided with a floor formed of nervous matter, in part of which, judging from WILDER's description, a well-marked commissure is placed. We have found in the larva a large commissure in this situation (Plate 24, figs. 44 and 45, *a.c.*); and it may be regarded as the homologue of the anterior commissure of higher types. This part of the ventricle is stated by WILDER to be without a roof. This appears to us highly improbable. We could not, however, determine the nature of the roof from our badly preserved specimens, but if present, there is no doubt that it is extremely thin, as indeed it is in the larva (Plate 24, fig. 46 B). In a dorsal direction the unpaired ventricle extends so as to separate the two posterior cerebral lobes. Anteriorly the ventricle is prolonged into two horns, which penetrate for a short distance, as *the lateral ventricles*, into the base of the anterior cerebral lobes. The front part of each anterior cerebral lobe, as well as of the whole of the posterior lobes, appears solid in our sections; but WILDER describes the anterior horns of the ventricle as being prolonged for the whole length of the anterior lobes.

In the embryos of all Vertebrates the cerebrum is not at first divided into two lobes, so that the fact of the posterior part of the cerebrum in *Lepidosteus* and probably other Ganoids remaining permanently in the undivided condition does not appear to us a sufficient ground for giving to the lobes of this part of the cerebrum the special name of prothalami, as proposed by WILDER, or for regarding them as a section of the brain peculiar to Fishes.

The thalamencephalon (*th.*) contains the usual parts, but is in some respects peculiar. Its lateral walls, forming the optic thalami, are thick, and are not sharply separated in front from the basal part of the cerebrum; between them is placed the third ventricle. The thalami are of considerable extent, though partially covered by the optic lobes and the posterior lobes of the cerebrum. They are not, however, relatively so large as in other Ganoid forms, more especially the Chondrostei and *Polypterus.*

On the roof of the thalamencephalon is placed a large thin-walled vesicle (Plate 25, figs. 47 A and B, *v.th.*), which undoubtedly forms the most characteristic structure connected with this part of the brain. Owing to the wretched state of preservation of the specimens, we have found it impossible to determine the exact relations of this body to the remainder of the thalamencephalon; but it appears to be attached to the roof of the thalamencephalon by a narrow stalk only. It extends forwards so as to overlap part of the cerebrum in front, and is closely invested by a highly vascular layer of the pia mater.

No mention is made by WILDER of this body; nor is it represented in his figures or in those of the other anatomists who have given drawings of the brain of *Lepidosteus.* It might at first be interpreted as a highly-developed pineal gland, but a comparison with the brain of the larva (vide p. 378) shows that this is not the case, but that the body in question is represented in the larva by a special outgrowth of the roof of the thalamencephalon. The vesicle of the roof of the thalamencephalon is therefore to be regarded as a peculiar development of the tela choroidea of the third ventricle.

How far this vesicle has a homologue in the brains of other Ganoids is not certain, since negative evidence on this subject is all but valueless. It is possible that a vesicular sac covering over the third ventricle of the Sturgeon described by STANNIUS,[*] and stated by him to be wholly formed of the membranes of the brain, is really the homologue of our vesicle.

WIEDERSHEIM[†] has recently described in *Protopterus* a body which is undoubtedly homologous with our vesicle, which he describes in the following way :—

"Dorsalwärts ist das Zwischenhirn durch ein tiefes, von Hirnschlitz eingenommenes Thal von Vorderhirn abgesetzt; dasselbe ist jedoch durch eine häutige, mit der Pia mater zusammenhängende Kuppel oder Kapsel überbrückt."

This "Kuppel" has precisely the same relations and a very similar appearance to our vesicle. The true pineal gland is placed behind it. It appears to us possible that the body found by HUXLEY[‡] in *Ceratodus*, which he holds to be the pineal gland, is in reality this vesicle. It is moreover possible that what has usually been regarded as the pineal gland in *Petromyzon* may in reality be the homologue of the vesicle we have found in *Lepidosteus*.

We have no observations on the pineal gland of the adult, but must refer the reader for the structure and relations of this body to the embryological section.

The infundibulum (Plate 25, fig. 47 A, *in.*) is very elongated. Immediately in front of it is placed the optic chiasma (Plate 25, figs. 47 A and C, *op.ch.*) from which the optic fibres can be traced passing along the sides of the optic thalami and to the optic lobes, very much as in MÜLLER's figure of the brain of *Polypterus*.

On the sides of the infundibulum are placed two prominent bodies, the lobi inferiores (*l.in.*), each of which contains a cavity continuous with the prolongation of the third ventricle into the infundibulum. The apex of the infundibulum is enlarged, and to it is attached a pituitary body (*pt.*).

The mid-brain is of considerable size, and consists of a basal portion connecting the optic thalami with the medulla, and a pair of large optic lobes (*op.l.*). The iter a tertio ad quartum ventriculum, which forms the ventricle of this part of the brain, is prolonged into each optic lobe, and the floor of each prolongation is taken up by a dome-shaped projection, the homologue of the torus semicircularis of Teleostei.

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[*] "Üb. d. Gehirns des Stors," MÜLLER's Archiv., 1843, and Lehrbuch d. vergl. Anat. d. Wirbelthiere. CATTIE (Archives de Biologie, vol. iii., 1882, has recently described in *Acipenser sturio* a vesicle on the roof of the thalamencephalon, whose cavity is continuous with the third ventricle. This vesicle is clearly homologous with that in *Lepidosteus*. (June 28, 1882.)

[†] R. WIEDERSHEIM, 'Morphol. Studien,' 1880, p. 71.

[‡] "On *Ceratodus Forsterei*," &c., Proc. Zool. Soc., 1876.

The hind-brain consists of the usual parts, the medulla oblongata and the cerebellum. The medulla presents no peculiar features. The sides of the fourth ventricle are thickened and everted, and marked with peculiar folds (Plate 25, figs. 47 A and B, *m.o.*).

The cerebellum is much larger than in the majority of Ganoids, and resembles in all essential features the cerebellum of Teleostei. In side views it has a somewhat S-shaped form, from the presence of a peculiar lateral sulcus (Plate 25, fig. 47 A, *cb.*). As shown by WILDER, its wall actually has in longitudinal section this form of curvature, owing to its anterior part projecting forwards into the cavity of the iter.* This forward projection is not, however, so conspicuous as in most Teleostei. The cerebellum contains a large unpaired prolongation of the fourth ventricle.

II. *Development.*

The early development of the brain has already been described; and, although we do not propose to give any detailed account of the later stages of its growth, we have thought it worth while calling attention to certain developmental features which may probably be regarded as to some extent characteristic of the Ganoids. With this view we have figured (Plate 24, figs. 44, 45) longitudinal sections of the brain at two stages, viz.: of larvæ of 15 and 26 millims., and transverse sections (Plate 24, figs. 46 A–G) of the brain of a larva at about the latter stage (25 millims.).

The original embryonic fore-brain is divided in both embryos into a cerebrum (*ce.*) in front and a thalamencephalon (*th.*) behind. In the younger embryo the cerebrum is a single lobe, as it is in the brains of all Vertebrate embryos; but in the older larva it is anteriorly (Plate 24, fig. 46 A) completely divided into two hemispheres. The roof of the undivided posterior part of the cerebrum is extremely thin (Plate 24, fig. 46 B). Near the posterior border of the base of the cerebrum there is a great development of nervous fibres, which may probably be regarded as in part equivalent to the anterior commissure (Plate 24, figs. 44, 45, *a.c.*).

Even in the oldest of the two brains the olfactory lobes are very slightly developed, constituting, however, small lateral and ventral prominences of the front end of the hemispheres. From each of them there springs a long olfactory nerve, extending for the whole length of the rostrum to the olfactory sac.

The thalamencephalon presents a very curious structure, and is relatively a more important part of the brain than in the embryo of any other form which we know of. Its roof, instead of being, as usual, compressed antero-posteriorly,† so as to be almost concealed between the cerebral hemispheres and the optic lobes (mid-brain), projects on the surface for a length quite equal to that of the cerebral hemispheres (Plate 24, figs. 44 and 45, *th.*). In the median line the roof of the thalamencephalon is thin and folded; at its posterior border is placed the opening of the small pineal gland. This body is a papilliform process of the nervous matter of the roof of this part of the brain, and instead of being directed forwards, as in most Vertebrate types, tends somewhat backwards, and rests on the mid-brain behind (Plate 24, figs. 44, 45, and 46 C and D, *pn.*). The roof of the thalamencephalon immediately in front of the pineal gland forms a sort of vesicle, the sides of which extend laterally as a pair of lobes, shown in transverse sections in Plate 24, figs. 46 C and D, as *th.l.* This vesicle becomes, we cannot doubt, the vesicle on the roof of the thalamencephalon which we have described in the adult brain. Immediately in front of the pineal gland the roof of the thalamencephalon contains a transverse commissure (Plate 24, fig. 46 C, *z.*), which is the homologue of a similarly situated commissure present in the Elasmobranch brain,[*] while behind the pineal gland is placed the posterior commissure. The sides of the thalamencephalon are greatly thickened, forming the optic thalami (Plate 24, figs. 46 C and D, *op.th.*), which are continuous in front with the thickened outer walls of the hemispheres. Below, the thalamencephalon is produced into a very elongated infundibulum (Plate 24, figs. 44, 45, 46 E, *in.*), the apex of which is trilobed as in Elasmobranchii and Teleostei. The sides of the infundibulum exhibit two lobes, the lobi inferiores (Plate 24, fig. 46 D, *l.in.*), which are continued posteriorly into the crura cerebri.

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* In WILDER's figure the walls of the cerebellum are represented as much too thin.

† Vide F. M. BALFOUR, 'Comparative Embryology,' vol. ii., figs. 243 and 250.

The pituitary body[†] (Plate 24, figs. 44, 45, 46 E, *pt.*) is small, not divided into lobes, and provided with a very minute lumen.

In front of the infundibulum is the optic chiasma (Plate 24, fig. 46 D, *op.ch.*), which is developed very early. It is, as stated by MÜLLER, a true chiasma.

The mid-brain (Plate 24, figs. 44 and 45, *m.b.*) is large, and consists in both stages of (1) a thickened floor forming the crura cerebri, the central canal of which constitutes the iter a tertio ad quartum ventriculum; and (2) the optic lobes (Plate 24, figs. 46 E, F, G, *op.l.*) above, each of which is provided with a cavity continuous with the median iter. The optic lobes are separated dorsally and in front by a well-marked median longitudinal groove. Posteriorly they largely overlap the cerebellum. In the anterior part of the optic lobes, at the point where the iter joins the third ventricle, there may be seen slight projections of the floor into the lumen of the optic lobes (Plate 24, fig. 46 E). These masses probably become in the adult the more conspicuous prominences of the floor of the ventricles of the optic lobes, which we regard as homologous with the tori semicirculares of the brain of the Teleostei.

The hind-brain is formed of the usual divisions, viz.: cerebellum and medulla oblongata (Plate 24, figs. 44 and 45, *cb.*, *md.*). The former constitutes a bilobed projection of the roof of the hind-brain. Only a small portion of it is during these stages left uncovered by the optic lobes, but the major part extends forwards for a considerable distance under the optic lobes, as shown in the transverse sections (Plate 24, figs. 46 F and G, *cb.*); and its two lobes, each with a prolongation of its cavity, are continued forwards beyond the opening of the iter into the fourth ventricle.

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[*] Vide F. M. BALFOUR, 'Comparative Embryology,' vol. ii., pp. 355–6, where it is suggested that this commissure is the homologue of the grey commissure of higher types.

[†] We have not been able to work out the early development of the pituitary body as satisfactorily as we could have wished. In Plate 24, fig. 40, there is shown an invagination of the oral epithelium to form it; in Plate 24, figs. 41 and 42, it is represented in transverse section in two consecutive sections. Anteriorly it is still connected with the oral epithelium (fig. 41), while posteriorly it is free. It is possible that an earlier stage of it is shown in Plate 23, fig. 35. Were it not for the evidence in other types of its being derived from the epiblast we should be inclined to regard it as hypoblastic in origin.

It is probable that the anterior horns of the cerebellum are equivalent to the prolongations of the cerebellum into the central cavity of the optic lobes of Teleostei, which are continuous with the so-called fornix of GÖTTSCHE.

III. *Comparison of the larval and adult brain of* Lepidosteus, *together with some observations on the systematic value of the characters of the Ganoid brain.*

The brain of the older of the two larvæ, which we have described, sufficiently resembles in most of its features that of the adult to render material assistance in the interpretation of certain of the parts of the latter. It will be remembered that in the adult brain the parts usually held to be olfactory lobes were described as the anterior cerebral lobes. The grounds for this will be apparent by a comparison of the cerebrum of the larva and adult. In the larva the cerebrum is formed of (1) an unpaired basal portion with a thin roof, and (2) of a pair of anterior lobes, with small olfactory bulbs at their free extremities.

The basal portion in the larva clearly corresponds in the adult with the basal portion, together with the two posterior cerebral lobes, which are merely special outgrowths of the dorsal edge of the primitive basal portion. The pair of anterior lobes have exactly the same relations in the larva as in the adult, except that in the former the ventricles are prolonged for their whole length instead of being confined to their proximal portions. If, therefore, our identifications of the larval parts of the brain are correct, there can hardly be a question as to our identifications of the parts in the adult. As concerns these identifications, the comparison of the brain of our two larvæ appears conclusive in favour of regarding the anterior lobes as parts of the cerebrum, as distinguished from the olfactory lobes, in that they are clearly derived from the undivided anterior portion of the cerebrum of the younger larva.

The comparison of the larval brain with that of the adult again appears to us to leave no doubt that the vesicle attached to the roof of the thalamencephalon in the adult is the same structure as the bilobed outgrowth of this roof in the larva; and since there is in addition a well-developed pineal gland in the larva with the usual relations, there can be no ground for identifying the vesicle in the adult with the pineal gland.

MÜLLER, in his often quoted memoir (No. 13), states that the brains of Ganoids are peculiar and distinct from those both of Teleostei and Elasmobranchii; but in addition to pointing out that the optic nerves form a chiasma he does not particularly mention the features, to which he alludes in general terms. More recently WILDER (No. 15) has returned to this subject; and though, as we have already had occasion to point out, we cannot accept all his identifications of the parts of the Ganoid brain, yet he has called attention to certain characteristic features of the cerebrum which have an undoubted systematic value.

The distinctive characters of the Ganoid brain are, in our opinion, (1) the great elongation of the region of the thalamencephalon; and (2) the unpaired condition of the posterior part of the cerebrum, and the presence of so thin a roof to the ventricle of this part as to cause it to appear open above.

The immense length of the region of the thalamencephalon is a feature in the Ganoid brain which must at once strike any one who examines figures of the brains of Chondrostrei, *Polypterus*, or *Amia*. It is less striking in the adult *Lepidosteus*, though here also we have shown that the thalamencephalon is really very greatly developed; but in the larva of *Lepidosteus* this feature is still better marked, so that the brain of the larva may be described as being more characteristically Ganoid than that of the adult.

The presence of a largely developed thalamencephalon at once distinguishes a Ganoid brain from that of a Teleostean Fish, in which the optic thalami are very much reduced; but *Lepidosteus* shows its Teleostean affinities by a commencing reduction of this part of the brain.

The large size of the thalamencephalon is also characteristic of the Ganoid brain in comparison with the brain of the Dipnoi; but is not however so very much more marked in the Ganoids than it is in some Elasmobranchii.

On the whole, we may consider the retention of a large thalamencephalon as a primitive character.

The second feature which we have given as characteristic of the Ganoid brain is essentially that which has been insisted upon by WILDER, though somewhat differently expressed by him.

The simplest condition of the cerebrum is that found in the larva of *Lepidosteus*, where there is an anterior pair of lobes, and an undivided posterior portion with a simple prolongation of the third ventricle, and a very thin roof. The dorsal edges of the posterior portion, adjoining the thin roof, usually become somewhat everted (cf. WILDER), and in *Lepidosteus* these edges have in the adult a very great development, and form (vide Plate 25, fig. 47 A–C, *ce′*.) two prominent lobes, which we have spoken of as the posterior cerebral lobes.

These characters of the cerebrum are perhaps even more distinctive than those of the thalamencephalon.

In Teleostei the cerebrum appears to be completely divided into two hemispheres, which are, however, all but solid, the lateral ventricles being only prolonged into their bases. In Dipnoi again there is either (*Protopterus*, WIEDERSHEIM[*]) a completely separated pair of oval hemispheres, not unlike those of the lower Amphibia, or the oval hemispheres are not completely separated from each other (*Ceratodus*, HUXLEY,[†]

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[*] 'Morphol. Studien,' iii. Jena, 1880.

[†] "On *Ceratodus Forsteri*," Proc. Zool. Soc., 1876.

Lepidosiren, HYRTL[*]); in either case the hemispheres are traversed for the whole length by lateral ventricles which are either completely or nearly completely separated from each other.

In Elasmobranchii the cerebrum is an unpaired though bilobed body, but traversed by two completely separated lateral ventricles, and without a trace of the peculiar membranous roof found in Ganoids.

Not less interesting than the distinguishing characters of the Ganoid brain are those cerebral characters which indicate affinities between *Lepidosteus* and other groups. The most striking of these are, as might have been anticipated, in the direction of the Teleostei.

Although the foremost division of the brain is very dissimilar in the two groups, yet the hind-brain in many Ganoids and the mid-brain also in *Lepidosteus* approaches closely to the Teleostean type. The most essential feature of the cerebellum in Teleostei is its prolongation forwards into the ventricles of the optic vesicles as the valvula cerebelli. We have already seen that there is a homologous part of the cerebellum in *Lepidosteus;* STANNIUS also describes this part in the Sturgeon, but no such part is represented in MÜLLER's figure of the brain of *Polypterus*, or described by him in the text.

The cerebellum is in most Ganoids relatively smaller, and this is even the case with *Amia;* but the cerebellum of *Lepidosteus* is hardly less bulky than that of most Teleostei.

The presence of tori semicirculares on the floor of the mid-brain of *Lepidosteus* again undoubtedly indicates its affinities with the Teleostei, and such processes are stated by STANNIUS to be absent in the Sturgeon, and have not, so far as we are aware, been described in other Ganoids. Lastly we may point to the presence of well-developed lobi inferiores in the brain of *Lepidosteus* as an undoubted Teleostean character.

On the whole, the brain of *Lepidosteus*, though preserving its true Ganoid characters, approaches more closely to the brain of the Teleostei than that of any other Ganoid, including even *Amia*.

It is not easy to point elsewhere to such marked resemblances of the Ganoid brain, as to the brain of the Teleostei.

The division of the cerebrum into anterior and posterior lobes, which is found in *Lepidosteus*, probably reappears again, as already indicated, in the higher Amphibia. The presence of the peculiar vesicle attached to the roof of the thalamencephalon has its parallel in the brain of *Protopterus*, and as pointing in the same direction a general similarity in the appearance of the brain of *Polypterus* to that of the Dipnoi may be mentioned.

There appears to us to be in no points a close resemblance between the brain of Ganoids and that of Elasmobranchii.

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[*] '*Lepidosiren paradoxa*.' Prag., 1845.

SENSE ORGANS.

Olfactory organ.

Development.—The nasal sacs first arise during the late embryonic period in the form of a pair of thickened patches of the nervous layer of the epiblast on the dorsal surface of the front end of the head (Plate 24, fig. 39, *ol.*). The patches very soon become partially invaginated; and a small cavity is developed between them and the epidermic layer of the epiblast (Plate 24, figs. 42 and 43, *ol.*). Subsequently, the roof of this space, formed by the epidermic layer of the epiblast, is either broken through or absorbed; and thus open pits, *lined entirely by the nervous layer of the epidermis,* are formed.

We are not acquainted with any description of an exactly similar mode of origin of the olfactory pits, though the process is almost identical with that of the other sense organs.

We have not worked out in detail the mode of formation of the double openings of the olfactory pits, but there can be but little doubt that it is caused by the division of the single opening into two.

The olfactory nerve is formed very early (Plate 24, fig. 39, I), and, as MARSHALL has found in Aves and Elasmobranchii, it arises at a stage prior to the first differentiation of an olfactory bulb as a special lobe of the brain.

The Eye.

Anatomy.—We have not made a careful histological examination of the eye of *Lepidosteus,* which in our specimens was not sufficiently well preserved for such a purpose; but we have found a vascular membrane enveloping the vitreous humour on its retinal aspect, which, so far as we know, is unlike anything which has so far been met with in the eye of any other adult Vertebrate.

The membrane itself is placed immediately outside the hyaloid membrane, *i.e.,* on side of the hyaloid membrane bounding the vitreous humour. It is easily removed from the retina, to which it is only adherent at the entrance of the optic nerve. In both the eyes we examined it also adhered, at one point, to the capsule of the lens, but we could not make out whether this adhesion was natural, or artificially produced by the coagulation of a thin layer of albuminous matter. In one instance, at any rate, the adhesion appeared firmer than could easily be produced artificially.

The arrangement of the vessels in the membrane is shown diagrammatically in Plate 25, fig. 49, while the characteristic form of the capillary plexus is represented in Plate 25, fig. 50.

The arterial supply appears to be derived from a vessel perforating the retina close to the optic nerve, and obviously homologous with the artery of the processus falciformis and pecten of Teleostei and Birds, and with the arteria centralis retinæ of Mammals. From this vessel branches diverge and pursue a course towards the periphery. They give off numerous branches, the blood from which enters a capillary plexus (Plate 25, figs. 49 and 50) and is collected again by veins, which pass outwards and finally bend over and fall into (Plate 25, fig. 49) a circular vein (*cr.v.*) placed at the outer edge of the retina along the insertion of the iris (*ir*). The terminal branches of some of the main arteries appear also to fall directly into this vein.

The membrane supporting the vessels just described is composed of a transparent matrix, in which numerous cells are embedded (Plate 25, fig. 50).

Development.—In the account of the first stages of development of *Lepidosteus*, the mode of formation of the optic cup, the lens, &c., have been described (vide Plates 22 and 23, figs. 23, 26, 35). With reference to the later stages in the development of the eye, the only subject with which we propose to deal is the growth of the mesoblastic processes which enter the cavity of the vitreous humour through the choroid slit.

Lepidosteus is very remarkable for the great number of mesoblast cells which thus enter the cavity of the vitreous humour, and for the fact that these cells are *at first unaccompanied by any vascular structures* (Plate 24, fig. 43, *v.h.*). The mesoblast cells are scattered through the vitreous humour, and there can be no doubt that during early larval life, at a period however when the larva is certainly able to see, every histologist would consider the vitreous humour to be a tissue formed of scattered cells, with a large amount of intercellular substance ; and the fact that it is so appears to us to demonstrate that KESSLER'S view of the vitreous humour being a mere transudation is not tenable.

In the larva five or six days after hatching, and about 15 millims. in length, the choroid slit is open for its whole length. The edges of the slit near the lens are folded, so as to form a ridge projecting into the cavity of the vitreous humour, while nearer the insertion of the optic nerve they cease to exhibit any such structure. The mesoblast, though it projects between the lips of the ridge near the lens, only extends through the choroid slit into the cavity of the vitreous humour in the neighbourhood of the optic nerve. Here it forms a lamina with a thickened edge, from which scattered cells in the cavity of the vitreous humour seem to radiate.

At a slightly later stage than that just described, blood-vessels become developed within the cavity of the vitreous humour, and form the vascular membrane already described in the adult, placed close to the layer of nerve-fibres of the retina, but separated from this layer by the hyaloid membrane (Plate 25, fig. 48, *v.sh.*). The artery bringing the blood to the above vascular membrane is bound up in the same sheath as the optic nerve, and passes through the choroid slit very close to the optic nerve. Its entrance into the cavity of the vitreous humour is shown in Plate 25, fig. 48 (*vs.*) ; its relation to the optic nerve in Plate 24, fig. 46, C and D (*vs.*).

The above sheath has, so far as we know, its nearest analogue in the eye of *Alytes*, where, however, it is only found in the larva.

The reader who will take the trouble to refer to the account of the imperfectly-developed processus falciformis of the Elasmobranch eye in the treatise 'On Comparative

Embryology,' by one of us,[*] will not fail to recognise that the folds of the retina at the sides of the choroid slit, and the mesoblastic process passing through this slit, are strikingly similar in *Lepidosteus* and Elasmobranchii; and that, if we are justified in holding them to be an imperfectly-developed processus falciformis in the one case, we are equally so in the other.

JOHANNES MÜLLER mentions the absence of a processus falciformis as one of the features distinguishing Ganoids and Teleostei. So far as the systematic separation of the two groups is concerned, he is probably perfectly justified in this course; but it is interesting to notice that both in Ganoids and Elasmobranchii we have traces of a structure which undergoes a very special development in the Teleostei, and that the processus falciformis of Teleostei is therefore to be regarded, not as an organ peculiar to them, but as the peculiar modification within the group of a primitive Vertebrate organ.

SUCTORIAL DISC.

One of the most remarkable organs of the larval *Lepidosteus* is the suctorial disc, placed at the front end of the head, to which we have made numerous allusions in the first section of this memoir.

The external features of the disc have been fully dealt with by AGASSIZ, and he also explained its function by observations on the habits of the larva. We have already quoted (p. 371) a passage from AGASSIZ' memoir showing how the young Fishes use the disc to attach themselves firmly to any convenient object. The discs appear in fact to be highly efficient organs of attachment, in that the young Fish can remain suspended by them to the sides of the jar, even after the water has been lowered below the level at which they are attached.

The disc is formed two or three days before hatching, and from AGASSIZ' statements, it appears to come into use immediately the young Fish is liberated from the egg membranes.

We have examined the histological structure of the disc at various ages of its growth, and may refer the reader to Plate 21, figs. 11 and 13, and Plate 24, figs. 40 and 44. The result of our examination has been to show that the disc is provided with a series of papillæ often exhibiting a bilateral arrangement. The papillæ are mainly constituted of highly modified cells of the mucous layer of the epidermis. These cells have the form of elongated columns, the nucleus being placed at the base, and the main mass of the cells being filled with a protoplasmic reticulum. They may probably be regarded as modified mucous cells. In the mesoblast adjoining the suctorial disc there are numerous sinus-like vascular channels.

It does not appear probable that the disc has a true sucking action. It is unprovided with muscular elements, and there appears to be no mechanism by which it could act as a sucking organ. We must suppose, therefore, that its adhesive power depends upon the capacity of the cells composing its papillæ to pour out a sticky secretion.

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[*] Vol. ii., p. 414.

Muscular System.

There is a peculiarity in the muscular system of *Lepidosteus*, which so far as we know has not been previously noticed. It is that the lateral muscles of each side are not divided, either in the region of the trunk or of the tail, into a dorso-lateral and ventro-lateral division.

This peculiarity is equally characteristic of the older larvæ as of the adult, and is shown in Plate 28, figs. 67, 72, and 73, and Plate 29, figs. 74–76. In the Cyclostomata the lateral muscles are not divided into dorsal and ventral sections; but except in this group such a division has been hitherto considered as invariable amongst Fishes.

This character must, without doubt, be held to be the indication of a very primitive arrangement of the muscular system. In the embryos of all Fishes with the usual type of the lateral muscles, the undivided condition of the muscles precedes the divided condition; and in primitive forms such as the Cyclostomata and Amphioxus the embryonic condition is retained, as it is in *Lepidosteus*.

Skeleton.

Part I.—*Vertebral column and ribs of the adult.*

A typical vertebra from the trunk of *Lepidosteus* has the following characters (Plate 29, figs. 80 and 81).

The centrum is slightly narrrower in the middle than at its two extremities. It articulates with adjacent vertebræ by a convex face in front and a concave face behind, being thus, according to Owen's nomenclature, opisthocœlous. It presents on its under surface a well-marked longitudinal ridge, which in many vertebræ is only united at its two extremities with the main body of the vertebra.

From the lateral borders of the centrum there project, at a point slightly nearer the front than the hind end, a pair of prominent hæmal processes (*h.a.*), to the ends of which are articulated the ribs. These processes have a nearly horizontal direction in the greater part of the trunk, though bent downwards in the tail.

The neural arches (*n.a.*) have a somewhat complicated form. They are mainly composed of two vertical plates, the breadth of the basal parts of which is nearly as great as the length of the vertebræ, so that comparatively narrow spaces are left between the neural arches of successive vertebræ for the passage of the spinal nerves. Some little way from its dorsal extremity each neural arch sends a horizontal process inwards, which meets its fellow and so forms a roof for the spinal canal. These processes appear to be confined to the posterior parts of the vertebræ, so that at the front ends of the vertebræ, and in the spaces between them, the neural canal is without an osseous roof. Above the level of this osseous roof there is a narrow passage, bounded laterally by the dorsal extremities of the neural plates. This passage is mainly filled up by a series of cartilaginous elements (Plate 29, figs. 80 and 81, *i.c.*) (probably fibro-cartilage), which rest upon the roof of the neural canal. Each element is situated *intervertebrally*, its anterior end being wedged in between the two dorsal processes of the neural arch of the vertebra in front, and its posterior end extending for some distance over the vertebra behind. The successive elements are connected by fibrous tissue, and are continuous dorsally with a fibrous band, known as the ligamentum longitudinale superius (Plate 29, figs. 80 and 81, *l.l.*), characteristic of Fishes generally, and running continuously for the whole length of the vertebral column. Each of the cartilaginous elements is, as will be afterwards shown, developed as two independent pieces of cartilage, and might be compared with the dorsal element which usually forms the keystone of the neural arch in Elasmobranchs, were not the latter vertebral instead of intervertebral in position. More or less similar elements are described by GÖTTE in the neural arches of many Teleostei, which also, however, appear to be vertebrally placed, and he has compared them and the corresponding elements in the Sturgeon with the Elasmobranch cartilages forming the keystone of the neural arch. GÖTTE does not, however, appear to have distinguished between the cartilaginous elements, and the osseus elements forming the roof of the spinal canal, which are true membrane bones; it is probable that the two are not so clearly separated in other types as in *Lepidosteus.*

The posterior ends of the neural plates of the neural arches are continued into the dorsal processes directed obliquely upwards and backwards, which have been somewhat unfortunately described by STANNIUS as rib-like projections of the neural arch. The dorsal processes of the two sides do not meet, but between them is placed a median free spinous element, also directed obliquely upwards and backwards, which forms a kind of roof for the groove in which the cartilaginous elements and the ligamentum longitudinale are placed.

The vertebræ are wholly formed of a very cellular osseous tissue, in which a distinction between the bases of the neural and hæmal processes and the remainder of the vertebra is not recognisable. The bodies of the vertebræ are, moreover, directly continuous with the neural and hæmal arches.

The ribs in the region of the trunk are articulated to the ends of the long hæmal processes. They envelop the body cavity, their proximal parts being placed immediately outside the peritoneal membrane, along the bases of the intermuscular septa. Their distal ends do not, however, remain close to the peritoneal membrane, *but pass outwards along the intermuscular septa till their free ends come into very close proximity with the skin.* This peculiarity, which holds good in the adult for all the free ribs, is shown in one of the anterior ribs of an advanced larva in Plate 28, fig. 72 (*rb.*). We are not aware that this has been previously noticed, but it appears to us to be a point not without interest in all questions which concern the homology of rib-like structures occupying different positions in relation to the muscles. Its bearings are fully dealt with in the section of this paper devoted to the consideration of the homologies of the ribs in Fishes.

As regards the behaviour of the ribs in the transitional region between the trunk and the tail, we cannot do better than translate the description given by Gegenbaur of this region (No. 6, p. 411):—"Up to the 34th vertebra the ribs borne by the laterally and posteriorly directed processes present nothing remarkable, though they have gradually become shorter. The ribs of the 35th vertebra exhibit a slight curvature outwards of their free ends, a peculiarity still more marked in the 36th. The last named pair of ribs converge somewhat in their descent backwards so that both ribs decidedly approach before bending outwards. The 37th vertebra is no longer provided with freely terminating ribs, but on the contrary, the same pair of processes which in front was provided with ribs, bears a short forked process as the hæmal arch. *The two, up to this point separated ribs, have here formed a hæmal arch by the fusion of their lower ends, which arch is movable just like the ribs, and, like them, is attached to the vertebral column.*"

In the region of the tail fin the hæmal arches supporting the caudal fin rays are very much enlarged.

Part II.—*Development of the vertebral column and ribs.*

The first development and early histological changes of the notochord have already been given, and we may take up the history of the vertebral column at a period when the notochord forms a large circular rod, whose cells are already highly vacuolated, while the septa between the vacuoles form a delicate wide-meshed reticulum. Surrounding the notochord is the usual cuticular sheath, which is still thin.

The first indications of the future vertebral column are to be found in the formation of a distinct mesoblastic investment of the notochord. On the dorsal aspect of the notochord, the mesoblast forms two ridges, one on each side, which are prolonged upwards so as to meet above the neural canal, for which they form a kind of sheath. On the ventral side of the notochord there are also two ridges, which are, however, except on the tail, much less prominent than the dorsal ridges.

The changes which next ensue are practically identical with those which take place in Teleostei. Around the cuticular sheath of the notochord there is formed an elastic membrane—the membrana elastica externa. At the same time the basal parts of the dorsal, or as we may perhaps more conveniently call them, the neural ridges of the notochord become enlarged at each intermuscular septum, and the tissue of these enlargements soon becomes converted into cartilage, thus forming a series of independent paired neural processes riding on the membrana elastica externa surrounding the notochord, and extending about two-thirds of the way up the sides of the medullary cord. They are shown in transverse section in Plate 28, fig. 67 (*n.a.*), and in a side view in fig. 68 (*n.a.*).

Simultaneously with the neural arches, the hæmal arches also become established, and arise by the formation of similar enlargements of the ventral or hæmal ridges. In the trunk they are very small, but in the region of the tail their condition is very different. At the front end of the anal fin the paired hæmal arches suddenly enlarge and extend ventralwards (Plate 28, fig. 67, *h.a.*).

Each succeeding pair of arches becomes larger than the one in front, and the two elements of each arch first nearly meet below the caudal vein (Plate 28, fig. 67) and finally actually do so, forming in this way a completely closed hæmal canal. At the point where they first meet the permanent caudal fin commences, and here (Plate 28, fig. 68) we find that not only do the hæmal arches meet and coalesce below the caudal vein, but they are actually produced into long spines supporting the fin rays of the caudal fin, which thus differs from the other fins in being supported by parts of the true vertebral column and not by independently formed elements of the skeleton.

Each of the large caudal hæmal arches, including the spine, forms a continous whole, and arises at an earlier period of larval life than any other part of the vertebral column. We noticed the first indications of the neural arches in the larva of about a week old, while they are converted into fully formed cartilage in the larva of three weeks.

The neural and hæmal arches, resting on the membrana elastica externa, do not at this early stage in the least constrict the notochord. They grow gradually more definite, till the larva is five or six weeks old and about 26 millims. in length, but otherwise for a long time undergo no important changes. During the same period, however, the true sheath of the notochord greatly increases in thickness, and the membrana elastica externa becomes more definite. So far it would be impossible to distinguish the development of the vertebral column of *Lepidosteus* from that of a Teleostean Fish.

Of the stages immediately following we have unfortunately had no examples, but we have been fortunate enough to obtain some young specimens of *Lepidosteus*,* which have enabled us to work out with tolerable completeness the remainder of the developmental history of the vertebral column. In the next oldest larva, of about 5·5 centims., the changes which have taken place are already sufficient to differentiate the vertebral column of *Lepidosteus* from that of a Teleostean, and to show how certain of the characteristic features of the adult take their origin.

In the notochord the most important and striking change consists in the appearance of a series of very well marked vertebral constrictions *opposite the insertions of the neural and hæmal arches.* The first constrictions of the notochord are thus, as in other Fishes, vertebral; and although, owing to the growth of the intervertebral cartilage, the vertebral constrictions are subsequently replaced by intervertebral constrictions, yet at the same time the primitive occurrence of vertebral constrictions demonstrates that the vertebral column of *Lepidosteus* is a modification of a type of vertebral column with biconcave vertebræ.

The structure of the gelatinous body of the notochord has undergone no important change. The sheath, however, exhibits certain features which deserve careful description. In the first place the attention of the observer is at once struck by the fact that, in the vertebral regions, the sheath is much thicker (·014 millim.) than in the intervertebral (·005 millim.), and a careful examination of the sheath in longitudinal sections shows that the thickening is due to the special differentiation of a superficial part (Plate 28, fig. 69, *sh.*) of the sheath in each vertebral region. This part is somewhat granular as compared to the remainder, especially in longitudinal sections. It forms a cylinder (the wall of which is about ·01 millim. thick) in each vertebral region, immediately within the membrana elastica externa. Between it and the gelatinous tissue of the notochord within there is a very thin unmodified portion of the sheath, which is continuous with the thinner intervertebral parts of the sheath. This part of the sheath is faintly, but at the same time distinctly, concentrically striated—a probable indication of concentric fibres. The inner unmodified layer of the sheath has the appearance in transverse sections through the vertebral regions of an inner membrane, and may perhaps be KÖLLIKER's "membrana elastica interna."

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* These specimens were given to us by Professor W. K. PARKER, who received them from Professor BURT G. WILDER.

We are not aware that any similar modification of the sheath has been described in other forms.

The whole sheath is still invested by a very distinct membrana elastica externa (*m.el*).

The changes which have taken place in the parts which form the permanent vertebræ will be best understood from Plate 28, figs. 69–71. From the transverse section (fig. 70) it will be seen that there are still neural and hæmal arches resting upon the membrana elastica externa; but longitudinal sections (fig. 69) show that laterally these arches join a cartilaginous tube, embracing the intervertebral regions of the notochord, and continuous from one vertebra to the next.

It will be convenient to treat separately the neural arches, the hæmal arches with their appendages, and the intervertebral cartilaginous rings.

The neural arches, except in the fact of embracing a relatively smaller part of the neural tube than in the earlier stage, do not at first sight appear to have undergone any changes. Viewed from the side, however, in dissected specimens, they are seen to be prolonged upwards so as to unite above with bars of cartilage directed obliquely backwards. An explanation of this appearance is easily found in the sections. The cartilaginous neural arches are invested by a delicate layer of homogeneous bone, developed in the perichondrium, and this bone is prolonged beyond the cartilage and joins a similar osseous investment of the dorsal bars above mentioned. The whole of these parts may, it appears to us, be certainly reckoned as parts of the neural arches, so that at this stage each neural arch consists of: (1) a pair of basal portions resting on the notochord consisting of cartilage invested by bone, (2) of a pair of dorsal cartilaginous bars invested in bone (*n.a'.*), and (3) of osseous bars connecting (1) and (2).

Though, in the absence of the immediately preceding stages, it is not perfectly certain that the dorsal pieces of cartilage are developed independently of the ventral, there appears to us every probability that this is so; and thus the cartilage of each neural arch is developed discontinuously, while the permanent bony neural arch, which commences as a deposit of bone partly in the perichondrium and partly in the intervening membrane, forms a continuous structure.

Analogous occurrences have been described by GÖTTE in Teleostei.

The dorsal portion of each neural arch becomes what we have called the dorsal process of the adult arch.

Between the dorsal processes of the two sides there is placed a median rod of cartilage (Plate 28, fig. 70, *i.s.*), which in its development is wholly independent of the true neural arches, and which constitutes the median spinous element of the adult. In tracing these backwards it becomes obvious that they are homologous with the interspinous elements supporting the dorsal fin, in that they are replaced by these interspinous elements in the region of the dorsal fin, and that the interspinous bones occupy the same position as the median spinous processes. This homology was first pointed out by GÖTTE in the case of the Teleostei.

Immediately beneath this rod is placed the longitudinal ligament (Plate 28, fig. 70, *l.l.*), but there is as yet no trace of a junction between the neural arches of the two sides in the space between the longitudinal ligament and the spinal cord.

The basal parts of the neural arches of the two sides are united dorsally by a thin cartilaginous layer resting on the sheath of the notochord, but they are not united ventrally with the hæmal arches.

The hæmal processes in the trunk are much more prominent than in the preceding stage, and their bases are united ventrally by a tolerably thick layer of cartilage. In the trunk they are continuous with the so-called ribs of the adult (Plate 28, fig. 70); but in order to study the nature of these ribs it is necessary to trace the modifications undergone by the hæmal arches in passing from the tail to the trunk.

It will be remembered that at an earlier stage the hæmal arches in the region of the tail-fin were fully formed, and that through the anterior part of the caudal region the hæmal processes were far advanced in development, and just in front of the caudal fin had actually met below the caudal vein.

The mode of development of the hæmal arches in the tail as *unjointed* cartilaginous bars investing the caudal arteries and veins is so similar to that of the caudal hæmal arches of Elasmobranchii, that it appears to us impossible to doubt their identity in the two groups.*

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* GEGENBAUR (No. 6) takes a different view on this subject, as is clear from the following passage in this memoir (pp. 369–370) :—"Each vertebra of *Lepidosteus* thus consists of a section of the notochord, and of the cartilaginous tissue surrounding its sheath, which gives origin to the upper arches for the whole length of the vertebral column, and in the caudal region to that of the lower arches also. *The latter do not however complete the enclosure of a lower canal, but this is effected by special independent elements*, which are to be interpreted as homologues of the ribs." (The italics are ours.) While we fully accept the homology between the ribs and the lower elements of the hæmal arches of the tail, the view expressed in the italicised section, to the effect that the lower parts of the caudal arches are not true hæmal arches but are independently formed elements, is entirely opposed to our observations, and has we

The changes which have taken place by this stage with reference to the hæmal arches of the tail are not very considerable.

In the case of a few more vertebræ the hæmal processes have united into an arch, and the spinous processes of the arches in the region of the caudal fin have grown considerably in length. A more important change is perhaps the commencement of a segmentation of the distal parts of the hæmal arches from the proximal. This process has not, however, as yet resulted in a complete separation of the two, such as we find in the adult.

If the hæmal processes are traced forwards (Plate 29, figs. 75 and 76) from the anterior segment where they meet ventrally, it will be found that each hæmal process consists of a basal portion, adjoining the notochord, and a peripheral portion. These two parts are completely continuous, but the line of a future separation is indicated by the structure of the cartilage, though not shown in our figures. As the true body cavity of the trunk replaces the obliterated body cavity of the caudal region, no break of continuity will be found in the structure of the hæmal processes (Plates 28 and 29, figs. 73 and 74), but while the basal portions grow somewhat larger, the peripheral portions gradually elongate and take the form of delicate rods of cartilage extending ventralwards, on each side of the body cavity, immediately outside the peritoneal membrane, and along the lines of insertion of the intermuscular septa. These rods obviously become the ribs of the adult.

As one travels forwards the ribs become continually longer and more important, and though they are at this stage united with the hæmal processes in every part of the trunk, yet they are much more completely separated from these processes in front than behind (Plate 28, fig. 72).

In front (Plate 28, fig. 72), each rib (*rb.*), after continuing its ventral course for some distance, immediately outside the peritoneal membrane, turns outwards, and passes along one of the intermuscular septa till it reaches the epidermis. This feature in the position of the ribs is, as has been already pointed out in the anatomical part of this section, characteristic of all the ribs of the adult.

It is unfortunate that we have had no specimens showing the ribs at an earlier stage of development; but it appears hardly open to doubt that *the ribs are originally continuous with the hæmal processes*, and that the indications of a separation between those two parts at this stage are not due to a secondary fusion, but to a commencing segmentation.

It further appears, as MÜLLER, GEGENBAUR and others have stated, that the ribs and hæmal processes of the tail are serially homologous structures; but that the view maintained by GÖTTE in his very valuable memoirs on the Vertebrate skeleton is also correct to the effect that *the hæmal arches of the tail are homologous throughout the series of Fishes.*

believe only arisen from the fact that GEGENBAUR had not the young larvæ to work with by which alone this question could be settled

To this subject we shall return again at the end of the section.

Before leaving the hæmal arches it may be mentioned that behind the region of the ventral caudal fin the two hæmal processes merge into one, and form an unpaired knob resting on the ventral side of the notochord, and not perforated by a canal.

There are now present well-developed intervertebral rings of cartilage, each of which eventually becomes divided into two parts, and converted into the adjacent faces of the contiguous vertebræ. These rings are united with the neural and hæmal arches of the vertebræ in front and behind.

Each ring, as shown by the transverse section (Plate 28, fig. 71), is not uniformly thick, but exhibits four projections, two dorsal and two ventral. These four projections are continuous with the bases of the neural and hæmal arches of the adjacent vertebræ, and afford presumptive evidence of the derivation of the intervertebral rings from the neural and hæmal arches; in that had they so originated, it would be natural to anticipate the presence of four thickenings indicating the four points from which the cartilage had spread, while if the rings had originated independently, it would not be easy to give any explanation of the presence of such thickenings. GEGENBAUR (No. 6), from the investigation of a much older larva than that we are now describing, also arrived at the conclusion that the intervertebral cartilages were derived from the neural and hæmal arches; but as doubts have been thrown upon this conclusion by GÖTTE, and as it obviously required further confirmation, we have considered it important to attempt to settle this point. From the description given above, it is clear that we have not, however, been able absolutely to trace the origin of this cartilage, but at the same time we think that we have adduced weighty evidence in corroboration of GEGENBAUR'S view.

As shown in longitudinal section (Plate 28, fig. 69, *iv.r.*), the intervertebral rings are thicker in the middle than at the two ends. In this thickened middle part the division of the cartilage into two parts to form the ends of two contiguous vertebræ is subsequently effected. The curved line which this segmentation will follow is, however, already marked out, and from surface views it might be supposed that this division had actually occurred.

The histological structure of the intervertebral cartilage is very distinct from that of the cartilage of the bases of the arches, the nuclei being much more closely packed. In parts, indeed, the intervertebral cartilage has almost the character of fibro-cartilage. On each side of the line of division separating two vertebræ it is invested by a superficial osseous deposit.

The next oldest larva we have had was 11 centims. in length. The filamentous dorsal lobe of the caudal fin still projected far beyond the permanent caudal fin (Plate 21, fig. 16).

The vertebral column was considerably less advanced in development than that dissected by GEGENBAUR, though it shows a great advance on the previous stage. Its features are illustrated by two transverse sections, one through the median plane of a vertebral region (Plate 29, fig. 78) and the other through that of an intervertebral region (Plate 29, fig. 79), and by a horizontal section (Plate 29, fig. 77).

In the last stage the notochord was only constricted vertebrally. Now, however, by the great growth of intervertebral cartilage there have appeared (Plate 29, fig. 77) very well-marked *intervertebral* constrictions, by the completion of which the vertebræ of *Lepidosteus* acquire their unique character amongst Fishes.

These constrictions still, however, coexist with the earlier, though at this stage relatively less conspicuous, vertebral constrictions.

The gelatinous body of the notochord retains its earlier condition. The sheath has, however, undergone some changes. In the vertebral regions there is present in any section of the sheath—(1) externally, the membrana elastica externa (*m.el.*); then (2) the external layer of the sheath (*sh.*), which is, however, less thick than before, and exhibits a very faint form of radial striation; and (3) internally, a fairly thick and concentrically striated layer. The whole thickness is, on an average, 0·18 millim.

In the intervertebral regions the membrana elastica externa is still present in most parts, but has become absorbed at the posterior border of each vertebra, as shown in longitudinal section in Plate 29, fig. 77. It is considerably puckered transversely. The sheath of the notochord within the membrana elastica externa is formed of a concentrically striated layer, continuous with the innermost layer of the sheath in the vertebral regions. It is puckered longitudinally. Thus, curiously enough, the membrana elastica externa and the sheath of the notochord in the intervertebral regions are folded in different directions, the folds of the one being only visible in transverse sections (Plate 29, fig. 79), and those of the other in longitudinal sections (Plate 29, fig. 77).

The osseous and cartilaginous structures investing the notochord may conveniently be dealt with in the same order as before, viz.: the neural arches, the hæmal arches, and the intervertebral cartilages.

The cartilaginous portions of the neural arches are still unossified, and form (Plate 29, fig. 78, *n.a.*) small wedge-shaped masses resting on the sheath of the notochord. They are invested by a thick layer of bone prolonged upwards to meet the dorsal processes (*n.a′.*), which are still formed of cartilage invested by bone.

It will be remembered that in the last stage there was no key-stone closing in the neural arch above. This deficiency is now however supplied, and consists of (1) two bars of cartilage repeated for each vertebra, but intervertebrally placed, which are directly differentiated from the ligamentum longitudinale superius, into which they merge above; and (2) two osseous plates placed on the outer sides of these cartilages, which are continuous with the lateral osseous bars of the neural arch. The former of these elements gives rise to the cartilaginous elements above the osseous bridge of the neural arch in the adult. The two osseous plates supporting these cartilages clearly form what we have called in our description of the adult the osseous roof of the spinal canal.

A comparison of the neural arch at this stage with the arch in the adult, and in the stage last described, shows that the greater part of the neural arch of the adult is formed of membrane-bone, there being preformed in cartilage only a small basal part, a dorsal process, and paired key-stones below the ligamentum longitudinale superius.

The hæmal arches (Plate 29, fig. 78) are still largely cartilaginous, and rest upon the sheath of the notochord. They are invested by a thick layer of bone. The bony layer investing the neural and hæmal arches is prolonged to form a continuous investment round the vertebral portions of the notochord (Plate 29, fig. 78). This investment is at the sides prolonged outwards into irregular processes (Plate 29, fig. 78), which form the commencement of the outer part of the thick but cellular osseous cylinder forming the middle part of the vertebral body.

The intervertebral cartilages are much larger than in the earlier stage (Plate 29, figs. 77 and 79), and it is by their growth that the intervertebral constrictions of the notochord are produced. They have ceased to be continuous with the cartilage of the arches, the intervening portion of the vertebral body between the two being only formed of bone. They are not yet divided into two masses to form the contiguous ends of adjacent vertebræ.

Externally, the part of each cartilage which will form the hinder end of a vertebral body is covered by a tube of bone, having the form of a truncated funnel, shown in longitudinal section in Plate 29, fig. 77, and in transverse section in Plate 29, fig. 79.

At each end, the intervertebral cartilages are becoming penetrated and replaced by beautiful branched processes from the homogeneous bone which was first of all formed in the perichondrium (Plate 29, fig. 77).

This constitutes the latest stage which we have had.

Gegenbaur (No. 6) has described the vertebral column in a somewhat older larva of 18 centims.

The chief points in which the vertebral column of this larva differed from ours are: (1) the disappearance of all trace of the primitive vertebral constriction of the notochord; (2) the nearly completed constriction of the notochord in the intervertebral regions; (3) the complete ossification of the vertebral portions of the bodies of the vertebræ, the terminal so-called intervertebral portions alone remaining cartilaginous; (4) the complete ossification of the basal portions of the hæmal and neural processes included within the bodies of the vertebræ, so that in the case of the neural arch all trace of the fact that the greater part was originally not formed in cartilage had become lost. The cartilage of the dorsal spinous processes was, however, still persistent.

The only points which remain obscure in the later history of the vertebral column are the history of the notochord and of its sheath. We do not know how far these are either simply absorbed or partially or wholly ossified.

Götte in his memoir on the formation of the vertebral bodies of the Teleostei attempts to prove (1) that the so-called membrana elastica externa of the Teleostei is not a homogeneous elastica, but is formed of cells, and (2) that in the vertebral regions ossification first occurs in it.

In *Lepidosteus* we have met with no indication that the membrana elastica externa is composed of cells; though it is fair to GÖTTE to state that we have not examined such isolated portions of it as he states are necessary in order to make out its structure. But further than this we have satisfied ourselves that during the earlier stage of ossification this membrane is not ossified, and indeed in part becomes absorbed in proximity to the intervertebral cartilages; and GEGENBAUR met with no ossification of this membrane in the later stage described by him.

Summary of the development of the vertebral column and ribs.

A mesoblastic investment is early formed round the notochord, which is produced into two dorsal and two ventral ridges, the former uniting above the neural canal. Around the cuticular sheath of the notochord an elastic membrane, the membrana elastica externa, is next developed. The neural ridges become enlarged at each intermuscular septum, and these enlargements soon become converted into cartilage, thus forming a series of neural processes riding on the membrana elastica externa, and extending about two-thirds of the way up the sides of the neural canal. The hæmal processes arise simultaneously with, and in the same manner as, the neural. They are small in the trunk, but at the front end of the anal fin they suddenly enlarge and extend ventralwards. Each succeeding pair of hæmal arches becomes larger than the one in front, each arch finally meeting its fellow below the caudal vein, thus forming a completely closed hæmal canal. These arches are moreover produced into long spines supporting the fin-rays of the caudal fin, which thus differs from the other impaired fins in being supported by parts of the vertebral column, and not by separately formed skeletal elements.

In the next stage which we have had the opportunity of studying (larva of $5\frac{1}{2}$ centims.), a series of very well-marked *vertebral* constrictions are to be seen in the notochord. The sheath is now much thicker in the vertebral than in the intervertebral regions: this is due to a special differentiation of a superficial part of the sheath, which appears more granular than the remainder. This granular part of the sheath thus forms a cylinder in each vertebral region. Between it and the gelatinous tissue of the notochord there remains a thin unmodified portion of the sheath, which is continuous with the intervertebral parts of the sheath. The neural and hæmal arches are seen to be continuous with a cartilaginous tube embracing the intervertebral regions of the notochord, and continuous from one vertebra to the next. A delicate layer of bone, developed in the perichondrium, invests the cartilaginous neural arches, and this bone grows upwards so as to unite above with the osseous investment of separately developed bars of cartilage, which are directed obliquely backwards. These bars, or dorsal processes, may be reckoned as parts of the neural arches. Between the dorsal processes of the two sides is placed a median rod of cartilage, which is developed separately from the true neural arches, and which constitutes the median spinous element of the adult. Immediately below this rod is placed the ligamentum longitudinale superius. There is now a commencement of separation between the dorsal and ventral parts of the hæmal arches, not only in the tail, but also in the trunk, where they pass ventralwards on each side of the body cavity, immediately outside the peritoneal membrane, along the lines of insertion of the intermuscular septa. These are obviously the ribs of the adult, and there is no break of continuity of structure between the hæmal processes of the tail and the ribs. In the anterior part of the trunk the ribs pass outwards along the intermuscular septa till they reach the epidermis. Thus the ribs are originally continuous with the hæmal processes. Behind the region of the ventral caudal fin the two hæmal processes merge into one, which is not perforated by a canal.

Each of the intervertebral rings of cartilage becomes eventually divided into two parts, and converted into the adjacent faces of contiguous vertebræ, the curved line where this will be effected being plainly marked out. These rings are united with the neural and hæmal arches of the vertebræ next in front and behind. As these rings are formed originally by the spreading of the cartilage from the primitive neural and hæmal processes, the intervertebral cartilages are clearly derived from the neural and hæmal arches. The intervertebral cartilages are thicker in the middle than at their two ends.

In our latest stage (11 centims.), the vertebral constrictions of the notochord are rendered much less conspicuous by the growth of the intervertebral cartilages giving rise to marked intervertebral constrictions. In the intervertebral regions the membrana elastica externa has become aborted at the posterior border of each vertebra, and the remaining part is considerably puckered transversely. The inner sheath of the notochord is puckered longitudinally in the intervertebral regions. The granular external layer of the sheath in the vertebral regions is less thick than in the last stage, and exhibits faint radial striations.

Two closely approximated cartilaginous elements now form a keystone to the neural arch above: these are directly differentiated from the ligamentum longitudinale superius, into which they merge above. An osseous plate is formed on the outer side of each of these cartilages. These plates are continuous with the lateral osseous bars of the neural arches, and also give rise to the osseous roof of the spinal canal of the adult.

Thus the greater part of the neural arches is formed of membrane bone. The hæmal arches are invested by a thick layer of bone, and there is also a continuous osseous investment round the vertebral portions of the notochord. The intervertebral cartilages become penetrated by branched processes of bone.

Comparison of the vertebral column of Lepidosteus *with that of other forms.*

The peculiar form of the articulatory faces of the vertebræ of *Lepidosteus* caused L. AGASSIZ (No. 2) to compare them with the vertebræ of Reptiles, and subsequent anatomists have suggested that they more nearly resemble the vertebræ of some Urodelous Amphibia than those of any other form.

If, however, GÖTTE's account of the formation of the amphibian vertebræ is correct, there are serious objections to a comparison between the vertebræ of *Lepidosteus* and Amphibia on developmental grounds. The essential point of similarity supposed to exist between them consists in the fact that in both there is a great development of intervertebral cartilage which constricts the notochord intervertebrally, and forms the articular faces of contiguous vertebræ.

In *Lepidosteus* this cartilage is, as we have seen, derived from the bases of the arches; but in Amphibia it is held by GÖTTE to be formed by a special thickening of a cellular sheath round the notochord which is probably homologous with the cartilaginous sheath of the notochord of Elasmobranchii, and therefore with part of the notochordal sheath placed within the membrana elastica externa.

If the above statements with reference to the origin of the intervertebral cartilage in the two types are true, it is clear that no homology can exist between structures so differently developed. Provisionally, therefore, we must look elsewhere than in *Lepidosteus* for the origin of the amphibian type of vertebræ.

The researches which we have recorded demonstrate, however, in a very conclusive manner that the vertebræ of *Lepidosteus* have very close affinities with those of Teleostei.

In support of this statement we may point: (1) To the structure of the sheath of the notochord; (2) to the formation of the greater part of the bodies of the vertebræ from ossification in membrane around the notochord; (3) to the early biconcave form of the vertebræ, only masked at a later period by the development of intervertebral cartilages; (4) to the character of the neural arches.

This latter feature will be made very clear if the reader will compare our figures of the sections of later vertebræ (Plate 29, fig. 78) with GÖTTE's* figure of the section of the vertebra of a Pike (plate 7, fig. 1). In GÖTTE's figure there are shown similar intercalated pieces of cartilage to those which we have found, and similar cartilaginous dorsal processes of the vertebræ. Thus we are justified in holding that whether or no the opisthocœlous form of the vertebræ of *Lepidosteus* is a commencement of a type of vertebræ inherited by the higher forms, yet in any case the vertebræ are essentially built on the type which has become inherited by the Teleostei from the bony Ganoids.

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* "Beiträge zur vergl. Morphol. d. Skeletsystems d. Wirbeltheire." Archiv. f. Mikr. Anat., vol. xvi., 1879.

Part III.—*The ribs of Fishes.*

The nature and homologies of the ribs of Fishes have long been a matter of controversy; but the subject has recently been brought forward in the important memoirs of Götte[*] on the Vertebrate skeleton. The alternatives usually adopted are, roughly speaking, these:—Either the hæmal arches of the tail are homologous throughout the piscine series, while the ribs of Ganoids and Teleostei are not homologous with those of Elasmobranchii; or the ribs are homologous in all the piscine groups, and the hæmal arches in the tail are differently formed in the different types. Götte has brought forward a great body of evidence in favour of the first view; while Gegenbaur[†] may be regarded as more especially the champion of the second view.

One of us held in a recent publication[‡] that the question was not yet settled, though the view that the ribs are homologous throughout the series was provisionally accepted.

It is admitted by both Gegenbaur and Götte that in *Lepidosteus* the ribs, in the transition from the trunk to the tail, bend inwards, and finally unite in the region of the tail to form the ventral parts of the hæmal arches, and our researches have abundantly confirmed this conclusion.

Are the hæmal arches, the ventral parts of which are thus formed by the coalescence of the ribs, homologous with the hæmal arches in Elasmobranchii? The researches recorded in the preceding pages appear to us to demonstrate in a conclusive manner that they are so.

The development of the hæmal arches in the tail in these two groups is practically identical; they are formed in both as simple elongations of the primitive hæmal processes, which meet below the caudal vein. In the adult there is an apparent difference between them, arising from the fact that in *Lepidosteus* the peripheral parts of the hæmal processes are only articulated with the basal portions, and not, as in Elasmobranchii, continuous with them. This difference does not, however, exist in the early larva, since in the larval *Lepidosteus* the hæmal arches of the tail are unsegmented cartilaginous arches, as they permanently are in Elasmobranchii. If, however, the homology between the hæmal arches of the two types should still be doubted, the fact that in both types the hæmal arches are similarly modified to support the fin-rays of the ventral lobe of the caudal fin, while in neither type are they modified to support the anal fin, may be pointed out as a very strong argument in confirmation of their homology.

The demonstration of the homology of the hæmal arches of the tail in *Lepidosteus* and Elasmobranchii might at first sight be taken as a conclusive argument in favour of Götte's view, that the ribs of Elasmobranchii are not homologous with those of Ganoidei. This view is mainly supported by two facts :—

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[*] "Beiträge z. vergl. Morph. d. Skeletsystems d. Wirbelthiere. II. Die Wirbelsäule u. ihre Anhänge." Archiv. f. Mikr. Anat., vol. xv., 1878, and vol. xvi., 1879.

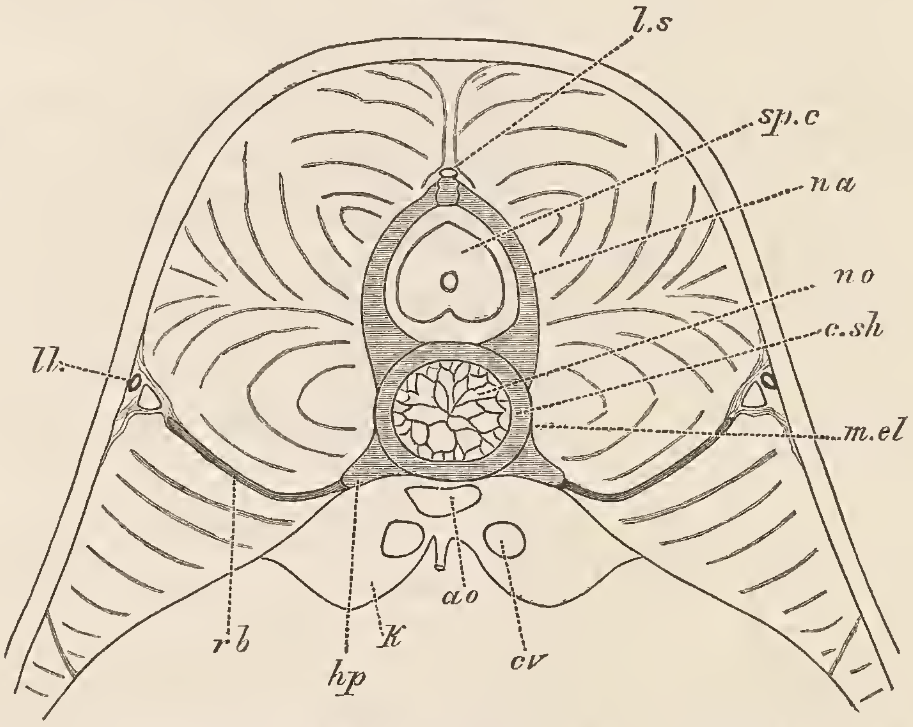
[†] "Ü. d. Entwick. d. Wirbelsäule d. *Lepidosteus*, mit. vergl. Anat. Bemerkungen." Jenaische Zeitschrift, Bd. iii., 1863.

[‡] 'Comparative Embryology,' vol. ii., pp. 462, 463.

(1) In the first place, the ribs in Elasmobranchii do not at first sight appear to be serially homologous with the ventral parts of the hæmal arches of the tail, but would rather seem to be lateral offshoots of the hæmal processes, while the hæmal arches of the tail appear to be completed by the coalescence of independent ventral prolongations of the hæmal processes.

(2) In the second place, the position of the ribs is different in the two groups. In Elasmobranchii they are situated between the dorso-lateral and ventro-lateral muscles (woodcut, fig. 1, *rb.*), while in *Lepidosteus* and other Ganoids they immediately girth the body-cavity.

Fig. 1.



Diagrammatic section through the trunk of an advanced embryo of *Scyllium*, to show the position of the ribs.

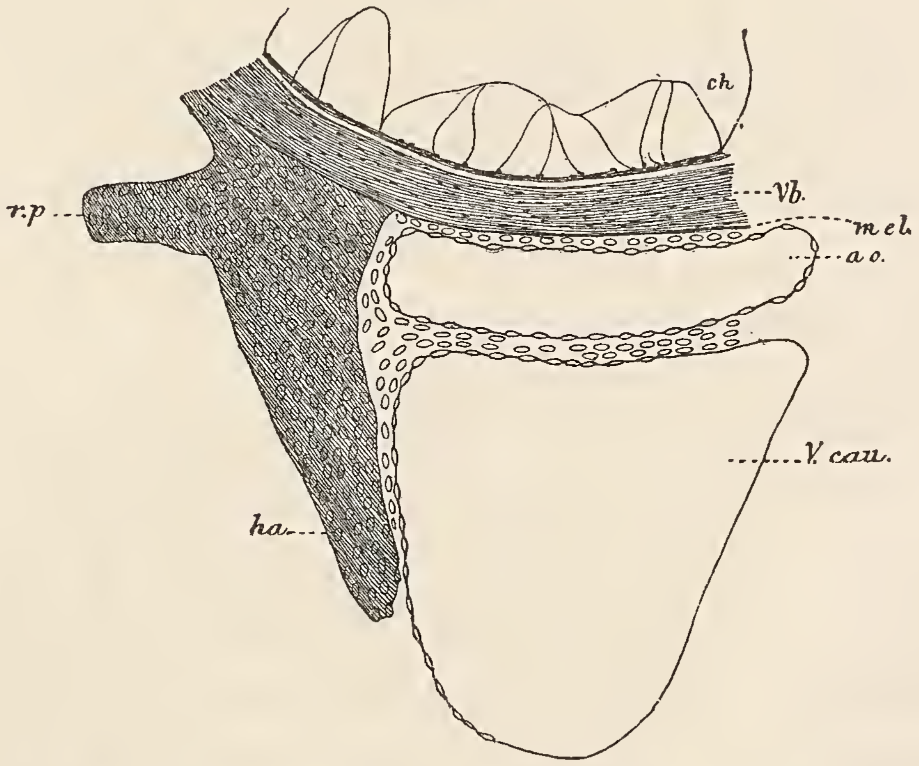
ao., aorta; *c.sh.*, cartilaginous notochordal sheath; *cv.*, cardinal vein; *hp.*, hæmal process; *k.*, kidney; *l.s.*, ligamentum longitudinale superius; *m.el.*, membrana elastica externa; *na.*, neural arch; *no.*, notochord; *ll.*, lateral line; *rb.*, rib; *sp.c.*, spinal cord.

There is much, therefore, to be said in favour of Götte's view. At the same time, there is another possible interpretation of the facts which would admit the homology of the ribs as well as of the hæmal arches throughout the Pisces.

Let us suppose, to start with, that the primitive arrangement of the parts is more or less nearly that found in *Lepidosteus*, where we have well-developed ribs in the region of the trunk, girthing the body-cavity, and uniting in the caudal region to form the ventral parts of the hæmal arches. It is easy to conceive that the ribs in the trunk might somewhat alter their position by passing into the muscles, along the intermuscular septa, till they come to lie between the dorso-lateral and ventro-lateral muscles, as in Elasmobranchii. *Lepidosteus* itself affords a proof that such a change in the position of the ribs is not impossible, in that it differs from other Ganoids and from Teleostei in the fact that the free ends of the ribs leave the neighbourhood of the body-cavity and penetrate into the muscles.

If it be granted that the mere difference in position between the ribs of Ganoids and Elasmobranchii is not of itself sufficient to disprove their homology, let us attempt to picture what would take place at the junction of the trunk and tail in a type in which the ribs had undergone the above change in position. On nearing the tail it may be supposed that the ribs would gradually become shorter, and at the same time alter their position, till finally they shaded off into ordinary hæmal processes. If, however, the hæmal canal became prolonged forwards by the formation of some additional complete or nearly complete hæmal arches, an alteration in the relation of the parts would necessarily take place. Owing to the position of the ribs, these structures could hardly assist in the new formation of the anterior part of the hæmal canal, but the continuation forwards of the canal would be effected by prolongations of the hæmal processes supporting the ribs. The new arches so formed would naturally be held to be homologous with the hæmal arches of the tail, though really not so, while the true nature of the ribs would also be liable to be misinterpreted, in that the ribs would appear to be lateral outgrowths of the hæmal processes of a wholly different nature to the ventral parts of the hæmal arches of the tail.

Fig. 2.



Transverse section through the ventral part of the notochord, and adjoining structures of an advanced *Scyllium* embryo at the root of the tail.

Vb., cartilaginous sheath of the notochord; *ha.*, hæmal process; *r.p.*, process to which the rib is articulated; *m.el.*, membrana elastica externa; *ch.*, notochord; *ao.*, aorta; *V.cau.*, caudal vein.

In some Elasmobranchii, as shown in the accompanying woodcut (fig. 2), in the transitional vertebræ between the trunk and the tail, the ribs are supported by lateral outgrowths of the hæmal processes, while the wholly independent prolongations of the hæmal processes appear to be about to give rise to the hæmal arches of the tail.

This peculiar state of things led GÖTTE, and subsequently one of us, to deny for Elasmobranchs all homology between the ribs and any part of the hæmal arches of the tail; but in view of the explanation just suggested, this denial was perhaps too hasty.

We are the more inclined to take this view because the researches of GÖTTE appear to show that an occurrence, in many respects analogous, has taken place in some Teleostei.

In Teleostei, JOHANNES MÜLLER, and following him GEGENBAUR, do not admit that the hæmal arches of the tail are in any part formed by the ribs. GEGENBAUR ('Elements of Comp. Anat.,' translation, p. 431) says, "In the Teleostei, the costiferous transverse processes" (what we have called the hæmal processes) "gradually converge in the caudal region, and form inferior arches, which are not homologous with those of Selachii and Ganoidei, although they also form spinous processes."

The opposite view, that the hæmal arches of the tail in Teleostei contain parts serially homologous with the basal parts of the hæmal processes as well as with the ribs, has been also maintained by many anatomists, *e.g.*, MECKEL, AUG. MÜLLER, &c., and has recently found a powerful ally in GÖTTE.

In many cases, the relations of the parts appear to be fundamentally those found in *Lepidosteus* and *Amia*, and GÖTTE has shown by his careful embryological investigations on *Esox* and *Anguilla*, that in these two forms there is practically conclusive evidence that the ribs as well as the hæmal costiferous processes of GEGENBAUR, which support them, enter into the formation of the hæmal arches of the tail.

In a great number of Teleostei, *e.g.*, the Salmon and most Cyprinoids, &c., the hæmal arches in the region of transition from the trunk to the tail have a structure which at first sight appears to support JOHANNES MÜLLER'S and GEGENBAUR'S view. The hæmal processes grow larger and meet each other ventrally; while the ribs articulated to them gradually grow smaller and disappear.

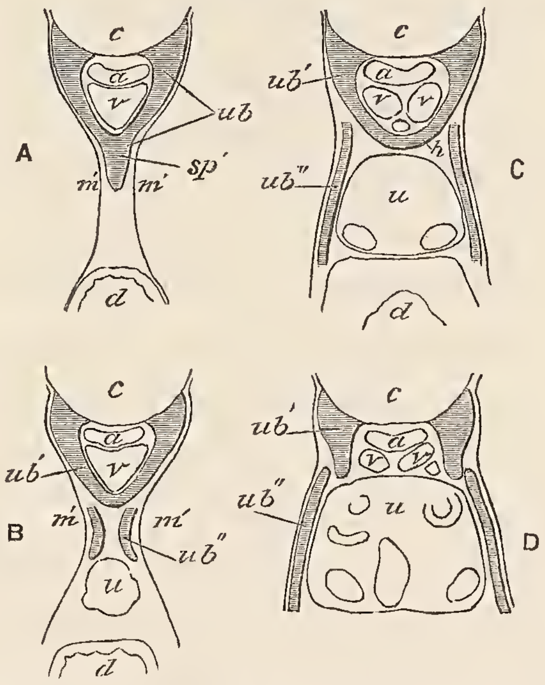
The Salmon is typical in this respect, and has been carefully studied by GÖTTE, who attempts to show (with, in our opinion, complete success) that the anterior hæmal arches are really not entirely homologous with the true hæmal arches behind, but that in the latter, the closure of the arch below is effected by the hæmal spine, which is serially homologous with a pair of coalesced ribs, while in the anterior hæmal arches, *i.e.*, those of the trunk, the closure of the arch is effected by a bridge of bone uniting the hæmal processes.

The arrangement of the parts just described, as well as the view of GÖTTE with reference to them, will be best understood from the accompanying woodcut (fig. 3), copied from GÖTTE'S memoir.

GÖTTE sums up his own results on this point in the following words (p. 138): "It follows from this, that the half rings, forming the hæmal canal in the hindermost trunk vertebræ of the Salmon, are not (with the exception of the last) completely homologous with those of the tail, but are formed by a connecting piece between the basal stumps (hæmal processes), which originates as a paired median process of these stumps."

The incomplete homology between the anterior hæmal arches and the true caudal hæmal arches which follow them is exactly what we suggest may be the case in Elasmobranchii, and if it be admitted in the one case, we see no reason why it should not also be admitted in the other.

Fig. 3.



Semi-diagrammatic transverse sections through the first caudal vertebra (A), the last trunk vertebra (B), and the two trunk vertebræ in front (C and D), of a Salmon embryo of 2–3 centims. (From Götte.)

ub., hæmal arch; *ub'.*, hæmal process; *ub''.*, rib; *c.*, notochord; *a.*, aorta; *v.*, vein; *h.*, connecting pieces between hæmal processes; *u.*, kidney; *d.*, intestine; *sp'.*, hæmal spine; *m'.*, muscles.

If this admission is made, the only ground for not regarding the ribs of Elasmobranchii as homologous with those of Ganoids is their different position, and we have already attempted to prove that this is not a fundamental point.

The results of our researches appear to us, then, to leave two alternatives as to the ribs of Fishes. One of these, which may be called Götte's view, may be thus stated: —The hæmal arches are homologous throughout the Pisces: in Teleostei, Ganoidei, and Dipnoi,* the ribs, placed on the inner face of the body-wall, are serially homologous with the ventral parts of the hæmal arches of the tail; in Elasmobranchii, on the other hand, the ribs are neither serially homologous with the hæmal arches of the tail nor homologous with the ribs of Teleostei and Ganoidei, but are outgrowths of the hæmal processes into the space between the dorso-lateral and ventro-lateral muscles, which may perhaps have their homologues in Teleostei and Ganoids in certain accessory processes of the vertebræ.

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* We find the serial homology of the ribs and ventral parts of the hæmal arches to be very clear in *Ceratodus*. Wiedersheim states that it is not clear in *Protopterus*, although he holds that the facts are in favour of this view.

The other view, which we are inclined to adopt, and the arguments for which have been stated in the preceding pages, is as follows:—The Teleostei, Ganoidei, Dipnoi, and Elasmobranchii are provided with homologous hæmal arches, which are formed by the coalescence below the caudal vein of simple prolongations of the primitive hæmal processes of the embryo. The canal enclosed by the hæmal arches can be demonstrated embryologically to be the aborted body cavity.

In the region of the trunk the hæmal processes and their prolongations behave somewhat differently in the different types. In Ganoids and Dipnoi, in which the most primitive arrangement is probably retained, the ribs are attached to the hæmal processes, and are placed immediately without the peritoneal membrane at the insertions of the intermuscular septa. These ribs are in many instances (*Lepidosteus*, *Acipenser*), and very probably in all, developed continuously with the hæmal processes, and become subsequently segmented from them. They are serially homologous with the ventral parts of the hæmal arches of the tail, which, like them, are in many instances (*Ceratodus*, *Lepidosteus*, *Polypterus*, and to some extent in *Amia*) segmented off from the basal parts of the hæmal arches.

In Teleostei the ribs have the same position and relations as those in Ganoids and Dipnoi, but their serial homology with the ventral parts of the hæmal processes of the tail, is often (*e.g.*, the Salmon) obscured by some of the anterior hæmal arches in the posterior part of the trunk being completed, not by the ribs, but by independent outgrowths of the basal parts of the hæmal processes.

In Elasmobranchii a still further divergence from the primitive arrangement is present. The ribs appear to have passed outwards along the intermuscular septa into the muscles, and are placed between the dorso-lateral and ventro-lateral muscles (a change of position of the ribs of the same nature, but affecting only their ends, is observable in *Lepidosteus*). This change of position, combined probably with the secondary formation of a certain number of anterior hæmal arches similar to those in the Salmon, renders their serial homology with the ventral parts of the hæmal processes of the tail far less clear than in other types, and further proof is required before such homology can be considered as definitely established.

This is not the place to enter into the obscure question as to how far the ribs of the Amphibia and Amniota are homologous with those of Fishes. It is to be remarked, however, that the ribs of the Urodela (1) occupy the same position in relation to the muscles as the Elasmobranch ribs, (2) that they are connected with the neural arches, and (3) that they coexist in the tail with the hæmal arches, and seem, therefore, to be as different as possible from the ribs of the Dipnoi.

Part IV.—*The skeleton of the ventral lobe of the tail fin, and its bearing on the nature of the tail fin of the various types of Pisces.*

In the embryos or larvæ of all the Elasmobranchii, Ganoidei, and Teleostei which have up to this time been studied, the unpaired fins arise as median longitudinal folds of the integument on the dorsal and ventral sides of the body, which meet at the apex of the tail. The tail at first is symmetrical, having a form which has been called diphycercal or protocercal. At a later stage, usually, though not always, parts of these fins atrophy, while other parts undergo a special development and constitute the permanent unpaired fins.

Since the majority of existing as well as extinct Fishes are provided with discontinuous fins, those forms, such as the Eel (*Anguilla*), in which the fins are continuous, have probably reverted to an embryonic condition: an evolutional process which is of more frequent occurrence than has usually been admitted.

In the caudal region there is almost always developed in the larvæ of the above groups a special ventral lobe of the embryonic fin a short distance from the end of the tail. In Elasmobranchii and Chondrostean Ganoids the portion of the embryonic tail behind this lobe persists through life, and a special type of caudal fin, which is usually called heterocercal, is thus produced. This type of caudal fin appears to have been the most usual in the earlier geological periods.

Simultaneously with the formation of the ventral lobe of the heterocercal caudal fin, the notochord with the vertebral tissues surrounding it, becomes bent somewhat dorsalwards, and thus the primitive caudal fin forms a dorsally directed lobe of the heterocercal tail. We shall call this part the dorsal lobe of the tail-fin, and the secondarily formed lobe the ventral lobe.

Lepidosteus and *Amia* (Wilder, No. 15) amongst the bony Ganoids, and, as has recently been shown by A. Agassiz,[*] most Teleostei acquire at an early stage of their development heterocercal caudal fins, like those of Elasmobranchii and the Chondrostean Ganoids; but in the course of their further growth the dorsal lobe partly atrophies, and partly disappears as such, owing to the great prominence acquired by the ventral lobe. A portion of the dorsally flexed notochord and of the cartilage or bone replacing or investing it remains, however, as an indication of the original dorsal lobe, though it does not project backwards beyond the level of the end of the ventral lobe, which in these types forms the terminal caudal fin.

The true significance of the dorsally flexed portion of the vertebral axis was first clearly stated by Huxley,[†] but as A. Agassiz has fairly pointed out in the paper already quoted, this fact does not in any way militate against the view put forward by

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[*] "On the Young Stages of some Osseous Fishes.—I. The Development of the Tail," Proc. of the American Academy of Arts and Sciences, vol. xiii., 1877.

[†] "Observations on the Development of some Parts of the Skeleton of Fishes," Quart. Journ. of Micr. Science, vol. vii., 1859.

L. AGASSIZ that there is a complete parallelism between the embryonic development of the tail in these Fishes and the palæontological development of this organ. We think that it is moreover convenient to retain the term homocercal for those types of caudal fin in which the dorsal lobe has atrophied so far as not to project beyond the ventral lobe.

We have stated these now well-known facts to enable the reader to follow us in dealing with the comparison between the skeleton supporting the fin-rays of the ventral lobe of the caudal fin, and that supporting the fin-rays of the remaining unpaired fins.

It has been shown that in *Lepidosteus* the unpaired fins fall into two categories, according to the nature of the skeletal parts supporting them. The fin-rays of the true ventral lobe of the caudal fin are supported by the spinous processes of certain of the hæmal arches. The remaining unpaired fins, including the anal fin, are supported by the so-called interspinous bones, which are developed independently of the vertebral column and its arches.

The question which first presents itself is, how far does this distinction hold good for other Fishes ? This question, though interesting, does not appear to have been greatly discussed by anatomists. Not unfrequently the skeletal supports of the ventral lobe of the caudal fin are assumed to be the same as those of the other fins.

DAVIDOFF,* for instance, in speaking of the unpaired fins of Elasmobranch embryos, says (p. 514): "The cartilaginous rays of the dorsal fins agreed not only in number with the spinous processes (as indeed is also found in the caudal fin of the full-grown Dog-fish)," &c.

THACHER,† again, in his memoir on the Median and Paired Fins, states at p. 284: "We shall here consider the skeleton of the dorsal and anal fins alone. That of the caudal fin has undergone peculiar modifications by the union of fin-rays with hæmal spines."

MIVART‡ goes into the question more fully. He points out (p. 471) that there is an essential difference between the dorsal and ventral parts of the caudal fin in Elasmobranchs, in that in the former the radials are more numerous than the vertebræ and unconformable to them, while in the latter they are equal in number to the vertebræ and continuous with them. "This," he goes on to say, "seems to point to a difference in nature between the dorsal and ventral portions of the caudal fin, in at least most Elasmobranchs." He further points out that *Polyodon* resembles Elasmobranchs. As to Teleostei, he does not express himself decidedly except in the case of *Murœna*, to which we shall return.

MIVART expresses himself as very doubtful as to the nature of the supports of the caudal fin, and thinks "that the caudal fin of different kinds of Fishes may have arisen in different ways in different cases."

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* "Beiträge z. vergl. Anat. d. hinteren Gliedmassen d. Fische," Morph. Jahrbuch, vol. v., 1879.

† Trans. of the Connecticut Acad., vol. iii., 1877.

‡ ST. GEORGE MIVART, "Fins of Elasmobranchs." Zool. Trans., vol. x.

An examination of the ventral part of the caudal fin in various Ganoids, Teleostei, and Elasmobranchii appears to us to show that there can be but little doubt that, in the majority of the members of these groups at any rate, and we believe in all, the same distinction between the ventral lobe of the caudal fin and the remaining unpaired fins is found as in *Lepidosteus*.

In the case of most Elasmobranchii, a simple inspection of the caudal fin suffices to prove this, and the anatomical features involved in this fact have usually been recognised; though, in the absence of embryological evidence, the legitimate conclusion has not always been drawn from them.

The difference between the ventral lobe of the caudal fin and the other fins in the mode in which the fin-rays are supported is as obvious in Chondrostean Ganoids as it is in Elasmobranchii; it would appear also to hold good for *Amia*. *Polypterus* we have had no opportunity of examining, but if, as there is no reason to doubt, the figure of its skeleton given by AGASSIZ ('Poissons Fossiles') is correct, there can be no question that the ventral lobe of the caudal fin is supported by the hæmal arches, and not by interspinous bones. In *Calamoicthys*, the tail of which we have had an opportunity of dissecting through the kindness of Professor PARKER, the fin-rays of the ventral lobe of the true caudal fin are undoubtedly supported by true hæmal arches.

There is no unanimity of opinion as to the nature of the elements supporting the fin-rays of the caudal fin of Teleostei.

HUXLEY,[*] in his paper on the development of the caudal fin of the Stickleback, holds that these elements are of the nature of interhæmal bones. He says (p. 39): "The last of these rings lay just where the notochord began to bend up. It was slightly longer than the bony ring which preceded it, and instead of having its posterior margin parallel with the anterior, it sloped from above downwards and backwards. Two short osseous plates, attached to the anterior part of the inferior surface of the penultimate ring, or rudimentary vertebral centrum, passed downwards and a little backwards, and abutted against a slender elongated mass of cartilage. Similar cartilaginous bodies occupy the same relation to corresponding plates of bone in the anterior vertebræ in the region of the anal fin; and it is here seen, that while the bony plates coalesce and form the inferior arches of the caudal vertebræ, the cartilaginous elements at their extremities become the interhæmal bones. The cartilage connected with the inferior arch of the penultimate centrum is therefore an 'interhæmal' cartilage. The anterior part of the inferior surface of the terminal ossification likewise has its osseous inferior arch, but the direction of this is nearly vertical, and though it is connected below with an element which corresponds in position with the interhæmal cartilage, this cartilage is five or six times as large, and constitutes a broad vertical plate, longer than it is deep, and having its longest axis inclined downwards and backwards. . . . .

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[*] "Observations on the Development of some parts of the Skeleton of Fishes." Quart. Journ. Micr. Science, vol. vii., 1859.

"Immediately behind and above this anterior hypural apophysis (as it may be termed) is another very much smaller vertical cartilaginous plate, which may be called the posterior hypural apophysis."

We have seen that MIVART expresses himself doubtful on the subject. GEGENBAUR* appears to regard them as hæmal arches.

The latter view appears to us without doubt the correct one. An examination of the tail of normal Teleostei shows that the fin-rays of that part of the caudal fin which is derived from the ventral lobe of the larva are supported by elements serially homologous with the hæmal arches, but in no way homologous with the interspinous bones of the anal fin. The elements in question formed of cartilage in the larva, become ossified in the adult, and are known as the hypural bones. They may appear in the form of a series of separate hæmal arches, corresponding in number with the primitive somites of this region, which usually, however, atrophy in the adult, or more often are from the first imperfectly segmented, and have in the adult the form of two or three or even of a single broad bony plate. The transitional forms between this state of things and that, for instance, in *Lepidosteus* are so numerous, that there can be no doubt that even the most peculiar forms of the hypural bones of Teleostei are simply modified hæmal arches.

This view of the hypural bones is, moreover, supported by embryological evidence, since AUG. MÜLLER† (p. 205) describes their development in a manner which, if his statements are to be trusted, leaves no doubt on this point.

There are a considerable number of Fishes which are not provided with an obvious caudal fin as distinct from the remaining unpaired fins, *i.e.*, Chimæra, Eels, and various Eel-like forms amongst Teleostei, and the Dipnoi. GEGENBAUR appears to hold that these Fishes ought to be classed together in relation to the structure of the caudal portion of their vertebral column, as he says on p. 431 of his 'Comparative Anatomy' (English translation): "In the Chimæræ, Dipnoi, and many Teleostei, the caudal portion of the vertebral column ends by gradually diminishing in size, but in most Fishes, &c."

For our purpose it will, however, be advisable to treat them separately.

The tail of Chimæra appears to us to be simply a peculiar modification of the typical Elasmobranch heterocercal tail, in which the true ventral lobe of the caudal fin may be recognised in the fin-fold immediately in front of the filamentous portion of the tail. In the allied genus *Callorhyncus* this feature is more distinct. The filamentous portion of the tail of Chimæra constitutes, according to the nomenclature adopted above, the true dorsal lobe, and may be partially paralleled in the filamentous dorsal lobe of the tail of the larval *Lepidosteus* (Plate 21, fig. 16).

The tail of the eel-like Teleostei is again undoubtedly a modification of the normal form of tail characteristic of the Teleostei, in which, however, the caudal fin has become very much reduced and merged into the prolongations of the anal and dorsal fins.

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* 'Elements of Comparative Anatomy.' (Translation), p. 431.

† "Beobachtungen zur vergl. Anat. d. Wirbelsäule." MÜLLER's Archiv., 1853.

This can be very clearly seen in Siluroid forms with an Eel-like tail, such as *Cnidoglanis*. Although the dorsal and ventral fins appear to be continuous round the end of the tail, and there is superficially no distinct caudal fin, yet an examination of the skeleton of *Cnidoglanis* shows that the end of the vertebral column is modified in the usual Teleostean fashion, and that the hæmal arches of the modified portion of the vertebral column support a small number of fin-rays; the adjoining ventral fin-rays being supported by independent osseous fin-supports (interspinous bones).

In the case of the Eel (*Anguilla anguilla*) Huxley (*loc. cit.*) long ago pointed out that the terminal portion of the vertebral column was modified in an analogous fashion to that of other Teleostei, and we have found that the modified hæmal arches of this part support a few fin-rays, through a still smaller number than in *Cnidoglanis*. The fin-rays so supported clearly constitute an aborted ventral lobe of the caudal fin.

Under these circumstances we think that the following statement by Mivart (Zool. Trans. vol. x., p. 471) is somewhat misleading :—

"As to the condition of this part (*i.e.*, the ventral lobe of the tail fin) in Teleosteans generally, I will not venture as yet to say anything generally, *except that it is plain that in such forms as Murœna, the dorsal and ventral parts of the caudal fin are similar in nature and homotypal with ordinary dorsal and anal fins.*"*

The italicized portion of this sentence is only true in respect to that part of the fringe of fin surrounding the end of the body, which is not only homotypal with, but actually part of, the dorsal and anal fins.

Having settled, then, that the tails of Chimæra and of Eel-like Teleostei are simply special modifications of the typical form of tail of the group of Fishes to which they respectively belong, we come to the consideration of the Dipnoi, in which the tail fin presents problems of more interest and greater difficulty than those we have so far had to deal with.

The undoubtedly very ancient and primitive character of the Dipnoi has led to the view, implicitly if not definitely stated in most text-books, that their tail-fin retains the character of the piscine tail prior to the formation of the ventral caudal lobe, a stage which is repeated embryologically in the pre-heterocercal condition of the tail in ordinary Fishes.

Through the want of embryological data, and in the absence of really careful histological examination of the tail of any of the Dipnoi, we are not willing to speak with very great confidence as to its nature; we are nevertheless of the opinion that the facts we can bring forward on this head are sufficient to show that the tail of the existing Dipnoi is largely aborted, so that it is more or less comparable with that of the Eel.

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* The italics are ours.

We have had opportunities of examining the structure of the tail of *Ceratodus* and *Protopterus* in dissected specimens in the Cambridge Museum. The vertebral axis runs to the ends of the tail without showing any signs of becoming dorsally flexed. At some distance from the end of the tail the fin-rays are supported by what are apparently segmented spinous prolongations of the neural and hæmal arches. The dorsal elements are placed above the longitudinal dorsal cord, and occupy therefore the same position as the independent elements of the neural arches of *Lepidosteus.* They are therefore to be regarded as homologous with the dorsal fin-supports or interspinous bones of other types. The corresponding ventral elements are therefore also to be regarded as interspinous bones.

In view of the fact that the fin-supports, whenever their development has been observed, are found to be formed independently of the neural and hæmal arches, we may fairly assume that this is also true for what we have identified as the interspinous elements in the Dipnoi.

The interspinous elements become gradually shorter as the end of the tail is approached, and it is very difficult from a simple examination of dissected specimens to make out how far any of the posterior fin-rays are supported by the hæmal arches only. To this question we shall return, but we may remark that, although there is a prolongation backwards of the vertebral axis beyond the last interspinous elements, composed it would seem of the coalesced neural and hæmal arches but without the notochord, yet by far the majority of the fin-rays which constitute the apparent caudal fin are supported by interspinous elements.

The grounds on which we hold that the tail of the Dipnoi is to be regarded as a degenerate rather than primitive type of tail are the following :—

(1) If it be granted that a diphycercal or protocercal form of tail must have preceded a heterocercal form, it is also clear that the ventral fin-rays of such a tail must have been supported, as in *Polypterus* and *Calamoicthys,* by hæmal arches, and not by interspinous elements; otherwise, a special ventral lobe, giving a heterocercal character to the tail, and provided with fin-rays supported only by hæmal arches, could never have become evolved from the protocercal tail fin. Since the ventral fin-rays of the tail of the Dipnoi are supported by interspinous elements and not by hæmal arches, this tail fin cannot claim to have the character of *that* primitive type of diphycercal or protocercal tail from which the heterocercal tail must be supposed to have been evolved.

(2) Since the nearest allies of the Dipnoi are to be found in *Polypterus* and the Crassopterygidæ of HUXLEY, and since in these forms (as evinced by the structure of the tail fin of *Polypterus,* and the transitional type between a heterocercal and diphycercal form of fin observable in fossil Crassopterygidæ) the ventral fin-rays of the caudal fin were clearly supported by hæmal arches and not by interspinous elements, it is rendered highly probable that the absence of fin-rays so supported in the Dipnoi is a result of degeneration of the posterior part of the tail.

[We use this argument without offering any opinion as to whether the diphycercal character of the tail of many Crassopterygidæ is primary or secondary.]

(3) The argument just used is supported by the degenerate and variable state of the end of the vertebral axis in the Dipnoi—a condition most easily explained by assuming that the terminal part of the tail has become aborted.

(4) We believe that in *Ceratodus* we have been able to trace a small number of the ventral fin-rays supported by hæmal arches only, but these rays are so short as not to extend so far back as some of the rays attached to the interspinous elements in front. These rays may probably be interpreted, like the more or less corresponding rays in the tail of the Eel, as the last remnant of a true caudal fin.

The above considerations appear to us to show with very considerable probability that the true caudal fin of the Dipnoi has become all but aborted like that of various Teleostei; and that the apparent caudal fin is formed by the anal and dorsal fins meeting round the end of the stump of the tail.

From the adult forms of Dipnoi we are, however, of opinion that no conclusion can be drawn as to whether their ancestors were provided with a diphycercal or a heterocercal form of caudal fin.

The general conclusions with reference to the tail fin at which we have arrived are the following:—

(1) The ventral lobe of the tail-fin of Pisces differs from the other unpaired fins in the fact that its fin-rays are directly supported by spinous processes of certain of the hæmal arches instead of independently developed interspinous bones.

(2) The presence or absence of fin-rays in the tail fin supported by hæmal arches may be used in deciding whether apparently diphycercal tail fins are aborted or primitive.

EXCRETORY AND GENERATIVE ORGANS.

I.—*Anatomy.*

The excretory organs of *Lepidosteus* have been described by MÜLLER (No. 13) and HYRTL (No. 11). These anatomists have given a fairly adequate account of the generative ducts in the female, and HYRTL has also described the male generative ducts and the kidney and its duct, but his description is contradicted by our observations in some of the most fundamental points.

In the female example of 100·5 centims. which we dissected, the kidney forms a paired gland, consisting of a narrow strip of glandular matter placed on each side of the vertebral column, on the dorsal aspect of the body cavity. It is covered on its ventral aspect by the oviduct and by its own duct, but is separated from both of these by a layer of the tough peritoneal membrane, through which the collecting tubes pass. It extends forwards from the anus for about three-fifths of the length of the body-cavity, and in our example had a total length of about 28 centims. (Plate 26, fig. 60, *k*).

Anteriorly the two kidneys are separated by a short interval in the median line, but posteriorly they come into contact, and are so intimately united as almost to constitute a single gland.

A superficial examination might lead to the supposition that the kidney extended forwards for the whole length of the body-cavity up to the region of the branchial arches, and HYRTL appears to have fallen into this error; but what appears to be its anterior continuation is really a form of lymphatic tissue, something like that of the spleen, filled with numerous cells. This matter (Plate 26, fig. 60, *ly.*) continues from the kidney forwards without any break, and has a colour so similar to that of the kidney as to be hardly distinguishable from it with the naked eye. The true anterior end of the kidney is placed about 3 centims. in front on the left side, and on the same level on the right side as the wide anterior end of the generative duct (Plate 26, fig. 60, *od.*). It is not obviously divided into segments, and is richly supplied with malpighian bodies.

It is clear from the above description that there is no trace of head-kidney or pronephros visible in the adult. To this subject we shall, however, again return.

As will appear from the embryological section, the ducts of the kidneys are probably simply the archinephric ducts, but to avoid the use of terms involving a theory, we propose in the anatomical part of our work to call them kidney ducts. They are thin-walled widish tubes coextensive with the kidneys. If cut open there may be seen on their inner aspect the numerous openings of the collecting tubes of the kidneys. They are placed ventrally to and on the outer border of the kidneys (Plate 26, fig. 60, *sg.*). Posteriorly they gradually enlarge, and approaching each other in the median line, coalesce, forming an unpaired vesicle or bladder (*bl.*)—about 6 centims. long in our example—opening by a median pore on a more or less prominent papilla (*u.g.*) behind the anus. The dilated portions of the two ducts are called by HYRTL the horns of the bladder.

The sides of the bladder and its so-called horns are provided with lateral pockets into which the collecting tubes of the kidney open. These pockets, which we have found in two female examples, are much larger in the horns of the bladder than in the bladder itself. Similar pockets, but larger than those we have found, have been described by HYRTL in the male, but are stated by him to be absent in the female. It is clear from our examples that this is by no means always the case.

HYRTL states that the wide kidney ducts, of which his description differs in no material point from our own, suddenly narrow in front, and, perforating the peritoneal lining, are continued forwards to supply the anterior part of the kidney. We have already shown that the anterior part of the kidney has no existence, and the kidney ducts supplying it are, according to our investigations, equally imaginary.

It was first shown by MÜLLER, whose observations on this point have been confirmed by HYRTL, &c., that the ovaries of *Lepidosteus* are continuous with their ducts, forming in this respect an exception to other Ganoids.

In our example of *Lepidosteus* the ovaries (Plate 26, fig. 60, *ov.*) were about 18 centims. in length. They have the form of simple sacs, filled with ova, and attached about their middle to their generative duct, and continued both backwards and forwards from their attachment into a blind process.

With reference to these sacs MÜLLER has pointed out—and the importance of this observation will become apparent when we deal with the development—that the ova are formed in the thickness of the inner wall of the sac. We hope to show that the inner wall of the sac is alone equivalent to the genital ridge of, for instance, the ovary of *Scyllium*. The outer aspect of this wall—*i.e.*, that turned towards the interior of the sac—is equivalent to the outer aspect of the Elasmobranch genital ridge, on which alone the ova are developed.* The sac into which the ova fall is, as we shall show in the embryological section, a special section of the body-cavity shut off from the remainder, and the dehiscence of the ova into this cavity is equivalent to their discharge into the body cavity in other forms.

The oviduct (Plate 26, fig. 60, *od.*) is a thin-walled duct of about 21 centims. in length in the example we are describing, continuous in front with the ovarian sac, and gradually tapering behind, till it ends (*od'.*) by opening into the dilated terminal section of the kidney duct on the inner side, a short distance before the latter unites with its fellow. It is throughout closely attached to the ureter and placed on its inner, and to some extent on its ventral, aspect. The hindermost part of the oviduct which runs beside the enlarged portion of the kidney duct—that portion called by HYRTL the horn of the urinary bladder—is so completely enveloped by the wall of the horn of the urinary bladder as to appear like a projection into the lumen of the latter structure, and the somewhat peculiar appearance which it presents in HYRTL's figure is due to this fact. In our examples the oviduct was provided with a simple opening into the kidney duct, on a slight papilla; the peculiar dilatations and processes of the terminal parts of the oviduct, which have been described by HYRTL, not being present.

The results we have arrived at with reference to the male organs are very different indeed from those of our predecessor, in that we find *the testicular products to be carried off by a series of vasa efferentia, which traverse the mesorchium, and are continuous with the uriniferous tubuli; so that the semen passes through the uriniferous tubuli into the kidney duct and so to the exterior. We have moreover been unable to find in the male a duct homologous with the oviduct of the female.*

This mode of transportation outwards of the semen has not hitherto been known to occur in Ganoids, though found in all Elasmobranchii, Amphibia, and Amniota. It is not, however, impossible that it exists in other Ganoids, but has hitherto been overlooked.

Our male example of *Lepidosteus* was about 60 centims. in length, and was no doubt mature. It was smaller than any of our female examples, but this according to GARMAN (vide, p. 361) is usual. The testes (Plate 26, fig. 58 A, *t.*) occupied a similar position to the ovaries, and were about 21 centims. long. They were, as is frequently the case with piscine testes, divided into a series of lobes (10–12), and were suspended by a delicate mesentery (mesorchium) from the dorsal wall of the abdomen on each side of the dorsal aorta. HYRTL (No. 11) states that air or quicksilver injected between the limbs of the mesentery, passed into a vas deferens homologous with the oviduct which joins the ureter. We have been unable to find such a vas deferens; but we have found in the mesorchium a number of tubes of a yellow colour, the colour being due to a granular substance quite unlike coagulated blood, but which appeared to us from microscopic examination to be the remains of spermatozoa.* These tubes to the number of 40–50 constitute, we believe, the vasa efferentia. Along the line of suspension of the testis on its inner border these tubes unite to form an elaborate network of tubes placed on the inner face of the testis—an arrangement very similar to that often found in Elasmobranchii (vide F. M. BALFOUR, 'Monograph on the Development of Elasmobranch Fishes,' plate 19, figs. 4 and 8).

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* 'Treatise on Comparative Embryology,' vol. i., p. 43.

We have figured this network on the posterior lobe of the testis (fig. 58 B), and have represented a section through it (fig. 59 A, *n.v.e.*), and through one of the vasa efferentia (*v.e.*) in the mesorchium. Such a section conclusively demonstrates the real nature of these passages: they are filled with sperm like that in the body of the testis, and are, as may be seen from the section figured, continuous with the seminal tubes of the testis itself.

At the attached base of the mesorchium the vasa efferentia unite into a longitudinal canal, placed on the inner side of the kidney duct (Plate 26, fig. 58 A, *l.c.*, also shown in section in Plate 26, fig. 59 B, *l.c.*). From this canal tubules pass off which are continuous with the tubuli uriniferi, as may be seen from fig. 59 B, but the exact course of these tubuli through the kidney could not be made out in the preparations we were able to make of the badly conserved kidney. HYRTL describes the arrangement of the vascular trunks in the mesorchium in the following way (No. 11, p. 6): "The mesorchium contains vascular trunks, viz., veins, which through their numerous anastomoses form a plexus at the hilus of the testis, whose efferent trunks, 13 in number, again unite into a plexus on the vertebral column, which is continuous with the cardinal veins." The arrangement (though not the number) of HYRTL's vessels is very similar to that of our vasa efferentia, and we cannot help thinking that a confusion of the two may have taken place; which, in badly conserved specimens, not injected with semen, would be very easy.

We have, as already stated, been unable to find in our dissections any trace of a duct homologous with the oviduct of the female, and our sections through the kidney and its ducts equally fail to bring to light such a duct. The kidney ducts are about 19 centims. in length, measured from the genital aperture to their front end. These ducts are generally similar to those in the female; they unite about 2 centims. from the genital pore to form an unpaired vesicle. Their posterior parts are considerably enlarged, forming what HYRTL calls the horns of the urinary bladder. In these enlarged portions, and in the wall of the unpaired urinary bladder, numerous transverse partitions are present, as correctly described by HYRTL, which are similar to those in the female, but more numerous. They give rise to a series of pits, at the blind ends of which are placed the openings of the kidney tubules. The kidney duct without doubt serves as vas deferens, and we have found in it masses of yellowish colour similar to the substance in the vasa efferentia identified by us as remains of spermatozoa.

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* The females we examined, which were no doubt procured at the same time as the male, had their oviducts filled with ova: and it is therefore not surprising that the vasa efferentia should be naturally injected with sperm.

II.—*Development.*

In the general account of the development we have already called attention to the earliest stages of the excretory system.

We may remind the reader that the first part of the system to be formed is the segmental or archinephric duct (Plate 23, figs. 28 and 29, *sg.*). This duct arises, as in Teleostei and Amphibia, by the constriction of a hollow ridge of the somatic mesoblast into a canal, which is placed in contiguity with the epiblast, along the line of junction between the mesoblastic somites and the lateral plates of mesoblast. Anteriorly the duct does not become shut off from the body-cavity, and also bends inwards towards the middle line. The inflected part of the duct is the first rudiment of the pronephros, and very soon becomes considerably dilated relatively to the posterior part of the duct.

The posterior part of each segmental duct acquires an opening into the cloacal section of the alimentary tract. Apart from this change, the whole of the ducts, except their pronephric sections, remain for a long time unaltered, and the next changes we have to speak of concern the definite establishment of the pronephros.

The dilated incurved portion of each segmental duct soon becomes convoluted, and by the time the embryo is about 10 millims. in length, but before the period of hatching, an important change is effected in the relations of their peritoneal openings.*

Instead of leading into the body-cavity, they open into an isolated chamber on each side (Plate 25, fig. 51, *pr.c.*), which we will call the *pronephric chamber.* The pronephric chamber is not, however, so far as we can judge, completely isolated from the body-cavity. We have not, it is true, detected with certainty at this stage a communication between the two; but in later stages, in larvæ of from 11 to 26 millims., we have found a richly ciliated passage leading from the body-cavity into the pronephros on each side (Plate 25, fig. 52, *p.f.p.*). We have not succeeded in determining with absolute certainty the exact relations between this passage and the tube of the pronephros, but we are inclined to believe that it opens directly into the pronephric chamber just spoken of.

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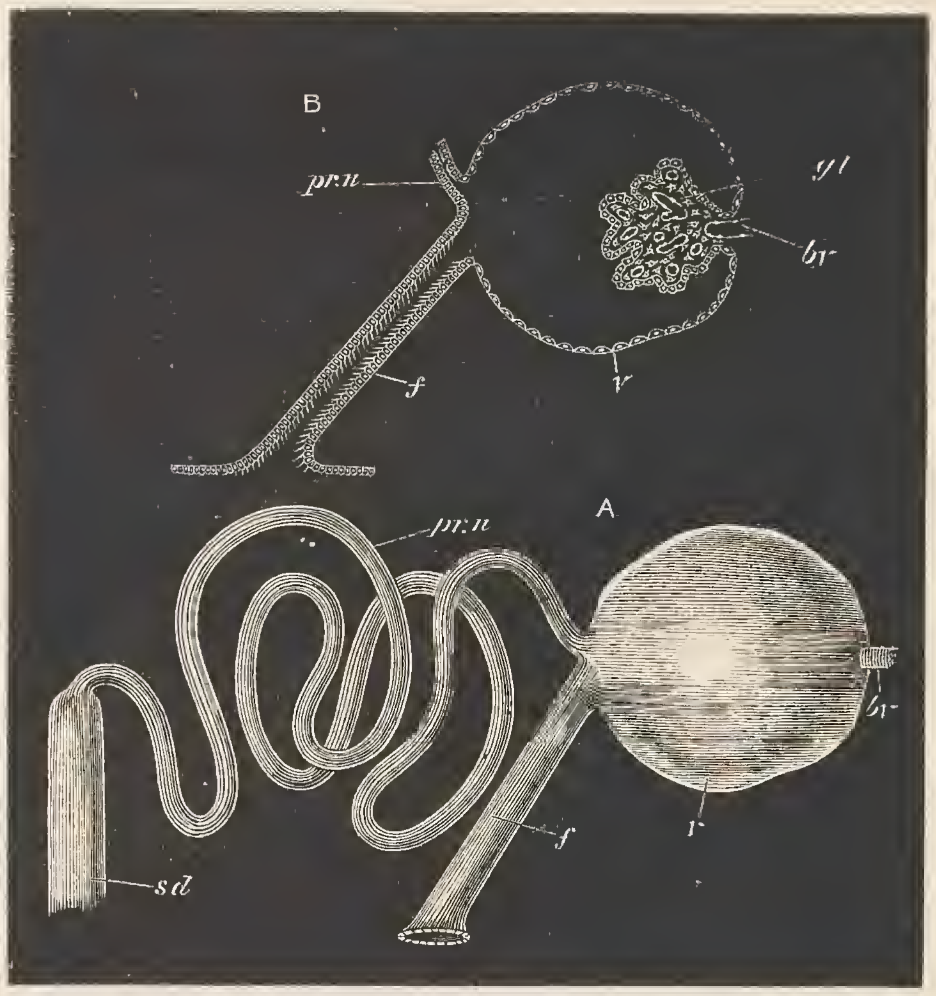
* The change is probably effected somewhat earlier than would appear from our description, but our specimens were not sufficiently well preserved to enable us to speak definitely as to the exact period.

As we hope to show, this chamber soon becomes largely filled by a vascular glomerulus. On the accomplishment of these changes, the pronephros is essentially provided with all the parts typically present in a segment of the mesonephros (woodcut, fig. 4). There is a peritoneal tube (*f.*),* opening into a vesicle (*v.*); from near the neck of the peritoneal tube there comes off a convoluted tube (*pr.n.*), forming the main mass of the pronephros, and ending in the segmental duct (*sd.*).

Fig. 4.



Diagrammatic views of the pronephros of *Lepidosteus*.

A, pronephros supposed to be isolated and seen from the side; B, section through the vesicle of the pronephros and the ciliated peritoneal funnel leading into it; *pr.n.*, coiled tube of pronephros; *sd.*, segmental or archinephric duct; *f.*, peritoneal funnel; *v.*, vesicle of pronephros; *bv.*, blood vessel of glomerulus; *gl.*, glomerulus.

The different parts do not, however, appear to have the same morphological significance as those in the mesonephros.

Judging from the analogy of Teleostei, the embryonic structure of whose pronephros is strikingly similar to that of *Lepidosteus,* the two pronephric chambers into which the segmental ducts open are constricted off sections of the body-cavity.

With the formation of the convoluted duct opening into the isolated section of the body-cavity we may speak of a definite pronephros as having become established. The pronephros is placed, as can be made out in later stages, on the level of the opening of the air-bladder into the throat.

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* We feel fairly confident that there is only one pronephric opening on each side, though we have no single series of sections sufficiently complete to demonstrate this fact with absolute certainty.

The pronephros increases in size, so far as could be determined, by the further convolution of the duct of which it is mainly formed; and the next change of importance which we have noticed is the formation of a vascular projection into the pronephric chamber, forming the glomerulus already spoken of (vide woodcut, fig. 4, *gl.*), which is similar to that of the pronephros of Teleostei. We first detected these glomeruli in an embryo of about 15 millims., some days after hatching (Plate 25, fig. 52, *gl.*), but it is quite possible that they may be formed considerably earlier.

In the same embryo in which the glomeruli were found we also detected for the first time a *mesonephros* consisting of a series of isolated segmental or nephridial tubes, placed posteriorly to the pronephros along the dorsal wall of the abdomen.

These were so far advanced at this stage that we are not in a position to give any account of their mode of origin. They are, however, formed independently of the segmental ducts, and in the establishment of the junction between the two structures, there is no outgrowth from the segmental duct to meet the segmental tubes. We could not at this stage find peritoneal funnels of the segmental tubes, though we have met with them at a later stage (Plate 25, fig. 53, *p.f.*), and our failure to find them at this stage is not to be regarded as conclusive against their existence.

A very considerable space exists between the pronephros and the foremost segmental tube of the mesonephros. The anterior mesonephric tubes are, moreover, formed earlier than the posterior.

In the course of further development, the mesonephric tubules increase in size, so that there ceases to be an interval between them, the mesonephros thus becoming a continuous gland. In an embryo of 26 millims. there was no indication of the formation of segmental tubes to fill up the space between the pronephros and mesonephros.

The two segmental ducts have united behind into an unpaired structure in an embryo of 11 millims. This structure is no doubt the future unpaired urinogenital chamber (Plate 26, figs. 58 A, and 60, *bl.*). Somewhat later, the hypoblastic cloaca becomes split into two sections, the hinder one receiving the coalesced segmental ducts, and the anterior remaining continuous with the alimentary tract. The opening of the hinder one forms the urinogenital opening, and that of the anterior the anus.

In an older larva of about 5·5 centims. the pronephros did not exhibit any marked signs of atrophy, though the duct between it and the mesonephros was somewhat reduced and surrounded by the trabecular tissue spoken of in connexion with the adult. In the region between the pronephros and the front end of the fully developed part of the mesonephros very rudimentary tubules had become established.

The latest stage of the excretory system which we have studied is in a young Fish of about 11 centims. in length. The special interest of this stage depends upon the fact that the ovary is already developed, and not only so, but the formation of the oviducts has commenced, and their condition at this stage throws considerable light on the obscure problem of their nature in the Ganoids.

Unfortunately, the head of the young Fish had been removed before it was put into our hands, so that it was impossible for us to determine whether the pronephros was still present; but as we shall subsequently show, the section of the segmental duct, originally present between the pronephros and the front end of the permanent kidney or mesonephros, has in any case disappeared.

In addition to an examination of the excretory organs *in situ*, which showed little except the presence of the generative ridges, we made a complete series of sections through the excretory organs for their whole length (Plate 26, figs. 54–57).

Posteriorly these sections showed nothing worthy of note, the excretory organs and their ducts differing in no important particular from these organs as we have described them in the adult, except in the fact that the segmental ducts are not joined by the oviducts.

Some little way in front of the point where the two segmental ducts coalesce to form the urinary bladder, the genital ridge comes into view. For its whole extent, except near its anterior part (of which more hereafter) this ridge projects freely into the body-cavity, and in this respect the young Fish differs entirely from the adult. As shown in Plate 26, figs. 56 and 57 (*g.r.*), it is attached to the abdominal wall on the ventral side of, and near the inner border of each kidney. The genital ridge itself has a structure very similar to that which is characteristic of young Elasmobranchii, and it may be presumed of young Fishes generally. The free edge of the ridge is swollen, and this part constitutes the true generative region of the ridge, while its dorsal portion forms the supporting mesentery. The ridge itself is formed of a central stroma and a germinal epithelium covering it. The epithelium is thin on the whole of the inner aspect of the ridge, but, just as in Elasmobranchii, it becomes greatly thickened for a band-like strip on the outer aspect. Here, the epithelium is several layers deep, and contains numerous primitive germinal cells (*p.o.*).

Though the generative organs were not sufficiently advanced for us to decide the point with certainty, the structure of the organ is in favour of the view that this specimen was a female, and, as will be shown directly, there can on other grounds be no doubt that this is so. The large size of the primitive germinal cells (primitive ova) reminded us of these bodies in Elasmobranchii.

In the region between the insertion of the genital ridge (or ovary, as we may more conveniently call it) and the segmental duct we detected the openings of a series of peritoneal funnels of the excretory tubes (Plate 26, fig. 57, *p.f.*), which clearly therefore persist till the young Fish has reached a very considerable size.

As we have already said, the ovary projects freely into the body-cavity for the greater part of its length. Anteriorly, however, we found that a lamina extended from the free ventral edge of the ovary to the dorsal wall of the body-cavity, to which it was attached on the level of the outer side of the segmental duct. A somewhat triangular channel was thus constituted, the inner wall of which was formed by the ovary, the outer by the lamina just spoken of, and the roof by the strip of the peritoneum of the abdominal wall covering that part of the ventral surface of the kidney in which the openings of the peritoneal funnels of the excretory tubes are placed. The structure of this canal will be at once understood by the section of it shown in Plate 26, fig. 55.

There can be no doubt that this canal is the commencing ovarian sac. On tracing it backwards we found that the lamina forming its outer wall arises as a fold growing upwards from the free edge of the genital ridge meeting a downward growth of the peritoneal membrane from the dorsal wall of the abdomen; and in Plate 26, fig. 56, these two laminæ may be seen before they have met. Anteriorly the canal becomes gradually smaller and smaller in correlation with the reduced size of the ovarian ridge, and ends blindly nearly on a level with the front end of the excretory organs.

It should be noted that, owing to the mode of formation of the ovarian sac, the outer side of the ovary with the band of thickened germinal epithelium is turned towards the lumen of the sac; and thus the fact of the ova being formed on the inner wall of the genital sac in the adult is explained, and the comparison which we instituted in our description of the adult between the inner wall of the genital sac and the free genital ridge of Elasmobranchs receives its justification.

It is further to be noticed that, from the mode of formation of the ovarian sac, the openings of the peritoneal funnels of the excretory organs ought to open into its lumen; and if these openings persist in the adult, they will no doubt be found in this situation.

Before entering on further theoretical considerations with reference to the oviduct, it will be convenient to complete our description of the excretory organs at this stage.

When we dissected the excretory organs out, and removed them from the body of the young Fish, we were under the impression that they extended for the whole length of the body-cavity. Great was our astonishment to find that slightly in front of the end of the ovary both excretory organs and segmental ducts grew rapidly smaller and finally vanished, and that what we had taken to be the front part of the kidney was nothing else but a linear streak of tissue formed of cells with peculiar granular contents supported in a trabecular work (Plate 26, fig. 54). This discovery first led us to investigate histologically what we, in common with previous observers, had supposed to be the anterior end of the kidneys in the adult, and to show that they were nothing else but trabecular tissue with cells like that of lymphatic glands. The interruption of the segmental duct at the commencement of this tissue demonstrates that if any rudiment of the pronephros still persists, it is quite functionless, in that it is not provided with a duct.

III.—*Theoretical considerations.*

There are three points in our observations on the urinogenital system which appear to call for special remark. The first of these concerns the structure and fate of the pronephros, the second the nature of the oviduct, and the third the presence of vasa efferentia in the male.

Although the history we have been able to give of the pronephros is not complete, we have nevertheless shown that in most points it is essentially similar to the pronephros of Teleostei. In an early stage we find the pronephros provided with a peritoneal funnel opening into the body-cavity. At a later stage we find that there is connected with the pronephros on each side, a cavity—the pronephric cavity—into which a glomerulus projects. This cavity is in communication on the one hand with the lumen of the coiled tube which forms the main mass of the pronephros, and on the other hand with the body-cavity by means of a richly ciliated canal (woodcut, fig. 4, p. 416).

In Teleostei the pronephros has precisely the same characters, except that the cavity in which the glomerulus is placed is without a peritoneal canal.

The questions which naturally arise in connexion with the pronephros are: (1) what is the origin of the above cavity with its glomerulus; and (2) what is the meaning of the ciliated canal connecting this cavity with the peritoneal cavity?

We have not from our researches been able to answer the first of these questions. In Teleostei, however, the origin of this cavity has been studied by ROSENBERG* and GÖTTE.† According to the account of the latter, which we have not ourselves confirmed but which has usually been accepted, the front end of the segmental duct, instead of becoming folded off from the body-cavity, becomes included in a kind of diverticulum of the body-cavity, which only communicates with the remainder of the body-cavity by a narrow opening. On the inner wall of this diverticulum a projection is formed which becomes a glomerulus. At this stage in the development of the pronephros we have essentially the same parts as in the fully formed pronephros of *Lepidosteus*, the only difference being that the passage connecting the diverticulum containing the glomerulus with the remainder of the body-cavity is short in Teleostei, and in *Lepidosteus* forms a longish ciliated canal. In Teleostei the opening into the body-cavity becomes soon closed. If the above comparison is justified, and if the development of these parts in *Lepidosteus* takes place as it is described as doing in Teleostei, there can, we think, be no doubt that the ciliated canal of *Lepidosteus*, which connects the pronephric cavity with the body-cavity, is a persisting communication between this cavity and the body-cavity; and that *Lepidosteus* presents in this respect a more primitive type of pronephros than Teleostei.

It may be noted that in *Lepidosteus* the whole pronephros has exactly the character of a single segmental tube of the mesonephros. The pronephric cavity with its

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* ROSENBERG, Untersuch. üb. d. Entwick. d. Teleostierniere. Dorpat, 1867.

† GÖTTE, Entwick. d. Unke, p. 826.

glomerulus is identical in structure with a malpighian body. The ciliated canal is similar in its relations to the peritoneal canal of such a segmental tube, and the coiled portion of the pronephros resembles the secreting part of the ordinary segmental tube. This comparison is no doubt an indication that the pronephros is physiologically very similar to the mesonephros, and so far justifies SEDGWICK'S[*] comparison between the two, but it does not appear to us to justify the morphological conclusions at which he has arrived, or to necessitate any modification in the views on this subject expressed by one of us.[†]

The genital ducts of Ganoids and Teleostei have for some time been a source of great difficulty to morphologists; and any contributions with reference to the ontogeny of these structures are of interest.

The essential point which we have made out is that the anterior part of the oviduct of *Lepidosteus* arises by a fold of the peritoneum attaching itself to the free edge of the genital ridge. We have not, unfortunately, had specimens old enough to decide how the posterior part of the oviduct is formed; and although in the absence of such stages it would be rash in the extreme to speak with confidence as to the nature of this part of the duct, it may be well to consider the possibilities of the case in relation to other Ganoids and Teleostei.

The simplest supposition would be that the posterior part of the genital duct had the same origin as the anterior, *i.e.*, that it was formed for its whole length by the concrescence of a peritoneal fold with the genital ridge, and that the duct so formed opened into the segmental duct.

The other possible supposition is that a true Müllerian duct—*i.e.*, a product of the splitting of the segmental duct—is subsequently developed, and that the open end of this duct coalesces with the duct which has already began to be formed in our oldest larva.

In attempting to estimate the relative probability of these two views, one important element is the relation of the oviducts of *Lepidosteus* to those of other Ganoids.

In all other Ganoids (vide HYRTL, No. 11) there are stated to be genital ducts in both sexes which are provided at their anterior extremities with a funnel-shaped mouth open to the abdominal cavity. At first sight, therefore, it might be supposed that they had no morphological relationship with the oviducts of *Lepidosteus*, but, apart from the presence of a funnel-shaped mouth, the oviducts of *Lepidosteus* are very similar to those of Chondrostean Ganoids, being thin-walled tubes opening on a projecting papilla into the dilated kidney ducts (horns of the urinary bladder, HYRTL). These relations seem to prove beyond a doubt that the oviduct of *Lepidosteus* is for its major part homologous with the genital ducts of other Ganoids.

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[*] SEDGWICK, "Early Development of the Wolffian Duct and anterior Wolffian Tubules in the Chick; with some Remarks on the Vertebrate Excretory System." Quart. Jour. of Micros. Science, vol. xxi., 1881.

[†] F. M. BALFOUR, 'Comparative Embryology,' vol. ii., pp. 600–603.

The relationship of the genital ducts to the kidney ducts in *Amia* and *Polypterus* is somewhat different from that in the Chondrostei and *Lepidosteus*. In *Amia* the ureters are so small that they may be described rather as joining the coalesced genital ducts than *vice versâ*, although the apparent coalesced portion of the genital ducts is shown to be really part of the kidney ducts by receiving the secretion of a number of mesonephric tubuli. In *Polypterus* the two ureters are stated to unite, and open by a common orifice into a sinus formed by the junction of the two genital ducts, which has not been described as receiving directly the secretion of any part of the mesonephros.

It has been usual to assume that the genital ducts of Ganoids are true Müllerian ducts in the sense above defined, on the ground that they are provided with a peritoneal opening and that they are united behind with the kidney ducts. In the absence of ontological evidence this identification is necessarily provisional. On the assumption that it is correct we should have to accept the second of the two alternatives above suggested as to the development of the posterior parts of the oviduct in *Lepidosteus*.

There appear to us, however, to be sufficiently serious objections to this view to render it necessary for us to suspend our judgment with reference to this point. In the first place, if the view that the genital ducts are Müllerian ducts is correct, the true genital ducts of *Lepidosteus* must necessarily be developed at a later period than the secondary attachment between their open mouths and the genital folds, which would, to say the least of it, be a remarkable inversion of the natural order of development. Secondly, the condition of our oldest larva shows that the Müllerian duct, if developed later, is only split off from quite the posterior part of the segmental duct; yet in all types in which the development of the Müllerian duct has been followed, its anterior extremity, with the abdominal opening, is split off from either the foremost or nearly the foremost part of the segmental duct.

Judging from the structure of the adult genital duct of other Ganoids they must also be developed only from the posterior part of the segmental duct, and this peculiarity so struck one of us that in a previous paper[*] the suggestion was put forward that the true Ganoid genital ducts were perhaps not Müllerian ducts, but enlarged segmental tubes with persisting abdominal funnels belonging to the mesonephros.

If the possibility of the oviduct of *Lepidosteus* not being a Müllerian duct is admitted, a similar doubt must also exist as to the genital ducts of other Ganoids, and we must be prepared to show that there is a reasonable ground for scepticism on this point. We would in this connexion point out that the second of the two arguments urged against the view that the genital duct of *Lepidosteus* is not a Müllerian duct applies with equal force to the case of all other Ganoids.

The short funnel-shaped genital duct of the Chondrostei is also very unlike undoubted Müllerian ducts, and could moreover easily be conceived as originating by a fold of the peritoneum, a slight extension of which would give rise to a genital duct like that of *Lepidosteus*.

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[*] F. M. BALFOUR, "On the Origin and History of the Urinogenital Organs of Vertebrates," Journ. of Anat. and Phys., vol. x., 1876.

The main difficulty of the view that the genital ducts of Ganoids are not Müllerian ducts lies in the fact that they open into the segmental duct. While it is easy to understand the genesis of a duct from a folding of the peritoneum, and also easy to understand how such a duct might lead to the exterior by coalescing, for instance, with an abdominal pore, it is not easy to see how such a duct could acquire a communication with the segmental duct.

We do not under these circumstances wish to speak dogmatically, either in favour of or against the view that the genital ducts of Ganoids are Müllerian ducts. Their ontogeny would be conclusive on this matter, and we trust that some of the anatomists who have the opportunity of studying the development of the Sturgeon will soon let us know the facts of the case. If there are persisting funnels of the mesonephric segmental tubes in adult Sturgeons, some of them ought to be situated within the genital ducts, if the latter are not Müllerian ducts; and naturalists who have the opportunity ought also to look out for such openings.

The mode of origin of the anterior part of the genital duct of *Lepidosteus* appears to us to tell strongly in favour of the view, already regarded as probable by one of us,* that the Teleostean genital ducts are derived from those of Ganoids; and if, as appears to us indubitable, the most primitive type of Ganoid genital ducts is found in the Chondrostei, it is interesting to notice that the remaining Ganoids present in various ways approximations to the arrangement typically found in Teleostei. *Lepidosteus* obviously approaches Teleostei in the fact of the ovarian ridge forming part of the wall of the oviduct, but differs from the Teleostei in the fact of the oviduct opening into the kidney ducts, instead of each pair of ducts having an independent opening in the cloaca, and in the fact that the male genital products are not carried to the exterior by a duct homologous with the oviduct. *Amia* is closer to the Teleostei in the arrangement of the posterior part of the genital ducts, in that the two genital ducts coalesce posteriorly; while *Polypterus* approaches still nearer to the Teleostei in the fact that the two genital ducts and the two kidney ducts unite with each other before they join; and in order to convert this arrangement into that characteristic of the Teleostei we have only to conceive the coalesced ducts of the kidneys acquiring an independent opening into the cloaca behind the genital opening.

The male genital ducts.—The discovery of the vasa efferentia in *Lepidosteus*, carrying off the semen from the testis, and transporting it to the mesonephros, and thence through the mesonephric tubes to the segmental duct, must be regarded as the most important of our results on the excretory system.

It proves in the first place that the transportation outwards of the genital products of both sexes by homologous ducts, which has been hitherto held to be universal in

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* F. M. Balfour, 'Comparative Embryology,' vol. ii., p. 605.

Ganoids, and which, in the absence of evidence to the contrary, must still be assumed to be true for all Ganoids except *Lepidosteus*, is a secondary arrangement. This conclusion follows from the fact that in Elasmobranchs, &c., which are not descendants of the Ganoids, the same arrangement of seminal ducts is found as in *Lepidosteus*, and it must therefore have been inherited from an ancestor common to the two groups.

If, therefore, the current statements about the generative ducts of Ganoids are true, the males must have lost their vasa efferentia, and the function of vas deferens must have been taken by the homologue of the oviduct, presumably present in the male. The Teleostei must, moreover, have sprung from Ganoidei in which the vasa efferentia had become aborted.

Considerable phylogenetic difficulties as to the relationships of Ganoidei and Elasmobranchii are removed by the discovery that Ganoids were originally provided with a system of vasa efferentia like that of Elasmobranchii.

THE ALIMENTARY CANAL AND ITS APPENDAGES.

I.—*Anatomy.*

AGASSIZ (No. 2) gives a short description with a figure of the viscera of *Lepidosteus* as a whole. VAN DER HŒVEN has also given a figure of them in his memoir on the air-bladder of this form (No. 8), and JOHANNES MÜLLER first detected the spiral valve and gave a short account of it in his memoir (No. 13). STANNIUS, again, makes several references to the viscera of *Lepidosteus* in his anatomy of the Vertebrata, and throws some doubt on MÜLLER's determination of the spiral valve.

The following description refers to a female *Lepidosteus* of 100·5 centims. (Plate 27, fig. 66).

With reference to the mouth and pharynx, we have nothing special to remark. Immediately behind the pharynx there comes an elongated tube, which is not divisable into stomach and œsophagus, and may be called the stomach (*st.*). It is about 44·6 centims. long, and gradually narrows from the middle towards the hinder or pyloric extremity. It runs straight backwards for the greater part of its length, the last 3·8 centims., however, taking a sudden bend forwards. For about half its length the walls are thin, and the mucous membrane is smooth; in the posterior half the walls are thick, and the mucous membrane is raised into numerous longitudinal ridges. The peculiar glandular structure of the epithelium of this part in the embryo is shown in Plate 27, fig. 62 (*st.*). Its opening into the duodenium is provided with a very distinct pyloric valve (*py.*). This valve projects into a kind of chamber, freely communicating with the duodenum, and containing four large pits (*c′.*), into each of which a group of pyloric cæca opens. These cæca form a fairly compact gland (*c.*) about 6·5 centims. long, which overlaps the stomach anteriorly, and the duodenum posteriorly.

Close to the pyloric valve, on its right side, is a small papilla, on the apex of which the bile duct opens (*b.d'.*).

A small, apparently glandular, mass closely connected with the bile duct, in the position in which we have seen the pancreas in the larva (Plate 27, figs. 62 and 63, *p.*), is almost certainly a rudimentary pancreas, like that of many Teleostei; but its preservation was too bad for histological examination. We believe that the pancreas of *Lepidosteus* has hitherto been overlooked.

The small intestine passes straight backwards for about 8 centims., and then presents three compact coils. From the end of these a section, about 5 centims. long, the walls of which are much thicker, runs forwards. The intestine then again turns backwards, making one spiral coil. This spiral part passes directly, without any sharp line of demarcation, into a short and straight tube, which tapers slightly from before backwards, and ends at the anus. The mucous membrane of the intestine for about the first 3·5 centims. is smooth, and the muscular walls thin: the rest of the small intestine has thick walls, and the mucous membrane is reticulated.

A short spiral valve (*sp.v.*), with a very rudimentary epithelial fold, making nearly two turns, begins in about the posterior half of the spiral coil of the intestine, extending backwards for slightly less than half the straight terminal portion of the intestine, and ending 4 centims. in front of the anus. Its total length in one example was about 4·5 centims.

The termination of the spiral valve is marked by a slight constriction, and we may call the straight portion of the intestine behind it the rectum (*rc.*).

The posterior part of the intestine, from the beginning of the spiral valve to the anus, is *connected with the ventral wall of the abdomen by a mesentery.*

The air-bladder (*a.b.*) is 45 centims. long, and opens into the alimentary canal by a slit-like aperture (*a.b'.*) on the median dorsal line, immediately behind the epipharyngeal teeth. Each lip of this aperture is largely formed by a muscular cushion, thickest at its posterior end, and extending about 6 millims. behind the aperture itself. A narrow passage is bounded by these muscular walls, which opens dorsally into the air-bladder.

The air-bladder is provided with two short anterior cornua, and tapers to a point behind: it shows no indication of any separation into two parts. A strong band of connective tissue runs along the inner aspect of its whole dorsal region, from which there are given off on each side—at intervals of about 12 millims. anteriorly, gradually increasing to 18 millims. posteriorly—bands of muscle, which pass outwards towards its side walls, and then spread out into the numerous reticulations with which the air-bladder is lined throughout. By the contraction of these muscles the cavity of the air-bladder can doubtless be very much diminished.

The main muscular bands circumscribe a series of more or less complete chambers, which were about twenty-seven in number on each side in our example. The chambers are confined to the sides, so that there is a continuous cavity running through the central part of the organ. The whole organ has the characteristic structure of a simple lung.

The liver (*lr.*) consists of a single elongated lobe, about 32 centims. long, tapering anteriorly and posteriorly, the anterior half being on the average twice as thick as the posterior half. The gall-bladder (*g.b.*) lies at its posterior end, and is of considerable size, tapering gradually so as to pass insensibly into the bile duct. The hepatic duct (*hp.d.*) opens into the gall-bladder at its anterior end.

The spleen (*s.*) is a large, compact, double gland, one lobe lying in the turn of the intestine immediately above the spiral valve, and the other on the opposite side of the intestine, so that the intestine is nearly embraced between the two lobes.

II.—*Development.*

We have already described in detail the first formation of the alimentary tract so far as we have been able to work it out, and we need only say here that the anterior and posterior ends of the canal become first formed, and that these two parts gradually elongate, so as to approach each other; the growth of the posterior part is, however, the most rapid. The junction of the two parts takes place a very short distance behind the opening of the bile duct into the intestine.

For some time after the two parts of the alimentary tract have nearly met, the ventral wall of the canal at this point is not closed; so that there is left a passage between the alimentary canal and the yolk-sac, which forms a vitelline duct.

After the yolk-sac has ceased to be visible as an external appendage it still persists within the abdominal cavity. It has, however, by this stage ceased to communicate with the gut, so that the eventual absorption of the yolk is no doubt entirely effected by the vitelline vessels. At these later stages of development we have noticed that numerous yolk nuclei, like those met with in Teleostei and Elasmobranchii,* are still to be found in the yolk.

It will be convenient to treat the history of sections of the alimentary tract in front of and behind the vitelline duct separately. The former gives rise to the pharyngeal region, the œsophagus, the stomach, and the duodenum.

The pharyngeal region, immediately after it has become established, gives rise to a series of paired pouches. These may be called the branchial pouches, and are placed between the successive branchial arches. The first or hyomandibular pouch, placed between the mandibular and hyoid arches, has rather the character of a double layer of hypoblast than of a true pouch, though in parts a slight space is developed between its two walls. It is shown in section in Plate 24, fig. 43 (*h.m.*), from an embryo of about 10 millims., shortly before hatching. It does not appear to undergo any further development, and, so far as we can make out, disappears shortly after the embryo is hatched, without acquiring an opening to the exterior.

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* For a history of similar nuclei, vide 'Comp. Embryol.,' vol. ii., chapters iii. and iv.

It is important to notice that this cleft, which in the cartilaginous Ganoids and *Polypterus* remains permanently open as the spiracle, is rudimentary even in the embryo of *Lepidosteus*.

The second pouch is the hyobranchial pouch: its outer end meets the epiblast before the larva is hatched, and a perforation is effected at the junction of the two layers, converting the pouch into a visceral cleft.

Behind the hyobranchial pouch there are four branchial pouches, which become perforated and converted into branchial clefts shortly after hatching.

The region of the œsophagus following the pharynx is not separated from the stomach, unless a glandular posterior region (vide description of adult) be regarded as the stomach, a non-glanular anterior region forming the œsophagus. The lumen of this part appears to be all but obliterated in the stages immediately before hatching, giving rise for a short period to a solid œsophagus like that of Elasmobranchii and Teleostei.*

From the anterior part of the region immediately behind the pharynx the air-bladder arises as a dorsal unpaired diverticulum. From the very first it has an elongated slit-like mouth (Plate 27, fig. 64, *a.b′.*), and is placed in the mesenteric attachment of the part of the throat from which it springs.

We have first noticed it in the stages immediately after hatching. At first very short and narrow, it grows in succeeding stages longer and wider, making its way backwards in the mesentery of the alimentary tract (Plate 27, fig. 65, *a.b.*). In the larva of a month and a half old (26 millims.) it has still a perfectly simple form, and is without traces of its adult lung-like structure; but in the larva of 11 centims. it has the typical adult structure.

The stomach is at first quite straight, but shortly after the larva is hatched its posterior end becomes bent ventralwards and forwards, so that the flexure of its posterior end (present in the adult) is very early established. The stomach is continuous behind with the duodenum, the commencement of which is indicated by the opening of the bile duct.

The liver is the first-formed alimentary gland, and is already a compact body before the larva is hatched. We have nothing to say with reference to its development, except that it exhibits the same simple structure in the embryo that it does in the adult.

A more interesting glandular body is the pancreas. It has already been stated that in the adult we have recognised a small body which we believe to be the pancreas, but that we were unable to study its histological characters.

In the embryo there is a well-developed pancreas which arises in the same position and the same manner as in those Vertebrata in which the pancreas is an important gland in the adult.

We have first noticed the pancreas in a stage shortly after hatching (Plate 27, fig. 61, *p.*). It then has the form of a funnel-shaped diverticulum of the *dorsal* wall of the duodenum, immediately behind the level of the opening of the bile duct. From the apex of this funnel numerous small glandular tubuli soon sprout out.

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* Vide 'Comp. Embryol.,' vol. ii., pp. 50–63.

The similarity in the development of the pancreas in *Lepidosteus* to that of the same gland in Elasmobranchii is very striking.*

The pancreas at a later stage is placed immediately behind the end of the liver in a loop formed by the pyloric section of the stomach (Plate 27, fig. 62, *p.*). During larval life it constitutes a considerable gland, the anterior end of which partly envelopes the bile duct (Plate 27, fig. 63, *p.*).

Considering the undoubted affinities between *Lepidosteus* and the Teleostei, the facts just recorded with reference to the pancreas appear to us to demonstrate that the small size and occasional absence (?) of this gland in Teleostei is a result of the degeneration of this gland; and it seems probable that the pancreas will be found in the larvæ of most Teleostei. These conclusions render intelligible, moreover, the great development of the pancreas in the Elasmobranchii.

We have first noticed the pyloric cæca arising as outgrowths of the duodenum in larvæ of about three weeks old, and they become rapidly longer and more prominent (Plate 27, fig. 62, *c.*).

The portion of the intestine behind the vitelline duct is, as in all the Vertebrata, at first straight. In Elasmobranchs the lumen of the part of the intestine in which a spiral valve is present in the adult, very early acquires a more or less semilunar form by the appearance of a fold which winds in a long spiral. In *Lepidosteus* there is a fold similar in every respect (Plate 25, fig. 53, *sp.v.*), forming an open spiral round the intestine. This fold is the first indication of the spiral valve, but it is relatively very much later in its appearance than in Elasmobranchs, not being formed till about three weeks after hatching. It is, moreover, in correlation with the small extent of the spiral valve of the adult, confined to a much smaller portion of the intestine than in Elasmobranchii, although owing to the relative straightness of the anterior part of the intestine it is proportionately longer in the embryo than in the adult.

The similarity of the embryonic spiral valve of *Lepidosteus* to that of Elasmobranchii shows that Stannius' hesitatation in accepting Müller's discovery of the spiral valve in *Lepidosteus* is not justified.

J. Müller ('Bau u. Entwick. d. Myxinoiden') holds that the so-called bursa entiana of Elasmobranchii (*i.e.*, the chamber placed between the part of the intestine with the spiral valve and the end of the pylorus) is the homologue of the more elongated portion of the small intestine which occupies a similar position in the Sturgeon. This portion of the small intestine is no doubt homologous with the still more elongated and coiled portion of the small intestine in *Lepidosteus* placed between the chamber into which the pyloric cæca, &c., open and the region of the spiral valve. The fact that the vitelline duct in the embryo *Lepidosteus* is placed close to the pyloric end of the stomach, and that the greater portion of the small intestine is derived from part of the alimentary canal behind this, shows that MÜLLER is mistaken in attempting to homologise the bursa entiana of Elasmobranchii, which is placed in front of the vitelline duct, with the coiled part of the small intestine of the above forms. The latter is either derived from an elongation of the very short portion of the intestine between the vitelline duct and the primitive spiral valve, or more probably by the conversion of the anterior part of the intestine, originally provided with a spiral valve into a coiled small intestine not so provided.

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* Vide F. M. Balfour, 'Monograph on Development of Elasmobranch Fishes,' p. 226.

We have already called attention to the peculiar mesentery present in the adult attaching the posterior straight part of the intestine to the ventral wall of the body. This mesentery, which together with the dorsal mesentery divides the hinder section of the body-cavity into two lateral compartments is, we believe, a persisting portion of the ventral mesentery which, as pointed out by one of us,[*] is primitively present for the whole length of the body-cavity. The persistence of such a large section of it as that found in the adult *Lepidosteus* is, so far as we know, quite exceptional. This mesentery is shown in section in the embryo in Plate 25, fig. 53 (*v.mt.*). The small vessel in it appears to be the remnant of the subintestinal vein.

The Gill on the Hyoid Arch.

It is well known that *Lepidosteus* is provided with a gill on the hyoid arch, divided on each side into two parts. An excellent figure of this gill is given by MÜLLER (No. 13, plate 5, fig. 6), who holds from a consideration of the vascular supply that the two parts of this gill represent respectively the hyoid gill and the mandibular gill (called by MÜLLER pseudobranch). MÜLLER'S views on this subject have not usually been accepted, but it is the fashion to regard the whole of the gill as the hyoid gill divided into two parts. It appeared to us not improbable that embryology might throw some light on the history of this gill, and accordingly we kept a look out in our embryos for traces of gills on the hyoid and mandibular arches. The results we have arrived at are purely negative, but are not the less surprising for this fact. The hyomandibular cleft as shown above, is never fully developed, and early undergoes a complete atrophy—a fact which is, on the whole, against MÜLLER'S view; but what astonished us most in connexion with the gill in question is that we have been unable to find any trace of it even in the oldest larva whose head we have had (26 millims.), and at a period when the gills on the hinder arches have reached their full development.

We imagined the gill in question to be the remnant of a gill fully formed in extinct Ganoid types, and therefore expected to find it better developed in the larva than in the adult. That the contrary is the fact appears to us fairly certain, although we cannot at present offer any explanation of it.

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[*] 'Comparative Embryology,' vol. ii., p. 514.

SYSTEMATIC POSITION OF LEPIDOSTEUS.

A. AGASSIZ concludes his memoir on the development of *Lepidosteus* by pointing out that in spite of certain affinities in other directions this form is "not so far removed from the bony Fishes as has been supposed." Our own observations go far to confirm AGASSIZ' opinion.

Apart from the complete segmentation, the general development of *Lepidosteus* is strikingly Teleostean. In addition to the general Teleostean features of the embryo and larva, which can only be appreciated by those who have had an opportunity of practically working at the subject, we may point to the following developmental features* as indicative of Teleostean affinities :—

(1.) The formation of the nervous system as a solid keel of the epiblast.

(2.) The division of the epiblast into a nervous and epidermic stratum.

(3.) The mode of development of the gut (vide pp. 369, 370).

(4.) The mode of development of the pronephros; though, as shown on p. 420, the pronephros of *Lepidosteus* has primitive characters not retained by Teleostei.

(5.) The early stages in the development of the vertebral column (vide p. 388).

In addition to these, so to speak, purely embryonic characters there are not a few important adult characters :—

(1.) The continuity of the oviducts with the genital glands.

(2.) The small size of the pancreas, and the presence of numerous so-called pancreatic cæca.

(3.) The somewhat coiled small intestine.

(4.) Certain characters of the brain, *e.g.*, the large size of the cerebellum; the presence of the so-called lobi inferiores on the infundibulum; and of tori semicirculares in the mid-brain.

In spite of the undoubtedly important list of features to which we have just called attention, a list containing not less important characters, both embryological and adult, separating *Lepidosteus* from the Teleostei, can be drawn up :—

(1.) The character of the truncus arteriosus.

(2.) The fact of the genital ducts joining the ureters.

(3.) The presence of vasa efferentia in the male carrying the semen from the testes to the kidney, and through the tubules of the latter into the kidney duct.

(4.) The presence of a well-developed opercular gill.

(5.) The presence of a spiral valve; though this character may possibly break down with the extension of our knowledge.

(6.) The typical Ganoid characters of the thalamencephalon and the cerebral hemispheres (vide pp. 381 and 382).

(7.) The chiasma of the optic nerves.

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* The features enumerated above are not in all cases confined to *Lepidosteus* and Teleostei, but are always eminently characteristic of the latter.

(8.) The absence of a pecten, and presence of a vascular membrane between the vitreous humour and the retina.

(9.) The opisthocœlous form of the vertebræ.

(10.) The articulation of the ventral parts of the hæmal arches of the tail with processes of the vertebral column.

(11.) The absence of a division of the muscles into dorso-lateral and ventro-lateral divisions.

(12.) The complete segmentation of the ovum.

The list just given appears to us sufficient to demonstrate that *Lepidosteus* cannot be classed with the Teleostei; and we hold that MÜLLER's view is correct, according to which *Lepidosteus* is a true Ganoid.

The existence of the Ganoids as a distinct group has, however, recently been challenged by so distinguished an Ichthyologist as GÜNTHER, and it may therefore be well to consider how far the group as defined by MÜLLER is a natural one for living forms,* and how far recent researches enable us to improve upon MÜLLER's definitions. In his classical memoir (No. 13) the characters of the Ganoids are thus shortly stated:—

"These Fishes are either provided with plate-like angular or rounded cement-covered scales, or they bear osseous plates, or are quite naked. The fins are often, but not always, beset with a double or single row of spinous plates or splints. The caudal fin occasionally embraces in its upper lobe the end of the vertebral column, which may be prolonged to the end of the upper lobe. Their double nasal openings resemble those of Teleostei. The gills are free, and lie in a branchial cavity under an operculum, like those of Teleostei. Many of them have an accessory organ of respiration, in the form of an opercular gill, which is distinct from the pseudobranch, and can be present together with the latter; many also have spiracles like Elasmobranchii. They have many valves in the stem of the aorta like the latter, also a muscular coat in the stem of the aorta. Their ova are transported from the abdominal cavity by oviducts. Their optic nerves do not cross each other. The intestine is often provided with a spiral valve, like Elasmobranchii. They have a swimming-bladder with a duct, like many Teleostei. Their pelvic fins are abdominal.

"If we include in a definition only those characters which are invariable, the Ganoids may be shortly defined as being those Fish with numerous valves to the stem of the aorta, which is also provided with a muscular coat; with free gills and an operculum, and with abdominal pelvic fins."

To these distinctive characters, he adds in an appendix to his paper, the presence of the spiral valve, and the absence of a processus falciformis and a choroid gland.

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* We do not profess to be able to discuss this question for extinct forms of Fish, though of course it is a necessary consequence of the theory of descent that the various groups should merge into each other as we go back in geological time.

To the distinctive set of characters given by MÜLLER we may probably add the following :—

(1.) Oviducts and urinary ducts always unite, and open by a common urinogenital aperture behind the anus.

(2.) Skull hyostylic.

(3.) Segmentation complete in the types so far investigated, though perhaps *Amia* may be found to resemble the Teleostei in this particular.

(5.) A pronephros of the Teleostean type present in the larva.

(6.) Thalamencephalon very large and well developed.

(7.) The ventricle in the posterior part of the cerebrum is not divided behind into lateral halves, the roof of the undivided part being extremely thin.

(8.) Abdominal pores always present.

The great number of characters just given are amply sufficient to differentiate the Ganoids as a group; but, curiously enough, the only characters amongst the whole series which have been given, which can be regarded as peculiar to the Ganoids, are (1) the characters of the brain, and (2) the fact of the oviducts and kidney ducts uniting together and opening by a common pore to the exterior.

This absence of characters peculiar to the Ganoids is an indication of how widely separated in organisation are the different members of this great group.

At the same time, the only group with which existing Ganoids have close affinities is the Teleostei. The points they have in common with the Elasmobranchii are merely such as are due to the fact that both retain numerous primitive Vertebrate characters,* and the gulf which really separates them is very wide.

There is again no indication of any close affinity between the Dipnoi and, at any rate, existing Ganoids.

Like the Ganoids, the Dipnoi are no doubt remnants of a very primitive stock; but in the conversion of the air-bladder into a true lung, the highly specialised character of their limbs,† their peculiar autostylic skulls, the fact of their ventral nasal openings leading directly into the mouth, their multisegmented bars (interspinous bars), directly prolonged from the neural and hæmal arches and supporting the fin-rays of the unpaired dorsal and ventral fins, and their well-developed cerebral hemispheres, very unlike those of Ganoids and approaching the Amphibian type, they form a very well-defined group, and one very distinctly separated from the Ganoids.

No doubt the Chondrostean Ganoids are nearly as far removed from the Teleostei as from the Dipnoi, but the links uniting these Ganoids with the Teleostei have been so fully preserved in the existing fauna of the globe, that the two groups almost run into each other. If, in fact, we were anxious to make any radical change in the ordinary classification of Fishes, it would be by uniting the Teleostei and Ganoids, or rather constituting the Teleostei into one of the sub-groups of the Ganoids, equivalent to the Chondrostei. We do not recommend such an arrangement, which in view of the great preponderance of the Teleostei amongst living Fishes would be highly inconvenient, but the step from *Amia* to the Teleostei is certainly not so great as that from the Chondrostei to *Amia*, and is undoubtedly less than that from the Selachii to the Holocephali.

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* As instances of this we may cite (1) the spiral valve; (2) the frequent presence of a spiracle; (3) the frequent presence of a communication between the pericardium and the body-cavity; (4) the heterocercal tail.

† Vide F. M. BALFOUR, "On the Development of the Skeleton of the Paired Fins of Elasmobranchs," Proc. Zool. Soc., 1881.

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List of Reference Letters.

a.	Anus.
a.b.	Air-bladder.
a.b′.	Aperture of air-bladder into throat.
a.c.	Anterior commissure.
a.f.	Anal fin.
al.	Alimentary canal.
ao.	Aorta.
ar.	Artery.
au.	Auditory pit.
b.	Brain.
b.c.	Body-cavity.
b.d.	Bile duct.
b.d′.	Aperture of bile duct into duodenum.
bl.	Coalesced portion of segmental ducts, forming urinogenital bladder.
br.a.	Branchial arches.
br.c.	Branchial clefts.
c.	Pyloric cæca.
c′.	Apertures of cæca into duodenum.
cb.	Cerebellum.
cd.v.	Cardinal vein.
ce.	Cerebrum : in figs. 47 A and B, anterior lobe of cerebrum.
ce′.	Posterior lobe of cerebrum.
c.f.	Caudal fin.
cn.	Centrum.
ch.	Choroidal fissure.
cr.v.	Circular vein of vascular membrane of eye.
c.sh.	Cuticular sheath of notochord.
c.v.	Caudal vein.
d.	Duodenum.
d.c.	Dorsal cartilage of neural arch.
d.f.	Dermal fin-rays.
d.l.	Dorsal lobe of caudal fin.
dl.f.	Dorsal fin.
e.	Eye.
ed.	Epidermis.
ep.	Epiblast.
f.b.	Fore-brain.
f.e.	Pyriform bodies surrounding the zona radiata of the ovum, probably the remains of epithelial cells.
g.b.	Gall-bladder.
g.d.	Genital duct.
gl.	Glomerulus.
g.r.	Genital ridge.
h.	Heart.
h.a.	Hæmal arch.
h.b.	Hind-brain.
h.c.	Head-cavity.
hp.d.	Hepatic duct.
h.m.	Hyomandibular cleft.
h.op.	Operculum.
hy.	Hypoblast ; in fig. 10, hyoid arch.
hyl.	Hyaloid membrane.
i.c.	Intercalated cartilaginous elements of the neural arches.
in.	Infundibulum.
ir.	Iris.
i.s.	Interspinous cartilage or bones.
i.v.	Sub-intestinal vein.
iv.r.	Intervertebral ring of cartilage.
k.	Kidney.
l.	Lens.
l.c.	Longitudinal canal, formed by union of the vasa efferentia.
l.in.	Lobi inferiores.
l.l.	Ligamentum longitudinale superius.

lr.	Liver.
lt.	Lateral line.
ly.	Lymphatic body in front of kidney.
m.	Mouth.
m.b.	Mid-brain.
m.c.	Medullary cord.
m.el.	Membrana elastica externa.
mes.	Mesorchium.
mn.	Mandible.
md. and *mo.*	Medulla oblongata.
ms.	Mesoblast.
n.a.	Neural arch.
n.a′.	Dorsal element of neural arch.
nc.	Notochord.
n.v.e.	Network formed by vasa efferentia on inner face of testis.
od.	Oviduct.
od′.	Aperture of oviduct into bladder.
ol.	Nasal pit or aperture.
olf.	Olfactory lobe.
op.	Optic vesicle.
op.ch.	Optic chiasma.
op.l.	Optic lobes.
op.th.	Optic thalami.
or.ep.	Oral epithelium.
ov.	Ovary.
p.	Pancreas.
pc.	Pericardium.
pc.f.	Pectoral fin.
p.ch.	Pigmented layer of choroid.
p.f.	Peritoneal funnel of segmental tube of mesonephros.
p.f.p.	Peritoneal funnel leading into pronephric chamber.
p.g.	Pectoral girdle.
pl.f.	Pelvic fin.
pn.	Pineal gland.
p.o.	Primitive germinal cells.
pr.	Mesoblastic somite.
pr.c.	Pronephric chamber.
pr.n.	Pronephros.
pr.n′.	Opening of pronephros into pronephric chamber.
pt.	Pituitary body.
py.	Pyloric valve.
p.z.	Parietal zone of blastoderm.
r.	Rostrum.
rb.	Rib.
rc.	Rectum.
s.	Spleen.
s.c.	Seminal vessels passing from the longitudinal canal into the kidney.
s.d.	Suctorial disc.
sg.	Segmental or archinephric duct.
sg.t.	Segmental tubules.
sh.	Granular outer portion of the sheath of the notochord in the vertebral regions.
s.mx.	Superior maxillary process.
s.nc.	Sub-notochordal rod.
so.	Somatic mesoblast.
sp.	Splanchnic mesoblast.
sp.n.	Spinal nerve.
sp.v.	Spiral valve.
st.	Stomach.
s.t.	Seminal tubes of the testis.
su.p.	Suctorial papillæ.
t.	Testis.
th.	Thalamencephalon.
th.l.	Lobes of the roof of the thalamencephalon.
tr.	Trabeculæ.
u.g.	Urinogenital aperture.
v.	Ventricle.
v.e.	Vasa efferentia.
v.h.	Vitreous humour.
v.l.	Ventral lobe of the caudal fin.
v.mt.	Ventral mesentery.
vn.	Vein.
vs.	Blood-vessel.

v.sh.	Vascular sheath between the hyaloid membrane and the vitreous humour.
v.th.	Vesicle of the thalamencephalon.
x.	Groove in epiblast, probably formed in process of hardening.
y.	Yolk.
z.	Commissure in front of pineal gland.
z.r.	Outer striated portion of investing membrane (zona radiata) of ovum.
z.r′.	Inner non-striated portion of investing membrane of ovum.
I.	Olfactory nerve.
II.	Optic nerve.
III.	Oculomotor nerve.
V.	Trigeminal nerve.
VIII.	Facial and auditory nerves.

Explanation of Figures.

PLATE 21.

- Figs. 1–4. Different stages in the segmentation of the ovum.
  - Fig. 1. Ovum with a single vertical furrow, from above.
  - Fig. 2. Ovum with two vertical furrows, from above.
  - Fig. 3. Side view of an ovum with a completely formed blastodermic disc.
  - Fig. 4. The same ovum as fig. 3, from below, showing four vertical furrows nearly meeting at the vegetative pole.
- Figs. 5–10. External views of embryos up to time of hatching.
  - Fig. 5. Embryo, 3·5 millims. long, third day after impregnation.
  - Fig. 6. Embryo on the fifth day after impregnation.
  - Fig. 7. Posterior part of same embryo as fig. 6, showing tail swelling.
  - Fig. 8. Embryo on the sixth day after impregnation.
  - Fig. 9. Embryo on the seventh day after impregnation.
  - Fig. 10. Embryo on the eleventh day after impregnation (shortly before hatching).
- Fig. 11. Head of embryo about the same age as fig. 10, ventral aspect.
- Fig. 12. Side view of a larva about 11 millims. in length, shortly after hatching.
- Fig. 13. Head of a larva about the same age as fig. 12, ventral aspect.
- Fig. 14. Side view of a larva about 15 millims. long, five days after hatching.
- Fig. 15. Head of a larva 23 millims. in length.
- Fig. 16. Tail of a larva 11 centims. in length.
- Fig. 17. Transverse section through the egg-membranes of a just-laid ovum.

We are indebted to Professor W. K. Parker for figs. 12, 14, and 15.

PLATE 22.

Figs. 18–22. Transverse sections of embryo on the third day after impregnation.
    Fig. 18. Through head, showing the medullary keel.
    Fig. 19. Through anterior part of trunk.
    Fig. 20. Through same region as fig. 19, showing a groove (*x.*) in the epiblast, probably artificially formed in the process of hardening.
    Fig. 21. Through anterior part of tail region, showing partial fusion of layers.
    Fig. 22. Through posterior part of tail region, showing more complete fusion of layers than fig. 21.
Figs. 23–25. Transverse sections of an embryo on the fifth day after impregnation.
    Fig. 23. Through fore-brain and optic vesicles.
    Fig. 24. Through hind-brain and auditory pits.
    Fig. 25. Through anterior part of trunk.
Figs. 26–27. Transverse sections of the head of an embryo on the sixth day after impregnation.
    Fig. 26. Through fore-brain and optic vesicles.
    Fig. 27. Through hind-brain and auditory pits.

PLATE 23.

Figs. 28–29. Transverse sections of the trunk of an embryo on the sixth day after impregnation.
    Fig. 28. Through anterior part of trunk (from a slightly older embryo than the other sections of this stage).
    Fig. 29. Slightly posterior to fig. 28, showing formation of segmental duct as a fold of the somatic mesoblast.
Fig. 30. Longitudinal horizontal section of embryo on the sixth day after impregnation, passing through the mesoblastic somites, notochord, and medullary canal.
Figs. 31–34. Transverse sections through an embryo on the seventh day after impregnation.
    Fig. 31. Through anterior part of trunk.
    Fig. 32. Through the trunk somewhat behind fig. 31.
    Fig. 33. Through tail region.
    Fig. 34. Further back than fig. 33, showing constriction of tail from the yolk.

Figs. 35–37. Transverse sections through an embryo on the eighth day after impregnation.

Fig. 35. Through fore-brain and optic vesicles.

Fig. 36. Through hind-brain, showing closed auditory pits, &c.

Fig. 37. Through anterior part of trunk.

Fig. 38. Section through tail of an embryo on the ninth day after impregnation.

PLATE 24.

Fig. 39. Section through the olfactory involution and part of fore-brain of a larva on the ninth day after impregnation, showing olfactory nerve.

Fig. 40. Section through the anterior part of the head of the same larva, showing pituitary involution.

Figs. 41–43. Transverse sections through an embryo on the eleventh day after impregnation.

Fig. 41. Through fore-part of head, showing the pituitary body still connected with the oral epithelium.

Fig. 42. Slightly further back than fig. 41, showing the pituitary body constricted off from the oral epithelium.

Fig. 43. Slightly posterior to fig. 42, to show olfactory involution, eye, and hyomandibular cleft.

Fig. 44. Longitudinal section of the head of an embryo of 15 millims. in length, a few days after hatching, showing the structure of the brain.

Fig. 45. Longitudinal section of the head of an embryo, about five weeks after hatching, 26 millims. in length, showing the structure of the brain. In the front part of the brain the section passes slightly to one side of the median line.

Figs. 46 A to 46 G. Transverse sections through the brain of an embryo 25 millims. in length, about a month after hatching.

Fig. 46 A. Through anterior lobes of cerebrum.

Fig. 46 B. Through posterior lobes of cerebrum.

Fig. 46 C. Through thalamencephalon.

Fig. 46 D. Through optic thalami and optic chiasma.

Fig. 46 E. Through optic lobes and infundibulum.

Fig. 46 F. Through optic lobes and cerebellum.

Fig. 46 G. Through optic lobes and cerebellum, slightly behind fig. 46 F.

PLATE 25.

Figs. 47 A, B, C. Figures of adult brain.

Fig. 47 A. From the side.
Fig. 47 B. From above.
Fig. 47 C. From below.

Fig. 48. Longitudinal vertical section through the eye of an embryo, about a week after hatching, showing the vascular membrane surrounding the vitreous humour.

Fig. 49. Diagram showing the arrangement of the vessels in the vascular membrane of the vitreous humour of adult eye.

Fig. 50. Capillaries of the same vascular membrane.

Fig. 51. Transverse section through anterior part of trunk of an embryo on the ninth day after impregnation, showing the pronephros and pronephric chamber.

Fig. 52. Transverse section through the region of the stomach of an embryo 15 millims. in length, shortly after hatching, to show the glomerulus and peritoneal funnel of pronephros.

Fig. 53. Transverse section through posterior part of the body of an embryo, about a month after hatching, showing the structure of the mesonephros, the spiral valve, &c.

PLATE 26.

Figs. 54, 55, 56, and 57 are a series of transverse sections through the genital ridge and mesonephros of one side from a larva of 11 centims.

Fig. 54. Section of the lymphatic organ which lies in front of the mesonephros.

Fig. 55. Section near the anterior end of the mesonephros, where the genital sac is completely formed.

Fig. 56. Section somewhat further back, showing the mode of formation of the genital sac.

Fig. 57. Section posterior to the above, the formation of the genital sac not having commenced, and the genital ridge with primitive germinal cells projecting freely into the body-cavity.

Fig. 58 A. View of the testis, mesorchium, and duct of the kidney of the left side of an adult male example of *Lepidosteus*, 60 centims. in length, showing the vasa efferentia and the longitudinal canal at the base of the mesorchium. The kidney ducts have been cut open posteriorly to show the structure of the interior.

Fig. 58 B. Inner aspect of the posterior lobe of the testis from the same example, to show the vasa efferentia forming a network on the face of the testis.

Figs. 59 A and B. Two sections showing the structure and relations of the efferent ducts of the testis in the same example.

Fig. 59 A. Section through the inner aspect of a portion of the testis and mesorchium, to show the network of the vasa efferentia (*n.v.e.*) becoming continuous with the seminal tubes (*s.t.*). The granular matter nearly filling the vasa efferentia and the seminal tubes represents the spermatozoa.

Fig. 59 B. Section through part of the kidney and its duct and the longitudinal canal (*l.c.*) at the base of the mesorchium. Canals (*s.c.*) are seen passing off from the latter, which enter the kidney and join the uriniferous tubuli. Some of the latter (as well as the seminal tubes) are seen to be filled with granular matter, which we believe to be the remains of spermatozoa.

Fig. 60. Diagram of the urinogenital organs of the left side of an adult female example of *Lepidosteus* 100 centims. in length. This figure shows the oviduct (*od.*) continuous with the investment of the ovary, opening at *od′.* into the dilated part of the kidney duct (segmental duct). It also shows the segmental duct and the junction of the latter with its fellow of the right side to form the so-called bladder, this part being represented as cut open. The kidney (*k.*) and lymphatic organ (*ly.*) in front of it are also shown.

PLATE 27.

Fig. 61. Transverse section through the developing pancreas (*p.*) of a larva 11 millims. in length.

Fig. 62. Longitudinal section through portions of the stomach, liver, and duodenum of an embryo about a month after hatching, to show the relations of the pancreas (*p.*) to the surrounding parts.

Fig. 63. External view of portions of the liver, stomach, duodenum, &c., of a young Fish, 11 centims. in length, to show the pancreas (*p.*).

Fig. 64. Transverse section through the anterior part of the trunk of an embryo, about a month after hatching, showing the connexion of the air-bladder with the throat (*a.b′.*).

Fig. 65. Transverse section through the same embryo as fig. 64 further back, showing the posterior part of the air-bladder (*a.b.*).

Fig. 66. Viscera of an adult female, 100 centims. in length, showing the alimentary canal with its appended glands in natural position, and the air-bladder with its aperture into the throat (*a.b′.*). The proximal part of the duodenum and the terminal part of the intestine are represented as cut open, the former to show the pyloric valve and the apertures of the pyloric cæca and bile duct, and the latter to show the spiral valve.

This figure was drawn for us by Professor A. C. Haddon.

PLATE 28.

Fig. 67. Transverse section through the tail of an advanced larva, showing the neural and hæmal processes, the independently developed interneural and interhæmal elements (*i.s.*), and the commencing dermal fin-rays (*d.f.*).

Fig. 68. Side view of the tail of a larva, 21 millims. in length, dissected so as to show the structure of the skeleton.

Fig. 69. Longitudinal horizontal section through the vertebral column of a larva, 5·5 centims. in length, on the level of the hæmal arches, showing the intervertebral rings of cartilage continuous with the arches, the vertebral constriction of the notochord, &c.

Figs. 70 and 71. Transverse sections through the vertebral column of a larva of 5·5 centims. The red represents bone, and the blue cartilage.

Fig. 70. Through the vertebral region, showing the neural and hæmal arches, the notochordal sheath, &c.

Fig. 71. Through the intervertebral region, showing the intervertebral cartilage.

Figs. 72 and 73. Transverse sections through the trunk of a larva of 5·5 centims. to show the structure of the ribs and hæmal arches.

Fig. 72. Through the anterior part of the trunk.

Fig. 73. Through the posterior part of the trunk.

PLATE 29.

Figs. 74–76. Transverse sections through the trunk of the same larva as figs. 72 and 73.

Fig. 74. Through the posterior part of the trunk (rather further back than fig. 73).

Fig. 75. Through the anterior part of the tail.

Fig. 76. Rather further back than fig. 75.

Fig. 77. Longitudinal horizontal section through the vertebral column of a larva of 11 centims., passing through the level of the hæmal arches, and showing the intervertebral constriction of the notochord, the ossification of the cartilage, &c.

Fig. 78. Transverse section through a vertebral region of the vertebral column of a larva 11 centims. in length.

Fig. 79. Transverse section through an intervertebral region of the same larva as fig. 78.

Fig. 80. Side view of two trunk vertebræ of an adult *Lepidosteus.*

Fig. 81. Front view of a trunk vertebra of adult.

In figures 80 and 81 the red does not represent bone as in the other figures, but simply the ligamentum longitudinale superius.

VIII. *On the Development of the Skull in* Lepidosteus osseus.

By William Kitchen Parker, *F.R.S.*

Received November 3,—Read December 8, 1881.

[Plates 30–38.]

In the early part of 1879 I received from Professor A. Agassiz a large series of eggs and embryos of *Lepidosteus*, which by the help of Mr. Garman he had obtained from the Black Lake the year before. These specimens, carefully preserved in fifty-four small bottles, Professor Agassiz put into my hands to be worked out by Mr. Balfour and myself. A few years before I had received from Professor Burt G. Wilder some larger young of this Fish, which extended the "stages" for us considerably; and several adults obtained by Mr. Balfour, and one which I received from Professor Flower, made up the total of our materials. The present communication is the result of my own researches into the growth of the skull and visceral arches. Mr. Balfour, assisted by my son, Mr. W. N. Parker, has prepared an elaborate memoir on the embryology of this important type, to which are added observations on the structure of various organs in the adult.

When our work was scarcely begun I prevailed upon Dr. Traquair to work out the *adult skull*, and that piece of research is, I believe, nearly finished, so that this *Holostean Ganoid* will soon have had a fair amount of attention given to it.

My observations extend over a series of embryos and young from one-third of an inch to four and a-half inches in length. The larger young are already quite like the adult; but as Dr. Traquair's paper has not yet been sent in to the Royal Society, I shall preface mine by a short description of the adult skull: it will give the same interpretation as the promised memoir by my talented friend, for we have had several discussions upon the nomenclature and meaning of the various parts, and are of one mind as to their names and nature.

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The skull of the adult Lepidosteus.

This skull, which belongs to the long-beaked variety, is more than a foot long, and the foremost two-thirds of its length belongs to the "rostrum," which is gradually attenuated from behind forwards, and then dilates gently at the fore end. The olfactory sacs are very small, and instead of being placed close in front of the antorbital region, are carried to the end of the snout, and have on each side a pair of holes bounded by small bony plates. The orbits are a little above half an inch across, and they are bounded by a perfect "circumorbital" series. The auditory capsules are impacted into the walls of the hind skull, which at the sides is only about one-twentieth the length of the head. The occipital condyle is behind the skull; the condyles of the quadrate are in front of the orbits. There is no fontanelle in the perfect skull—that space is entirely covered by large ganoid scutes.

A. *On the superficial plates or "scutes."*

Beginning at the occipital roof, we find the hind skull covered with two large plates that represent the parietals; here, at once, we see how variable these scutes are, which answer to *much more* than the investing bones in the higher types, for there is, on the left side (in my specimen), an irregularly four-sided "dermo-occipital," the inner edge of which passes over the mid-line a little. Thus the right parietal is much larger than the left. Outside each parietal there is a somewhat smaller scute—the squamosal; this bone finishes the roof. The two or three small, irregular scutes behind the squamosal are "post-temporals," and serve as fixing-points to the *clavicular series* of the shoulder-girdle; they are post-cranial.

Outside the hinder part of the squamosal there are two smallish additional "temporal" scutes, wedged in above the "opercular."

In front of the parietals and squamosals we see the frontals; they interdigitate by large, sharp, sutural teeth, with those bones, and then run on over the orbital region, and over the hinder two-fifths of the rostrum. They are elegantly narrow-waisted (taken together) in the orbital region, and they contract into sharp styles in front, where they embrace the next pair.

These next bones are the "ethmo-nasals," or superficial splints of the ethmoidal region; they are close together along the mid line, sharp and narrow between the fore part of the frontals, and then wider; they reach, behind, within an inch and a-half of the orbits, and then run forward up to the nasals (proper). These latter are small, short, widely-crescentic plates, that lie on the small, distal, olfactory sacs; they are the *ossa terminalia* of the upper *mucous* series. The nasals finish the skull-beak above; below, an oblique transverse plate of bone supports the prenasal pad or remnant of the larval sucking-disk; it may be called the "prenasal bone." The nasal bone sends a process obliquely across the nasal opening; this bone lies in the membranous bar, which was developed early, dividing the "nostril" into two holes. Behind and under the prenasal and nasal bones we have the two short thick premaxillaries, which carry some very large, sharp teeth, as well as a rasp of smaller denticles. These bones have a palatal as well as a dentary region; and between the two the bone is punched with holes by the foremost teeth of the mandible, as in the Crocodile; the palatine processes diverge a little, behind, to embrace the vomers.

The vomers are very long, thin styles of bone, closely co-adapted along the mid line of the palatal region of the rostrum up to its hinder fourth; they have their lower face covered with a rasp of very small teeth.

Outside these there is on each side a larger splint, which reaches from the premaxillary to the antorbital space. The outside of each bone is bevelled and helps to carry the large sub-marginal teeth; these are the *superficial* palatines. Outside these, on the margin, we have the "maxillary chain" of mucous bones, about fifteen on each side, but some of them are ankylosed together; they increase in length from before backwards. The preorbital (prequadrate) space is overlapped by the last two of this chain; these are free; the larger front bone answers to the free part of the "os mystaceum" of an Acanthopterous Teleostean; the lesser scale to its jugal. A large splint passes along by its sharp fore end, on the inside of the parosteal palatine, nearly to its middle; it broadens rapidly at the prequadrate space and turns downwards, as a rounded wing. It is notched below close in front of the quadrate condyle, and then passes backwards as a large lanceolate splint, which is closely applied to nearly all the inner face of the mandibular suspensorium; this is the pterygoid bone, the osseous counterpart of the whole palato-quadrate arcade. Above the broad hind part of that arcade, close in front of the basi-pterygoid articulation, there is a much smaller lanceolate splint, the meso-pterygoid; it lies inside, but rises somewhat above the obliquely directed suspensorium. An oblong splint, rather larger than the last, lies on the outside of the lower face of the suspensorium, and is dilated where it fits to the enlargement at the quadrate condyle; this is the "preopercular": a very different bone from its counterpart in the Teleostei, and like the lower part of a Frog's squamosal. These four—the superficial palatine, the pterygoid, the mesopterygoid, and the preopercular—are the splints of the palato-quadrate arcade; the rest of the mandibular splints are on the free ramus, and will be described soon. There is a perfect "circumorbital" series of small, thick ganoid scutes, which run before and behind into other tracts, and are not separated as a mere ring. The upper region is composed of a chain of larger bones than the rest, and this tract is continued forwards as three bones, narrowing forwards, in the *preorbital* region, over the "prequadrate space." Behind and above, these larger bones run backwards under the squamosal until they become the "post-temporals" to which the clavicular bones are attached. They are the direct cephalic continuation of what in the Teleostei are known as the "lateral line" series. A row of small scutes runs straight down in front of the orbit from the preorbital band; the lowest of these binds on the quadrate, over the hinge, and over the foot of the preopercular. From this angular bone there is an increasing number of scutes under the orbit, and those behind the orbit become a solid tesselated pavement (part of which I have just described as lying directly beneath the squamosal), and this pavement lies on the edge of the lower part of the interopercular and covers all its ascending part.

This most remarkable "interopercular" is much like the preopercular of the Teleosteans, and might easily be mistaken for it. It is a huge plate bent upon itself at a right angle; the ascending part is half the size of the lower region, and whilst covered by the facial "pavement," itself covers the hyomandibular and symplectic; that part is pointed above. The lower region of the bone is a large externally ganoid tract, pointed in front, ear-shaped behind, hollow within, and coiled inwards. Behind the facial pavement, and articulated by a cup-like facet to the "opercular process" of the hyomandibular, is the opercular bone; it is four-sided, but narrow above where it articulates with the post-temporal. Its broad lower edge is toothed and overlaps the subopercular; its hind face is free and forms the upper half of the free edge of the *operculum*. The "subopercular" forms the lower half of that edge; it is a broad plate with a rounded lower and hinder margin, and is uncinate in front and above, where it is wedged in between the interopercular and opercular. These three bones—the interopercular, the opercular, and the subopercular—belong to the hyomandibular region. The splints of the mandible are ganoid where they are exposed. The main bone is the "dentary"; it covers nearly all the outside of the ramus and the upper and lower edges within. On the inside, between those edges, MECKEL's cartilage is hidden in its foremost two-thirds by a long, thin, narrow splint, the "splenial." The rami of the mandible are close together, half-way, backwards, and then gently diverge. The coronoid region is very high, large, and incurved. The "coronoid" bone flanks the front of this part as an oblique splint, and the "supra-angulare" covers the smooth, convex outside; a short, thick, wedge of bone, the "angulare," is set on to the angle of the ramus. These five bones—the dentary, splenial, coronoid, supra-angulare, and angulare—are the normal investing bones of the mandible. In the lower part of the hyoid arch there are three narrow flattened rays, from an inch to an inch and a-half in length, attached to the outer face of the "epi-hyal"; these are the "branchiostegals," and they correspond in number to what we find in the Cyprinoid Teleosteans.

Under the skull there is a long beam of bone, reaching to the middle of the vomers, lying there somewhat above them, and then escaping, showing a free keel; this is the "parasphenoid." It runs backwards, gently broadening, and then narrowing again, and gradually losing its keel, up to the "basipterygoid processes," it flanks them with a small pair of wings, and is then alate again, more extensively, under the "sacculus" of the ear, and ends as two flat processes under the basioccipital, and nearly reaching to its end.

B. *On the endo-cranial bones.*

In the hind skull I fail to find a distinct supraoccipital under the roof, for the exoccipitals meet there, and are separated only by a narrow tract of cartilage. The basioccipital is large, and projects beyond the arch; its hollow articular face is transverse, and notched above and below. The exoccipitals are large, and finish the arch above. The auditory capsule is invested by *four* cartilage bones on each side, viz.: a large prootic, a middle sized epiotic, and a small opisthotic, and the sphenotic bone which is of the average size, and ossifies the post-orbital angle of the endocranium, where the fore part of the auditory capsules are imbedded. I can find no "pterotic" under the squamosal. There are two alisphenoids, and they ossify the basipterygoid processes, but I find no distinct basisphenoid, presphenoid, nor orbito-sphenoids; although after the brain cavity has retreated, losing much of its front part, a bony plate is found in the presphenoidal region. This plate is thin, fenestrate, and rests upon the parasphenoid. I believe that it is formed by the coalescence of the once distinct lateral ethmoids, which were at first formed in the sides of the fore part of the chondrocranium, like the primary elements of the Frog's "girdle-bone" (see Plate 38, figs. 2, 3, *l.eth.*). The rostral part of the endocranium is, I believe, unossified, even in the adult.

The quadrate and metapterygoid bones occupy only part of the mandibular suspensorium; the rest remains unossified; the metapterygoid bone takes up the part which answers to the pedicle and rudimentary otic process; a facet of cartilage covers it where it articulates with the "basipterygoid process." The hyomandibular is fenestrate and well ossified; it is only half as large as the mandibular suspensorium. A short tract of cartilage separates its upper from the slender long symplectic.

The articulare is a strong solid mass of bone, capped by a saddle-shaped condyle for articulation with the quadrate.

The small inter-hyal piece is not ossified, but the epi-, cerato-, and hypo-hyals are; the long *double* basi-hyal ("ento-glossal") does not become solid bone, but it is *endosteally* hardened. The fore part of the basi-branchial series is slightly ossified, the arches are as well ossified as in the ordinary Teleostei; thus there are in the first four arches a pharyngo, epi-, cerato-, and hypo-branchial piece, largely converted into bone; the fifth is ossified, but it only has the cerato-branchial element in it.

First Stage.—Recently hatched young of Lepidosteus, $2\frac{3}{4}$ *to* $4\frac{1}{2}$ *lines long.*

At this stage (Plate 30, figs. 1, 2) the mesocephalic flexure is at its fullest development, but the angle formed by the meeting of the *para*-chordal and *pro*-chordal regions is very open. Moreover, the bend appears less than it is, for the snout is very large and dilated, and its axis is coincident with that of the cerebral hemispheres (C^{1a}.) and of the mid brain (C^2.).

The fissure, however, between the fore and hind brain (C^3.), and below the mid brain, is very large, and is turned backwards considerably, above; the notochord (*nc.*) seeks to ascend into this space, but only rises one-third of the way to the top; the third nerve descends in this cavity to reach the orbital muscles.

In the sectional figures given to illustrate this stage, very little of the primordial cranium comes into view; it is not yet formed into consistent cartilage, but the "embryonic cartilage" is sufficiently developed to show the cranial and facial rudiments.

The pituitary body (*py.*) is distinguishable from the palatal skin, and it is quite free above, the infundibulum (*inf.*) lying over it.

In both figures (Plate 30, figs. 1 and 2, cl^{1-4}.) the clefts and visceral folds are visible on their inner or *hypoblastic* aspect; in the more advanced specimen a rod of solidifying tissue was seen as cut through in six of these, namely, the mandible, hyoid, and four of the branchial arches (*mn.*, *hy.*, br^{1-4}.)—all but the *fifth*, which remains rudimentary.

The heart (*h.*) is seen impacted between the mandibular rudiment (*mn.*) and the yolk (*y.*); it thus lies as far forwards as the snout.

The notochord (*nc.*) is very large, and in the more advanced specimen (Plate 30, fig. 2) the terminal hook of this cephalic end of the rod is fixed in between the infundibulum (*inf.*) and the swelling antero-inferior part of the hind brain (C^3.).

Second Stage.—Recently hatched young of Lepidosteus, 5 *to* $5\frac{1}{2}$ *lines long.*

In somewhat larger specimens (about 11 millims. long) I was able to make preparations of the whole chondrocranium (Plate 30, fig. 3): a sectional view is given of a somewhat larger specimen (Plate 30, fig. 4).

The section shows that the skull is rapidly straightening, but of course the great *clinoid fissure* is never obliterated. This is formed by the primordial flexure, and is always permanent in the Vertebrata; but the flexure itself is not so great in this type as in the *Amphibia* and the "Amniota."

The notochord (Plate 30, fig. 4, *nc.*) is not traceable so far up in the clinoid region as it was; the pituitary body (*py.*) is still independent of the infundibulum (*inf.*).

In the dissected skull (Plate 30, fig. 3) the basal and facial parts alone are chondrified, the sides and roof are entirely membranous; the cartilage is still very fragile, but quite differentiated from the surrounding connective-fibre cells. The chondrocranium proper is at present composed of two bands of cartilage, which run from the atlas to the large suctorial snout (*s.d.*); the lateral structures are the auditory capsules and the visceral arches.

The basal pair of bands (Plate 30, fig. 3, *iv.*, *tr.*) are thickest behind and at the fore end; for two-fifths of their length they embrace the notochord by their inner edge; they then diverge from each other so as to form a large spindle-shaped space (*py.*) in front of which, for an eighth of their whole length, they approximate, but do not come into contact with each other.

These dilated fore ends of the basal bands are as broad as the parachordal part behind (*iv.*), but are not so thick; they are conjugated by a tract of embryonic cartilage, the rudiment of the "intertrabecula" (*i.tr.*); they themselves are the rudiments of the cornua trabeculæ (*c.tr.*). The bowed interorbital part of the basal bands are the trabeculæ (*tr.*), they are only half the width of the hind and fore parts; these out-bent bars do not merely fence in the small pituitary body, they embrace the base of the fore-brain.

The free, ascending, blunt end of the cranial notochord (*nc.*) is twisted a little to the left—it may curve either way, but the appearance is partly due to artificial pressure; it is not invested by a large ascending "posterior clinoid" cartilage, as in the *Amniota*, for that tract is very small and very late in appearance in *Lepidosteus*. The *pro*-chordal part of the basal bands is bent somewhat upon the *para*-chordal part. Behind the exit of the 9th and 10th nerves and the hind face of the auditory capsules the investing bars (*iv.*) thin out, and end in a sharp edge in front of the 1st vertebra.

The auditory capsules (*au.*) are of a short oval shape; they are largely chondrified, but the layer of cartilage is very thin; below, there is an oval fenestra, still membranous (*au.f.*); it is about two-fifths the length of the capsule. The capsules and the investing bars are only connected together by embryonic cartilage at present; the convexity of the capsules has produced a corresponding concavity in the side of each bar. The horizontal canal bulges over the lower convex part, which contains the otolith; thus there is a rudimentary "tegmen tympani." Rudiments of all the framework of the visceral arches are now present, but the segmentation of these parts is imperfect. The skeleton of the first and second arches—mandibles and hyoid—is massive; that of the branchial arches is very delicate. The pier of the mandibular arch (*p.pg.*, *q.*, *pd.*) is not a mere "pterygo-quadrate," as in the Selachians, Teleostei, and Urodeles, but is a "palato-quadrate," as in the Anura. This arises from the primary continuity of the ethmo-palatine cartilage with the pterygoid fore-growth of the mandibular suspensorium; and this is not all, for the *palatine region* of the bar is also *primarily continuous* with the trabecula at its dilated fore end.

The "pedicle" of this compound suspensorium is not *fixed*, as in the Tadpole, but *free*, as in the metamorphosed Frog; as soon as it is sufficiently developed to articulate with the basal bar, it will then, at that part, correspond very accurately with what we find in the adult Frog. At present this palato-quadrate, or suspensorial cartilage is roughly scythe-shaped, the "blade" being behind, and lying between the auditory capsule and the narrowing basal band, whilst the "handle" is in front, and is glued to the trabecula. The blade is shorter than the handle, and on its heel it has an oval, condyloid facet, which looks forwards and a little outwards and downwards; it is pinched a little behind this facet, and then becomes an apiculated lanceolate blade. The handle is oval in section, narrow at first, and then wider as it turns inwards at its junction with the trabecula. A free bar, rather shorter than the "handle," and having its thick end reversed, or behind, articulates by a flat facet with the flat facet of the suspensorium. This is MECKEL'S cartilage, or the free mandible (*mk.*).

Already, in the coronoid region, this short lower jaw is elevated into a crest; its lower edge is convex, its upper concave, and its end is blunt; it reaches nearly as far forwards as the palatine region of the suspensorium. Between the convex hind margin of the first arch (suspensorium and mandible) and the concave fore margin of the next (the hyoid) arch there is a *hypoblastic pouch* (see Plate 30, figs. 1 and 2, *cl*^{1}.), but this first cleft does not open on the outside, like the hyo-branchial and four branchials.

The hyoid arch consists already of *four* segments on each side, with the rudiment of a median conjugating bar, composed of embryonic cartilage. The pier (*hm.*) of the hyoid arch, like its counterpart in the Mammal, is anvil-shaped; this is the hyomandibular, with its symplectic process, the counterpart of the pterygoid fore-growth of the mandibular pier or suspensorium.*

At present it is only a fourth less than the pier in front of it; but it becomes relatively much less, instead of much larger, as in the Teleostei. The hyomandibular is now a solid mass of cartilage, concave above, bilobate below, and sending upwards, to the auditory capsule, a thick process, and downwards, to the articular process of the suspensorium, a sharp process. The upper process is its proximal part or head, the lower is its symplectic prolongation.

The middle part has two lobes, below, and behind; the upper of these afterwards gives off the opercular process, the lower is already scooped on its inside where it articulates with the next joint, the inter-hyal (*i.hy.*). This latter piece is a small pyriform segment, with its lesser end upwards and hooked inwards, for articulation with the hyomandibular. This is, so to speak, a supernumerary segment placed between the *antero-superior* pier and the *postero-inferior* arch. That arch is already composed of a main bar above, the cerato-hyal (*c.hy.*),—a thick rounded rod, and a short, almost globular segment below, the hypo-hyal (*h.hy.*); the two fit together by flat facets; the right and left hypo-hyals touch each other below. In front of and between these lower segments there is a tract of tissue which will harden into the basi-hyal (*b.hy.*).

The branchial arches (*br.*) are slender rods, bent in a sigmoid manner so as to form round in-turned hooks, above. The last (*br*^{5}.) is only one-fourth as large as the other four, and is permanently arrested as the "inferior pharyngeal;" it is simply a short cerato-branchial.

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* Mr. BALFOUR informs me that this bar is primarily continuous with the skull, and that after chondrification it is still unsegmented.

The four well-developed rods are not yet segmented across into the normal pieces; they are oval in section, pointed above, and rounded below, where they articulate with the common basi-branchial (*b.br.*), which is one-third thicker than the arches. This conjugating rod is somewhat flat; it is thickest in front, where it fits by a rounded end between the hypo-hyals, and flat behind, where it projects beyond the arches.

Third Stage.—Young Lepidostei, *$7\frac{1}{4}$ to 8 lines long.*

In young individuals about two-thirds of an inch (16 to 17 millims.) long, the cartilage has become quite consistent, and some parts not chondrified in the last stage have become solid.

The head (Plate 30, figs. 5, 6) has straightened out considerably, and the basis cranii bulges below; but the sharp, recurved fold under the mid brain, where the fore and hind brain approximate, although a mere chink, is ineradicable.

The round and curiously mammillated snout is much smaller now, relatively; the notochord (*nc.*) has retreated considerably; the pituitary body (*py.*) is still separate from the infundibulum (*inf.*).

The azygous trabecular bar (figs. 7, 8, *i.tr.*) now shows itself in sections taken vertically through the head, but all the rest of the basal cartilage is far from the mid line. The notochord (*nc.*) is very large, and descends considerably beneath the swelling hind brain (figs. 5, 6, C^3.). Below the mouth and throat, the fore end of the mandible is seen in the *right* section (Plate 30, fig. 5, *mk.*); it contained more than half of the head above, and less than half below; thus that rod, and the rest of the post-orals (*hy.*, br^{1-4}.), are cut through a little on the right side of the middle. But in the *left* half (Plate 30, fig. 6), the median rods (*b.hy.*, br^{1-4}.) are brought into view, below; above, the junction of the trabecula with the palato-quadrate (*tr.*, *p.pg.*) is seen; this, however, had to be exposed by dissection; the sub-ocular space is seen as a crescentic gap with its convexity downwards.

The chondrocranium is figured as seen from above and below in a dissection of a somewhat larger specimen (8 lines long); it is much more perfect than in the last stage (Plate 30, figs. 3, 7, 8).

A full third of the cranial floor is membranous; the side walls are membranous in front, and are made by the auditory capsules behind; but a rudiment of the roof or "tegmen cranii" is now found, right and left.

The thick cranial notochord (*nc.*) is only half the length of the chondrocranium, now; it is somewhat moniliform, lessening by three successive stages, and is bent a little in this specimen to the right.

The narrow part is rounded at its end, and ascends but little into the clinoid fissure (Plate 30, fig. 5, *nc.*); a third part at least is not invested by the parachordal cartilage (*iv.*).

The huge so-called pituitary space (*py.*) is pyriform, has an indented outline behind, and is apiculate in front. The concave inner side of the basal bands embrace the notochord closely, and end behind in bevelled flaps; in front, where the last narrowing of the notochord takes place, they end almost transversely, but give off from their outer edge the trabecular outgrowths. They are quite confluent, externally, with the well-chondrified auditory capsules (*au.*). In their broad, proximal part the trabeculæ (*tr.*) are dilated, and are perforated by the internal carotid artery;* thence they converge steadily as scarcely arcuate bands, only one-third as large as their parachordal roots (*iv.*).

Their front confluent part is as long as their hinder separate part; and there the bands double their width, have a convex outline externally, and narrowing inwards at their extremity, are continuous then with the palato-quadrate bars (*p.pg.*). The front margin of these four bands is crenate, with three convexities, the middle enlargement being the largest; this is not, however, formed by the trabeculæ themselves, but by a pyriform wedge of newer cartilage, which has developed between and above them. This new element is the intertrabecula (*i.tr.*); in the last stage (Plate 30, fig. 3, *i.tr.*) it was merely composed of embryonic cartilage—a small, inconspicuous tract of tissue, lying between the dilated ends of the trabeculæ (*tr.*).

This new tract does not reach to the pituitary space, but already, in the upper view, can be seen projecting beyond the confluent paired bands (*tr.*, *p.pg.*). The simple, ovoidal form of the auditory capsules (*au.*) is now lost, for the semi-circular canals (*a.s.c.*, *h.s.c.*, *p.s.c.*) have developed greatly, and they have given the form of their curves and swellings to the cartilaginous capsule.

The floor of the sacculus is still largely membranous (fig. 7, *au.f.*); this circular fenestra is inside the centre of the floor; within, also, the capsule is not cartilaginous. Outside, the bulging of the horizontal canal (*h.s.c.*) has formed a rudiment of the "tegmen tympani" (fig. 7); this, however, is simply used to form the large oblong concavity for the hyomandibular (*hm.*), and never acquires any tympanic function.

The large cranial nerves have free course over the basal plate, and in front of and behind the auditory capsules they have only membrane to pass through; the space for these (Plate 30, figs. 7, 8, IX., X.) is seen to be very wide.

From the front of the auditory capsule cartilage is creeping along the superorbital region (fig. 8, *s.ob.*); this new tract is styliform, and at present only reaches one-third of the distance to the end of the chondrocranium. Below this rudiment, right and left, of the "tegmen cranii" the proximal part of each trabecula has developed an oblong facet of cartilage for articulation with the *pedicle* of the suspensorium (*pd.*); this is the first appearance of the paired "basipterygoid" processes of the basicranii. I take this to be not only the first appearance of these important processes in the development of this particular skull, but also in the order of the types; in the Amphibia, Reptiles, Birds, and Mammals these processes are seldom absent.

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* This passage is seen inside the superorbital cartilage (*s.ob.*); the line leading from the letters to this part in fig. 8 is not long enough.

The dorsal end of the suspensorium or mandibular pier has already become oblique; the true extremity is now lateral, and is the pedicle of the pier; this is an oblong tract or facet articulating with the basi-pterygoid face on the basal band.

The second part is the otic process (behind *pd.*); it is a subtriangular "ear" of cartilage, running backwards towards the pier of the hyoid arch (*hm.*), under the superorbital rudiment (*s.ob.*). Thence the suspensorium narrows gently, and a little behind the middle of the bar forms, on its outside, a jutting step; this is the gently concave quadrate hinge which articulates with the mandible (*mk.*).

The rest of the suspensorium lessens to one-half its hinder width, and is *f*-shaped, bending inwards at first and then outwards before it makes its last inward bend to join the trabecula (Plate 30, figs. 7, 8, *p.pg.*, *tr.*). The extent of the coalesced part is greater than the width of the bar; both that part and the fore part of the free bar belong, not to the proper pier of the mandible, but to the palato-maxillary arcade—it is the proper "ethmo-palatine." In the Tadpole of all, and the adult of most, Batrachia, it is not differentiated from the pterygoid band. The mandible (*mk.*) has already a flattish articular facet, an angular and a coronoid process; the main bar is gently arcuate, and lessens gradually to its distal end, which is rounded, and does not touch its fellow of the opposite side.

The hyoid arch is large and highly subdivided; it has now a large forwardly-projecting basal piece (Plate 30, fig. 8, *b.hy.*).

The pier is the hyomandibular (*hm.*), with its styloid symplectic fore-growth (*sy.*); this part is free, and not, like its serial homologue the pterygoid process of the suspensorium, concrescent with the bar in front of it. This hyoid pier is still three-fourths as large as the pier of the mandible; its dorsal condyle is large and rounded, its body is swollen behind, ready to form the "opercular process," and below this knob, on the inner side, it is scooped for articulation with the inter-hyal (*i.hy.*). Here, as in the *hyostylic* types of fishes, the hyoid arch is subdivided primarily into an antero-superior and a postero-inferior bar; but in this type, as in the Teleostei, the latter is subdivided again into three segments—the inter-hyal, the cerato-hyal, and the hypo-hyal (*i.hy.*, *c.hy.*, *h.hy.*).*

The inter-hyal (*i.hy.*) is a small, short, but thickish segment which articulates with the inner face of the hyomandibular (*hm.*) above, and with the cerato-hyal (*c.hy.*) below. The cerato-hyal is nearly as long as the mandible, and is twice as thick as its distal part; it is oval in section, rather pinched in at the middle, and rounded at both ends. The lower convex end fits into the shallow cavity on the top of the hypo-hyal

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* In the Sturgeon and its congeners the symplectic is segmented off from the hyomandibular, and the same thing often occurs in the diminished and modified hyoid pier of the Batrachia, and even amongst some of the *Sauropsida*, as in the Chelonia (see 'On the Skull of *Chelone viridis*,' "Challenger" series, vol. i., part 5, plate 6, figs. 6, 6*a*).

(*h.hy.*), the thick, semi-globular, distal segment of the arch. The basal piece, *glosso-* vel *basi-hyal* (*b.hy.*), is wide, oblong, and three-fourths the height of the cerato-hyal; its rounded fore end is rather blunt; it is, in reality, a double bar, and its tissue is much lighter and more spongy than that of the side bars; its posterior end is doubly scooped for articulation with the hypo-hyals.

The branchial arches (Plate 30, fig. 8, *br'.*) are now beginning to divide across into four pieces on each side, namely, a pharyngo-, an epi-, a cerato-, and a hypo-branchial (see Plate 31, figs. 12, 13); a rod of cartilage of about the same thickness as the arches runs along the mid line, connecting them together; this is the basi-branchial (*b.br.*).

Transverse sections of a somewhat younger specimen ($7\frac{1}{4}$ lines long) show much that is instructive, and corroborate the observations made upon *dissected* embryos.

Section 1.—The first of these (Plate 31, fig. 1) is in front of the chondrocranium, through the fore part of the nasal capsule (*ol.*). The skin is very thick, and the mucous membrane of the nasal sac is composed of large columnar cells. The upper part is much flatter than the lower, in which three of the suctorial disks (*s.d.*) are seen in section.

Section 2.—The next section (Plate 31, fig. 2), is through the middle of the nasal sacs (*ol.*) and the fore-part of the hemispheres (C^{1a}.); here the lower face is less convex and the upper more so; the fore end of each trabecula (*c.tr.*) is cut through; it is a rounded projection, the rudiment of the free cornu.

Section 3.—The third section (Plate 31, fig. 3) is through the back of the nasal sacs (*ol.*), the first third of the cerebral hemispheres (C^{1a}.), and the solid coalesced end of the triple trabecular outgrowths of the basis cranii. The outline of the trabeculæ (*tr.*) is clearly seen below; but above, these bars—which are oval in section at this part and very thick—are confluent with an upper median mass, the intertrabecula (*i.tr.*); this is wider than the paired bars, fits in and on them, and rises over them as a dilated and concave floor to the membranous floor of the cranium.

Section 4.—The fourth section (Plate 31, fig. 4) is through the hemispheres (C^{1a}.) and barely misses the eye-ball (for *ol.* read *e.*). Here the fore end of the palato-quadrate cartilages (*p.pg.*) is cut through, where these bars run into the trabeculæ (*tr.*). The intertrabecula (*i.tr.*) is wider and flatter, and the shallow sulcus between the paired trabeculæ is gone; they together form a convex mass below, at this part. The section of the palatines would be circular, but the conjugational band is very thick, and obscures their real form.

Section 5.—The fifth section (Plate 31, fig. 5) runs through the fore part of the eye-balls (*e.*). This section is at the end of the intertrabecula (*i.tr.*), and the trabeculæ and palatine bars (*tr.*, *p.pg.*) are only half as large as in the last, and this section is seen to be oval, for the connecting cells are much reduced in quantity.

Section 6.—The sixth section (Plate 31, fig. 6), a little farther back, shows an evident fissure between the trabeculæ (*tr.*), which are now flatter, like bricks, and the connecting band is so thin as almost to set the palatine band (*p.pg.*) free.

Section 7.—Here (Plate 31, fig. 7) those bands (*p.pg.*) are free, and the trabeculæ (*tr.*) look like two flat, partly confluent bricks, gently bent round the base of the membrano-cranium. The section of the palatine bars, now very slender, is circular.

Section 8.—In this section (Plate 31, fig. 8) the eye (*e.*) is cut through its middle, and the mandible, very short as yet, has come into view; here, of course, the mouth cavity (*m.*) is shown. The pineal body (*pnl.*) and the fore part of the mid brain can just be seen, as well as the back part of the hemisphere (C^{1a}.), running into the thalamencephalon below. The trabeculæ (*tr.*) do not now follow the convexity of the membrano-cranium, but are flat and horizontally placed; they are distinct here, and oval in section. The palatine bars (*p.pg.*) are now round in section, and are more than twice their own diameter from the trabeculæ. Below the mouth cavity (*m.*) the mandibles (*mk.*) are cut through obliquely; they are rounded rods, similar to the palatines at the same part.

Section 9.—The ninth section (Plate 31, fig. 9) is through the mid brain (C^2.) and the thalamencephalon (C^1.). The trabeculæ (*tr.*) are now nearly cylindrical, and are a distance apart equal to twice their own thickness. Between them a thin lamina of bone is cut through; this is the parasphenoid (*pa.s.*). Outside the oral cavity (*m.*) the palato-quadrate cartilages (*p.pg.*) are seen to be flattened and concavo-convex, the convex side being turned inwards. On each side, below the mouth, the mandibles (*mk.*) are cut through obliquely. Above, in the superorbital region, the rudiment of the tegmen cranii is seen as a narrow band of cartilage—flat outside and convex within.

Section 10.—In the next section (Plate 31, fig. 10) the swelling mid brain (C^2.) reaches the superorbital bands on each side; the trabeculæ (*tr.*) are nearly twice as far apart as in the last, and are quite circular in section. Between and below them the parasphenoid (*pa.s.*) is twice as wide as in the last. This section shows the articular end of the mandible (*ar.c.*) joined to the hinge on the quadrate end (*q.*) of the palato-quadrate pier; this part is now high, flattened, and convexo-concave, with the concavity on the outside.

Section 11.—This is through the widest part of the mid brain (Plate 31, fig. 11, C^2.), the thalamencephalon (C^1.), the infundibulum (*inf.*), and the distinct pituitary body (*py.*). The superorbital bands are flatter and nearly vertical in position; the trabeculæ (*tr.*) are now furthest from the mid line; the parasphenoid bone is still visible below the pituitary body; and the convexo-concave quadrates (*q.*) are now cut through *behind* the mandible at the mid line below the mouth (*m.*). The tongue (*tg.*) is cut through in front of its cartilaginous core.

Section 12.—The next slice is through the hind brain (Plate 31, fig. 12, C^3.), the front part of the auditory capsule (*vb.*), and the fore end of the notochord (*nc.*). On each side of the hind brain there is a large mass of nerve-cells, the rudiment of the ganglia of the trigeminal and facial nerves (V., VII.).

The investing mass (*iv.*), wide up to its fore end, and not embracing the ascending apex of the notochord (*nc.*), is a rather thick plate of cartilage; the inner edge of each slab does not touch the notochord at this part. These basal bands are quite confluent with the auditory capsules, which are very solid on their outer side, but remain membranous to a considerable extent on the inner.

The anterior semicircular canal (*a.s.c.*) is cut through, and also the main cavity (*vb.*) with the horizontal canal opening into it; this canal bulges out the capsule, and under the "tegmen" thus formed we see the hyomandibular (*hm.*) as a continuous ray, thick and bulbous, below. Below it, at a little distance, the joint being further back, we see the second segment, or inter-hyal (*i.hy*); it is short and semi-elliptical, with a truncated end below. That end articulates with the rounded top of the phalangiform cerato-hyal (*c.hy.*), which is between it and the hypo-hyal; this latter segment was in front of this section (see Plate 30, fig. 8, *h.hy.*)

The foremost branchial arch is cut through near its ventral end, through the lower part of the cerato-branchial, as well as the distil and basal pieces (*c.br.*, *h.br.*, *b.br.*); these parts lie behind the tongue.

Section 13.—The next section (Plate 31, fig. 13) is behind the junction of the auditory capsules with the basal plates. These latter are here very massive, and almost square; the notochord (*nc.*) between them is very large; the ganglia of the vagus and glosso-pharyngeal (IX., X.) fill up much of the space below, between these bars and the capsules; but above, the open space is for the auditory nerve (VIII.). The anterior and posterior canals (*p.s.c.*) are cut through at their junction, and the horizontal canal where it opens behind into the vestibule (*vb.*). The hyomandibular (*hm.*) is severed behind its upper, or articular head, and that is the only part of the hyoid arch which comes into view here. The lower part of two of the branchial arches (*br.*) is cut through close above the heart (*h.*).

Bony matter is forming in the opercular fold, a growth from the hyoid region, and its great size, wrapping over the gill-arches and heart (*h.*), is well shown.

Section 14.—In this section (Plate 31, fig. 14*), the auditory capsule is seen to approach the investing mass (*iv.*) behind the large membranous deficiency in the inner wall. The basal plates cut through are here at their thickest part, and the posterior canal (*p.s.c.*) is most of it seen, as it becomes bulbous below. Here the notochord (*nc.*) has almost its full (*spinal*) thickness; the hyomandibular is cut through in its hinder part, or "opercular process;" only the upper part of this section is figured.

Section 15.—This (Plate 31, fig. 15*), which is behind the ear-capsule, shows that the occipital ring is still very incomplete, only the basal and lateral parts (*e.o.*) being developed. The basal cartilage (*iv.*) is thinning out towards the first vertebra; the pharyngo-branchial of the last functional arch (*p.br.*) and its gills (*br.p.*) are displayed; the notochord (*nc.*) is now full-sized. A comparison of these sections with the upper and lower views of the chondrocranium at this stage (Plate 30, figs. 7, 8) will make all plain.

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* Fig. 14 is lettered 15, and fig. 15, 14 by mistake.

Fourth Stage.—Young Lepidosteus, *$11\frac{1}{2}$ to $12\frac{1}{2}$ lines long: average size 1 inch.*

In these, the largest of those reared by Professor AGASSIZ and Mr. GARMAN, the chondrocranium is perfect; the occipital arch is beginning to ossify, and the investing bones are very numerous and quite distinct.

The cranium at this stage (Plate 32) corresponds very closely with that of a young Sturgeon *five inches* long, but has much larger membranous tracts, and is altogether a much lighter structure; in having rudimentary basi- and ex-occipital bony centres, it has already gone beyond the skull of an adult Sturgeon.

The fissure between the fore and hind brain (Plate 32, fig. 4, C^1., C^3.) is very distinct, and reaches to the base of the mid brain (C^2.); but there is no "posterior clinoid wall," such as would exist and be very massive in the skull of an embryo Sauropsidan or Mammal at the same stage.

Already the notochord (Plate 32, fig. 4, *nc.*) has retreated to a considerable distance behind the pituitary body (*py.*), which is now an appendage to the infundibulum (*inf.*); the brain well fills the whole cranial cavity up to this stage, but the hemispheres (C^{1a}.) are relatively very small.

Another thing to be noticed is this, namely, that the *pre-cerebral* growth of cartilage is almost as long as the whole cranial cavity, although it is only a fraction of the length to which it will attain. At first sight it might be thought that the mesocephalic flexure was gone, but the up-throwing of the mid brain, and the meeting of the fore and hind brain, show that the bend is very large and very sharp *at one point.* The four faces of the skull are all largely membranous, and but for the notochord (*nc.*), the floor would be open along nearly its whole length, for the cartilage only closes in at the mid line beneath the front end of the hemispheres and the olfactory lobes (C^{1a}., C^{1b}.). The thick cranial notochord (*nc.*) is receiving a bony investment between the thin, post-auditory ends of the investing mass (*iv.*); this will be seen better in the transverse sections (Plate 33). The fore ends of the basal bars (figs. 2, 3, *iv.*) diverge from the notochord (*nc.*) some distance behind its apex; in the middle part they are completely confluent with the auditory sacs.

The narrowed, diverging bars that retreat from, and then shoot on far in front of the notochord, are the trabeculæ (*tr.*); they approximate gently, and their interspace in front is sharply pointed. But the trabeculæ have not merely approximated, they are united together by the intertrabecular wedge (see Plate 31, figs. 7, 8, *i.tr.*); and this has now become a large rod, running forwards to the end of the narrow snout.

Outside this thick rod, but little of which is formed by the lateral bars, those bars grow externally into a large lanceolate leaf of cartilage, which reaches right and left nearly to the small, *distant* nasal sacs (fig. 5, *ol.*). These peculiarly *Acipenserine* outgrowths of the trabeculæ are the familiar "cornua" (*c.tr.*) curiously modified; both ends of each leafy growth are free, as rounded ears of cartilage; on the inside there is a sulcus, deepest, on both surfaces, between each cornu and the coalesced bars in the middle.

There is a squarish ear of cartilage on each side the pituitary body, growing from the trabeculæ, and looking outwards and forwards; these are the "basi-pterygoid processes" (*b.pg.*), so familiar to us in the Amniota; to these the pedicles of the suspensorium (*pd.*) are articulated. Behind these the auditory capsules are seen as very large masses, completely confluent with the chondrocranium. Below (Plate 32, fig. 2, *vb.*), the vestibule forms an almost hemispherial projection; this is caused by the sacculus, which contains the otolith (fig. 3, *ot.*).

Behind this there is a lesser eminence caused by the ampulla of the posterior canal (*p.s.c.*), between it and the larger cavity we see a shallow fossa. On the outside of this fossa the capsule projects, where it contains the horizontal canal (*h.s.c.*), and under this projection there is an oblong articular cavity for the hyomandibular.

Outside this, and between it and the superorbital band (*s.ob.c.*), there is a rounded projection. This corresponds with the lateral pre-auditory mass seen in Teleostei and ossified as the so-called post-frontal—my sphenotic ("On the Salmon's Skull," Phil. Trans., 1873, Plate 7, figs. 1–3).

Above (Plate 32, figs. 1, 5, *sp.o.*, *a.s.c.*, *h.s.c.*, *p.s.c.*), the outline of the large ear capsules is sinuous, the sinuosities being caused by the bulgings of the horizontal and posterior canals, and by the sphenotic process. Between the capsules, above, the tegmen cranii (*s.o.*, *t.cr.*) is developed both over the occipital and the post-sphenoidal regions. This roof is rather pointed behind, over the foramen magnum, and has an evenly concave margin in front; there it forms the hinder boundary of the large fontanelle (*fo.*), which is a short oval, emarginate in its narrow fore end. In front of the fontanelle there is a considerable *ethmoidal tegmen* (*t.cr.*), which covers the olfactory lobes and the small hemispheres (Plate 32, figs. 4, 5, C^{1a}., C^{1b}.); this is pointed behind in the middle, and laterally runs into the narrow arcuate superorbital band. The sides of the skull are oblique, the roof being more than thrice the width of the floor in the orbital region; these sides are mainly membranous; thus the orbito-sphenoidal cartilages are only represented by so much of the superorbital bands as belong to their territory; the alisphenoidal cartilage is merely so much of the chondrocranium as projects beyond the auditory capsule, laterally, between the basipterygoid and the sphenotic processes (*b.pg.*, *sp.o.*).

The small nasal capsules (fig. 5, *ol.*) have no separate cartilaginous roof; they are carried to the front of the snout.

The suspensorium of the mandible has retained its *primary* continuity with the ethmo-palatine cartilage, so that it is still a palato-quadrate (Plate 32, fig. 2, *p.pg.*, *pd.*); but this is quite free now from the other primary connexion, namely, that with the trabecula (*tr.*). This large arch, with its pier, foregrowth, and free mandibular bar, has undergone a similar lengthening to that of the cranium.

The pier or suspensorium is a large, oblongo-arcuate plate, ending in front in an oval sub-convex condyle, and a long, terete, pterygo-palatine process; this latter is considerably longer than the main part, and reaches as far forwards as the cornu trabeculæ (Plate 32, figs. 1, 2, *p.pg.*, *c.tr.*) ; it is slightly arcuate.

The main part of the suspensorium has a thick convex lower, and a sharp concave upper, margin. The upper edge has a convex enlargement behind. This is the rounded pedicle, which articulates with the basipterygoid process of the trabecula (*pd.*, *b.pg.*). The lower margin becomes concave towards the end ; a postero-external, triangular process—the otic process—finishes the dorsal end of the suspensorium. The sub-convex, oval condyle (*q.c.*) looks forward and outward, and fits into the scooped hinder face of the articular region of the free mandible. Above its articular concavity the mandible sends forwards a large rounded ear of cartilage, convex outside (Plate 32, fig. 1, *cr.c.*) and concave within (fig. 2, *cr.c.*); this is the coronoid process of the mandible. The angular process is a free rounded spur below and behind the articular concavity. The rest of the rod (*mk.*) is terete, and almost straight; it is only slightly arched upwards and reaches nearly to the end of the snout.

The hyoid arch (Plate 32, figs. 1 and 2) has a pier which is curiously and suggestively like the suspensorium of the mandible; but, already, it is relatively much less, being now about half as large. It has a pedicle, a free posterior process, a fore-growing rounded rod, and an articular facet for the free, inferior arch; the only difference, here, is the absence of any *borrowed* addition at the fore end, such as the pterygoid cartilage has in the concrescent palatine.

The *pedicle* of the hyoid pier (*hm.*) is the oblong, articular head, fitting inside the oblong concavity under the auditory "tegmen;" it has no definite neck.

The free posterior process (*op.p.*) is for the opercular bone; it is short and rounded. The fore-growing rounded rod is the "symplectic" region (*sy.*); it is terete, gently curved downwards, blunt at its fore end and enlarged near its origin; it lies anteriorly behind (under) the convex edge of the suspensorium. The body of the hyomandibular is gently bilobate and fenestrate in the middle (*hm.f.*); the articular facet for the "inter-hyal" (fig. 6, *i.hy.*) is a scooping between the two convexities of the hind margin (Plate 32, fig. 2).

The rest of the hyoid arch is not in one piece like the mandible, but in *three*, and these have, also, a large double median bar conjugating them. The first of these is a small, unciform segment of cartilage, the inter-hyal (fig. 6, *i.hy.*) ; it is articulated to the inner face of the hyomandibular by its hooked end, obliquely, and obliquely also to the top of the cerato-hyal by its base. The latter (*c.hy.*) is half the length and twice the thickness of the mandible; it is a rounded rod, swollen near the top, and then thickened gradually to its distal end. All but the top and lower concave face is ossified.

The distal concave end is articulated to the top of a globular segment—the hypo-hyal (*h.hy.*) ; this is not ossified. The right and left segments fit into a pair of concavities on the hind face of the glossal piece, or basihyal (*b.hy.*). This is tongue-shaped, the sides are parallel, the fore end rounded ; it is moderately thick, is essentially double, and is as long as the cerato-hyal. The subdivided, ossifying branchial arches will be described in the next stage.

The uniformity of the rapidly elongating intertrabecula is shown in the vertical section of the skull (Plate 32, fig. 4, *i.tr.*). The three trabeculæ are shown inside the lengthening snout, with its four rows of mucous glands in a second upper view (Plate 32, fig. 5); and in it, also, the position and relation of the sense-capsules and brain are displayed, and also how that the tegmen cranii (*t.cr.*) leaves the large mid brain (C^2.) unprotected.

A partial view of the chondrocranium, namely, the floor, from its upper face up to the end of the cranial cavity (Plate 32, fig. 3) shows the huge notochord (*nc.*), whose bony sheath is incomplete above, and the fore end of which is free, and but little attenuated; that part is curved but little upwards (see Plate 32, fig. 4). The fusion of the basal bands and auditory capsules is shown to be perfect, and the cupped tracts for the ampulla of the posterior canal (*p.s.c.*) and for the sacculus (*vb.*) are also seen. In front of these, on each side of the diverging parachordals—now to be called trabeculæ (*tr.*)—the basipterygoid peduncles (*b.pg.*) are shown. In the emargination behind these, in the fore part of the ear-capsules and in the occipital ring, thin films of bony matter are forming, which will become the alisphenoids, prootics, and exoccipitals; these will be shown better in the sections. The huge lanceolate pituitary fenestra (*py.*) is floored by the parasphenoid (*pa.s.*), which wedges in, in front, between the converging trabeculæ. In front of that part the chondrocranium is complete. The olfactory nerves (I.) escape from the bulbs (Plate 32, figs. 4, 5, C^{1b}.) and run along to the distant nasal sacs (*ol.*) between the intertrabecula and cornua trabeculæ in the deep groove between them above. The nasal branch of the ophthalmic (5′) runs forwards outside these. Some of the bony plates are shown on the chondrocranium; the foremost of these is the first of the *maxillary chain* (figs. 1, 2, *mx′.*); and on the palato-pterygoid there are three "parastoses," namely: the palatine, pterygoid, and mesopterygoid (Plate 32, figs. 1, 2, *pa′.*, *pg.*, *ms.pg.*).

Transversely vertical sections show much that is instructive in this stage also (see Plate 33, figs. 1–13).

Section 1.—In this (Plate 33, fig. 1) the fore end of the long face is seen to be convex above and somewhat concave below. The skin is very thick and glandular; the nasal sacs (*ol.*) are simple pouches, with a thick epithelium, the tissue beneath the skin has now become osseous; in this way we get the premaxillaries, nasals, maxillaries, palatines, &c.; the plates directly over the nasal sacs are the nasals, and the palatine part of the premaxillaries is shown below the sacs. In this section we see the fore end of the prenasal cartilage or intertrabecula (*i.tr.*); it is a long oval in section, with the narrow part below and the sides compressed.

Section 2.—Here (Plate 33, fig. 2) we have the prenasal (*i.tr.*) cut through behind the nasal sacs; palatine teeth attached to bony laminæ (*pa′.*) are seen below, and similar bony tracts (*p.ob.*) are seen, right and left above, enclosing the olfactory nerve (I.). Here the cartilage (*i.tr.*) is alate, each sharp wing being nearer the base than the top.

Section 3.—In this (Plate 33, fig. 3) the parts to be described are numerous, for the cornua trabeculæ (*c.tr.*) are cut through, and also the palato-quadrate (*p.pg.*) near its fore end. The intertrabecula (*i.tr.*) is oval in section, but it grows out, right and left, into wings, which thicken towards their outer edge and are as wide as the median bar; these are the cornua trabeculæ (*c.tr.*). Under their rounded end we see a small oval section of cartilage placed obliquely; this is the palato-quadrate (*p.pg.*). The fossa over each cornu trabeculæ is more scooped than that beneath it; in this lies the olfactory nerve (I.).

Beneath the intertrabecula, and following its curve, there is a thin lamina of bone; this is the parasphenoid; the oblique laminæ right and left of this are the vomers. The palatines (*pa'.*) are seen in the submarginal ridge, and one of the maxillary chain of bones (*mx'.*) in the lesser, outer ridge. Above, some of the superior (or ethmo-nasal, *et.n.*) and supero-lateral (or preorbital, *p.ob.*) scutes are cut through; the former protect the olfactory nerves (I.).

Part of this section (fig. 3A) is separate from the rest; it is through the lower jaw, in front of the tongue.

Here Meckel's cartilage (*mk.*) has a short oval section; outside it we see the dentary (*d.*) as a larger and a lesser lamina.

Section 4.—This (Plate 33, fig. 4) is behind the angle of the mouth, and close in front of the cranial cavity, where the three bars (*i.tr.*, *c.tr.*) are thickest. The middle part is one-half higher than in the last section, and is broader above and below. The side bars (*c.tr.*) are twice as thick here, and are shorter, and upturned; the olfactory nerve (I.) grooves both the bars, and is more than half enclosed in cartilage. Under these thick rounded wings each palato-quadrate (*p.pg.*) is seen; it is twice as thick as in the last section, is circular, and is its own width below the trabecular cornu.

As this is close behind the gape the mandible is in two sections; the upper is small—it is the fore end of the coronoid process; at a good distance below this part the main rod (*mk.*) is severed; it is oval, with the narrow end above. Bony laminæ belonging to the preorbital series of scutes are seen supero-laterally; below the intertrabeculæ the parasphenoid (*pa.s.*) is shown; a hooked, zigzag line of bone is seen propping up the palato-quadrate cartilage; this is the pterygoid, whilst outside the mandible the dentary (*d.*) is visible.

Section 5.—This (Plate 33, fig. 5) was made through the fore part of the hemispheres (C^{1a}.) and of the tegmen cranii. The base is formed almost entirely by the trabeculæ (*tr.*) for the middle bar dies out in this region; here they are at their thickest part; they form a crescentic mass, the horns of which grow upwards and a little outwards as the lateral ethmoidal wall. These walls pass above into the convex roof; there is a superorbital enlargement where these thickish laminæ pass into each other.

The palato-quadrate cartilages (*p.pg.*) are lesser again, flattened obliquely, and are still further from the basal bars. Behind the angle of the mouth the mandible thickens rapidly; this part (*ar.c.*) shows a section oblique and sub-reniform, close in front of the articular condyle. The large mouth-cavity (*m.*) is partly filled here with a transversely oval mass—the tongue; it has a large *double* core of a soft spongy kind of cartilage; this is the exceptionally symmetrical basi-hyal (*b.hy.*).

The same kind of surface bones are cut through above; the outer film is a superorbital (*s.ob.*)—a continuation of the same chain as the preorbitals; and the inner piece is the pointed fore end of the frontal (*f.*). The pterygoids (*pg.*) are still in section, but the mandible is cut through behind its splints.

Section 6.—The next section (Plate 33, fig. 6) is through the middle of the eye-ball (*e.*), and through the hinder wide part of the hemispheres (C^{1a}.). Here the cranium is largely membranous, for the only cartilages cut through are the superorbitals (*s.ob.c.*), mere bands running superolaterally, and the trabeculæ below. The latter (*tr.*) are just distinct at this point, and are brick-shaped—a little turned up at their outer ends. Thus at this part we have the fore end of the great fontanelle (see Plate 32, fig. 1, *fo.*), and the wide orbito-sphenoidal fenestra (fig. 2, *os.f.*).

The quadrate region (*p.pg.*) is cut through here, behind the hinge of the mandible; this is still faced on the inside by the pterygoid (*pg.*), whilst another film of bone is cut through above, namely, the mesopterygoid (*ms.pg.*). Above, the frontals (*f.*) are seen in section, and below, the tongue and basi-hyal (*b.hy.*).

Section 7.—This (Plate 33, fig. 7) is post-orbital, and is through the mid brain (C^2.), and the thalamencephalon (C^1.), near the pituitary body.

The trabeculæ (*tr.*) are very small, nearly circular in section, and at their greatest distance apart. The flattened superorbital bands (*s.ob.c.*) are thrown to the side of the membrano-cranium, which is at its weakest point here. The quadrate (*q.*), which was thin above and thick below in the last, is now thick above and thin below; the long pterygoid is still on its inside, and below its out-turned thin edge a small round rod of cartilage is seen: this is the symplectic (*sy.*).

In the root of the tongue there are three cartilages cut through, the middle bar is nearly circular, the others are flattened; these are the basi-branchial and the first hypo-branchials (*b.br.*, *h.br.*). In a fold outside and below these, a thicker round rod is seen; this is the cerato-hyal (*c.hy.*). Protecting the pituitary body, the parasphenoid (*pa.s.*) has here widened out considerably; for a short space, shown in this and the next section, the roof shows scarcely any osteoblasts.

Section 8.—This section (Plate 33, fig. 8) is through the fore part of the auditory capsules and the widest part of the mid brain (C^2.), where it turns down to join the hind brain. The anterior canal (*a.s.c.*) is cut through; here the capsule (*au.*) is imperfect within, and is beginning to ossify as the prootic. The investing mass (*iv.*) is cut through where it is shooting out into the trabeculæ, and the bands are wide apart. The superorbital band has now passed into the antero-superior angle of the auditory capsule. The pharyngeal cavity is open below, for here the great opercular fold is cut through; in it bony plates (*op.*) are forming; in its upper part the root of the symplectic (*sy.*) is severed, and below that a much larger bar with an ectosteal sheath; this is the cerato-hyal (*c.hy.*); below, the fore part of the heart (*h.*) is cut through, and above and across it the basal, distal, and cornual elements of one or two of the middle branchial arches (*b.br.*, *h.cr.*, *c.br.*). Here the parasphenoid (*pa.s.*) is of great width, undergirding not only the open basal fontanelle, but the basal plates also. The fore part of the Gasserian ganglion (5) is cut through.

Section 9.—This (Plate 33, fig. 9) is through the widest part of the hind brain (C^3.), and behind the fontanelle, for here we have the hind "tegmen" (*t.cr.*), as a thinnish layer of roof-cartilage, passing externally into the auditory capsules. At this part there is a thick band of cartilage on the inner side, in front of the large "meatus internus." Here the large ampulla of the anterior canal (*a.s.c.*) and the beginning of the horizontal canal are exposed, and outside it there is a ledge—tegmen tympani—with which the hyomandibular (*hm.*) is continuous. This is a flat cartilage; the whole of the hyoid pier is relatively small in this type.

Here the notochord is cut through, where it is enclosed by the broad, fore margin of the investing mass (*iv.*), which thins out, and curves outwards and upwards, to join the capsule. The opercular fold (*op.*) is cut through behind the lower part of the hyoid arch, which is projected forwards; the branchials (*br.*) are similar to the last, but further back in the series, and the heart (*h.*) is now cut across at its fore part.

There are several bony plates cut through in this, viz.: parietal, squamosal, opercular (*op.*).

Section 10.—Here (Plate 33, fig. 10) only the upper or main part is figured; it shows a section of the middle or widest part of the auditory capsule, where the inner wall is membranous. The anterior canal (*a.s.c.*) is cut through close to its junction with the posterior, and also the horizontal canal (*h.s.c.*) with the subdivisions of the vestibule (*vb.*). This section is behind the junction of the capsule with the investing mass (*iv.*), and shows the ganglion of the 7th and 8th nerves (VII., VIII.). The flat hyomandibular (*hm.*) is still in section, but behind its articular head; the broad parasphenoid (*pa.s.*), and the outer bony plates, are similar to those of the last section.

Section 11.—This (Plate 33, fig. 11) shows the hind part of the auditory capsule; the whole course of the posterior canal (*p.s.c.*) is seen through this back wall of cartilage. Here the capsules are some distance from the investing mass (*iv.*); above, the roof (*t.cr.*) has become deficient again, and at this part the hyomandibular (*hm.*) is cut through some distance below the condyle, and in this specimen a film of bone is seen investing the cartilage. The large notochord (*nc.*) has also a sheath of bone, and under each basal bar there is a section of the forked hinder part of the parasphenoid (*pa.s.*). One of the branchial arches (? the third) is seen in its whole extent (*p.br.*, *e.br.*, *c.br.*, *h.br.*, *b.br.*).

Section 12.—In this section (Plate 33, fig. 12) the occipital arch (*e.o.*) is cut through a little behind its upper part, and quite behind the auditory capsules. Here the 9th and 10th nerves and their ganglia (IX., X.) are laid bare, and the heart (*h.*) is cut through at its thickest part. The investing mass has become narrow and hour-glass shaped in section; it sends upwards a cartilage which thins out gradually upwards; this is the exoccipital (*e.o.*), which becomes supraoccipital above. The exoccipital bones are forming as embracing ectostoses; the notochord has its sheath, the rudiment of the basioccipital (*b.o.*), and under the investing mass the parasphenoidal forks are seen. Here there is one pharyngo-branchial (*p.br.*) and the distal parts of two or three arches, with the basal piece (*b.br.*) lying over the heart (*h.*).

Section 13.—This last section (Plate 33, fig. 13) is close in front of the first vertebra; it catches the last pharyngo-branchial (for *h.br.* read *p.br.*), and shows some pharyngeal teeth. Only half the arch comes in here, but it shows well the outer and inner laminæ of the exoccipital bone (*e.o.*). Here the notochordal bony sheath (*nc.*, *b.o.*) is thicker, and the splintery ends of the parasphenoid narrower than in the last section.

In the last three sections bony laminæ are forming beneath the skin; these and their relations will be better understood by reference to the figures and descriptions of more advanced stages.

Fifth Stage.—Young Lepidostei *2 to 2½ inches long.*

In this stage the skull is rapidly acquiring its permanent character (Plate 34, figs. 1, 2); the rostral part is now twice as long as the cranial cavity; in the last the pre-cranial part was not quite so long as the cranial.

This is mainly due to the growth of the intertrabecula (*i.tr.*), which already is considerably more than twice as long as the very long cornua (*c.tr.*).

The endocranium is now in this specimen (which is 2 inches long) nearly as perfect as it will be; the upper fontanelle (*fo.*) is a short ellipse, with the long diameter axial; it is relatively much less, through the growth of the tegmen cranii (*t.cr.*), fore and aft. The whole endocranium may be said to be pyriform, with an extremely long stalk; the "nose" of the *pear* is represented by the basioccipital (*b.o.*). The occipito-auditory region is semicircular above, with an apiculation over the foramen magnum; the orbital region is suddenly narrowed, and this gently lessens into the ethmoidal, which as gently becomes rostral, and the rostrum slowly lessens to its fore end, where it has two small wings. Near its root, however, it has two large wings—these are the cornua trabeculæ (*c.tr.*). The floor of the skull is still largely open (fig. 3), and the sides are occupied by the large orbital fenestræ (*os.f.*). The sheath of the notochord (*nc.*) is most completely ossified below; above (fig. 3), it is still membranous in front; the bony sheath is the basioccipital (*b.o.*); it runs a little, right and left, into the basal cartilage.

The auditory capsules are completely confluent with the chondrocranium; and their canals (*a.s.c.*, *h.s.c.*, *p.s.c.*) are to be seen through the cartilaginous wall. Above, the even form of the skull is not much altered by the canals within; below, the succulus (fig. 2, *vb.*) largely swells the vestibular region; the form of this cavity is a short oval (figs. 2 and 3, *vb.*). The condition of the hind skull is best studied from above, in a preparation of the base (fig. 3); here the exoccipitals (*e.o.*) are seen to be forming broad borders of bone to the cartilage of the arch, and these are approaching the cephalostyle (basioccipital, *b.o.*).

Further forwards, in the angle between the capsule and the basal plate, under the Gasserian ganglion, the cartilage is being ossified as a prootic (*pr.o.*), and the wider wings which stand out from the front of the capsule are becoming the sphenotics (*sp.o.*). Also, still further forwards, the lower wings or basipterygoids (*b.pg.*) are getting a coating of bone; this ectostosis runs upwards into the side wall, in front of the capsule—it is the alisphenoid (*al.s.*).

In front of these last wings the trabeculæ (*tr.*) become bent, first outwards and then inwards, ready to join the median bar (*i.tr.*). They are rounded, solid rods. The basal fontanelle is now divided across, near its hinder end, by a narrow band, which is cartilaginous at its roots and fibrous in the middle; this is the *late* and feeble "post-pituitary wall" (*p.cl.*)—here a mere *bridge.*

The small triangular space behind is the posterior basi-cranial fontanelle (*p.b.c.f.*); the large, pinched, pyriform space in front is the pituitary space (*py.*) or anterior basi-cranial fontanelle. Its narrow anterior third runs up to the intertrabecula (*i.tr.*), which goes further back than the ethmoidal wall (*l.eth.*), and is, indeed, the rudiment of the "perpendicular ethmoid." Infero-laterally, the ethmoidal wall (Plate 34, fig. 2) is very restricted, for the orbito-sphenoidal fenestral (*os.f.*) is of great length, being extended equally in front of and behind the optic nerve (II.). But above (fig. 1, *t.cr.*) the tegmen cranii, in front, is as large as the long oval fontanella (*fo.*); behind that space the spheno-occipital tegmen is one-half longer, axially, and twice as wide across.

The three confluent trabecular bars combine to close in the fore part of the cranial cavity (Plate 34, figs. 1 and 3), only leaving an opening right and left for the long olfactory nerves (I.). When they have escaped from the skull they lie for the hinder half of their course in a deep rounded sulcus between the cornual extensions of the trabeculæ (*c.tr.*) and the huge middle bar (*i.tr.*). The three bars are at first nearly of the same width; the cornua are rounded where they first project as longitudinal wings, but they soon become narrow rods, and end in a pointed manner behind the middle of the precranial region. Thence the intertrabecula is a gently compressed rod, only slowly lessening forwards, and ending as a slightly winged lobe in the end of the beak.

The first and second visceral arches (Plate 34, fig. 4) are elongated forwards like the skull, but the "pier" of the hyoid arch (*hm.*, *sy.*) is less than half the size now of the mandibular suspensorium (*p.pg.*, *pd.*). I have shown these with their splints attached as seen from above (Plate 34, fig. 1) and from below (fig. 2); also without the "parostoses" from their inner side (fig. 4). Their intrinsic bony centres or "ectostoses" are now clearly seen, but they are very small in proportion to the cartilage in which they appear. The suspensorium and its free bar reach from the auditory region to a small distance behind the end of the snout; these cartilages, moreover, largely overlap each other; end to end, they would be considerably longer than the entire skull. The quadrate hinge (*q.c.*) is opposite the point where the olfactory nerves (I.) emerge, a little in front of the cranial cavity. The dorsal end of the suspensorium is bilobate; the pedicle (*pd.*) is an oblong, oblique facet, looking backwards and inwards, and articulating with a similar facet on the basipterygoid (*b.pg.*); the outer lobe is free; it is the triangular otic process (*ot.p.*); it reaches almost to the ear-capsule, but is too short to articulate with it. The main part of the suspensorium runs from these hind lobes to the quadrate hinge (*q.c.*); the inner margin is first hollow and then arched; the arch runs along the free anterior process; that edge is sharp. The outer edge is thick and ribbed on its inner face (Plate 34, figs. 2, 4); it is nearly parallel with the upper, being convex behind and concave in front. The width of this large plate is equal to half its length, and it is very elegantly sigmoid.

The quadrate hinge (*q.c.*) is a small oblong saddle, the main direction of which is forwards and a little downwards; it is convex outwardly, but has a rising inner crest (Plate 34, fig. 4), which fits closely to the articular condyle (*ar.c.*). Beyond the hinge the pterygo-palatine bar is first half as wide as its root, and then losing its inner crest, it becomes a rounded, straight, slender style, which ends very close to the point of the cornu trabeculæ (Plate 34, fig. 1, *p.pg.*, *c.tr.*).

The lower or inner face of the suspensorium is gently convex; the upper or outer face, gently concave. On the convex inner face there is a large splint-bone — the pterygoid (Plate 34, fig. 2, *pg.*); it covers more than half of the broad part of the cartilage—its antero-superior part. For some distance in front of the quadrate condyle it is continued forwards, undiminished in size; it then lessens gradually into a pointed style, which runs parallel with, and a little on the outside of, the rostrum (*i.tr.*) for three-fifths of its length.

A very narrow dentigerous bone underlies the narrow fore half of the pterygoid, and then goes beyond it up to the premaxillary, or nearly to the end of the snout. It is rather broadened in its diverging hind part, and then the right and left bones gently converge forwards; these are the *parosteal* palatines (*pa′.*).

Along the inner edge of the suspensorium there is a narrow, thin, falcate splint, which reaches from the top of the broad part to the neck of the pedicle; this is the "mesopterygoid" (*ms.pg.*). The front of the neck of the pedicle is ossified as a small ectosteal patch; this is the "metapterygoid" (*mt.pg.*). The neck of the quadrate condyle also is ossified; this is the small quadrate ectostosis (*q.*).

From the dilated end of the intertrabecula (Plate 5, fig. 2, *i.tr.*) to the fore end of the palato-quadrate styles, there is an extremely delicate pair of bony threads; these are the vomers (*v.*). In their hinder third these bones underlie the styloid end of a bone more than twice their breadth; this is the parasphenoid (*pa.s.*); it is carinate below, widening as it approaches the basis cranii; it then narrows till it nearly reaches the optic nerve (for I. read II.); thence it widens, loses its keel, and becomes grooved along its middle below. This bone then trebles its width, and sends out two pairs of angular projections, the lesser to support the basipterygoid processes (*b.pg.*), and the larger farther back to support the vestibular swellings of the ear-capsules (*vb.*). The bone then becomes divided into two sharp styles, which embrace the lower part of the basioccipital. The parasphenoid is three-fourths the length of the head; it forms the only floor to the skull in the pituitary region (fig. 3, *py.*, *pa.s.*).

The bones that invest the skull above, and postero-laterally, will be described in the next stage, but partly also in the sections illustrating this. The splints of the first arch also will be described hereafter, but there is one which is figured on this Plate, namely, the preopercular. This bone (Plate 34, figs. 1, 2, *p.op.*) is a very narrow, but rather sharp splint, which is applied to, and takes the curves of, the lower edge of the suspensorium; it is the normal splint of the mandibular suspensorium, and is more like that of a Frog than that of an osseous Fish.

The length of the free mandible (Plate 34, fig. 4), as compared with that of the palato-quadrate, is as 20 to 19; it is, therefore, already a very long jaw. In its hind part it is nearly as broad as its pier, and then runs to its end as a somewhat stouter rod than the pterygo-palatine above it.

The condyle (*ar.c.*) is cylindroidal, concave as seen from the side, but somewhat convex across; there is an ectosteal "articulare" (*ar.*) in the broad part near the condyle. The angle is scarcely produced; but in front of the condyle, above, the cartilage grows into a pedunculated crescent of cartilage, the coronoid crest (*cr.c.*) The notch between this part and the lessening rod (*mk.*) is very deep; this leafy coronoid is convex outside and concave within. The long Meckelian rod (*mk.*) is gently arcuate and pointed at its fore end, where it nearly meets its fellow of the other side.

This skull is scarcely *amphistylic* even, much less *hyostylic*, for the pedicle of the mandibular pier is strong and well articulated, but the hyomandibular (*hm.*) is feeble, and the binding process or symplectic (*sq.*), feebler still.

The hyoid pier or hyomandibular (Plate 34, fig. 4, *hm.*), with its symplectic foregrowth (*sy.*), is about one-third the size of the palato-quadrate in front of it. The articular facet is a long, arched, convex condyle, with scarcely a perceptible neck; behind it is the knob for the opercular; below, this multilobate mass is sinuous, and scooped on its inner face to form the oblique condyloid facet for the interhyal. The front margin is concave, and the whole bulk suddenly lessens into a sigmoid style, bent first upwards, and then downwards, as it runs obliquely forwards to bind inside the hinder angle of the suspensorium, which is scooped to receive it. A small oval fenestra is seen in front of the middle of the hyomandibular; and around this, below the condyle, there is an ectosteal sheath. All but the front fourth of the symplectic also is ossified as a delicate shaft-bone.

The free or postero-inferior part of the hyoid arch is more evenly massive than its pier or antero-superior part. The conjugational piece, or inter-hyal (*i.hy.*), is pyriform;

it forms a loose joint inside the hyomandibular by its narrow upper end; and below, its inner face is scooped for the backwardly bent head of the cerato-hyal (*c.hy.*).

That segment is seven times the size of the inter-hyal joint; its head is bent back, its shoulder thickened, and has a short, separate ossification, the epi-hyal (*e.hy.*), and its shaft is rounded. The inferior condyle is hemispherical, and the main part of the thickest rod is ossified as the cerato-hyal (*c.hy.*).

The hypo-hyals (Plate 34, fig. 4, *h.hy.*) are sub-globular nodules, scooped above for the cerato-hyal, and ossified on their outside. They fit against the paired concavities of the basi-hyal (*b.hy.*)—a double, oblong, tongue-shaped, interglossal segment, as long, and twice as wide, as the cerato-hyal. This thickish plate is grooved above and below, is rounded and emarginate in front, and is composed of a spongy kind of cartilage, full of fibrous septa, which form a network in it.

There are four perfect, and one imperfect, branchial arches; they are less than half as solid as the hyoid (Plate 34, fig. 4, *br.*). In the mandibular arch, when the mouth is closed, the axes of the suspensorium and its free cartilage become coincident. In the hyoid arch they are *parallel*, not coincident; here, in the branchial arches, they are continuous, the upper being superimposed upon the lower element.

The relative size of these parts is greatly altered, and the subdivision does not exactly correspond with that of the mandibular and hyoid. The part answering to the suspensorium and hyomandibular is less than a fourth the size of that which corresponds to the mandible and cerato-hyal. Each "pier" is subdivided into two pieces—a pharyngo-branchial (*p.br.*) above, and an epi-branchial (*e.br.*) below. The upper piece is a little tongue of cartilage, turned inwards and forwards, and the lower has a short, bony shaft, and is a little rod turned directly downwards. These two segments *do not* correspond with the hyomandibular and symplectic; the upper piece has a double counterpart in the hyoid arch, namely, the hyomandibular and symplectic—one cartilage, with two bony centres in it.

Also, the counterpart of the cerato-hyal, the cerato-branchial (*c.br.*) articulates directly with its pier, the epibranchial; so that there is nothing in the branchial arches corresponding to the inter-hyal.

Instead of the hypo-branchials (*h.br.*) being short nodules, they are in the first two arches nearly as long as the cerato-branchials; are thicker than them below, but less ossified. The three joints between the four pieces seem to show no distinct joint-cavity, but are fibrous. Below, the rounded rods of the hypo-branchials fit into depression on the basal bar—basi-branchial (*b.br.*). This is, in front, a thickish, rounded rod of cartilage; it then thins out, and behind it is flat and emarginate. The first and second arch unite with the long, first basi-branchial segment; the third nearly reaches the short second piece, and the fourth is loosely attached to the side of the third piece, which is as long as the first. None of these pieces are ossified, and the first does not reach the basi-hyal, for the hypo-hyals are thrust between them.

The lessening third and fourth hypo-branchials (*h.br.*) are not ossified. The fifth cerato-branchial (*c.br* ^{5}.) is less than those in front of it ; it is not ossified, and there is no other segment in that arch.

The branchial arches are but little modified after this ; they merely increase in size, and are always small as compared with the arches in front of them. A series of sectional views will complete the illustrations of this stage.

I have now to illustrate this stage by a series of sections made from a specimen $2\frac{1}{2}$ inches long.

Section 1.—This section (Plate 33, fig. 14) is taken from near the end of the snout in front of the nasal sacs. The intertrabecula (*i.tr.*) is seen to be oval, with the larger end below. The whole snout is elliptical, with a slight convexity above, and a slight concavity below, at the mid line. The premaxillaries (*px.*) are cut through, both in their body and their palatine process (*p.px.*) ; they are reticulations of thin laminæ, and both above and below enclose a mucous gland.

Section 2.—The nasal sacs are cut through here (Plate 33, fig. 15, *ol.*) and the bony laminæ are more complex ; the intertrabecula (*i.tr.*) has the same shape as in the last, but the whole snout is rounder above and flatter below.

Section 3.—Behind the nasal sacs other bones come into view (Plate 33, fig. 16) ; here the evenly elliptical rostrum (*i.tr.*) has the laminæ of the ethmo-nasal (*et.n.*) surrounding it, and on the side a mucous gland is seen in the upper half of one of the maxillary chain (*mx′.*) ; here the lower face of the snout is becoming convex, with a median groove ; above this, right and left, vomerine teeth (*v.t.*) are seen.

Section 4.—In this section (Plate 35, fig. 1) the lower jaw also is cut through ; the snout is flatter here, has a ridged lip, right and left, with sub-marginal grooves. The intertrabecula (*i.tr.*), half-way between its fore end and the cornua (Plate 34, figs. 1, 2, *c.tr.*) is circular in section, and is flanked by the layers of the thickening ethmo-nasal (*et.n.*) ; outside, one of the maxillary chain (*mx′.*) is seen lying over the palatine (*pa′.*), with its large tooth, and under the rostrum the two vomers (*v.*), each with a small tooth, are cut through. Below, the mandibular rods (*mk.*) are cut through at their front part, and right and left we see the solidifying substance of the dentaries (*d.*) ; over the cartilage towards the mid line, a small, separated style is cut through—this is the splenial (*spl.*).

Section 5.—Another section in front of the angles of the mouth (Plate 35, fig. 2) brings the tongue into view. Here the rostrum (*i.tr.*) is deep, and twice the size it had in the last section ; it is flattish above, and more convex below. Here the loose reticulation of the extremely thin bony laminæ would seem to defy interpretation ; but it can be classified into groups, and these groups named. Under the flat top of the beak, on each side of the rostrum, several of these thin plates are seen to be connected together, overarching a mucous gland (*m.g.*) above, and the olfactory nerve (I.) lower down. Above these is a sub-marginal groove, right and left ; outside this groove the beak is convex, and from the convex part there runs inwards a thin bony flake towards the deep, palatal chink; this is the fore part of the frontal. Below this there is one of the maxillary chain (*mx′.*) cut through, protecting a gland; and inside this, on the lateral lobe of the beak, there is a wedge-shaped tract of fine diplöe, the lower part of which carries a large tooth; this tract is the palatine (*pa′.*) cut through. In the triangular median keel of the beak there are three thin plates cut through; two of them are superficial and the third is deeper, taking the form of the rostrum somewhat, but diverging externally, and having a short crus below—this is the parasphenoid (*pa.s.*). The paired laminæ running downwards and inwards, outside it, are the vomers (*v.*) in their widest part; they are overlapped by the inner end of the frontals, externally; which, at their outer end, overlap, obliquely, the sharp end of both the cornua trabeculæ and the pterygoid cartilages (*p.pg.*). Below, the thick lower jaws have in them the section of the MECKEL'S cartilage (*mk.*), large and almost circular. There is here the flat, double tongue, with its soft basi-hyal (*b.hy.*), also double. On each side of the tongue there is a deep sulcus. The splenial bone (*spl.*), over and within the cartilage, is here at its largest size, and the dentary (*d.*) takes up a large space by its reticulations; it encloses a mucous gland below.

Section 6.—This section (Plate 35, fig. 3) is through the angle of the mouth, and thus the upper and lower tracts are continuous; the lower or mandibular region is of great height, being cut through close in front of its huge coronoid region. Here the cornua trabeculæ (*c.tr.*) are at their thickest part, or middle (Plate 34, figs. 1, 2, *c.tr.*) and are continuous by a thin oblique tract, with the rostrum (*i.tr.*) which is at the thickest part in this and the next section. Here it is semi-elliptical above, and sub-carinate below, and the thin edges of the cornua ascend to their thick outer part; the olfactory nerves (I.) lie in the hollow between the bars. A little below the cornua the pterygo-palatines (*p.pg.*) are cut through; they are oval in section, their oblique position is parallel with that of the cornua, and their size is nearly as great. The mandibles (*mk.*) are oval in section, here, and twice as thick as the pterygo-palatines; the basi-hyal (*b.hy.*) is here at its widest part. The laminæ of the ethmo-nasals (*et.n.*), and of the frontals (*f.*) run close to each other, and below the pterygo-palatine there are two tracts of reticulated bone; these are the palatines (see fig. 2, *pa′.*) below, and the pterygoid (*pg.*) above. The splenial (*spl.*) is here at its widest part, and the dentary (*d.*) is composed of a large strip of bone, externally, and of a wide network, below.

Section 7.—The position of this section (Plate 35, fig. 4) is evident, for it is through the thickest part of the coronoid process of the mandible (Plate 34, fig. 4 *cr.c.*) where this remarkable crest is separated from the main rod by a large rounded notch. Here the cornua trabeculæ (*c.tr.*) are thinning out, behind, and the intertrabecula (*i.tr.*) is most solid, it is quite round above, and sub-angulate below. The pterygo-palatines (*p.pg.*) are oval and are further from the narrowed cornua (*c.tr.*)—they are twice as near to the coronoid cartilage (*cr.c.*) Below that crest,—which is placed obliquely across the face a little tilted upwards, and the section of which is oblong, but hooked inside—the main bar (*mk.*) is a large nearly vertical ellipse; here the basi-hyal (*b.hy.*) is very wide. The ethmo-nasals (*et.n.*) are less, and the frontals (*f.*) are larger; the palatine is gone, and the pterygoid (*pg.*) is becoming a much thicker bone. Over the coronoid process (*cr.c.*) the supra-angular bone (*s.ag.*) is seen, the dentary (*d.*) is very extensive, and is helping the supra-angulare to cover the coronoid cartilage, and growing down the outside of and coming beneath the main rod (*mk.*) The coronoid bone (under *cr.c.*) now appears inside the mandible.

Section 8.—This slice (Plate 35, fig. 5) is a little in front of the hinge of the lower jaw, and behind the outspread wings of the trabeculæ; hence, the rostrum appears to be single, although it has the trabeculæ confluent with it in its lower half. About the middle there is a slight hollow; above, it is rounded, and at its base somewhat mammillate in section; this part has the parasphenoid (*pa.s.*) fitting to it, which is thus convex instead of carinate.

The pterygo-palatine (*p.pg.*) is oblique and oval; it is nearer the rostrum than the mandible; that part (*ar.c.*) is larger than the rostrum, and is irregularly spindle-shaped in section, with its upper half slightly incurved; in its inner face the "articular" centre (*ar.*) has appeared. The basi-hyal (*b.hy.*) is now wider, and thicker; flat above, and convex behind. The ethmo-nasal bone (*et.n.*) is narrower, and the frontal (*f.*) wider; the parasphenoid (*pa.s.*) has lost its keel, and become convex and alate. The pterygoid (*pg.*) has now more diplöe above, is growing far down as a thin lamina inside the angle of the mouth; the dentary (*d.*) lies on both sides of the lower half of the cartilage; above, the supra-angulare (*s.ag.*) lies over it, and below, the angulare (*ag.*) flanks it.

Section 9.—A little further back (Plate 35, fig. 6) we get a similar section to the last, but the pterygoid (below *p.pg.*) is still more complex, above, and the articular cavity of the hinge of the lower jaw (*q.c.*, *ar.c.*) is laid open, and has a piece of the quadrate in its hinder face.

Section 10.—This is through the fore part of the eye-ball (Plate 35, fig. 7, *e.*); and here, the upper part of the chondrocranial mass is thicker; for it is in the ethmoidal region, and the olfactory nerves (I.) now run through tunnels in the closed-in skull. At this part the suspensorium is cut through in the quadrate region (*q.c.*); it appears as a sigmoid tract; thin above, thicker and rounded below, and with its upper, slightly out-turned, edge not far from the cranial axis. Below, the basi-hyal (*b.hy.*) is becoming more solid. Here each frontal is mainly a flat lamina, becoming complex externally; some of the complex outer part, however, belongs to a circumorbital. Inside the suspensorium the pterygoid (*pg.*) is a sigmoid tract of diplöe, and below the cartilage a small, triradiate tract of bone is cut through; this is the preopercular (*p.op.*).

Section 11.—Here (Plate 35, fig. 8) the cranium is cut through where the olfactory lobes (C^{1b}.) lie; it is therefore behind the proper septal portion of the intertrabecula, and shows the beginning of the tegmen cranii (*t.cr.*). The other parts are similar to those exposed in the last section, but, here, the quadrate bone (*q.c.*) is seen in the lower part of the suspensorium, as an enclosing ectosteal plate.

Section 12.—A little further back (Plate 35, fig. 9) the section is through the hemispheres (C^{1a}.), and is behind the quadrate bone; the other parts are similar to those in the last two sections.

Section 13.—Here (Plate 35, fig. 10) the hemispheres (C^{1a}.) are wider, and with them the cranial cavity, whose walls are thinner, and laterally are partly ossified; these bones right and left, are the lateral ethmoids (*l.eth.*); they were not seen in the dissection of the lesser specimen of this stage; (2 inches long, Plate 34; the one sectioned was $2\frac{1}{2}$ inches); but these bones are figured in the dissection of the next stage (Plate 38, figs. 2, 3).

Looking at the base of the cranial axis in this and the last three or four sections, we see that the parasphenoid (*pa.s.*) fits to a cartilaginous mass having a *trilobate outline* below; this arises from the fact that it was formed by the coalescence of three cartilages, viz.: the trabeculæ and the intertrabecula.

Section 14.—The hemispheres in this section (Plate 35, fig. 11, C^{1a}.) are rapidly widening, and the cranial walls are now deficient, the sides being membranous—the orbito-sphenoidal fenestra—in the lower half. The tegmen cranii (*t.cr.*) runs down the sides half way, and is grooved above; the lower edge of the cartilage just touches the optic nerve (II.). Here the base of the skull has lost its height; it is concave above, and scooped below. Here is the hinder end of the intertrabecula, and the trabeculæ (*tr.*) are each of them crested below. The suspensorium is very flat here, especially towards the top; it thickens out again above, and is surmounted there by a small extraneous bony plate—the mesopterygoid (*ms.pg.*, see also Plate 34, figs. 1, 2); here the pterygoid (*pg.*) has become a thin plate. The essentially double nature of the basi-hyal (*b.hy.*) is clearly seen in this section.

Section 15.—The hinder part of the hemispheres (Plate 35, fig. 12, C^{1a}.) are now cut through, and the tegmen cranii (*t.cr.*) is now a thin, sinuous awning thrown over the brain-cavity; it is hollow above, and convex at the sides. The frontals (*f.*) are becoming thin, and the hinder superorbitals (*s.ob.*) are thick and large. The trabeculæ (*tr.*) only are seen in this section, which is through the fore part of the long pituitary space (see Plate 34, fig. 3, *py.*); they are oval in section, and a space equal to their width is filled up between them by the parasphenoid (*pa.s.*).

The suspensorium (*q.c.*) has thickened again, and still the same bones are applied to it, namely: the mesopterygoid, pterygoid, and preopercular (*ms.pg.*, *pg.p.*, *p.op.*). Below, the section was made behind the basi-hyal, and through the first basi- and hypo-branchials (*b.br.*, *h.br′.*); outside, we see the cerato-hyal (*c.hy.*), with its bony sheath, cut through.

Section 16.—In this section (Plate 36, fig. 1) the skull is cut through close in front of the basi-pterygoids (Plate 34, figs. 1–3, *b.pg.*); in this specimen there must have been some little projection backwards from the front tegmen (*t.cr.*), not seen in the one dissected; this would have made the fontanelle heart-shaped, instead of circular.*

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* This projection from the front "tegmen" was seen in the last stage (Plate 32, fig. 4, *t.cr.*), where the fontanelle has a similar shape to that of a young Salmon of the 2nd week. ("Salmon's Skull," Plate 4, figs. 1, 2.)

Here the cavity of the skull is at its widest part; further back the width of the head is due to the addition of the auditory capsules, but the hind brain (C^3.) is only half as wide as the mid brain (C^2.). Here the razor passed from the front of the fontanelle, above, to the middle of the fontanelle, below, just where the trabeculæ pass into the investing mass (*iv.*), and behind the optic foramina (see Plate 35, figs. 11, 12); the section is therefore somewhat oblique, backwards and downwards. The slight projection from the front tegmen (*t.cr.*) is wide apart from the alisphenoidal region, or lateral band of cartilage (*al.s.*), which is thickish, convexo-concave, and occupies more than half of the side wall.

Below, under the thalamencephalon, the parasphenoid (*pa.s.*) is thick and narrow, and is strongly wedged in between the narrowest part of the basal bars (see also Plate 34, fig. 3, *tr.*, *iv.*, *pa.s.*), which are oval in section, and slightly tilted outside. Between them and the alisphenoidal tract (*al.s.*), part of the trigeminal nerve (V.) is seen. Above, the parietals (*p.*) are cut through, and also a lateral bone—the squamosal (*sq.*). Here the suspensorium (*q.*) is, in section, like a drumstick, but feeble below and out-turned, for it thins down towards its lower thickening, and there bends outwards over the small, round symplectic (*sy.*). Above, it is round, very solid, and sheathed at the very top with a bony tract; this is the "metapterygoid" (*mt.pg.*) which is cut through.

Below, the preopercular (*p.op.*) is seen as a small tract of diplöe, enclosing a mucous gland; inside it, the cerato-hyal, with its ectostosis (*c.hy.*), is severed, and further inwards the basi-branchial and the first and second hypo-branchials (*b.br.*, $h.br^1$., $b.br^2$.).

Section 17.—We have now a section (Plate 36, fig. 2) close in front of the pituitary body, but missing it; it is through the infundibulum (*inf.*) below, and the mid brain (C^2.) above, and catches the terminal point of the small tegminal projection; this thin slice was the next to the last, which is not always the case in those which are figured.

Here the basipterygoid processes (for *q.* read *b.pg.*) are seen as thick wings growing from the front part of the investing mass;* they are partly ossified by the lower edge of the alisphenoidal centre; they are thin at their root, thick outside, and they dip a little; above them, the 5th nerve (V.) is cut through. Here the parasphenoid (*pa.s.*) sends out its first pair of angular projections (Plate 34, fig. 2), so that it forms a bony floor to the skull. Above, the parietals and squamosals (*p.*, *sq.*) are seen in section, and also one of the numerous post-orbital scales. The symplectic (*sy.*) is cut off where it has a bony sheath, as it passes forwards to the suspensorium. The cerato-hyal (*c.hy.*) is trilobate here in section; outside, it is the interopercular (*i.op.*); the branchial sections are of the same bars as in the last.

Section 18.—This (Plate 36, fig. 3) is through the fore part of the hind tegmen (*t.cr.*), and where the mid brain passes into the hind brain (C^3.). Here the head is slightly concave above; the roof-cartilage is rather thin, and forms part of a nearly complete cincture, for it is confluent here with the fore part of the auditory capsule (*au.*), which in turn passes into the investing mass (*iv.*) below. There the cartilage is a thick wedge, right and left, and the space between the two wedges is filled in by the parasphenoid (*pa.s.*); in front of the apex of the notochord. Where these basal plates pass into the capsules there the facial nerve (VII.) is severed, and further outwards the capsule has a groove under it in which the fore part of the head of the hyomandibular (*hm.*) is seen; it is partly ossified above and also below as the symplectic. A neat lip, the rudimentary "tegmen tympani," is seen outside the rod, and above it the ampulla of the anterior canal (*a.s.c.*), whose arch also is severed higher up. The bony tracts below are parts of the large, infolded, angular interopercular (*i.op.*, see also Plate 37, fig. 4, *i.op.*).

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* Two errors escaped me in the lettering of this figure,—for *q.* read *b.pg.*, and for *mt.pg.* read *iv.*

The second, third, and fourth hypo-branchials and the middle of the basi-branchial (*h.br* $^{2-4}$., *b.br.*) are severed, and the gills are seen depending over the fore part of the heart (*h.*).

Section 19.—The roof is now in the superoccipital region (Plate 36, fig. 4, *t.cr.*); it is twice as thick and only two-thirds the width at this part, as compared with the last. Here the hind brain (C^3.) has only membranous sheets interposed between it and the auditory labyrinth, which is deficient in its cartilaginous wall in this, the region of the "meatus internus," where the auditory nerves (VIII.) enter. The section of the hyomandibular (*hm.*) is here behind the symplectic; above its head the horizontal canal (*h.s.c.*) is cut through and the end of the arch of the anterior canal (*a.s.c.*) is seen to come close to the great inner "fenestra" of the capsule. Here, as in Sharks and Skates, the basal plate (*iv.*) projects beyond the capsules, and in the angular space the 7th nerve (VII.) is cut through.

Below, this section is still in front of the notochord, but the parachordals (*iv.*) bend down very near to each other; they are supported by the parasphenoid (*pa.s.*). Here the branchial arches (*h.br.*, *b.br.*) are cut through near their hinder part; below, their gills (*g.p.*) are seen hanging over the heart (*h.*), and the interopercular is seen outside the hyomandibular (*hm.*).

Section 20.—In this slice (Plate 36, fig. 5) the apex of the notochord (*nc.*) is cut through, and here the auditory capsules have recovered their inner wall; the end of the horizontal (*h.s.c.*) and the part leading to the common sinus of the anterior (*a.s.c.*) and posterior canals are cut across. The vestibule (*vb.*) is lessening here, where the back of the "sacculus" is shown. The hyomandibular is severed behind its head, and outside it the interopercular and subopercular are shown in section in the fold (*op.*). Here a ganglionic mass belonging to the 9th and 10th nerves (X.) is brought into view; and above, the parietals and one of the temporal series (*s.t.*) are severed.

Section 21.—The back of the auditory capsule, with the ampulla and hind part of the posterior canal (Plate 36, fig. 6, *p.s.c.*), are now severed; here the double passage for the 9th and 10th nerves and the nerves themselves (IX., X.) are seen; the bony laminæ are severed that are finding their way into the back of the auditory capsule; in the cavity of the ampulla, and from the foramen over the top of the oblique lower part of the occipital arch, there are parts of the spreading exoccipitals (*e.o.*). The opisthotic and epiotic are formed over the posterior canal *later* (see Plate 38).

The two halves of the investing mass, below, form a rest for the oblique plates of the upper part or arch, and are only slightly tilted upwards outside; they are thick inside and flattish on the outside, and have the notochord (*nc.*) between them; this rod is, here, ensheathed in bone, and this thick bony sheath has sent a wing, right and left, over the basal part of the moieties of the investing mass, which are curling over their external edge, as far as to the edges of the underlying parasphenoid (*pa.s.*); this ectosteal growth is the basioccipital (*b.o.*). The top of two of the branchial arches (*p.br.*), the parietal (at its end), and a supra-temporal bone are also seen. In this section the occipital arch is seen to be two-winged, right and left; in the last stage (Plate 33, figs. 12, 13, *iv.*) the section, here, was like an hour-glass.

Section 22.—The roof is now (Plate 36, fig. 7, *s.o.*) very thick, and here the vagus nerve only (X.) is seen with its ganglion; the back wall of the auditory capsules nearly meet above; the basal plate (*iv.*), between the halves of which the osseous sheath of the notochord (*nc.*) is seen, with its right and left basioccipital wings (*b.o.*), is surmounted by the side plates of the arch (*e.o.*). Behind the horizontal canal the combined occipital arch and auditory capsules form large thick shoulders of cartilage; whilst, above, the arch has three roundish crests (*s.o.*). Here the basal plate (*iv.*) is thicker, and as in the last, the parasphenoid is corrugated; the wings of the basioccipital bone (*b.o.*) do not yet invest the lower face of the cartilage; the opercular (*op.*), and a post-temporal scute, are seen in section, as also some parts of the hinder branchial arches (*br.*), with their pectinate gills and grooving vessels. Here the peculiar *four-fold* nature of the occipital arch is well seen, the oblique sides resting on a projecting threshold, through which the notochord, with its bony sheath, runs.

Section 23.—The last of the sections (Plate 36, figs. 8, 9) figured is through the thinner hind edge of the occipital ring, which is lozenge-shaped, and somewhat winged, right and left, for the side-walls have the same obliquity as the halves of the archway above; here the threshold is not so wide as in the last section; it is narrowing towards the end of the projecting basioccipital (see Plate 34, figs. 1–3, *b.o.*).

The exoccipital ectosteal plate (*e.o.*) is seen inside the converging arch, the right and left plates nearly meet above, and there is no key-stone piece or supraoccipital bone in the rounded median part. Here the notochord (*nc.*) lies impacted between the basal plates and their ascending arch; it is the core of a strong basioccipital bone (*b.o.*), which strongly encloses it, the soft tissue spreading in radiating lobes in the thickening bone-substance. Laterally, the bone has spread so as to enclose the halves of the investing mass, and runs beyond these parts; it grows as a right and left sharp plate. Here the wide corrugated parasphenoid (*pa.s.*) is in two parts, for it is forked behind (see also Plate 34, fig. 2, *pa.s.*).

Sixth Stage.—Young Lepidosteus, 4 *inches* 5 *lines long.*

In this stage the dermal scutes are so well developed that they can be named and classified ; I shall describe them first, and the endocranium afterwards.

Notwithstanding the size of this specimen it had still the remains of the embryonic suctorial disk at the end of the snout (Plate 37, figs. 1–3), forming a pad on the end of the premaxillaries (*px.*), and lying in a horizontal plane. The lower jaw just reaches this part, the disk, itself, overlapping it.

The rostral region of the head is twice as long as the cranial ; the opercular bones (*op.*, *s.op.*), pass behind the projecting basioccipital (*b.o.*)

The mandibles are two-thirds the length of the head, and as in embryo Frogs, are articulated to the quadrate *in front of the eye-ball* (*e.*) ; in old Frogs the condyle of the quadrate may reach as far backwards as the opercular folds do in this Fish.

The bony scutes of the hinder part of the head and face do not differ much from those covering the body, except in size ; but in the rostral region, both above and below, but especially below, many of the bony plates are styles of great length and tenuity ; this is a specialization quite like that which is seen in the skulls of longirostral Birds, and in some extinct Sauropsida, *e.g.*, the *Ichthyosaurus.*

Most of the bones of the roof are not difficult to decipher, for the eye detects quickly the parietals, frontals, and squamosals (Plate 37, fig. 1, *p.*, *f.*, *sq.*) ; but nearly the whole extent of the rostrum has to be traversed before we reach the true nasals. These bones (*n.*) are small crescentic scutes that cover the small, distal olfactory capsules (*ol.*) But along the top of the rostrum, from the ethmoidal region, where the frontals diverge nearly to the nasal roofs, two long, narrow styles of bone are seen ; these I propose to call " ethmo-nasals " (*et.n.*) ; they are manifestly separate centres that correspond to the elongated hinder part of the nasals of a Bird.

The olfactory sac, in both Ganoids and Teleosteans, is devoid of a proper paraneural roof, and the bone covering it is merely one of the many "slime-bones" seen in the skulls of these Fishes ; still, that scute which directly covers the olfactory organ has the first claim to be called the nasal. The frontals (*f.*) in their foremost third are divarcated and styloid, embracing the hind end of the ethmo-nasals (*et.n.*) ; they are wide where they meet over the antorbital region, become pinched up to their hinder fourth, and then widen most where they are overlapped by the parietals. These latter bones (fig. 2, *p.*) are large and oblong, covering the skull well from the middle of the eye-balls to the back of the ear capsule ; they are flanked and overlapped by the temporal series.

The principal temporal bone is the squamosal (*sq.*) ; it is a long and irregular triangle, with its sharp end foremost ; its broad end is overlapped by the second large temporal (*s.t.*), which covers the hinder part of the parietal as a rounded scale. Under it there is a lesser pair, and under these upper, larger bones, there is a considerable patch of small scutes margined by the circumorbital series in front, and the angulated interopercular (*i.op.*), behind. The circumorbital series is a very perfect ring of small scutes round the eye-socket; of these the antero-superior scales are the largest. A short chain of three or four small scutes runs forwards from the *superorbital* part of the ring; these may be called *preorbitals* (*p.ob.*); they are tilted up and form a sort of "eave" to the large convex coronoid part of the mandible. There, indeed, in front of the eye-balls, the skull is pinched inwards, and set, as in a vice, between the high hinder part of the lower jaw, whose steep, almost vertical, hind margin chafes, so to speak, right and left, against each circumorbital ring (Plate 37, fig. 1).

Below the tilted preorbitals there is another short, feeble chain of three or four scutes; the last but one of these (*m.x″.*) is as long as the others together, and has all the relations of the free part of the edentulous "os mystacum," or maxillary of typical Teleosteans; the little scute behind it (*j.*) shows the same relations as the small malar (jugal) of many Teleostei.

Outside of and protecting the sub-marginal row of mucous glands, there is a long chain of bones (see Plate 32, fig. 5, *m.c.g.*, and Plate 37, figs. 1, 2, *m.x′.*); this series of scutes is the continuation of the *mystaceum* series (*m.x″.*, *j.*) but thrice their width; this may be called the *maxillary chain.* This is composed of about fourteen or fifteen very similar scutes; they are oblong, their width being about half their length. In front of these the small premaxillaries (*px.*) are seen to be distinct, right and left. Each moiety (or centre) is pointed in front, has a small palatine plate and a dentary edge with sharp teeth; these rows of teeth (fig. 3, *px.*) meet in front at an acute angle.*

Behind the palatine plate of the premaxillaries, right and left, there is a long bone in close contact with its fellow of the opposite side, and so slender that the two together are not so wide as a single ethmo-nasal (fig. 2, *et.n.*); these "needles" are the vomers (fig. 3, *v.*); they become covered with a very fine rasp of teeth, and are nearly half the length of the entire skull.

Bounding these, along the palatal face of the rostrum, there is a pair of bones one seventh longer than the vomers, and twice as wide; these are the "parosteal palatines" (*pa′.*). These bones become invested with a rasp of teeth a degree coarser than that on the vomers.

In the long valley between these palatine splints and the maxillary chain there is a row of *large* sharp teeth, and on the edge of the chain an outermost row of *small* sharp teeth. A very long carinate, trough-like bone runs over the hind part of the two vomers for a considerable distance, and then extends to the end of the skull; on escaping from them it appears also rough, with a fine rasp of teeth. Further back these teeth cease, but the bone is carinate up to the basi-pterygoid (*b.pg.*); this is the parasphenoid (Plate 37, fig. 3, *pa.s.*) This bone is wider in the ethmoidal than in the post-orbital region, but it then widens to thrice its breadth even in the ethmoidal region. It flanks the basipterygoids (*b.pg.*) with a pair of small wings, is spread out under the auditory capsules (*au.*), and applies itself as a forked splint to the under face of the basioccipital (*b.o.*). Behind, for three-fifths of their length, the palatine splints (*pa'.*) are bound on their inner edge by a larger, but similar bone; this is the pterygoid (fig. 3, *pg.*); both these bones are seen in their relation to the suspensorium in other figures (Plate 37, fig. 4, and Plate 38, fig. 5, *pa'.*, *pg.*). The pterygoids acquire a fine bony rasp when they lie close to the palatal surface. Each bone is a long style in front, and then widens gradually so as to become a broad spatula in the orbital region.

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* In an old specimen I find a flat sub-arcuate scute binding across in front of the distant premaxillaries; this latter bone might be thought to be an edentulous azygous premaxillary and the two next behind it, not premaxillaries, but the foremost of the maxillary chain; I incline to call it a prenasal.

The styloid palato-pterygoid cartilage is applied to its outer face above, but the bone passes backwards, and invests three-fourths of the inner face of the broad suspensorium (see Plate 38, *pg.*), ending behind, with a thin, rounded margin.

When these parts are removed from the rest of the skull (Plate 37, fig. 4, and Plate 38, fig. 5) their parosteal relation to the prognathous suspensorium is clearly seen.

Over the edge of the suspensorium, in its broadest part, there is a *third* parostosis; this is the mesopterygoid (Plate 37, fig. 4, and Plate 38, fig. 5, *ms.pg.*). It is a thin, falcate bone above, one-sixth the length, and one-sixth the width, of the pterygoid.

A *fourth* splint is applied to the suspensorium, and this, like the last, is extremely small as compared with what is seen in the Teleostei; this is the preoperculum (*p.op.*); this bone is falcate, narrow, gently curved downwards, pointed behind, where it lies on the interopercular, and roughly notched in front, where it binds on the outside of the quadrate bone (*q.*); it is only one-third the length and one-third the width of the succeeding bone—the interopercular (*i.op.*); whereas in the Teleostei it is much the larger bone, as a rule.

The free part of the lower jaw, or mandible, is of great length, and the dentary bone covers it from end to end (Plate 37, fig. 4, *d.*); on the inside (Plate 38, fig. 5, *d.*) it is only seen at the edges of the jaw. Under its upper edge, on the inside, a much smaller bone, three-fourths its length, binds on the upper edge of the cartilaginous axis (*mk.*); this is the splenial (*spl.*).

Behind the splenial, on the inner side (Plate 38, fig. 5 *cr.*) the coronoid is seen as a pedate tract of bone with its heel behind; it binds on the inside of the fore part of the large cartilaginous coronoid (*cr.c.*). Under the short angular process of the articular cartilage there is a small angulare (*ag.*), and outside the large, ear-shaped coronoid cartilage, on its convex face, there is a considerable scale of an oval shape, and placed obliquely forwards and upwards; this is the supra-angulare (Plate 37, fig. 4, *s.ag.*).

The specialised "scutes" just described belong to the mandibular arch; those next to be noticed belong to the hyoid. On the knob of the hyomandibular a large oval scute is articulated by its capped fore end: this is the operculare (Plate 37, fig. 1, *op.*); below this a similar bone is seen, but of an uncinate or semi-crescentic form, with its sharp end behind, and its upper edge inside the operculare; this is the sub-operculare (*s.op.*).

Binding on the fore edge of the two last, we see an angulated sub-crescentic bone, with its concave edge above, its angulated margin below, and its front point binding under the hind point of the preoperculare. This is the interoperculare (Plate 37, fig. 4, and Plate 38, fig. 5, *i.op.*). These three bones are the hyomandibular splints; those now to be described belong to the cerato-hyal; these are the branchiostegals: they are beneath and within the great operculum (Plate 37, fig. 1, *br.s.*), and are attached in front to the cerato-hyal. Generally *seven* in number in the Teleostei, there are only *three* here, as in the Carp. These bones are narrow and falcate, with their concave margin above; the uppermost is the largest; the lower is the least.

The remainder of the bones which I have to treat of are *intrinsic* centres, or "ectostoses;" they will come under notice, now, in a description of the endocranium, most of which, however, is cartilaginous.

The endocranium at this stage differs but little from that of the adult, in which, although the bony centres become dense and relatively larger, are yet not altered, either in their number or relations, to any appreciable degree.

In this skull, the "prenasal rostrum," or intertrabecula, is as much developed as in the most specialised of the Selachians—namely, the "Pristidæ," or *Saw-fishes*—much more than in the ordinary Skate ("Raiidæ"). Here the length of this precranial region is, as compared with the cranial cavity, as 14 to 5, or nearly three times as long. In an old specimen the cranial cavity is only 2 inches long, and the whole skull 12½ inches, or 2 to 10½; the brain has, relatively, retreated. Measured from the quadrate condyle, in this young stage, we get the same proportion as the measurement of the precranial to the cranial; in the old the prequadrate region is 9 inches long, and the post-quadrate 3½ inches.

This remarkable pyriform, long-stalked skull owes its greatest expansion to the superaddition behind of the large ovoidal auditory capsules, and next to them to the greater size, in the young, of the mid brain; it soon narrows in over the small hemispheres. As in the last stage, the roof of the mid brain is membranous—this is the large circular fontanelle (Plate 38, fig. 1, *fo.*), the margins of which are very moderate bands of cartilage—the postorbital part of the superorbital bands (see Plate 30, fig. 8, *so.b.*).

Below (Plate 38, fig. 3, *py.*, *pa.s.*) there is a rather large and lanceolate pituitary fontanelle; and inside the orbits (Plate 38, fig. 2, *o.s.f.*) there are the long "orbitosphenoidal fenestræ." The upper fontanelle (*fo.*) takes up about a third of the roof, but it is not so long as either the fore or the hind part of the tegmen cranii (*t.cr.*). Thus although this is rather a well-developed chondrocranium it has four large membranous deficiencies in it.

The basioccipital (Plate 38, figs. 1–3, *bo.*) might be taken for the centrum of the first vertebra—it projects so far behind the exoccipitals (*e.o.*). The bony matter, which did occupy much of the sheath of the cranial notochord, is now mainly confined to its hind part, and forms a four-sided mass; this mass does not run forwards into the rest of the notochord—at least on its upper surface (Plate 9, fig. 3, *nc.*).

This rugous bone is broadest behind, where it is scooped for the first vertebra; above, it is flattened; its fore end is emarginate, and its two oblique antero-lateral faces are joined by suture to the exoccipitals (*e.o.*)

These bones are small lunate tracts behind the passages for the 9th and 10th nerves (IX., X.); the rest of the occipital arch is devoid of bone, for the superoccipital is absent, as in the Amphibia; that region projects, as an obtuse angle of cartilage, over the foramen magnum.

The auditory capsules project well into the basal plate (*iv.*); their canals (*a.s.c.*, *h.s.c.*, *p.s.c.*) are large, and easily seen through the transparent cartilage; both above and below they send their diverticula inwards towards the mid line, so as to make the roof and floor of the skull into the shape of an hour-glass. The upper part spreads outwards over the horizontal canal, covering the facet for the hyomandibular (*hm.c.*); further inwards, below, the swelling "sacculus" on each side makes a notable bulging, which is partly floored by the parasphenoid (*pa.s.*). Postero-laterally, the capsules are but slightly angulated; but in front they grow outwards and forwards into an ear-shaped projection, which is separated by a round notch from the root of the superorbital band (*s.ob.c.*). That process is the "sphenotic" outgrowth of the chondrocranium in front of the capsule; it is becoming bony (Plate 38, figs. 1, 2, *sp.o.*); also below, in front of the capsule, and surrounding the chinks and openings for the 5th and 7th nerves (V., VII.), the prootic centre (*pr.o.*) is spreading into the cartilage; in front it runs into the back of the corresponding basipterygoid (Plate 38, fig. 3, *b.pg.*); and behind it has reached the concavity for the sacculus (*vb.*)

Also above (figs. 1 and 4), an irregular bony tract is seen imperfectly divided into two patches, which lie over the ampulla of the posterior and the end of the horizontal canal (*p.s.c.*, *h.s.c.*). The upper part is the rudiment of the epiotic (*ep.*), the lower portion of the opisthotic (*op.*).

In front of the basipterygoid (figs. 2, 3, *b.pg.*), and rising upwards from it into the limited tract of cartilaginous wall between the orbito-sphenoidal fenestra and the ear-capsule, there is a bony tract, smaller than the prootic and next in front of it; this is the alisphenoid (*al.s.*).

Along the skull base, in front of the projecting basioccipital, there is no intrinsic bony centre, and laterally, the whole orbito-sphenoidal region is membranous.

But where the skull is closing in, in front, the rapidly narrowing cranium has a short tract of cartilage in its sides; this is the lateral ethmoidal region; the free border of this cartilage in front of the fenestra is ossified as a crescentic patch (Plate 38, figs. 2, 3, *o.s.f.*, *l.eth.*); this answers to the so-called prefrontal of the Teleostei, but it does not grow out into ethmoidal wings, as in those types.

A very important change has taken place inside the basis cranii, for now there is a very definite "posterior clinoid" *bridge*, not *wall*, of cartilage (Plate 38, fig. 3, *p.cl.*); it is small, very narrow in the middle, and runs straight across, joining the roots of the trabeculæ (*tr.*) together, but lying only at a small height above them.

This is a very feeble rudiment of the thick and high wall, which is developed in this part in most of the "Amniota," where it runs up in the deep fissure under the mid brain. However, even here it divides the basi-cranial fontanelle into two parts, a large anterior (*py.*) and a small posterior space (*p.b.c.f.*).

Thus it is evident that in this, as in other kinds of Ichthyopsida, the basis-cranii is much less affected by the mesocephalic flexure than it is in the Sauropsida and Mammalia.

The main pituitary space (Plate 38, fig. 3, *py.*) is lessened by the ingrowth of the trabeculæ (*tr.*); but in front, it is filled in by the hind part of the long intertrabecula. The trabeculæ are bowed out right and left, between the 5th and 2nd nerves (V., II.); the 1st nerve (I.) escapes from the front of the enclosed end of the cranium, and runs all the distance to the nasal sacs close to the sides of the intertrabecula (*i.tr.*)

The paired trabeculæ (*tr.*) do not end where the skull has closed in; in front of the narrowed tegmen cranii (Plate 38, figs. 1, 2, *t.cr.*) the intertrabecula is seen to be narrow above, and to have narrow wings running along its sides.

These wings soon dilate, so as to give the rostral part of the skull an oval widening along the front *two-fifths* of its *hinder fifth.* These parts are the cornua trabeculæ, and although they are so short now, they were (Plate 30), once, the main part of the skull in front, and for some time came little short of the end of the snout. Now, they are like the right and left sides of a lanceolate leaf, with a huge mid-rib; only their terminal point is free, and the 1st nerve runs in a groove between them and the long rostrum.

The rostrum (*i.tr.*) is very uniform up to near the front end; it then becomes slightly alate before ending in a blunt and somewhat decurved point (*p.n.*); its section is nearly oval, the thicker end below.

The suspensorium (Plate 37, fig. 4, and Plate 38, fig. 5) retains the form it had in the last stage (Plate 34), but it is twice as large, and its bony centres are now perfect. The upper bone is the metapterygoid (*mt.pg.*), it occupies the neck of the suspensorium, leaving cartilage, however, on the concave articular facet—for the basipterygoid—and also on the short round "trochanter," below the joint; this spur is the arrested otic process (*ot.p.*).*

The quadrate (*q.*) is a bony quadrant running, at its angle, close to the articular condyle (*q.c.*); this latter is an elegant convexo-concave trochlea, with its largest convexity on the outside. The main part of the body of the suspensorium is unossified; it is a large oblong tract, with its postero-inferior angle rounded off; it is rather hollow outside and convex within, where it is invested by the pterygoid bone (*pg.*).

The pterygo-palatine rod (*p.pg.*) is unaltered since the last stage; it never ossifies, and reaches as far forwards as the cornua trabeculæ (*c.tr.*).

The articulo-Meckelian rod (*mk.*, *ar.*) has increased in size (both actually and relatively); its intrinsic centre also, the articulare, has become two separate points of bone (Plate 38, fig. 5, *ar.*). The rest of this long, subarcuate, terete rod (*mk.*), runs along the grooved inner face of the dentary (*d.*) nearly to its distal end. The coronoid crest (*cr.c.*) is a very large "ear" of cartilage; it is convex outside and hollow within; its fore part is a free lobe.

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* In Plate 38, fig. 5, below, for *pa.* read *pd.*

The hyomandibular (Plate 37, fig. 4, and Plate 38, fig. 5, *hm.*) is a remarkable bar, about half the size of its "serial homologue"—the suspensorium. Its arched, extended head is a convexo-concave condyle for articulation under the horizontal canal; behind this there is a cartilaginous knob for the opercular bone. The bony shaft is short, pinched in the middle, and has an oval fenestra near its front third. Below the shaft it swells out into a solid bilobate mass, the lesser lobe being behind. In front of the fore lobe, on the inside, there is a concavity for the inter-hyal (*i.hy.*). The bar from that point becomes the small sigmoid symplectic (*sy.*); it is bent downwards suddenly, and then runs straight forwards to lie along the inside of the hinder third of the lower edge of the suspensorium.

Its bony shaft occupies its hinder two-thirds; where it becomes straight, there it has a small bony elbow; its fore end is a blunt point (Plate 38, fig. 5, *sy.*).

The inter-hyal (*i.hy.*) is a small pyriform cartilage, its narrow end fits into the concavity in the hyomandibular, and its broad end has a cup on its inner side for the head of the cerato-hyal (*c.hy.*).

The latter segment has a "trochanter" behind its small rounded head; its shoulder is ossified as a separate epi-hyal (*e.hy.*); the main shaft (*c.hy.*) has its own centre; it is narrower in the middle, and is only separated from the stylo-hyal by a tract of cartilage.

The rounded lower end of the cerato-hyal fits into the oblique shallow cup of the sub-globular hypo-hyal (*h.hy.*); this short segment is ossified on its outer face; at present, at any rate it has no second centre, as in the Teleostei; but in these, as in *Acipenser*, it is completely segmented off from the cerato-hyal.

The basal piece (*b.hy.*) is a large "inter-glossal" plate as long as all these three segments above it; it is oblong, rather pinched in the middle, emarginate in front, thickish, and somewhat fibro-cartilaginous, having cross-bands and reticulating, connective fibres, wrought into it on its upper surface, and its hyaline cartilage somewhat softer than in the other parts.

The basal piece of the branchial system, and part of the first part of hypobranchials (*b.br.*, *h.br′.*) are figured. For the rest, I must refer to the figures and descriptions of the last stage; these parts have not altered in any important degree—except in size.

Comparison with other types, and Summary.

As soon as the primordial cranium becomes sufficiently differentiated—as hyaline cartilage—to be distinguished from the rest of the cephalic mesoblast, we find a peculiarly simple foundation for all the aftergrowths. In Stage 1, in embryos $10\frac{1}{2}$ m.m. long, nearly all the parts of the chondrocranium—including in this term the visceral arches—are present; the hinder arches become broken up, afterwards, but the two first, and largest, the mandible and hyoid, are already as much segmented as they will be in the adult.

The skull-floor, only, is developed, as yet, and the rostral part, in front, is not chondrified, but its outlines can be traced, and the roof and walls of the skull are merely developments from the basal bands.

Those bands in this type lend no support to the theory of the visceral (or ventral) nature of the *pro*-chordal tracts or *trabeculæ;* they are, manifestly, mere continuations of the undivided *para*-chordal cartilages, which expand and contract in relation to the parts around and over them. They diverge from the front third of the notochord, as though their relation to it was not intimate, and show—for a long while at least—no tendency to grow up, with that axis, into the hollow of the mid brain.

I see nothing in this lyriform basal skeleton of the skull but an undivided *basi-neural* structure comparable to, and a *primary cephalic variation* of, the tracts that form the paired rudiments of the *neuro-central* cartilages of the spine. The cessation at the end of the notochord (mesially), and close behind the oral opening, laterally, of the hypo-blastic layer, causes all the pre-oral and pre-pituitary parts to be, in a sense, imperfect; they are developed as porches and outworks to the full and complete structure further back, but this does not destroy their homology, nor break their continuity with the parts formed from their own embryonic layer, of which they are the direct ongrowths.

Yet all parts growing out,—forwards, upwards, or downwards,—in front of the perfect axis, which ends close in front of the infundibulum, must be very cautiously named as " serial homologues " of the perfect base and its upper and lower arched growths; they are probably mere *outgrowths;* at most they are only *rudiments.*

The *primary* trabeculæ are merely direct *on*-growths of the parachordals; the cornua trabeculæ are *out*-growths of the trabeculæ.

The intermediate element, or intertrabecula, is a fresh outbreak, so to speak, of the median mesoblast of the axis, which is tubular, behind, where it encloses the notochord, but, re-appearing in front, beyond it, it shoots forth as a *solid process* of the skeletal axis.

Close to the fore end of the primary trabeculæ there arises a similar but rather smaller bar, and the two parts are so close together that they chondrify continuously; these side bars are the palatine cartilages, evidently rudimentary structures.

Here they are not distinct from the long spur (pterygoid cartilage), which shoots forwards from the dorsal element (suspensorium) of the mandibular arch; this is like what we see in the Tadpole, but unlike that which is found in Skates, Teleosteans, and Urodeles. Thus, with the palatine included, the suspensorium here is a *palato-quadrate;* in the Skate, Teleostean, and Urodele the suspensorium is a *pterygo-*

quadrate. As in the Tadpole, the fore end of this bar is fixed; as in the Teleostean Urodele, and adult Frog, the hind part, or pedicle, is free.

As in the Tadpole, the suspensorium is sub-parallel with the axis of the skull, and the free mandible (MECKEL'S cartilage) grows forwards and inwards; that condition is temporary in the Batrachian, it is permanent in *Lepidosteus.*

As to the development of the basal bands of the skull, this type agrees with the Selachians and Teleosteans (*Salmo*) in the synchronism of the *para-* and *pro*-chordal tracts; but in Batrachians, Urodeles, and Marsipobranchs, the *trabeculæ are developed first;* they embrace the fore end of the notochord closely, and are both *para-* and *pro*-chordal; afterwards the hinder parachordal region is chondrified, separately in Urodeles, and continuously in Batrachians and Marsipobranchs.

The development of the complex hyoid arch is very different in this *Holostean* Ganoid, and in the *Chondrostean* Sturgeon, from what we find in Teleosteans, Batrachians, and Urodeles.

In the Salmon the primary bar breaks up into two long bands, with a short segment below; the foremost is the larger, retains its connexion with the ear-capsule, widens above as the hyomandibular, and narrows, antero-inferiorly, as the symplectic region.

The narrower, hind band becomes postero-inferior in position, keeps the small distal segment, and acquires a new, small segment, above, by which it becomes attached to the space between the hyomandibular and symplectic; the *late,* small upper segment is the inter-hyal, the long bar the epi-ceratohyal, and the short, distal segment the hypo-hyal.

In the lowest Urodele, *Proteus,* the hyoid arch is composed of two massive segments, one short, the upper or hyomandibular, and the other, the long, lower bar, the cerato-hyal; this is like that which obtains in Sharks.

In the larger Urodeles (*Menopoma, Cryptobranchus*) there is but little difference (apparently) in the time of development of the segments, but the upper part breaks up into two segments corresponding to the hyomandibular and symplectic segments in the Sturgeon; the cerato-hyal is large, and the hypo-hyal breaks up into three pieces.

In many of the Caducibranchiate Urodeles, and in some kinds of Anura (*Salamandra, Triton, Pseudophryne, Bombinator*), all but the uppermost part of the hyoid arch is suppressed; but in the Batrachia, generally, it is developed, as *two, three,* or even *four* segments; these, with the exception of the uppermost, as a rule, do not appear until two or three months after transformation, and are only developed in the Tadpole in rare cases, as in *Pseudis* and *Pipa.*

In *Lepidosteus* and in *Acipenser* the formation of the segments of the hyoid arch takes place at once during chondrification; *Lepidosteus* has the same number of cartilages as the Teleostei, but *Acipenser* has a distinct symplectic piece—a kind of segmentation which is not equivalent to the subdivision of the upper part of a branchial arch into a *pharyngo-* and an *epi*-branchial, but the epi-hyal is segmented at its distal fourth.

The basi-hyal of *Lepidosteus* is remarkable for being very long and essentially double. There are only four perfect and one imperfect branchial arches; the auditory capsules, at first, are as distinct as in the Tadpole, their basal region remains membranous for a good while, as in the Salmon.

In embryos two-thirds of an inch in length, more than one-half larger than our first stage, the chondrocranium is larger and stronger, but has few fresh things in it.

The trabeculæ and palato-quadrate cartilages are still confluent, but the former are now some distance apart, the binding cells of the former stage being converted into a pyriform mass of true cartilage, with its broad end in front and projecting beyond the paired bands or trabeculæ; this is the intertrabecula. The pedicle of the suspensorium has applied the inner side of its apex to the most curved part of the trabecula, and an oblong joint is forming.

A spike of cartilage has grown forwards from the auditory capsule over the hinder part of the superorbital region; this structure is seen temporarily in large larvæ of *Triton,* and permanently in *Siren.*

The divergence of the basal bands is now at its fullest, and the apex of the cranial notochord—one-third of the rod—is twisted, curves a little upwards, and is far from the moieties of the investing mass.

In young *Lepidostei,* one-half larger than the last (1 inch long), the chondrocranium may be said to be complete, and free from intrinsic ossification, except in the sheath of the notochord, the cerato-hyal, and some parts of the branchial arches.

The whole structure is much longer, but most of the increase in length is due to the development of the three basal cartilages in front of the cranial cavity. The occipital arch is perfect, and the tegmen from it runs well forward.

The superorbital band is now perfect, and in front it passes into an anterior tegmen round the olfactory lobes, and the hemispheres, thus the cranial box is perfect there. But there is a large pyriform fontanelle below, a larger oval fontanelle on each side in the orbital region, and a still larger membranous space, the great fontanelle, above.

The sudden and immense development of the precranial bars in so short a time is very remarkable; their relative massiveness makes this skull like that of a young Sturgeon five or six times as large. In that type the solid rostrum is formed of the two large trabecular cornua, which flank the still larger intertrabecula, like decurrent leaves. In the Sturgeon there is an antorbital expansion of the lateral ethmoidal region at the end of the rostrum, and each olfactory capsule lies close in front of the antorbital wall, as in a crypt. But in *Lepidosteus* the two capsules are carried away to near the end of the snout, and have no cartilage near them except the rostral bar, on each side of which they lie.

Whilst the fore part of the chrondrocranium is like that of a young Sturgeon 5 or 6 inches long, the cranium proper is like that of a Salmon ten or twelve days after hatching, when its length agrees with that of this stage of *Lepidosteus,* namely, about 1 inch.

In the Salmon "fry" there is a hinder and a front tegmen cranii, a pair of superorbital bands running from the auditory capsules to the anterior tegmen, a largely open roof between, open orbito-sphenoidal spaces, and an open pituitary fontanelle. Moreover, this is the "norma" according to which the skull of *Polypterus* is formed; but of course during growth it becomes more solid, and partly ossified.

The suspensorium suggests a very mixed relationship in this type; it runs forwards, parallel with the skull, as in the Tadpole, but its pedicle is now well articulated with a basipterygoid process as in the metamorphosed Frog, and more clearly than in that type prefigures the cranio-facial relation of the Sauropsida, where, as in Lizards and many Birds, the pterygoid portion of the suspensorial apparatus articulates with a basipterygoid outgrowth of the skull. The open orbito-sphenoidal spaces are seen again in Batrachians, *e.g.*, in *Acris Pickeringii* and *Rappia bicolor.*

The palatine portion of the suspensorium (or palato-quadrate cartilage) loses its ethmoidal conjugation, but retains its continuity with the pterygoid cartilage. The primarily and permanently separate palatine of the "Siluroids" runs forwards in the same manner, with no ethmo-palatine joint, such as is seen in the Salmon.

In adult Batrachians of the genus *Bufo* the continuity of the palatine cartilage is lost both with the ethmoid and the pterygoid cartilage, but it articulates with the former by a raised process as in the Salmon. The small size of the hyomandibular of *Lepidosteus,* and its distance from the mandibular pier, prepare us for the transformations of that part in the Batrachia where it becomes the "columella."

In a further stage, when it has doubled the size it had in the last instance—has become 2 inches long—the young *Lepidosteus* has fairly attained to its own characteristic type of skull, and most of the very limited osseus centres are now apparent. All the fore face is now greatly drawn out, twice as much as in the last, and the suspensorium, mandible, and lingual cartilages, have shot on forwards in like manner. The membranous spaces are only different from the last by the upper fontanelle being relatively less, and neatly circular, whilst the lower space is being divided by a *late* and *feeble* "post-pituitary" bar with a large anterior, and a small posterior space. But the type of skull seen in the young "fry" of the Salmon, and in such minute arrested Frogs as the Nearctic *Acris Pickeringii,* and the Australian *Rappia bicolor* and *Camariolius tasmaniensis,* is not departed from, nor, indeed, will be.

But even what is seen in young *Lepidostei* 2 *inches* long scarcely prepares us for what we find in specimens a little more than twice that size. At this stage, as in Sawfishes (*Pristis*), this prenasal cartilage (intertrabecula) has become *three times* as long as the whole cranial cavity, and *six times* as long as its associated cornua trabeculæ—now mere delicate leafy appendages to its base. The cranium proper has not altered in any important degree since the last stage, but the bony centres are nearly all there; all those that are seen in the Salmon, or in Teleostei, generally are found, with the exception of the super-occipital and a bone not found, I believe, in the Ganoids, namely, the "pterotic;" its suppression is correlated with the development of a special temporal scute over the same region; the "squamosal," a bone only, exceptionally present in the Teleostei among the Siluroids.

The post-pituitary band of cartilage, a feeble promise of the solid post-clinoid wall of the "Amniota," is now complete. Some things characterising this type are now well seen, namely, the small lateral ethmoidal bone in the thin, closing-in, skull wall, without any prefrontal (ali-ethmoidal) wing, the double "articular" centre, and the huge coronoid crest to the articulo-Meckelian rod; these two latter characters are also seen in *Amia calva*, as shown by Professor BRIDGE.

The very small size of the preoperculars, the form and size of the interopercular, which here so strongly resembles the preopercular of the Teleostei, are very noticeable in this skull, as also the long chain of bones interspersed between the "os mystaceum" or edentulous maxillary and the premaxillary.

The adult condition of the skull in *Polypterus* and *Amia* (TRAQUAIR, Jour. of Anat. and Phys., vol. 5, plate 6, pp. 166–182; and BRIDGE, ibid., vol. 9, plate 23, pp. 605–622) presents so many things, both in likeness and contrast, that they must be noticed in conclusion.

In *Polypterus*, as in *Lepidosteus*, the four fontanelles are permanently open; the basioccipital projects far beyond the oblique foramen magnum, and the occipital bone is single, made up evidently of a basal and two lateral pieces, without a supraoccipital; then there are no pterotics, and the olfactory capsules are sub-terminal. But in this type there are no epiotics distinct from the opisthotics.

There is a large sphenotic bone, right and left, which takes up the antero-posterior sphenoidal regions and part of the lateral ethmoidal, besides which there is a pair of lateral ethmoidals which project outwards, and an ethmo-septal bone in front.

The metapterygoid in *Polypterus* is far from the skull, in which there are no basipterygoid processes; the palatine is a small ectosteal bone; the hyomandibular has only one centre; and the preopercular is continuous with the squamosal, as in the Amphibia; there is no interopercular.

But *Amia calva* has a skull which comes much nearer to that of *Lepidosteus* in several respects, but the lateral and inferior fontanelles are filled in, in this solid skull, which comes nearer that of the Physostomous Teleosteans.

The basioccipital projects behind the oblique foramen magnum; there is no supraoccipital, nor any pterotics, and the epiotics are distinct from the opisthotics.

It has a pair of bones which are not seen in *Lepidosteus*, namely, the orbitosphenoids; and its so-called prefrontals or lateral ethmoids project, as in the Teleostei. It has a distinct pedicle to the suspensorium, capped with cartilage, but not forming a definite joint with any distinct basipterygoid.

Its palatine cartilage is ossified both endosteally and ectosteally; and the whole palato-pterygoid is almost Teleostean in its solidity.

There is a large coronoid crest, and there are *two* articular bones on each side, as in *Lepidosteus*. There is a cartilaginous inter-hyal, articulated between a distinct hyomandibular and symplectic; and the upper or styloid end of the cerato-hyal has, as in Osseous Fishes, a separate centre, whilst the hypo-hyal has *only one*, as in *Lepidosteus;* in Osseous Fishes it has *two*.

There are the four normal operculars; the "os mystaceum" is dentigerous and carries a jugal in its hinder half. Above the skull, the scutes, which seem to me to be a little misunderstood by Professor Bridge, correspond, in essentials, to those of *Lepidosteus*. I should propose the term "azygous parietal" for his *dermo*-superoccipital; "squamosals" for his *parietals;* and "prerostral" for the azygous transverse bone, which, as in *Lepidosteus*, furnishes the snout in front, and which is called by him *ethmoid*. The olfactory capsules are sub-terminal, and the large *nasals*, which cover them by their notched fore margin, are evidently the nasals and "ethmo-nasals" of *Lepidosteus*, in one piece, right and left.

Amia is a true Ganoid, and it has several unmistakable diagnostics even in its skull; but it comes very near to the Physostomous Teleosteans.

Description of the Plates.

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Plate.	Fig.	Stage.		Number of times magnified.
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,,	4	4	Section of the head of an embryo of the same stage .	24
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,,	2	6	Hind part of same; lower view	$6\frac{3}{4}$
,,	3	6	Part of basis-cranii of same; upper view . . .	$13\frac{1}{2}$
,,	4	6	Part of chondrocranium of same; upper view . .	$13\frac{1}{2}$
,,	5	6	Inferior arches of the same skull; inner view . .	$7\frac{1}{2}$

List of Abbreviations.

The Roman figures indicate nerves or their foramina.

ag.	Angular.
al.s.	Alisphenoid.
ar.	Articular.
ar.c.	Articular cartilage.
a.s.c.	Anterior semicircular canal.
au.	Auditory capsule.
au.f.	Auditory fenestra.
b.a.	Basilar artery.
b.br.	Basi-branchial.
b.hy.	Basi-hyal.
b.o.	Basioccipital.
b.pg.	Basipterygoid.
C^1.	Fore brain.
C^2.	Mid brain.
C^3.	Hind brain.
c.br.	Cerato-branchial.
c.hy.	Cerato-hyal.
cl.	Cleft.
cr.	Coronoid.
cr.c.	Coronoid cartilage.
c.tr.	Cornua trabeculæ.
d.	Dentary.
e.	Eye.
e.br.	Epi-branchial.
e.hy.	Epi-hyal.
ep.	Epiotic.
et.n.	Ethmonasal.
f.	Frontal.
fo.	Fontanelle.
g.p. and *br.p.*	Gill processes.
h.	Heart.
h.br.	Hypo-branchial.
h.hy.	Hypo-hyal.
hm.	Hyomandibular.
hm.c.	Hyomandibular facet and condyle.
hm.f.	Hyomandibular fenestra.
h.s.c.	Horizontal semicircular canal.
i.hy.	Inter-hyal.
inf.	Infundibulum.
i.op.	Interopercular.
i.tr.	Intertrabecula.
j.	Jugal.
l.eth.	Lateral ethmoid.
m.	Mouth.
m.c.g. and *m.g.*	Mucous gland.
mk.	Meckel's cartilage.
mn.	Mandible.
ms.pg.	Mesopterygoid.
mt.pg.	Metapterygoid.
mx'., *mx.''*	Maxillary.
my.	Myelon.
n.	Nasal.
nc.	Notochord.
ol.	Olfactory capsule.
op.	Opercular and opisthotic.
op.p.	Opercular process.
os.f.	Orbito-sphenoidal fenestra.
ot.p.	Otic process.
p.	Parietal.
pa'.	Superficial palatine.
pa.s.	Parasphenoid.
p.br.	Pharyngo-branchial.
p.cl.	Posterior clinoid.
pd.	Pedicle.
pg.	Pterygoid.
pnl.	Pineal gland.
p.ob.	Preorbital.
p.op.	Preopercular.
p.pg.	Palato-pterygoid.
p.px.	Palatine process of premaxillary.
pr.o.	Prootic.

pt.o.	Postorbital.
px.	Premaxillary.
py.	Pituitary body and space.
q.	Quadrate.
q.c.	Quadrate condyle.
s.ag.	Supra-angular.
s.d.	Sucking disk.
spl.	Splenial.
s.ob.	Supraorbital.
s.ob.c. and *s.ob.*	Supraorbital cartilage.
s.op.	Subopercular.
sp.o.	Sphenotic.
s.t.	Supratemporal.
su.ob.	Suborbital.
sy.	Symplectic.
t.cr.	Tegmen cranii.
tr.	Trabecula.
v.	Vomer.
vb.	Vestibule.

IX. *On the Vibrations of a Vortex Ring, and the Action upon each other of Two Vortices in a Perfect Fluid.*

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Communicated by Lord RAYLEIGH, *F.R.S.*

Received November 16,—Read December 8, 1881.*

THE following paper contains (1) a discussion of the vibrations which take place in the axis of the core of a vortex ring whose section is very small in comparison with its aperture when the axis is made to deviate slightly from the circular form; and (2) a discussion of the action upon each other of two vortex rings which move in such a way that they never approach nearer than a large multiple of the diameter of either.

The fluid in which these vortices exist is supposed to be frictionless and incompressible.

The method which I have employed is the same in both cases, and is purely kinematical. It is merely the application of the fact that if $F(x, y, z, t)=0$ be any equation to a surface which always consists of the same particles then

$$\frac{dF}{dt}+u\frac{dF}{dx}+v\frac{dF}{dy}+w\frac{dF}{dz}=0$$

where u, v, w are the velocities of the particle at (x, y, z) along the axis of x, y, z respectively, and where the differential coefficients are partial.

The surface of a vortex ring is evidently a surface of this kind, and the equation just written is the condition that $F(x, y, z, t)=0$ should be the equation to the surface of a vortex ring. I have found that this condition, joined to the ordinary expressions for the velocity due to a vortex element, is sufficient to solve the problems discussed in this paper. This is an instance of the large number of problems in vortex motion which are capable of purely kinematical solution; indeed, a vortex theory of gases would be entirely kinematical so long as we only considered the molecules of gas themselves and not their effects upon the containing vessel, &c. For example, in this theory when two atoms clash, the problem of finding their subsequent motion must be capable of solution by purely kinematical considerations, but in the ordinary theory of gases the "clash of atoms" involves dynamical considerations of very considerable complexity. This is a consequence of the vortex theory being of a much more fundamental character than the ordinary one that atoms consist of small pieces of solid matter.

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* Since the paper was sent into the Society it has been copied by the author with changes in the notation, introduced chiefly to facilitate the printing, but no change of any importance has been introduced into the substance of the paper.

Problem I. To find the vibrations of the circular axis of a vortex ring.

Using cylindrical coordinates let the equations to the axis of the core be

$$\rho=a+\Sigma\alpha_n \cos n\theta$$
$$z=\zeta+\Sigma\beta_n \cos n\theta$$

where α_n and β_n are small compared with a the radius of the core when undisturbed; the summation over all integer values of n between zero and infinity. The axis of z is perpendicular to the plane of the vortex, and θ is measured from the axis of x as initial line.

The velocity due to a distribution of vortices is proportional to the magnetic force produced by a system of currents arranged in exactly the same way as the vortices and of the same strength.

Now the vortex filaments we are considering are distributed uniformly (or very approximately so)* in a ring the radius of whose transverse section is very small in comparison with the radius of the aperture. Now if electric currents flow uniformly through a conductor of such a shape the magnetic action at a point outside or on the surface of the conductor is the same as if all the currents were condensed into one flowing along the axis.† Hence when finding the velocities outside the vortex ring we may suppose the vortices condensed into one at the axis of the core. If ω be the angular velocity of molecular rotation, e the radius of the transverse section of the core, then $\pi e^2\omega$ is the strength of the vortex which we must suppose placed at the axis of the core. We shall for brevity denote $\pi e^2\omega$ by m.

The components (u, v, w) of the velocity at the point (x, y, z) are given by

$$u=\frac{m}{2\pi}\int_0^{s'}\frac{1}{r^3}\left\{\frac{dz'}{ds'}(y'-y)-\frac{dy'}{ds'}(z'-z)\right\}ds'$$
$$v=\frac{m}{2\pi}\int_0^{s'}\frac{1}{r^3}\left\{\frac{dx'}{ds'}(z'-z)-\frac{dz'}{ds'}(x'-x)\right\}ds'$$
$$w=\frac{m}{2\pi}\int_0^{s'}\frac{1}{r^3}\left\{\frac{dy'}{ds'}(x'-x)-\frac{dx'}{ds'}(y'-y)\right\}ds'$$

where r is the distance of the point (x, y, z) from the point (x', y', z'), a point on the vortex whose polar coordinates are (ρ', θ); s' is an arc of the vortex ring.

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* See a note by Sir W. THOMSON at the end of HELMHOLTZ's paper on "Vortex Motion," Phil. Mag., 1867.

† MAXWELL's 'Electricity and Magnetism,' 2nd edition, §. 683.

Now from the equation to the axis of the vortex we have

$$dx'=d(\rho'\cos\theta)=d\theta\Sigma(-n\alpha_n\sin n\theta\cos\theta-a\sin\theta-\alpha_n\cos n\theta\sin\theta)$$
$$dy'=d(\rho'\sin\theta)=d\theta\Sigma(-n\alpha_n\sin n\theta\cos\theta+a\cos\theta+\alpha_n\cos n\theta\cos\theta)$$
$$dz'=-d\theta\Sigma(n\beta_n\sin n\theta)$$

neglecting α_n^2 and β_n^2

$$ds'=d\theta(a+\Sigma\alpha_n\cos n\theta)$$

and

$$\frac{dx'}{ds'}=-\sin\theta-\frac{1}{a}\Sigma n\alpha_n\cos\theta\sin n\theta$$
$$\frac{dy'}{ds'}=\cos\theta-\frac{1}{a}\Sigma n\alpha_n\sin\theta\sin n\theta$$
$$\frac{dz'}{ds'}=-\frac{1}{a}\Sigma n\beta_n\sin n\theta$$

If ρ, ψ, $\mathfrak{z}+\zeta$ be the cylindrical coordinates of the point x, y, z

$$r^2=\rho^2+\rho'^2+(\zeta-\Sigma\beta_n\cos n\theta)^2-2\rho\rho'\cos(\theta-\psi)$$

say

$$r^2=\rho^2+\rho'^2+\zeta'^2-2\rho\rho'\cos(\theta-\psi)$$

Let

$$\frac{1}{\{\rho^2+\rho'^2+\zeta'-2\rho\rho'\cos(\theta-\psi)\}^{\frac{3}{2}}}=\mathrm{C}_0+\mathrm{C}_1\cos(\theta-\psi)+\ldots\mathrm{C}_n\cos n(\theta-\psi)$$

where the C's are functions of ρ, ρ', and ζ'.

Since ρ' and ζ' are functions of θ, C_0, C_1, . . . C_n will be functions of θ, but since θ only enters into ρ' and ζ' in the form $\alpha_n\cos n\theta$, $\beta_n\cos n\theta$, the terms in the C's which involve θ will be multiplied by α_n or β_n and so will be small.

If

$$\frac{1}{\{a^2+\rho^2+\zeta^2-2a\rho\cos(\theta-\psi)\}^{\frac{3}{2}}}=\mathrm{A}_0+\mathrm{A}_1\cos(\theta-\psi)+\ldots\mathrm{A}_n\cos n(\theta-\psi)$$

then

$$\mathrm{C}_m=\mathrm{A}_m+\Sigma\alpha_n\cos n\theta\frac{d\mathrm{A}_m}{da}$$
$$+2\zeta\Sigma\beta_n\cos n\theta\frac{d\mathrm{A}_n}{d\zeta^2}$$

+ terms of higher dimensions in α_n and β_n.

We shall only require those expressions for a point nearly in the plane of the vortex where ζ is very small, so that in this case

$$\mathrm{C}_m=\mathrm{A}_m+\Sigma\alpha_n\cos n\theta\frac{d\mathrm{A}_m}{da}$$

We can determine the components of the velocity in terms of the quantities we have denoted by $A_0 \ldots A_n$, we shall for the sake of clearness divide the determination up into several steps.

To determine the parts of u, v, w independent of α_n and β_n, in which we shall not suppose ζ small,

$$u=\frac{m}{2\pi}\int_0^{2\pi}\cos\theta\zeta\{A_0+A_1\cos(\theta-\psi)+\ldots A_n\cos n(\theta-\psi)\}ad\theta$$

$$=\frac{ma}{2\pi}\zeta\int_0^{2\pi}A_1\cos^2\theta\cos\psi d\theta=\tfrac{1}{2}ma\zeta A_1\cos\psi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$v=\frac{m}{2\pi}\int_0^{2\pi}\sin\theta\zeta(A_0+A_1\cos(\theta-\psi)+\ldots)ad\theta=\tfrac{1}{2}ma\zeta A_1\sin\psi \quad . \quad . \quad . \quad . \quad (2)$$

$$w=\frac{m}{2\pi}\int_0^{2\pi}\{\cos\theta(a\cos\theta-\rho\cos\psi)+\sin\theta(a\sin\theta-\rho\sin\psi)\}(A_0+A_1\cos\overline{\theta-\psi}+\ldots)ad\theta$$

$$=\frac{ma}{2\pi}\int_0^{2\pi}\{a-\rho\cos(\theta-\psi)\}(A_0+A_1\cos(\theta-\psi)+\ldots)d\theta$$

$$=\frac{ma}{2\pi}(A_0a2\pi-A_1\rho\pi)$$

$$=\tfrac{1}{2}ma\{2A_0a-A_1\rho\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

These are the velocities due to the undisturbed vortex, and in using them in the second half of the paper we require A_0, A_1 determined without supposing ζ to be small.

2nd. The values of u, v, w arising from small terms in ds'.

As far as now concerned,

$$ds'=d\theta\alpha_n\cos n\theta$$

$u=0$, $v=0$ because they involve $\zeta\alpha_n$.

$$w=\frac{m}{2\pi}\int_0^{2\pi}(a+\rho\cos(\theta-\psi))[A_0+A_1\cos(\theta-\psi)+\ldots]\alpha_n\cos n\theta d\theta$$

$$=\frac{m}{2\pi}\int_0^{2\pi}[aA_n\cos n(\theta-\psi)-\tfrac{1}{2}\rho A_{n+1}\cos n(\theta-\psi)-\tfrac{1}{2}\rho A_{n-1}\cos n(\theta-\psi)]\alpha_n\cos n\theta d\theta$$

$$=\tfrac{1}{2}m\alpha_n[aA_n-\tfrac{1}{2}\rho(A_{n+1}+A_{n-1})]\cos n\psi$$

3rd. Small terms in $\frac{dx'}{ds'}, \frac{dy'}{ds'}, \frac{dz'}{ds'}$.

$$u=\frac{m}{2\pi}\int_0^{2\pi}-\frac{n\beta_n}{a}\sin n\theta(a\sin\theta-\rho\sin\psi)[\mathrm{A}_0+\mathrm{A}_1\cos(\theta-\psi)+\ldots]ad\theta$$

$$=\frac{ma}{2\pi}\int_0^{2\pi}\tfrac{1}{2}u\beta_n(\cos(n+1)\theta-\cos(n-1)\theta)+\frac{n\beta_n}{a}\rho\sin\psi\sin n\theta[\mathrm{A}_0+\mathrm{A}_1\cos(\theta-\psi)]d\theta$$

$$=\tfrac{1}{2}ma\left\{n\beta_n(\mathrm{A}_{n+1}\cos(n+1)\psi-\mathrm{A}_{n-1}\cos(n-1)\psi)+\frac{n\beta_n}{a}\rho\sin\psi\sin n\psi\mathrm{A}_n\right\}$$

$$=\tfrac{1}{4}man\beta_n\left\{\cos(n+1)\psi\left(\mathrm{A}_{n+1}-\frac{\rho}{a}\mathrm{A}_n\right)-\cos(n-1)\psi\left(\mathrm{A}_{n-1}-\frac{\rho}{a}\mathrm{A}_n\right)\right\}$$

$$v=\frac{m}{2\pi}\int_0^{2\pi}n\beta_n\sin n\theta(a\cos\theta-\rho\cos\psi)[\mathrm{A}_0+\mathrm{A}_1\cos(\theta-\psi)-]ad\theta$$

$$=\tfrac{1}{4}man\beta_n\left\{\sin(n+1)\psi\left(\mathrm{A}_{n+1}-\frac{\rho}{a}\mathrm{A}_n\right)+\sin(n-1)\psi\left(\mathrm{A}_{n-1}-\frac{\rho}{a}\mathrm{A}_n\right)\right\}$$

$$w=\frac{m}{2\pi}\int_0^{2\pi}\left\{-\frac{n\alpha_n}{a}\sin\theta\sin n\theta(a\cos\theta-\rho\cos\psi)\right.$$
$$\left.+\frac{n\alpha_n}{a}\cos\theta\sin n\theta(a\sin\theta-\rho\sin\psi)\right\}(\mathrm{A}_0+\mathrm{A}_1\cos(\theta-\psi)-)ad\theta$$

$$=\frac{m}{2\pi}n\alpha_n\int_0^{2\pi}\{\tfrac{1}{2}\rho\cos\psi(\cos(n-1)\theta-\cos(n+1)\theta)$$
$$-\tfrac{1}{2}\rho\sin\psi(\sin(n+1)\theta+\sin(n-1)\theta)(\mathrm{A}_0+\mathrm{A}_1\cos(\theta-\psi))ad\theta\}$$

$$=\tfrac{1}{2}mn\alpha_n[\tfrac{1}{2}\rho\cos\psi(\mathrm{A}_{n-1}\cos(n-1)\psi-\mathrm{A}_{n+1}\cos(n+1)\psi)$$
$$-\tfrac{1}{2}\rho\sin\psi(\mathrm{A}_{n+1}\sin(n+1)\psi+\mathrm{A}_{n-1}\sin(n-1)\psi)]$$

$$=\tfrac{1}{4}mn\alpha_n\rho(\mathrm{A}_{n-1}-\mathrm{A}_{n+1})\cos n\psi.$$

4th. Small terms in $x'-x$, $y'-y$.

$$u=-\frac{m}{2\pi}\int_0^{2\pi}a\cos\theta\cos n\theta\beta_n(\mathrm{A}_0+\mathrm{A}_1\cos(\theta-\psi)+\ldots)d\theta$$

$$=-\frac{ma}{2\pi}\int_0^{2\pi}\tfrac{1}{2}\beta_n\{\cos(n-1)\theta+\cos(n+1)\theta\}(\mathrm{A}_0+\mathrm{A}_1\cos(\theta-\psi)+\ldots)d\theta$$

$$=-\tfrac{1}{4}ma\beta_n(\mathrm{A}_{n-1}\cos(n-1)\psi+\mathrm{A}_{n+1}\cos(n+1)\psi)$$

$$v=\frac{m}{2\pi}\int_0^{2\pi}-a\sin\theta\cos n\theta\beta_n(\mathrm{A}_0+\mathrm{A}_1\cos(\theta-\psi)+\ldots)d\theta$$

$$=\tfrac{1}{4}ma\beta_n(\mathrm{A}_{n-1}\sin(n-1)\psi-\mathrm{A}_{n+1}\sin(n+1)\psi)$$

$$w=\frac{m}{2\pi}\int_0^{2\pi}(\cos\theta(\alpha_n\cos n\theta\cos\theta)+\sin\theta(\alpha_n\cos n\theta\sin\theta)\}(\mathrm{A}_0+\mathrm{A}_1\cos(\theta-\psi))ad\theta$$

$$=\frac{m}{2\pi}\int_0^{2\pi}\alpha_n\cos n\theta(\mathrm{A}_0+\mathrm{A}_1\cos(\theta-\psi)+\ldots)ad\theta$$

$$=\tfrac{1}{2}ma\alpha_n\mathrm{A}_n\cos n\psi.$$

5th. Small terms arising from C_n containing $\cos n\theta$. These are

$$u=0,\ v=0$$

$$w=\frac{m}{2\pi}\int_0^{2\pi}(a-\rho\cos(\theta-\psi))\left(\alpha_n\cos n\theta\frac{d\mathrm{A}_0}{da}+-\alpha_n\cos n\theta\frac{d\mathrm{A}_m}{da}\cos m(\theta-\psi)\ldots\right)ad\theta$$

$$=\tfrac{1}{2}m\left\{a^2\alpha_n\frac{d\mathrm{A}_n}{da}\cos n\psi-\frac{\rho a\alpha_n}{2}\left(\frac{d\mathrm{A}_{n+1}}{da}+\frac{d\mathrm{A}_{n-1}}{da}\right)\cos n\psi\right\}$$

$$=\tfrac{1}{2}m\alpha_n\left\{a^2\frac{d\mathrm{A}_n}{da}-\tfrac{1}{2}\rho a\left(\frac{d\mathrm{A}_{n+1}}{da}+\frac{d\mathrm{A}_{n-1}}{da}\right)\right\}\cos n\psi.$$

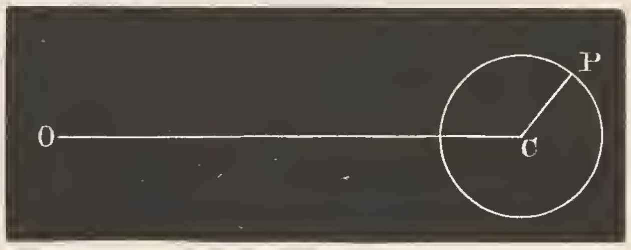
Collecting the terms we find

$$u=\tfrac{1}{2}ma\Big\{\zeta\mathrm{A}_1\cos\psi+\tfrac{1}{2}\beta_n(n-1)\mathrm{A}_{n+1}\cos(n+1)\psi-(n+1)\mathrm{A}_{n-1}\cos(n-1)\psi)$$
$$+\tfrac{1}{2}n\beta_n\mathrm{A}_n\frac{\rho}{a}(\cos(n-1)\psi-\cos(n+1)\psi)\Big\}\quad.\quad(4)$$

$$v=\tfrac{1}{2}ma\Big\{\zeta\mathrm{A}_1\sin\psi+\tfrac{1}{2}\beta_n((n-1)\mathrm{A}_{n+1}\sin(n+1)\psi+(n+1)\mathrm{A}_{n-1}\sin(n-1)\psi)$$
$$-\tfrac{1}{2}n\beta_n\mathrm{A}_n\frac{\rho}{a}(\sin(n+1)\psi+\sin(n-1)\psi)\Big\}\quad.\quad(5)$$

$$w=\tfrac{1}{2}ma\Big\{2a\mathrm{A}_0-\rho\mathrm{A}_1+\left(2\alpha_n\mathrm{A}_n+\tfrac{1}{2}\alpha_n\frac{\rho}{a}((n-1)\mathrm{A}_{n-1}-(n+1)\mathrm{A}_{n+1})\right)\cos n\psi$$
$$+\alpha_n\left[a\frac{d\mathrm{A}_n}{da}-\rho\left(\frac{d\mathrm{A}_{n+1}}{da}+\frac{d\mathrm{A}_{n-1}}{da}\right)\right]\cos n\psi\Big\}\quad.\quad(6)$$

Fig. 1.



Let the figure represent a section of the vortex ring by a plane through its straight axis. Let ϕ be the angle which the radius vector drawn from C the centre of the section of the core to any point P on the surface of the ring makes with the straight axis of the ring. Let C P$=e$.

Then the equations to the surface of the core are

$$\rho=a+\Sigma\alpha_n\cos n\psi+e\sin\phi$$
$$z=\mathfrak{z}+\Sigma\beta_n\cos n\psi+e\cos\phi$$

Since the vortex rings always consists of the same particles if $F(\rho, \theta, \phi)=0$ be an equation to its surface, we must have

$$\frac{dF}{dt}+\frac{dR}{d\rho}.R+\frac{dF}{d\psi}.\Psi+\frac{dF}{d\phi}.\Phi=0$$

when the differential coefficients are partial.

R is the velocity in the direction in which ρ is measured, Ψ the angular velocity round the axis of z, and Φ the angular velocity of C P round a normal to the plane containing the axis of z and O C.

Applying this equation to the first of the equations to the core, we get

$$\Sigma\dot{\alpha}_n \cos n\psi-R-\Sigma n\alpha_n \sin n\psi.\Psi+e \cos \phi.\Phi=0$$

or

$$R=\Sigma\dot{\alpha}_n \cos n\psi-\Sigma n\alpha_n \sin n\psi.\Psi+e \cos \phi.\Phi$$

and in a similar way we find

$$w=\dot{z}+\Sigma(\dot{\beta}_n \cos n\psi-n\beta_n \sin n\psi\Psi)-e \sin \phi.\Phi$$

where w is the velocity of a point on the surface of the core parallel to the axis of z. Now Ψ is zero when α_n and β_n are both zero, and it will be small in this case since α_n and β_n are both small, hence neglecting the squares of small quantities, these equations become

$$R=\Sigma\dot{\alpha}_n \cos n\psi+e \cos \phi.\Phi \quad \ldots \ldots \quad (7)$$

$$w=\dot{z}+\Sigma\dot{\beta}_n \cos n\psi-e \sin \phi.\Phi \quad \ldots \ldots \quad (8)$$

But $R=u \cos \psi+v \sin \psi$.

Substituting for u and v the values given in equations (4) and (5), we get

$$R=\tfrac{1}{2}ma\{\zeta A_1+\tfrac{1}{2}\Sigma\beta_n \cos n\psi((n-1)A_{n+1}-(n+1)A_{n-1})\}$$

Since the A's are multiplied by the small quantities ζ, α_n, β_n, we may suppose since we neglect quantities of the order α_n^2 that the A's are found on the supposition that α_n and β_n are zero, or that the A's are the same as if the vortex was undisturbed.

Let us denote the value of the A's for the undisturbed vortex by German letters.

Equating the two expressions for R and putting

$$\zeta=\Sigma\beta_n \cos n\psi=e \cos \phi$$

we get

$$\tfrac{1}{2}ma\Sigma\{(\beta_n \cos n\psi+e \cos \phi)\mathfrak{A}_1+\tfrac{1}{2}\beta_n \cos n\psi((n-1)\mathfrak{A}_{n+1}-(n+1)\mathfrak{A}_{n-1})\}$$
$$=\Sigma\dot{\alpha}_n \cos n\psi+e \cos \phi.\Phi$$

equating the coefficients of $\cos \phi$ and $\cos n\psi$ we get

$$\tfrac{1}{2}ma\mathfrak{A}_1=\Phi \quad \ldots \ldots \quad (9)$$

$$\tfrac{1}{2}ma\beta_n\{\mathfrak{A}_1+\tfrac{1}{2}((n-1)\mathfrak{A}_{n+1}-(n+1)\mathfrak{A}_{n-1})\}=\dot{\alpha}_n \quad \ldots \ldots \quad (10)$$

If we equate the value of w from equation (6) to that given by equation (7) we get

$$\tfrac{1}{2}ma\Big(2a\mathrm{A}_0-\rho\mathrm{A}_1+\Sigma\tfrac{1}{2}\frac{\rho}{a}.\alpha_n[(n-1)\mathrm{A}_{n-1}-(n+1)\mathrm{A}_{n+1}]\cos n\psi$$

$$+\left\{2\Sigma a_n\mathrm{A}_n+\Sigma\alpha_n\left[\frac{ad\mathrm{A}_n}{da}-\rho\left(\frac{d\mathrm{A}_{n+1}}{da}+\frac{d\mathrm{A}_{n-1}}{da}\right)\right]\right\}\cos n\psi\Big)$$

$$=\dot{\mathfrak{z}}+\Sigma\dot{\beta}_n\cos n\psi-e\sin\phi.\Phi$$

Since the term $2a\mathrm{A}_0-\rho\mathrm{A}_1$ is not multiplied by any small quantity we cannot suppose the A's to have the same value as for the undisturbed vortex, we must substitute for $2a\mathrm{A}_0-\rho\mathrm{A}_1$

$$2a\mathfrak{A}_0-a\mathfrak{A}_1-\alpha_n\cos n\psi\mathfrak{A}_1+\alpha_n\cos n\psi\frac{d}{d\rho}(2a\mathfrak{A}_0-a\mathfrak{A}_1)-e\sin\phi\mathfrak{A}_1$$

Since the other terms are multiplied by small quantities, we may substitute for the A's their undisturbed values. Making these substitutions we get

$$\tfrac{1}{2}ma\Big\{2a\mathfrak{A}_0-a\mathfrak{A}_1-e\sin\phi\mathfrak{A}_1+\Sigma\alpha_n\cos n\psi\Big[\frac{d}{d\rho}(2a\mathfrak{A}_0-a\mathfrak{A}_1)$$

$$+2\mathfrak{A}_n-\mathfrak{A}_1+\tfrac{1}{2}((n-1)\mathfrak{A}_{n-1}-(n+1)\mathfrak{A}_{n+1})+\frac{ad\mathfrak{A}_n}{da}-\frac{a}{2}\left(\frac{d\mathfrak{A}_{n+1}}{da}+\frac{d\mathfrak{A}_{n-1}}{da}\right)\Big]\Big\}$$

$$=\dot{\mathfrak{z}}-e\sin\phi\Phi+\Sigma\dot{\beta}_n\cos n\psi$$

Equating constant terms and the coefficients of $\sin\phi$ and $\cos n\psi$, we get

$$\tfrac{1}{2}ma^2(2\mathfrak{A}_0-\mathfrak{A}_1)=\dot{\mathfrak{z}}\quad.\quad.\quad.\quad.\quad.\quad.\quad.\quad.\quad.\quad.\quad(11)$$

$$\tfrac{1}{2}ma\mathfrak{A}_1=\Phi$$

$$\tfrac{1}{2}ma\alpha_n\Big\{\frac{d}{d\rho}(2a\mathfrak{A}_0-a\mathfrak{A}_1)+2\mathfrak{A}_n-\mathfrak{A}_1+\tfrac{1}{2}((n-1)\mathfrak{A}_{n-1}-(n+1)\mathfrak{A}_{n+1}$$

$$+a\frac{d\mathfrak{A}_n}{da}-\tfrac{1}{2}a\left(\frac{d\mathfrak{A}_{n+1}}{da}+\frac{d\mathfrak{A}_{n-1}}{da}\right)\Big\}=\dot{\beta}_n\,.\quad.\quad.\quad.\quad.\quad.\quad.\quad.\quad.\quad(12)$$

The first of these equations gives the velocity of translation of an undisturbed circular vortex ring, the second is the same as the one we previously obtained for Φ.

We must now proceed to find the values of the $\mathfrak{A}$'s supposing the transverse section of the vortex core to be small compared with its aperture.

Since

$$\frac{1}{\{a^2+\rho+\zeta^2-2\rho a\cos(\theta-\psi)\}^{\frac{3}{2}}}=\mathfrak{A}_0+\mathfrak{A}_1\cos(\theta-\psi)+\ldots\mathfrak{A}_n\cos n(\theta-\psi)$$

$$\mathfrak{A}_n=\frac{1}{\pi}\int_0^{2\pi}\frac{\cos n\chi.d\chi}{(a^2+\rho^2+\zeta^2-2\rho a\cos\chi)^{\frac{3}{2}}}$$

except when $n=0$ when

$$\mathfrak{A}_0=\frac{1}{2\pi}\int_0^{2\pi}\frac{d\chi}{(a^2+\rho^2+\zeta^2-2\rho a\cos\chi)^{\frac{3}{2}}}$$

Now

$$a^2+\rho^2+\zeta^2-2\rho a\cos\chi=(a+\rho)^2+\zeta^2-4\rho a\cos^2\frac{\chi}{2}=((a+\rho)^2+\zeta^2)\left(1-\kappa^2\cos^2\frac{\chi}{2}\right)$$

where

$$\kappa^2=\frac{4\rho a}{(a+\rho)^2+\zeta^2}$$

Now in the case we are considering ρ is very nearly equal to a and ζ is very small, hence κ is very nearly equal to unity

$$1-\kappa^2\cos^2\frac{\chi}{2}=\kappa_1^{\,2}+\kappa^2\sin^2\frac{\chi}{2}$$

where

$$\kappa_1^{\,2}=1-\kappa^2=\frac{(\rho-a)^2+\zeta^2}{(\rho+a)^2+\zeta^2}$$

and is very small

Therefore
$$\mathfrak{A}_n=\frac{1}{\pi}\int_0^{2\pi}\frac{\cos n\chi d\chi}{((a+\rho)^2+\zeta^2)^{\frac{3}{2}}\left(\kappa_1^{\,2}+\kappa^2\sin^2\frac{\chi}{2}\right)^{\frac{3}{2}}}$$

Now since κ_1 is very small

$$\int_0^{2\pi}\frac{\cos n\chi d\chi}{\left(\kappa_1^{\,2}+\kappa^2\sin^2\frac{\chi}{2}\right)^{\frac{3}{2}}}$$

will be very large, and the large part will arise from very small values of χ, or from values of χ very nearly equal to 2π, the parts arising from small values of χ and from values near to 2π will evidently be equal; hence the integral will be approximately

$$\int_0^{\epsilon}\frac{\left(1-\frac{n^2\chi^2}{2}\right)d\chi}{\left(\kappa_1^{\,2}+\frac{\chi^2}{4}\right)^{\frac{3}{2}}}$$

where ϵ is large compared with κ_1, this integral

$$=\frac{4(1+2n^2\kappa_1^{\,2})}{\kappa_1^{\,2}}\left[\frac{\chi}{(\kappa_1^{\,2}+\chi^2)^{\frac{1}{2}}}\right]_0^{\epsilon}-8n^2\left[\log\left(\chi+\sqrt{\kappa_1^{\,2}+\chi^2}\right)\right]_0^{\epsilon}$$

or, since ϵ is large compared with κ_1, this is approximately

$$\frac{4(1+2n^2\kappa_1^{\,2})}{\kappa_1^{\,2}}-8n^2\log\frac{2\epsilon}{\kappa_1}$$

or retaining only the more important terms

$$=\frac{4}{\kappa_1^{\,2}}+8n^2\log\kappa_1$$

hence

$$\mathfrak{A}_n=\left(\frac{4}{\pi{\kappa_1}^2}+\frac{8n^2}{\pi}\log\kappa_1\right)\div((a+\rho)^2+\zeta^2)^{\frac{3}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

except when $n=0$ and

$$\mathfrak{A}_0=\frac{2}{\pi{\kappa_1}^2}\div((a+\rho)^2+\zeta^2)^{\frac{3}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

If we substitute these values for the $\mathfrak{A}$'s the equation

$$\tfrac{1}{2}ma\mathfrak{A}_1=\Phi$$

gives

$$\tfrac{1}{2}ma\left(\frac{4}{\pi{\kappa_1}^2}+\frac{8}{\pi}\log\kappa_1\right)\div 8a^3=\Phi$$

or, since κ_1 is approximately $e^2/4a^2$, we get if we substitue this value for κ_1 and $\pi e^2\omega$ for m

$$\omega-\tfrac{1}{2}\omega\frac{e^2}{a^2}\log\frac{2a}{e}=\Phi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

The second term on the left-hand side of this equation being small compared with the first, we get as a rougher approximation

$$\omega=\Phi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

The equation

$$\tfrac{1}{2}ma\beta_n\{\mathfrak{A}_1+\tfrac{1}{2}((n-1)\mathfrak{A}_{n+1}-(n+1)\mathfrak{A}_{n-1})\}=\dot{\alpha}_n$$

gives on substitution

$$-\frac{\omega e^2}{2a^2}n^2\log\frac{2a}{e}\beta_n=\dot{\alpha}_n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

Substituting for $\mathfrak{A}_0$, $\mathfrak{A}_1$ in equation (11) we find

$$\dot{\mathfrak{z}}=\frac{\omega e^2}{2a}\log\frac{2a}{e} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

This agrees to the degree of approximation we are working to with the value for the velocity of translation of a circular vortex found by Sir W. THOMSON and given in Professor TAIT's translation of HELMHOLTZ's paper on "Vortex Motion" (Phil. Mag., June, 1867).

The value of ϕ given by equation (16) is also the same as that obtained by Sir W. THOMSON.

Substituting for the $\mathfrak{A}$'s in equation (9) we find

$$\tfrac{1}{2}ma\alpha_n\left\{-\left(\frac{d}{da}+\frac{d}{d\rho}\right)\left[\frac{4}{\pi}\log\frac{(a-\rho)^2+\zeta^2}{(a+\rho)^2+\zeta^2}\Big/((a+\rho)^2+\zeta^2)^{\frac{3}{2}}\right]\right.$$
$$\left.-\frac{4}{\pi}(n^2+2)\log\frac{(a-\rho)^2+\zeta^2}{(a+\rho)^2+\zeta^2}\Big/((a+\rho)^2+\zeta^2)^{\frac{3}{2}}\right\}=\dot{\beta}_n$$

or neglecting terms on the left-hand side which are not multiplied by the large quantity $\log \frac{\overline{a-\rho}^2+\zeta^2}{(a+\rho)^2+\zeta^2}=\log \frac{e^2}{4a^2}$ we find

$$\tfrac{1}{2}ma\alpha_n \frac{8}{\pi}\,\frac{(n^2-1)}{8a^2}\log\frac{2a}{e}=\dot{\beta}_n$$

or

$$\tfrac{1}{2}\omega\,\frac{e^2}{a^2}(n^2-1)\log\frac{2a}{e}\cdot\alpha_n=\dot{\beta}_n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

Differentiating equation (15) with respect to the time and substituting for $\dot{\beta}_n$ from (16) we get

$$-\left(\tfrac{1}{2}\omega\,\frac{e^2}{a^2}\log\frac{2a}{e}\right)^2(n^2-1)n^2\alpha_n=\alpha''_n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

or

$$\alpha_n=\mathrm{A}\cos\left\{\frac{\omega e^2}{2a^2}\log\frac{2a}{e}\,n.\sqrt{n^2-1}t+\beta\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

$$\beta_n=\mathrm{A}\,\frac{\sqrt{n^2-1}}{n}\sin\left\{\frac{\omega e^2}{2a^2}\log\frac{2a}{e}\cdot n\sqrt{n^2-1}.t+\beta\right\} \quad . \quad . \quad . \quad . \quad . \quad (22)$$

where A and β are arbitrary constants.

These equations show that the circular vortex ring of indefinitely small section is stable for all displacements of its circular axis, and that the time of vibration for a displacement expressed by

$$\rho=a+\alpha_n\cos n\theta$$

is

$$2\pi\Big/\frac{\omega e^2}{2a^2}\log\frac{2a}{e}.n.\sqrt{n^2-1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

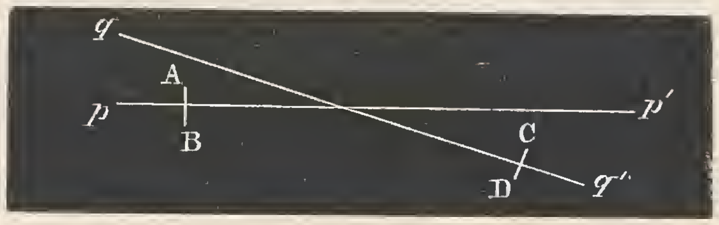
If V be the velocity of translation of the vortex, viz.: $\omega e^2\log\frac{2a}{e}\Big/2a$, the time of vibration is $2\pi a/\mathrm{V}.n\sqrt{n^2-1}$.

Sir W. THOMSON has proved that the circular vortex ring is stable for all alterations in the shape of the cross section. If we combine this with the result just obtained we see that the circular vortex ring is stable for all possible displacements. Sir W. THOMSON has also proved that for a displacement of the n^{th} order in the shape of the cross section of the vortex arc the time of vibration $=2\pi/(n-1)\omega$; hence these vibrations begin by being much quicker than those we have been considering, but since for large values of n the latter are proportional to n^2 whilst the former are only proportional to n, the vibrations of a higher order will be quicker for the circular axis than for the core. When n is very great $\sqrt{n^2-1}=n$, thus the amplitude of α_n is equal to the amplitude of β_n and $\alpha_n^2+\beta_n^2=\mathrm{A}^2$ a constant quantity; thus each point on the arc describes a circle about its mean position with an angular velocity $\omega e^2n^2\log 2a/e\Big/a^2$.

Problem II. To find the action upon each other of two vortex rings which move so as never to approach nearer than a large multiple of the diameter of either.

For the sake of simplicity we shall suppose that the normals to the planes of the vortices intersect.

Fig. 2.



Let the plane of the paper contain $p\,p'$ and $q\,q'$ the normals to the two vortices, let A B be the vortex moving along $p\,p'$, C D the vortex moving along $q\,q'$.

Let the figure of the circular axis of the vortex A B be given by

$$\rho'=a'+\Sigma\alpha_n'\cos n\theta'$$
$$z'=\mathfrak{z}'+\Sigma\beta_n'\cos n\theta'$$

where z' is measured along and ρ' perpendicular to $p\,p'$. Since the vortices never approach near to one another α_n' and β_n' will be small compared with a'; they will be functions of the time which we shall have to find.

Let the figure of the circular axis of C D be given by

$$\rho=a+\Sigma\alpha_n\cos n\theta$$
$$z=\mathfrak{z}+\Sigma\beta_n\cos n\theta$$

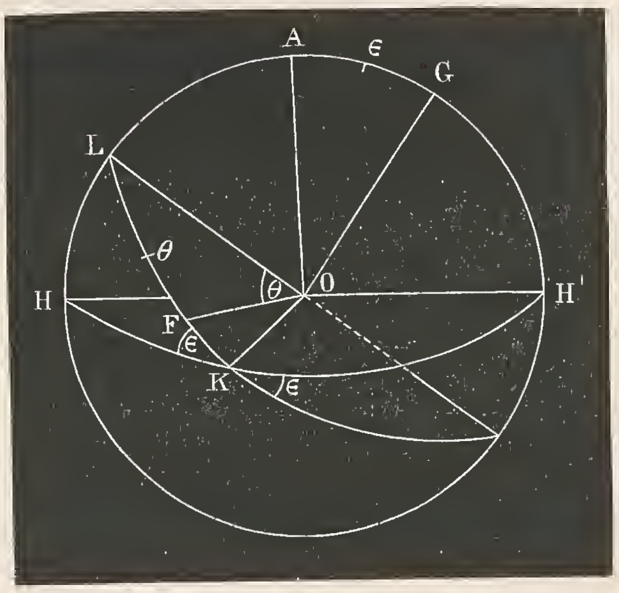
where z and ρ are measured respectively along and perpendicular to $q\,q'$. For the same reason as before α_n and β_n will be small compared with a. To find how the vortex C D is affected by the vortex A B we shall have to find the velocities of the fluid along z and ρ due to the vortex A B; in doing this we may as a first approximation assume that the axis of A B is circular and in one plane, *i.e.*, we may calculate the velocities as if α_n' and β_n' were both zero.

Let ϵ denote the angle between $p\,p'$ and $q\,q'$. Let $p\,p'$ be taken as the axis of z', the perpendicular to $p\,p'$ drawn upwards through the centre of the vortex A B, being the axis of x'.

Let l, m, n be the direction cosines referred to these axes of a radius vector in the plane of the vortex ring C D, drawn from the centre of the vortex ring and making an angle θ with C D the intersection of the plane of the vortex ring with the plane of the paper.

To find l, m, n through the centre of a sphere draw planes parallel to the two vortex rings and let these be H K H′, L K H′, the former being parallel to the ring A B and the latter to C D. Let A G be the poles of these great circles.

Fig. 3.



Then O H, K O, O A are parallel to our axes of x', y', z' respectively. The angle A G or K is ϵ, and if F is parallel to the radius vector above referred to, F L is equal to θ. Then

$$
\begin{aligned}
l &= \cos \mathrm{HF} = \cos\theta\cos\epsilon \\
m &= -\cos \mathrm{FK} = -\sin\theta \\
n &= \cos \mathrm{FA} = \cos\theta\sin\epsilon
\end{aligned}
$$

The velocity γ along the axis of z' due to the vortex A B is by formula 3 given by

$$\gamma=\tfrac{1}{2}m'(2a'^2\mathrm{A}_0-a'\rho'\mathrm{A}_1)$$

where m' is the strength of the vortex A B

$$\mathrm{A}_0=\frac{1}{2\pi}\int_0^{2\pi}\frac{d\theta}{(a'^2+\rho'^2+\zeta'^2-2\rho'a'\cos\theta)^{\frac{3}{2}}}$$

$$\mathrm{A}_1=\frac{1}{2\pi}\int_0^{2\pi}\frac{\cos\theta.d\theta}{(a'^2+\rho'^2+\zeta'^2-2\rho'a'\cos\theta)^{\frac{3}{2}}}$$

Now since the vortex rings never approach nearer than a large multiple of their diameter, a'^2 will be small compared with $\rho'^2+\zeta'^2$, if we neglect small quantities of a higher order than $a'^2/\rho'^2+\zeta'^2$, we find

$$\mathrm{A}_0=\frac{1}{(\rho'^2+\zeta'^2)^{\frac{3}{2}}}+\frac{3}{4}\frac{a'^2(3\rho'^2-2\zeta'^2)}{(\rho'^2+\zeta'^2)^{\frac{7}{2}}}$$

$$\mathrm{A}_1=\frac{3a'\rho'}{(\rho'^2+\zeta'^2)^{\frac{5}{2}}}$$

Let the coordinates of the centre of the vortex ring C D be f, 0, h, then for a point on the vortex ring

$$
\begin{aligned}
x &= f+al = f+a\cos\epsilon\cos\theta \\
y &= am = -a\sin\theta \\
z &= h+an = h+a\sin\epsilon\cos\theta
\end{aligned}
$$

Substituting in the expressions for A_0 and A_1, we find on neglecting small quantities of an order higher than a'^2/f^2+h^2

$$A_0=\frac{1}{(f^2+h^2)^{\frac{3}{2}}}-\frac{3}{2}\frac{a^2}{(f^2+h^2)^{\frac{5}{2}}}+\frac{15}{4}\frac{a^2(h\sin\epsilon+f\cos\epsilon)^2}{(f^2+h^2)^{\frac{7}{2}}}+\frac{3}{4}\frac{a'^2(3f^2-2h^2)}{(f^2+h^2)^{\frac{7}{2}}}$$

$$-\cos\theta\,\frac{3a(h\sin\epsilon+f\cos\epsilon)}{(f^2+h^2)^{\frac{5}{2}}}+\cos 2\theta\,\frac{15}{4}\frac{a^2(h\sin\epsilon+f\cos\epsilon)^2}{(f^2+h^2)^{\frac{7}{2}}}$$

$$\rho' A_1=\frac{3a'\rho'^2}{(\rho'^2+\zeta'^2)^{\frac{5}{2}}}$$

$$=\frac{3a'f^2}{(f^2+h^2)^{\frac{5}{2}}}-\frac{15}{2}\frac{a'a^2f^2}{(f^2+h^2)^{\frac{7}{2}}}+\frac{105}{4}\frac{a'a^2f^2(h\sin\epsilon+f\cos\epsilon)^2}{(f^2+h^2)^{\frac{9}{2}}}+\frac{3a'a^2(1-\frac{1}{2}\sin^2\epsilon)}{(f^2+h^2)^{\frac{5}{2}}}$$

$$-\frac{15a'a^2f(h\sin\epsilon+f\cos\epsilon)}{(f^2+h^2)^{\frac{7}{2}}}+\cos\theta\left\{\frac{6aa'f\cos\epsilon}{(f^2+h^2)^{\frac{5}{2}}}-\frac{15aa'f^2(h\sin\epsilon+f\cos\epsilon)}{(f^2+h^2)^{\frac{7}{2}}}\right\}$$

$$+\cos 2\theta\left\{\frac{105}{4}\frac{a'a^2f^2(h\sin\epsilon+f\cos\epsilon)^2}{(f^2+h^2)^{\frac{9}{2}}}-\frac{15a'a^2f\cos\epsilon(h\sin\epsilon+f\cos\epsilon)}{(f^2+h^2)^{\frac{7}{2}}}-\frac{3}{2}\frac{a'a^2\sin^2\epsilon}{(f^2+h^2)^{\frac{5}{2}}}\right\}$$

Although for reference we give the complete values of A_0 and A_1 to the order of approximation we are working to, yet when we have in the expressions for the velocities a coefficient consisting of terms of differerent orders, we shall only retain the largest term.

If we do this we find

$$\gamma=\frac{1}{2}\frac{m'a'^2}{(f^2+h^2)^{\frac{5}{2}}}(2h^2-f^2)$$

$$+\frac{3}{2}\cos\theta\frac{m'a'^2}{(f^2+h^2)^{\frac{7}{2}}}\left(f^2(f\cos\epsilon+3h\sin\epsilon)-2h^2(2f\cos\epsilon+h\sin\epsilon)\right)$$

$$+\frac{3}{2}\cos 2\theta\frac{m'a'^2a^2}{(f^2+h^2)^{\frac{7}{2}}}\left\{\frac{5}{2}(f\cos\epsilon+h\sin\epsilon)^2-\frac{35}{4}\frac{f^2(f\cos\epsilon+h\sin\epsilon)^2}{(f^2+h^2)}+5f\cos\epsilon(f\cos\epsilon+h\sin\epsilon)\right.$$

$$\left.+\frac{1}{2}\sin^2\epsilon(f^2+h^2)\right\}$$

From the formulæ (1) and (3) we find the velocity along ρ'

$$=\frac{1}{2}m'a'\zeta' A_1$$

$$=\frac{3}{2}\frac{m'a'^2\rho\zeta'}{(\rho'^2+\zeta'^2)^{\frac{5}{2}}}$$

Hence α, the velocity along x' at the vortex C D due to the vortex A B,

$$=\frac{3}{2}\frac{m'a'^2\zeta'(f+a\cos\epsilon\cos\theta)}{(\rho'^2+\zeta'^2)^{\frac{5}{2}}}$$

$$=\frac{3m'a'^2}{2}\frac{fh}{(f^2+h^2)^{\frac{5}{2}}}$$

$$+\cos\theta.\frac{3}{2}\frac{m'a'^2a}{(f^2+h^2)^{\frac{5}{2}}}.\left\{(h\cos\epsilon+f\sin\epsilon)-\frac{5fh(h\sin\epsilon+f\cos\epsilon)}{f^2+h^2}\right\}$$

$$+\cos 2\theta.\frac{3}{2}\frac{m'a'^2a^2}{(f^2+h^2)^{\frac{5}{2}}}\left\{\frac{35}{4}\frac{(h\sin\epsilon+f\cos\epsilon)^2fh}{(f^2+h^2)^2}-\frac{5}{4}\frac{(h\cos\epsilon+f\sin\epsilon)(h\sin\epsilon+f\cos\epsilon)}{(f^2+h^2)}+\frac{1}{2}\sin\epsilon\cos\epsilon\right\}$$

In this expression for α we have only retained the highest terms in each coefficient. If β be the velocity parallel to the axis of y', we have

$$\begin{aligned}\beta&=-\tfrac{3}{2}\frac{m'a'^2a\zeta'\sin\theta}{(\rho'^2+\zeta^2)^{\frac{5}{2}}}\\&=-\tfrac{3}{2}\frac{m'a'^2a}{(f^2+h^2)^{\frac{5}{2}}}\left(h\sin\theta-\frac{5ah\sin\theta\cos\theta(h\sin\epsilon+f\cos\epsilon)}{f^2+h^2}+a\sin\theta\cos\theta\sin\epsilon\right)\end{aligned}$$

The velocity perpendicular to the plane of the vortex C D

$$=\gamma\cos\epsilon-\alpha\sin\epsilon$$

$$\left.\begin{aligned}&=\tfrac{1}{2}\frac{m'a'^2}{(f^2+h^2)^{\frac{5}{2}}}\{(2h^2-f^2)\cos\epsilon-3fh\sin\epsilon\}\\&+\cos\theta.\tfrac{3}{2}\frac{m'a'^2a}{(f^2+h^2)^{\frac{7}{2}}}\{\cos 2\epsilon(f^3-4fh^2)+\tfrac{1}{2}\sin 2\epsilon(7f^2h-3h^3)\}\\&+\cos 2\theta.\tfrac{3}{2}\frac{m'a'^2a^2}{(f^2+h^2)^{\frac{7}{2}}}\left\{\tfrac{5}{2}(h\sin\epsilon+f\cos\epsilon)(h\sin 2\epsilon+f\cos 2\epsilon+2f)-\tfrac{35}{4}\frac{f^2(h\sin\epsilon+f\cos\epsilon)^2}{f^2+h^2}\right\}\\&+\ldots\end{aligned}\right\}(24)$$

The velocity along the radius vector of the vortex ring C D due to the vortex A B

$$\begin{aligned}&=\alpha l+\beta m+\gamma n\\&=\alpha\cos\epsilon\cos\theta-\beta\sin\theta+\gamma\sin\epsilon\cos\theta\end{aligned}$$

or substituting for α, β, γ their values

$$\left.\begin{aligned}&=\tfrac{3}{4}\frac{m'a'^2a}{(f^2+h^2)^{\frac{5}{2}}}\left\{\tfrac{1}{2}h+\tfrac{1}{2}\cos 2\epsilon\frac{(3h^3-7hf^2)}{f^2+h^2}+\sin 2\epsilon\frac{(f^3-4fh^2)}{f^2+h^2}\right\}\\&+\tfrac{1}{2}\cos\theta\frac{m'a'^2}{(f^2+h^2)^{\frac{5}{2}}}\{(2h^2-f^2)\sin\epsilon+3fh\cos\epsilon\}\\&+\tfrac{3}{4}\cos 2\theta\frac{m'a'^2a}{(f^2+h^2)^{\frac{5}{2}}}\left\{-\tfrac{3}{2}h+\tfrac{1}{2}\cos 2\epsilon\frac{(3h^3-7f^2h)}{f^2+h^2}+\sin 2\epsilon\frac{(f^3-4fh^2)}{f^2+h^2}\right\}\end{aligned}\right\}.\quad(25)$$

These expressions will enable us to find the effect of one vortex on another. We have, for example, expressed the velocity perpendicular to the plane of C D due to the vortex ring A B in the form A+B cos θ+C cos 2θ+ . . . , in Problem I. we expressed the velocity in the same direction due to the vortex C D itself in the same form, hence the total velocity perpendicular to C D can be expressed in this form, but by formula (8) the velocity perpendicular to the plane of C D is

$$\Sigma\frac{d\beta_n}{dt}\cos n\theta$$

hence $\frac{d\beta_n}{dt}$= coefficient of $\cos n\theta$ in the expression for the velocity perpendicular to the plane of the vortex C D. To find β_n we must therefore express the velocity due to the vortex A B as a function of the time. In order to make the work as simple as possible we shall suppose that the vortices in their undisturbed states had equal strengths and radii. In the small terms which express the velocity at the vortex C D due to the vortex A B we may as a first approximation calculate the quantities on the supposition that the motion is undisturbed. In order to make the expressions as simple as possible, let us measure the time from the instant when the distance between the centres of the vortices has its least value (it is easy to see that this will be when the line joining the centres of the vortices is parallel to the line bisecting the angle between their directions of motion), then the square of the distance between their centres will be expressible in the form $c^2+\beta t^2$ when c is the least distance between the centres, t the time that has elapsed since the centres were this distance apart; let v be the velocity of translation of either vortex when undisturbed, then

$$\begin{aligned} f&=-c\sin\tfrac{1}{2}\epsilon-v\sin\epsilon.t \\ h&=\ \ c\cos\tfrac{1}{2}\epsilon-v(1-\cos\epsilon)t \end{aligned}$$

Therefore

$$f^2+h^2=c^2+4v^2\sin^2\tfrac{1}{2}\epsilon.t^2$$

Making these substitutions we find that the velocity perpendicular to the plane of the vortex C D

$$\begin{aligned} &=\tfrac{1}{2}\frac{m'a'^2}{(c^2+4v^2\sin^2\frac{1}{2}\epsilon.t^2)^{\frac{5}{2}}}\{\tfrac{1}{2}c^2(3+\cos\epsilon)+2v^2\sin^2\tfrac{1}{2}\epsilon(1-3\cos\epsilon)t^2\} \\ &\quad+\cos\theta\tfrac{3}{2}\frac{m'a'^2a}{(c^2+4v^2\sin^2\frac{1}{2}\epsilon.t^2)^{\frac{7}{2}}}\{\mathrm{A}t^3+\mathrm{B}t^2+\mathrm{C}t+\mathrm{D}\} \\ &\quad+\cos 2\theta\tfrac{3}{2}\frac{m'a'^2a^2}{(c^2+4v^2\sin^2\frac{1}{2}\epsilon\, t^2)^{\frac{9}{2}}}\{\mathrm{A}'t^4+\mathrm{B}'t^3+\mathrm{C}'t^2+\mathrm{D}'t+\mathrm{E}'\} \end{aligned}$$

where

$$\left.\begin{aligned} \mathrm{A}&=8v^3\sin^3\tfrac{1}{2}\epsilon(\cos\tfrac{3}{2}\epsilon-5\cos\tfrac{1}{2}\epsilon) \\ \mathrm{B}&=cv^2\sin^2\tfrac{1}{2}\epsilon(15\sin\tfrac{1}{2}\epsilon-\sin\tfrac{3}{2}\epsilon) \\ \mathrm{C}&=c^2v\sin\tfrac{1}{2}\epsilon(15\cos\tfrac{1}{2}\epsilon+\cos\tfrac{3}{2}\epsilon) \\ \mathrm{D}&=-\tfrac{1}{4}c^3(5\sin\tfrac{1}{2}\epsilon+\sin\tfrac{3}{2}\epsilon) \end{aligned}\right\}\quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

$$\left.\begin{aligned} \mathrm{A}'&=-10v^4\sin^4\tfrac{1}{2}\epsilon(2\cos 3\epsilon+\tfrac{30}{8}\cos 2\epsilon+7\cos\epsilon+\tfrac{42}{8}) \\ \mathrm{B}'&=\ \ 10cv^3\sin^3\tfrac{1}{2}\epsilon(3\sin\epsilon-\tfrac{1}{2}\sin 2\epsilon) \\ \mathrm{C}'&=-10c^2v^2\sin^2\tfrac{1}{2}\epsilon\cos 3\epsilon \\ \mathrm{D}'&=-\tfrac{5}{8}c^3v\sin\epsilon(14\sin\epsilon+\sin 2\epsilon) \\ \mathrm{E}'&=\ \ \tfrac{5}{8}c^4\sin^2\tfrac{1}{2}\epsilon(11+\cos\epsilon+8\cos 2\epsilon) \end{aligned}\right\}\quad . \quad . \quad . \quad (27)$$

The velocity along the radius vector

$$
\begin{aligned}
&=\tfrac{3}{4}\frac{m'a'^2a}{(c^2+4v^2\sin^2\frac{1}{2}\epsilon.t^2)^{\frac{7}{2}}}(\mathrm{F}t^3+\mathrm{G}t^2+\mathrm{H}t+\mathrm{K})\\
&+\cos\theta\tfrac{1}{2}\frac{m'a'^2}{(c^2+4v^2\sin^2\frac{1}{2}\epsilon.t^2)^{\frac{5}{2}}}\{\tfrac{1}{2}c^2\sin\epsilon-6cv\sin\tfrac{1}{2}\epsilon.t+2\sin^2\tfrac{1}{2}\epsilon\sin\epsilon.v^2t^2\}\\
&+\cos 2\theta\tfrac{3}{4}\frac{m'a'^2a}{(c^2+4v^2\sin^2\frac{1}{2}\epsilon.t^2)^{\frac{7}{2}}}(\mathrm{F}'t^3+\mathrm{G}'t^2+\mathrm{H}'t+\mathrm{K}')
\end{aligned}
$$

where

$$
\left.\begin{aligned}
\mathrm{F}&=v^3\sin^3\tfrac{1}{2}\epsilon(2\sin\tfrac{3}{2}\epsilon-14\sin\tfrac{1}{2}\epsilon)\\
\mathrm{G}&=cv^2\sin^2\tfrac{1}{2}\epsilon(\cos\tfrac{3}{2}\epsilon-13\cos\tfrac{1}{2}\epsilon)\\
\mathrm{H}&=\tfrac{1}{2}c^2v\sin\tfrac{1}{2}\epsilon(\sin\tfrac{3}{2}\epsilon+13\sin\tfrac{1}{2}\epsilon)\\
\mathrm{K}&=\tfrac{1}{4}c^3(\cos\tfrac{3}{2}\epsilon+7\cos\tfrac{1}{2}\epsilon)
\end{aligned}\right\}\quad\ldots\ldots\ldots\quad(28)
$$

$$
\left.\begin{aligned}
\mathrm{F}'&=2v^3\sin^3\tfrac{1}{2}\epsilon(\sin\tfrac{3}{2}\epsilon+\sin\tfrac{1}{2}\epsilon)\\
\mathrm{G}'&=cv^2\sin^2\tfrac{1}{2}\epsilon(\cos\tfrac{3}{2}\epsilon-21\cos\tfrac{1}{2}\epsilon)\\
\mathrm{H}'&=\tfrac{1}{2}c^2v\sin\tfrac{1}{2}\epsilon(\sin\tfrac{3}{2}\epsilon+21\sin\tfrac{1}{2}\epsilon)\\
\mathrm{K}'&=\tfrac{1}{4}c^3(\cos\tfrac{3}{2}\epsilon-\cos\tfrac{5}{2}\epsilon)
\end{aligned}\right\}\quad\ldots\ldots\ldots\quad(29)
$$

We can now write down the differential equations giving α_n and β_n. We shall begin with that giving $\beta_{,}$ as the simplest, as a reference to equation (19) will show that the vortex ring C D contributes nothing to this term, so that

$$
\frac{d\beta_{,}}{dt}=\tfrac{3}{2}\frac{m'a'^2a}{(c^2+4v^2\sin^2\frac{1}{2}\epsilon.t^2)^{\frac{7}{2}}}(\mathrm{A}t^3+\mathrm{B}t^2+\mathrm{C}t+\mathrm{D})
$$

integrating and writing for brevity κ instead of $4v^2\sin^2\frac{1}{2}\epsilon$ we find

$$
\begin{aligned}
\beta_{,}=\tfrac{3}{2}m'a'^2a\Bigg\{&\tfrac{1}{5}\frac{\frac{\mathrm{A}c^2}{\kappa^4}-\frac{\mathrm{C}}{\kappa^2}}{(c^2+\kappa^2t^2)^{\frac{5}{2}}}-\tfrac{1}{3}\frac{\frac{\mathrm{A}}{\kappa^4}}{(c^2+\kappa^2t^2)^{\frac{3}{2}}}+\tfrac{1}{5}\frac{\mathrm{D}-\frac{\mathrm{B}c^2}{\kappa^2}}{c^2}\frac{t}{(c^2+\kappa^2t^2)^{\frac{5}{2}}}+\tfrac{1}{15}\frac{\left(\frac{4\mathrm{D}}{c^4}+\frac{\mathrm{B}}{c^2\kappa^2}\right)}{(c^2+\kappa^2t^2)^{\frac{3}{2}}}.t\\
&+\tfrac{1}{15}\left(\frac{8\mathrm{D}}{c^6}+\frac{2\mathrm{B}}{c^4\kappa^2}\right)\left(\frac{t}{(c^2+\kappa^2t^2)^{\frac{1}{2}}}+\frac{1}{2v\sin\frac{1}{2}\epsilon}\right)\Bigg\}
\end{aligned}
$$

where the arbitrary constant arising from the integration has been determined so as to make $\beta_{,}=0$ when $t=-\infty$.

Substituting for A, B, C, D their values we find

$$
\begin{aligned}
\beta_{,}=\tfrac{3}{2}m'a'^2a\Bigg\{&\tfrac{1}{20}\frac{c^2(\cos\frac{3}{2}\epsilon-\frac{35}{2}\cos\frac{1}{2}\epsilon)}{v\sin\frac{1}{2}\epsilon}\frac{1}{(c^2+\kappa^2t^2)^{\frac{5}{2}}}-\frac{\cos\frac{3}{2}\epsilon-5\cos\frac{1}{2}\epsilon}{6v\sin\frac{1}{2}\epsilon}\frac{1}{(c^2+\kappa^2t^2)^{\frac{3}{2}}}\\
&-\frac{c\sin\frac{1}{2}\epsilon}{(c^2+\kappa^2t^2)^{\frac{5}{2}}}t-\frac{(\sin\frac{1}{2}\epsilon+\sin\frac{3}{2}\epsilon)\epsilon}{12c(c^2+\kappa^2t^2)^{\frac{3}{2}}}-\frac{(\sin\frac{1}{2}\epsilon+\sin\frac{3}{2}\epsilon)}{6c^3}\left(\frac{\epsilon}{(c^2+\kappa^2t^2)^{\frac{1}{2}}}+\frac{1}{2v\sin\epsilon}\right)\Bigg\}\quad(30)
\end{aligned}
$$

This expression vanishes when $t=-\infty$; it begins by being positive but soon changes sign; it is negative when $t=0$ and remains negative for all greater values of t.

The last terms are the only ones that do not vanish when $t=\infty$. Putting $t=\infty$ we find

$$\beta_{,}=-\tfrac{1}{4}\frac{m'a'^2a}{c^3v\sin\frac{1}{2}\epsilon}(\sin\tfrac{1}{2}\epsilon+\sin\tfrac{3}{2}\epsilon)$$

$$=-\frac{m'a'^2a}{c^3v}\cos^2\tfrac{1}{2}\epsilon$$

Now $\beta_{,}/a$ is the angle through which the plane of the vortex is turned, and since the vortex moves at right angles to its plane this will be the angle through which the direction of motion of the vortex ring is turned.

Since $\beta_{,}$ is negative, the part of the vortex ring C D where $\cos\theta$ is positive is tilted backwards; now as we have taken it, $\cos\theta$ is positive for the upper part of the vortex, hence this part of the vortex is tilted backwards, and the normal to its plane, which is the direction in which the vortex ring moves, is bent towards pp', the direction of motion of the vortex ring A B through an angle whose circular measure

$$=\frac{m'a^3}{vc^3}\cos^2\tfrac{1}{2}\epsilon.$$

Thus the deflection, other things being the same, varies inversely as the cube of the least distance between the vortices.

Let us now consider the effect of the vortex ring C D on the vortex A B. Let us take the perpendicular to the plane of C D as the new axis of z, the perpendicular to this drawn upwards in the plane of the paper as the new axis of x. The work we went through before consisted in finding expressions for the velocities along the axes of coordinates due to one vortex at a point on the other in terms of f, h, l, m, n, and then finding the velocities perpendicular to the plane of the vortex and along its radius vector in terms of the time by substituting from the equations

$$\left.\begin{aligned} f&=-c\sin\tfrac{1}{2}\epsilon-v\sin\epsilon.t\\ h&=c\cos\tfrac{1}{2}\epsilon-v(1-\cos\epsilon)t \end{aligned}\right\}$$

$$\left.\begin{aligned} l&=\cos\epsilon\cos\theta\\ m&=-\sin\theta\\ n&=\sin\epsilon\sin\theta \end{aligned}\right\}$$

velocity perpendicular to the plane of the vortex $=\gamma\cos\epsilon-\alpha\sin\epsilon$, velocity along the radius vector $=\alpha\cos\epsilon\cos\theta-\beta\sin\theta+\gamma\sin\epsilon\cos\theta$.

Now in finding the effect of the vortex ring C D on the vortex A B, the general expressions giving the velocities will be the same as before, as we have taken corresponding axes of coordinates. The difference in the work will come in when we substitute for the quantities involved their expressions in terms of the time; if we denote corresponding quantities in this case by affixing dashes to the symbols, denoting them in the previous one, we easily find

$$\left.\begin{aligned} f' &= -c\sin\tfrac{1}{2}\epsilon + v\sin\epsilon . t \\ h' &= -c\cos\tfrac{1}{2}\epsilon - v(1-\cos\epsilon)t \end{aligned}\right\}$$

$$\left.\begin{aligned} l' &= \quad\cos\epsilon\cos\theta' \\ m' &= -\sin\theta' \\ n' &= -\sin\epsilon\cos\theta' \end{aligned}\right\}$$

velocity perpendicular to the plane of the vortex A B

$$=\gamma\cos\epsilon+\alpha\sin\epsilon$$

velocity along the radius vector $=\alpha\cos\epsilon\cos\theta'-\beta\sin\theta'-\gamma\sin\epsilon\cos\theta'$.

It will be seen that we can get the expressions for the quantities denoted by the accented letters from those for the quantities denoted by the unaccented letters by writing $2\pi-\epsilon$ instead of ϵ, hence the value of $\beta_{,}'$ will be got by writing in the expression for $\beta_{,}$ (equation (30)) $2\pi-\epsilon$ instead of ϵ, and interchanging a and a'.

Hence the value of $\beta_{,}'$ when $t=\infty$

$$=-\frac{ma^2a'}{vc^3}\cos^2\frac{2\pi-\epsilon}{2}=-\frac{ma^2a'}{vc^3}\cos^2\tfrac{1}{2}\epsilon$$

Now this being negative shows that the parts of the vortex ring A B where $\cos\theta'$ is positive are tilted backwards, now $\cos\theta'$ is positive in the upper half of the vortex ring A B, therefore the direction of motion of the vortex A B, which is perpendicular to the plane of the vortex, is turned away from the direction of motion of the vortex C D through an angle whose circular measure is $ma^2\cos^2\frac{1}{2}\epsilon/vc^3$; but since in the case we are considering $a=a'$, $m=m'$, this angle is the same as that through which the path of the vortex C D is turned towards the path of the vortex A B. We may express the results we have obtained by saying that the direction of motion of the vortex which is in front when the vortices are nearest together, is bent towards the direction of motion of the one which is behind, that the direction of motion of the latter is bent through an equal amount in the same direction, and that the amount of this bending is $ma^2\cos^2\frac{1}{2}\epsilon/vc^3$.

Let us now consider the effect the collision has on the size of the vortices.

The equation giving the increase in radius is

$\frac{d\alpha_0}{dt}=$the part independent of θ in the expression for the velocity along the radius vector of C D.

A reference to equation (17) will show that the vortex ring C D itself contributes nothing to this term, therefore

$\frac{d\alpha_0}{dt}$=the part independent of θ in the expression for the velocity due to the vortex ring A B along the radius vector of C D.

Hence from equation (28) we have

$$\frac{d\alpha_0}{dt}=\frac{3}{4}\frac{m'a'^2a}{(c^2+4v^2\sin^2\tfrac{1}{2}\epsilon.t^2)^{\frac{7}{2}}}\{\mathrm{F}t^3+\mathrm{G}t^2+\mathrm{H}t+\mathrm{K}\}$$

If we integrate this equation, substitute for F, G, H, K their values as given in equation (28) and determine the arbitrary constant introduced by the integration, so that $\alpha_0=0$ when $t=-\infty$, we find

$$\alpha_0=\frac{3}{4}m'a'^2a\left\{\frac{-c^2}{2v(c^2+4v^2\sin^2\tfrac{1}{2}\epsilon.t^2)^{\frac{5}{2}}}+\frac{(1+\sin^2\tfrac{1}{2}\epsilon)}{6v(c^2+4v^2\sin^2\tfrac{1}{2}\epsilon.t^2)^{\frac{3}{2}}}+\frac{c\cos\tfrac{1}{2}\epsilon.t}{(c^2+4v^2\sin^2\tfrac{1}{2}\epsilon.t^2)^{\frac{5}{2}}}\right.$$
$$\left.+\frac{(\cos\tfrac{3}{2}\epsilon+3\cos\tfrac{1}{2}\epsilon)t}{12c(c^2+4v^2\sin^2\tfrac{1}{2}\epsilon.t^2)^{\frac{3}{2}}}+\frac{3\cos\tfrac{1}{2}\epsilon+\cos\tfrac{3}{2}\epsilon}{6c^3}\left(\frac{\epsilon}{(c^3+4v^2\sin^2\tfrac{1}{2}\epsilon.t^2)^{\frac{1}{2}}}+\frac{1}{2v\sin\tfrac{1}{2}\epsilon}\right)\right\}$$

This expression vanishes when $t=-\infty$; it begins by being negative, so that the radius of C D is diminished at first when $t=0$, the sign of α_0 depends upon the value of ϵ, if ϵ be less than 60° it is certainly positive when $t=0$; when $t=\infty$ α_0 is positive, and its value is

$$\frac{m'a'^2a}{8vc^3\sin\tfrac{1}{2}\epsilon}(3\cos\tfrac{1}{2}\epsilon+\cos\tfrac{3}{2}\epsilon)$$
$$=\frac{1}{2}\frac{m'a'^2a}{vc^3\sin\tfrac{1}{2}\epsilon}\cos^3\tfrac{1}{2}\epsilon.$$

As this is positive, the vortex ring C D is bigger after the collision. The effect of the vortex ring C D on the vortex A B can be got as we saw before by writing $2\pi-\epsilon$ for ϵ in the formula given above. We have thus the ultimate increase α_0' in the radius of A B given by

$$\alpha_0'=\frac{1}{2}\frac{ma^2a'\cos^3\tfrac{1}{2}(2\pi-\epsilon)}{c^3\sin\tfrac{1}{2}(2\pi-\epsilon)}$$

or since $a=a'$, $m=m'$

$$\alpha_0=-\frac{1}{2}\frac{m'a'^2a}{c^3\sin\tfrac{1}{2}\epsilon}\cos^3\tfrac{1}{2}\epsilon$$

Hence the radius of the vortex ring A B is diminished by the collision. The effect of the collision on the size of the vortices is thus to increase the radius of the one which is in front when the vortex rings are nearest together, and decrease that of the one in the rear by $m'a^3\cos^3\tfrac{1}{2}\epsilon/2vc^3\sin\tfrac{1}{2}\epsilon$. Hence the alteration in the radius is, *cœteris paribus*, inversely proportional to the cube of the shortest distance between the vortices.

We can now find the force resultant of the impulse after collision. The impulse for a vortex ring with a very fine core equals the strength multiplied by the area. Let ω be the small angle through which the direction of motion of the vortices is deflected; let δa be the alteration in the radius of either, for the vortex ring in front δa will be positive, for the one in the rear it will be negative but of equal numerical value.

The resolved part of the force resultant of the impulse along the line bisecting their original direction of motion after collision

$$\begin{aligned}&=\pi m(a+\delta a)^2\cos\left(\frac{\epsilon}{2}-\omega\right)+\pi m(a-\delta a)^2\cos\left(\frac{\epsilon}{2}+\omega\right)\\&=2\pi ma^2\cos\tfrac{1}{2}\epsilon\end{aligned}$$

the same as before collision.

The component perpendicular to the bisector of the angle between their directions of motion

$$\begin{aligned}&=\pi m(a+\delta a)^2\sin(\tfrac{1}{2}\epsilon-\omega)-\pi m(a-\delta a)^2\sin(\tfrac{1}{2}\epsilon+\omega)\\&=\pi m(4a\delta a\sin\tfrac{1}{2}\epsilon-2\omega a^2\cos\tfrac{1}{2}\epsilon)\end{aligned}$$

Substituting for δa and ω the values $ma^3\cos^3\frac{1}{2}\epsilon/2vc^3\sin\frac{1}{2}\epsilon$, and $ma^2\cos^2\frac{1}{2}\epsilon/vc^3$ respectively, we find that the component of the impulse perpendicular to the bisector of the angle between the directions of motion vanishes, as it did before the collision; hence we see that the force resultant of the impulse is not altered by the collision, a result which we know is true.

We pass on to consider the terms α_2 and β_2. We know that

$\frac{d\alpha_2}{dt}=$ coefficient of $\cos 2\theta$ in the expression for the velocity along the radius vector of the vortex ring C D.

Now the vortex ring C D itself, as we see from equation (17), contributes to the expression for the velocity along its radius vector the term

$$-\cos 2\theta.\frac{2m'}{a^2}\log\frac{2a}{c}.\beta_2$$

The vortex ring A B contributes as we see from equation (29) the term

$$-\cos 2\theta.\tfrac{3}{4}\frac{m'a'^2a}{(c^2+4v^2\sin^2\frac{1}{2}\epsilon.t^2)^{\frac{7}{2}}}(\mathrm{F}'t^3+\mathrm{G}'t^2+\mathrm{H}'t+\mathrm{K}')$$

say

$$\cos 2\theta f(t)$$

Thus

$$\frac{d\alpha_2}{dt}=-\frac{2m}{a^2}\log\frac{2a}{c}.\beta_2+f(t)$$

Now

$\frac{d\beta_2}{dt}=$ the coefficient of $\cos 2\theta$ in the expression for the velocity perpendicular to the plane of the vortex C D.

The vortex C D itself contributes to this coefficient the term

$$\tfrac{3}{2}\frac{m}{a^2}\log\frac{2a}{c}.\alpha_2$$

The vortex A B contributes the term

$$\tfrac{3}{2}\frac{m'a^2a'^2}{(c^2+4v^2\sin^2\tfrac{1}{2}\epsilon.t^2)^{\frac{9}{2}}}(\mathrm{A}'t^4+\mathrm{B}'t^3+\mathrm{C}'t^2+\mathrm{D}'t+\mathrm{E}')$$

say

$$\mathrm{F}(t)$$

Thus

$$\frac{d\beta_2}{dt}=\tfrac{3}{2}\frac{m}{a^2}\log\frac{2a}{e}\alpha_2+\mathrm{F}(t)$$

Eliminating β_2 we find

$$\frac{d^2\alpha_2}{dt^2}+\frac{3m^2}{a^4}\left(\log\frac{2a}{e}\right)^2\alpha_2=f'(t)-\frac{2m}{a^2}\log\frac{2a}{e}.\mathrm{F}(t)$$

say

$$=\chi(t)$$

or writing n^2 for

$$\frac{3m^2}{a^4}\left(\log\frac{2a}{e}\right)^2$$

the equation takes the form

$$\frac{d^2\alpha_2}{dt^2}+n^2\alpha_2=\chi(t)$$

The solution of this differential equation is

$$\alpha_2=\mathrm{A}\cos nt+\mathrm{B}\sin nt+\frac{\cos nt}{n}\int^t\chi(t')\sin nt'dt'$$
$$-\frac{\sin nt}{n}\int^t\chi(t')\cos nt'dt'$$

or choosing the arbitrary constants so that α_2 and $\frac{d\alpha_2}{dt}$ both vanish when $t=-\infty$ we find

$$\alpha_2=\frac{\cos nt}{n}\int_{-\infty}^t\chi(t')\sin nt'dt'-\frac{\sin nt}{n}\int_{-\infty}^t\chi(t')\cos nt'dt'$$

The complete value of $\chi(t)$ is given by the equation

$$\chi(t)=\tfrac{3}{4}\frac{m'a'^2(3\mathrm{F}'t^2+2\mathrm{G}'t+\mathrm{H}')}{(c^2+4v^2\sin^2\tfrac{1}{2}\epsilon.t^2)^{\frac{7}{2}}}$$
$$-\frac{21m'a'^2av^2\sin^2\tfrac{1}{2}\epsilon(\mathrm{F}'t^4+\mathrm{G}'t^3+\mathrm{H}'t^2+\mathrm{K}'t)}{(c^2+4v^2\sin^2\tfrac{1}{2}\epsilon.t^2)^{\frac{9}{2}}}$$
$$-\frac{2m}{a^2}\log\frac{2a}{e}.\tfrac{3}{2}\frac{m'a'^2a^2(\mathrm{A}'t^4+\mathrm{B}'t^3+\mathrm{C}'t^2+\mathrm{D}'t+\mathrm{E}')}{(c^2+4v^2\sin^2\tfrac{1}{2}\epsilon.t^2)^{\frac{9}{2}}}$$

Thus the coefficients of $\cos nt$ and $\sin nt$ in this expression for α_2 will involve integrals of the type

$$\int_{-\infty}^{t} \frac{\cos nt dt}{(a^2+t^2)^{\frac{2P+1}{2}}}$$

I have not succeeded in evaluating this integral; it is evident however that the more important part of these integrals will be produced during the time the vortices are nearly at their minimum distance apart. During the time they are far apart they will not contribute anything appreciable to this integral, so that soon after the vortex rings have passed their minimum distance the equation may without sensible error be written

$$\alpha_2 = \mathrm{P}\frac{\cos nt}{n} - \mathrm{Q}\frac{\sin nt}{n}$$

where P and Q are constants and

$$\mathrm{P} = \int_{-\infty}^{+\infty} \chi(t) \cos nt.dt$$

$$\mathrm{Q} = \int_{-\infty}^{+\infty} \chi(t) \sin nt.dt$$

There will be a similar expression for β_2. Thus the vortex rings are thrown by the collision into a state of vibration about their circular form.

We can find the action of two unequal vortices on each other by means of work of a very similar character to that just given. The only difference is that instead of the former values for f and h we must substitute the values

$$f = -c \sin\alpha - v \sin\epsilon.t$$
$$h = c\cos\alpha + (v\cos\epsilon - w)t$$

where v is the velocity of the vortex which is in front when they are nearest together, w the velocity of the one in the rear; α is the angle between the line joining their centres when they are nearest together, and the direction of motion of the vortex in the rear, β is the angle between this line and the direction of motion of the vortex in front, ϵ is the angle between the direction of motion of the vortices; α and β are given by the equations

$$w\cos\alpha = v\cos\beta$$
$$\alpha + \beta = \epsilon$$

I shall not trouble the reader with the expressions for the velocities perpendicular to the plane of either vortex and along the radius vector, but confine myself to quoting the most important consequences to be got from these expressions.

I find that after the collision the direction of motion of C D (the vortex which is in front when they are nearest together) is deflected towards the direction of motion of the other vortex A B through an angle whose circular measure is

$$\frac{m'a'^2 \cos \alpha \sin 2\beta}{\kappa c^2} = \frac{2m'a'^2 \sin^2 \epsilon vw(v - w \cos \epsilon)}{\kappa^4 c^3}$$

where m', m are the strengths of the vortices A B and C D respectively, and a' and a their radii, κ is the relative velocity of the two vortices, viz.:

$$(v^2 + w^2 - 2vw \cos \epsilon)^{\frac{1}{2}}$$

The direction of motion of A B is deflected from that of C D through an angle whose circular measure is

$$\frac{ma^2 \cos \beta \sin 2\alpha}{\kappa c^3} = \frac{2ma^2 \sin^2 \epsilon vw(w - v \cos \epsilon)}{\kappa^4 c^3}$$

The radius of the vortex C D is increased by

$$\frac{m'a'^2 a \cos \alpha \cos^2 \beta}{\kappa c^3} = \frac{m'a'^2 a \sin^3 \epsilon vw^2}{\kappa^4 c^3}$$

The radius of the vortex A B is diminished by

$$\frac{ma^2 a' \cos \beta \cos^2 \alpha}{\kappa c^3} = \frac{ma^2 a' \sin^3 \epsilon v^2 w}{\kappa^4 c^3}$$

The velocity of the vortex C D is diminished by

$$\frac{mm'}{2\pi a} \frac{\cos \alpha \cos {}^2\beta}{\kappa c^3} a'^2 \log \frac{2a}{e}$$

$$= \frac{mm'a'^2 \sin^3 \epsilon vw^2}{2\pi a \kappa^4 c^3} \log \frac{2a}{e}$$

The velocity of the vortex A B is increased by

$$\frac{mm'a^2 \sin^3 \epsilon wv^2}{2\pi a' \kappa^4 c^3} \log \frac{2a'}{e'}$$

where e, e' are the radii of the cross sections of the vortices C D, A B respectively.

The kinetic energy of the vortex C D is increased by

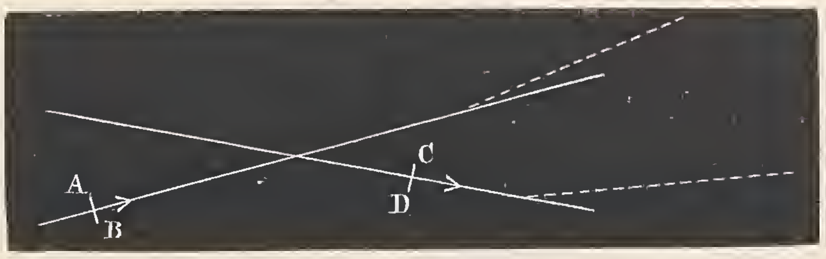
$$\frac{2\pi \rho mm'a'^2 a^2 v \cos \alpha \cos^2 \beta}{\kappa c^3} = \frac{2\pi \rho mm'a'^2 a^2 \sin^3 \epsilon . v^2 w^2}{\kappa^4 c^3}$$

The kinetic energy of A B is diminished by the same amount.

With the help of these results we may find the way in which two vortices affect each other in all cases.

Case I.—Vortices moving in the same direction.

Fig 4.



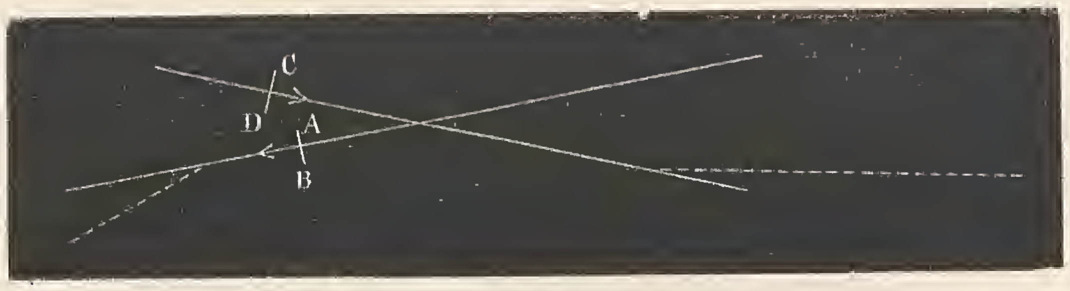
1. When the positions of the vortices when nearest together are as represented in fig. 4.

The way in which their paths are deflected is indicated by the dotted lines.

The vortex C D increases in radius and energy and its velocity is decreased.

Case II.—Vortices moving in opposite directions.

Fig 5.



The position of the vortices when nearest together is represented in fig. 5.

The way their paths are deflected is indicated by the dotted lines in the figure.

The vortex C D decreases in radius and energy and its velocity is increased.

The vortex A B increases in radius and energy and its velocity is decreased.

These results may be summed up in the following rule. The vortex which first passes through the point of intersection of the directions of motion of the vortices is deflected towards the direction of motion of the other: it increases in radius and energy and its velocity is decreased; the other vortex is deflected in the same direction: it decreases in radius and energy and its velocity is increased.

(Note added March 21st, 1882.)

Sir William Thomson has pointed out to me that when n is very great the time of vibration of the single vortex which for this case is (equation 23)

$$2\pi \Big/ \frac{\varpi 2\pi^2 e^2}{l^2} \log \frac{nl}{\pi e}$$

when l is the wave-length $2\pi a/n$, does not agree infinitely nearly as it ought with the value obtained by him for the rapidity of the transverse vibrations of a straight columnar vortex, which by formula 61 of his paper "On the Vibrations of a Columnar Vortex," is

$$2\pi\Big/\frac{\omega 2\pi^2 e^2}{l^2}\left\{\log\frac{l}{2\pi e}+\cdot 1159\right\}$$

The results would agree approximately if $l/2e$ were indefinitely great compared with n, but it obliges us to neglect the factor $\log n$, and thus the agreement instead of getting better as it ought gets worse as n increases. The way I determined A_n is only suitable when $n\kappa_{\prime}$ is small, as it is only allowable to expand $\cos n\phi$ as $1-\frac{n^2\phi^2}{2}$ when $n\phi$ remains small within the limits of integration. I have therefore endeavoured to determine A_n in a way which shall not be open to these objections.

With the notation of the paper since

$$A_n=\frac{1}{\pi}\int_0^{2\pi}\frac{\cos n\phi . d\phi}{(a^2+\rho^2+\zeta^2-2a\rho\cos\phi)^{\frac{3}{2}}}$$

we may easily prove that if $a-\rho$ be small compared with a, that

$$a\left(\frac{dA_{n-1}}{da}-\frac{dA_n}{da}\right)=(n-1)A_{n-1}-(n-3)A_n$$

and

$$a\left(\frac{dA_{n+1}}{da}-\frac{dA_n}{da}\right)=(n+3)A_n-(n+1)A_{n+1}$$

If we make these substitutions we find from equations (10) and (12)

$$\frac{d\alpha_n}{dt}=\tfrac{1}{2}ma\beta_n\{\mathfrak{A}_1+\tfrac{1}{2}((n-1)\mathfrak{A}_{n+1}-(n+1)\mathfrak{A}_{n-1})\}$$

$$\frac{d\beta_n}{dt}=\tfrac{1}{2}ma\alpha_n\{\mathfrak{A}_1-\mathfrak{A}_n\}$$

Hence we have only to find an expression for $\mathfrak{A}_n$

$$\mathfrak{A}_n=\frac{1}{\pi((a+\rho)^2+\zeta^2)^{\frac{3}{2}}}\int_0^{2\pi}\frac{\cos n\phi}{\left(\kappa_1^2+\kappa^2\sin^2\frac{\phi}{2}\right)^{\frac{3}{2}}}$$

Now as κ_1^2 is very small this integral will be very large, and as the large part arises when ϕ is small or nearly 2π, we may write as the approximate value of $\mathfrak{A}_n$

$$\frac{2}{\pi\{(a+\rho)^2+\zeta^2\}^{\frac{3}{2}}}\int_0^{\infty}\frac{\cos n\phi d\phi}{\left(\kappa_1^2+\frac{\phi^2}{4}\right)^{\frac{3}{2}}}$$

$$=\frac{4}{\pi\{(a+\rho)^2+\zeta^2\}^{\frac{3}{2}}}\int_0^{\infty}\frac{\cos 2n\phi . d\phi}{(\kappa_1^2+\phi^2)^{\frac{3}{2}}}$$

Let us consider the more general integral

$$u=\int_0^\infty \frac{\cos s\phi d\phi}{(a^2+\phi^2)^{\frac{2p+1}{2}}}$$

Considered as a function of s it is easy to prove that the integral satisfies the differential equation

$$\frac{d^2u}{ds^2}-\frac{2p-1}{2s}\frac{du}{ds}-a^2u=0$$

or if $u=s^pv$

$$\frac{d^2v}{ds^2}+\frac{1}{s}\frac{dv}{ds}-v\left(\frac{p^2}{s^2}+a^2\right)=0$$

The solution of this differential equation may be expressed in terms of BESSEL'S functions of the first and second kinds; we shall, however, only solve here the special case which we require.

It is easily proved that

$$\int_0^\infty \frac{\cos s\phi}{(a^2+\phi^2)^{\frac{3}{2}}}d\phi=-\frac{s}{a^2}\frac{d}{ds}\int_0^\infty \frac{\cos s\phi}{(a^2+\phi^2)^{\frac{1}{2}}}d\phi$$

Since $p=0$ for $\int_0^\infty \frac{\cos s\phi}{(a^2+\phi^2)^{\frac{1}{2}}}d\phi$ it satisfies the differential equation

$$\frac{d^2u}{ds^2}+\frac{1}{s}\frac{du}{ds}-a^2u=0$$

This is the equation solved by Professor STOKES in his paper "On the effect of Internal Friction on the Motion of Pendulums," Camb. Phil. Trans., 1850, and quoted by Sir WILLIAM THOMSON, Phil. Mag., September, 1880.

The solution is there shown to be

$$u=\left(\mathrm{E}+\mathrm{D}\log\frac{1}{as}\right)\left(1+\frac{a^2s^2}{2^2}+\frac{a^4s^4}{2^24^2}+\dots\right)$$

$$+\mathrm{D}\left(\frac{a^2s^2}{2^2}\mathrm{S}_1+\frac{a^4s^4}{2^24^2}\mathrm{S}_2+\dots\right)$$

where

$$\mathrm{S}_n=1^{-1}+2^{-1}+\dots n^{-1}$$

and if as for $\int_0^\infty \frac{\cos s\phi}{(a^2+\phi^2)^{\frac{1}{2}}}d\phi$, $u=0$ when s is infinite E/D is shown to be

$$\log 8+\pi^{-\frac{1}{2}}\Gamma'\tfrac{1}{2}=\cdot 11593$$

so that

$$\int_0^\infty \frac{\cos s\phi}{(a^2+\phi^2)^{\frac{1}{2}}}d\phi = \mathrm{D}\Big[\Big(\cdot 11593 + \log\frac{1}{as}\Big)\Big(1+\frac{a^2s^2}{2^2}+\frac{a^4s^4}{2^24^2}+\dots$$

$$+\Big(\frac{a^2s^2}{2^2}\mathrm{S}_1+\frac{a^4s^4}{2^24^2}\mathrm{S}_2+\dots\Big)$$

Therefore

$$\int_0^\infty \frac{\cos s\phi}{(a^2+\phi^2)^{\frac{3}{2}}}d\phi = -\frac{s}{a^2}\mathrm{D}\Big[\Big(\cdot 11593 + \log\frac{1}{as}\Big)\Big(\frac{a^2s}{2}+\frac{a^4s^3}{2^24}+\dots\Big)$$

$$+\frac{1}{s}\Big(1+\frac{a^2s^2}{2^2}+\frac{a^4s^4}{2^24^2}+\dots\Big)$$

$$+\Big(\frac{a^2s^2}{2}\mathrm{S}_1+\frac{a^4s^3}{2^24}\mathrm{S}_2+\dots\Big)$$

Now since

$$\int_0^\infty \frac{d\phi}{(a^2+\phi^2)^{\frac{1}{2}}}=\frac{1}{a^2}$$

putting $s=0$ we find $\mathrm{D}=-1$.

Therefore

$$\int_0^\infty \frac{\cos s\phi . d\phi}{(a^2+\phi^2)^{\frac{3}{2}}}=\frac{1}{a^2}\Big\{1+\frac{a^2s^2}{2^2}+\frac{a^4s^4}{2^24^2}+\dots\Big\}$$

$$+\Big(\cdot 11593 + \log\frac{1}{as}\Big)\Big(\frac{s^2}{2}+\frac{a^2s^4}{2^24}+\dots\Big)$$

$$+\Big(\frac{s^2}{2}\mathrm{S}_1+\frac{a^2s^4}{2^24}\mathrm{S}_2+\dots\Big)$$

Now approximately

$$\mathfrak{A}_n=\frac{1}{2\pi a^3}\int_0^\infty\frac{\cos 2n\phi d\phi}{({\kappa_1}^2+\phi^2)^{\frac{3}{2}}}$$

hence A_n is found, and if we substitute these values in the expressions for $\frac{d\alpha_n}{dt}$, $\frac{d\beta_n}{dt}$ we shall be able to find the time of vibration in any particular case.

If we suppose $n\kappa_1$ is small, then approximately

$$\mathfrak{A}_n=\frac{1}{2\pi a^3}\Big\{\frac{1}{{\kappa_1}^2}+n^2\Big\{2\Big(\log\frac{1}{2n{\kappa_1}^2}+\cdot 11593\Big)+1\Big\}\Big\}$$

or if we neglect 1.23 in comparison with $2\log\frac{1}{2n\kappa_1}$

$$\mathfrak{A}_n=\frac{1}{2\pi a^3}\Big\{\frac{1}{\kappa^1}+2n^2\log\frac{1}{2n\kappa_1}\Big\}$$

Now

$$\frac{d\alpha_n}{dt}=\tfrac{1}{2}ma\beta_n\{\mathfrak{A}_1+\tfrac{1}{2}\}(n-1)\mathfrak{A}_{n+1}-(n+1)\mathfrak{A}_{n-1})$$

$$\frac{d\beta_n}{dt}=\tfrac{1}{2}ma\alpha_n\{\mathfrak{A}_1-\mathfrak{A}_n\}$$

Substituting we find

$$\frac{d\alpha_n}{dt}=-\frac{m\beta_n}{2\pi a^2}(n^2\log 2\kappa_1+(n^2-1)(n+1)\log n+1-(n-1)\log n-1)$$

$$=-\frac{m}{2\pi a^2}f\beta_n \quad \text{say}$$

$$\frac{d\beta_n}{dt}=\frac{m\alpha_n}{2\pi a^2}(n^2\log 2n\kappa_1-\log 2\kappa_1)$$

$$=\frac{m}{2\pi a^2}g\alpha_n \quad \text{say}$$

Thus

$$\frac{d^2\beta_n}{dt}+\left(\frac{m}{2\pi a^2}\right)^2 fg\beta_n=0$$

or the time of vibration

$$=2\pi\Big/\frac{\omega e^2}{2a^2}\sqrt{fg}$$

Now if n be not very large

$$f=n^2\log 2\kappa_1=-n^2\log\frac{a}{e}$$

$$g=(n^2-1)\log 2\kappa_1=-(n^2-1)\log\frac{a}{e}$$

and the time of vibration

$$=2\pi\Big/\frac{\omega e^2}{2a^2}n\sqrt{n^2-1}\log\frac{a}{e}$$

If n be large

$$f=n^2\log 2\kappa_1+n^2\log n=-n^2\log\frac{a}{ne}$$

$$g=n^2\log 2n\kappa_1=-n^2\log\frac{a}{ne}$$

and the time of vibration

$$=2\pi\Big/\frac{\omega e^2}{a^2}n^2\log\frac{a}{ne}$$

or if l be the wave length $=2\pi a/n$

$$=2\pi\Big/\frac{2\pi^2\omega e^2}{l^2}\log\frac{l}{2\pi e}$$

which agrees approximately with Sir WILLIAM THOMSON'S result.

X. THE BAKERIAN LECTURE.—*Chemical Theory of Gunpowder.*

By H. DEBUS, *Ph.D., F.R.S.*

Received February 8,—Read February 23, 1882.

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ACCORDING to BELLANI, the English army used cannon at the Battle of Crecy in the year 1346. The correctness of this report has been doubted since English and French writers in their description of the battle do not mention the use of cannon. However this may be, it is certain, that from the middle of the 14th century the application of powder to the purposes of the art of war became more and more general, until towards the close of the Middle Ages heavy ordnance was used by all European armies. The effect of this new application of gunpowder upon the civilisation of our race is usually considered to have been of the same importance as the invention of the art of printing or the discovery of America. And, although 536 years have passed away since the Battle of Crecy was fought, we have to this day no satisfactory account of the chemical reactions which occur during the combustion of gunpowder, no theory to enable us to determine the quantitative relations of the products of combustion *à priori* from the composition of the powder. The attempts, which have been made from time to time by eminent men to supply solutions of the problems indicated, have been, as is well known, unsuccessful.

In the following pages I propose to describe a theory which explains in a satisfactory manner the chemical reactions which occur during and after the explosion, not only of a powder of normal composition, but, generally, of a mixture of x molecules of saltpetre, y atoms of carbon, and z atoms of sulphur.

Until about the year 1856 the metamorphosis of gunpowder was represented in chemical text-books to take place according to the equation

$$2KNO_3+3C+S=K_2S+N_2+3CO_2$$

This equation is obviously not correct, because besides sulphide, also sulphate and carbonate of potassium are formed.

After the publication of BUNSEN'S and SCHISCHKOFF'S[*] classical investigation in 1857, the incorrectness of the equation was generally recognised, and the view expressed that the explosion of gunpowder could not be represented by a chemical equation on account of its complex nature. Passing by for the present the papers published between the years 1858 and 1874, I propose to take at once into consideration the most recent and important investigation by NOBLE and ABEL;[†] I do so because we receive from the pages of their papers a very complete account of our present knowledge of the combustion of gunpowder.

Five different descriptions of powder were used in their experiments.

1. Pebble powder (P.); 2. Rifle large grain (R. L. G.); 3. Rifle fine grain (R. F. G.); 4. Fine grain (F. G.); and 5. Spanish pebble powder.

The first four descriptions were manufactured at Waltham Abbey.

It will be convenient for the purposes of reference to give in the following table the composition of these powders.

TABLE I.—Showing the composition of the powders used by NOBLE and ABEL.

Constituents of powder.		Powders from Waltham Abbey.								Spanish.	
		P.		R. L. G.		R. F. G.		F. G.			
Saltpetre			74·67		74·95		75·04		73·55		75·30
Potassic sulphate .			0·09		0·15		0·14		0·36		0·27
Potassic chloride .			. .		. .		. .		. .		0·02
Sulphur			10·07		10·27		9·93		10·02		12·42
Charcoal	Carbon . . .	12·12	14·22	10·86	13·52	10·67	14·09	11·36	14·59	8·65	11·34
	Hydrogen . .	0·42		0·42		0·52		0·49		0·38	
	Oxygen . . .	1·45		1·99		2·66		2·57		1·68	
	Ash	0·23		0·25		0·24		0·17		0·63	
Water			0·95		1·11		0·80		1·48		0·65

NOBLE and ABEL burnt from 100 to 700 grms. of powder in hermetically closed steel cylinders.[‡] The analyses of the products of combustion obtained in 31 experiments have been published, and of these it will be desirable to reproduce a few representative cases in Table II.

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[*] Pogg. Ann., Bd. cii., p. 321.

[†] Phil. Trans., Vol. 165 (1875), p. 49, Vol. 171 (1880), p. 203.

[‡] Ibid., Vol. 165, p. 61.

TABLE II.—Containing the results of nine experiments calculated for 1 grm. of powder.

Products of combustion.	F. G.			R. L. G.			P.		
K_2CO_3	0·2429	·2615	·3255	·3007	·3017	·3635	·2879	·3098	·3680
$K_2S_2O_3$	·1851	·1666	·0780	·1166	·0740	·0369	·1845	·0338	·0761
K_2SO_4	·1288	·1268	·1204	·1171	·1395	·0625	·0733	·0658	·0523
K_2S	·0000	·0196	·0252	·0230	·0337	·0565	·0128	·1055	·0220
KCNS	·0009	·0004	·0004	·0000	·0003	·0015	·0022	·0013	·0033
KNO_3	·0010	·0005	·0005	·0032	·0002	·0000	·0014	·0011	·0025
$(NH_4)_4H_2(CO_3)_3$.	·0186	·0002	·0005	·0003	·0002	·0006	·0003	·0004	·0007
S	·0026	·0068	·0306	·0041	·0262	·0480	·0111	·0340	·0484
H_2S	·0090	·0080	·0088	·0041	·0127	·0067	0129	·0084	·0086
CO	·0316	·0339	·0343	·0303	·0390	·0472	·0419	·0473	·0362
CO_2	·2689	·2678	·2650	·2597	·2610	·2677	·2630	·2770	·2710
CH_4	·0003	·0000	·0005	·0006	·0000	·0007	·0007	·0012	·0013
H	·0006	·0008	·0007	·0005	·0007	·0005	·0005	·0005	·0005
N	·1096	·1071	·1096	·1201	·1108	·1077	·1075	·1139	·1090
No. of experiment	XL.	XIX.	LXXV.	I.	IV.	XXXIX.	XXXVIII.	XIV.	LXXVII.

From this table it is clear that not only powders of different description, but also mixtures of the same nature, will yield during combustion the products in variable quantities. The salts of potassium, especially the hyposulphite and sulphide, vary considerably in different experiments.

NOBLE and ABEL draw the following conclusions from the results of their investigations.

1. "The variations in the composition of the products of explosion furnished in close chambers by one and the same powder, under different conditions as regards pressure, and by two powders of similar composition under the same conditions as regards pressure, are so considerable, that no value whatever can be attached to any attempt to give a general chemical expression to the metamorphosis of a gunpowder of normal composition (p. 137).

"Any attempt to express, even in a comparatively complicated chemical equation, the nature of the metamorphosis which a gunpowder of average composition may be considered to undergo when exploded in a confined space, would therefore only be calculated to convey an erroneous impression as to the simplicity or the definite nature of the chemical results and their uniformity under different conditions, while it would, in reality, possess no important bearing upon an elucidation of the theory of explosion of gunpowder (p. 85).

2. "The proportions in which the several constituents of solid powder-residue are formed, are quite as much affected by slight accidental variations in the conditions which attend the explosion of one and the same powder in different experiments, as

by decided differences in the composition as well as in the size of grain of different powders (p. 137).

3. "Very small grain powder, such as F. G. and R. F. G. furnish decidedly smaller proportions of gaseous products than a large grain powder (R. L. G.), while the latter again furnishes somewhat smaller proportions than a still larger powder (pebble), though the difference between the gaseous products of these two powders is comparatively inconsiderable.

4. "In all but very exceptional results, the solid residue furnished by the explosion of gunpowder contains as important constituents, potassium carbonate, sulphate, hyposulphite and sulphide, the proportion of carbonate being very much higher, and that of sulphate very much lower than stated by recent investigators."

The view of NOBLE and ABEL may be briefly stated as follows :—

One and the same description of powder, exploded several times in succession, will yield the products of combustion, in the different experiments, in variable proportions; hence: the metamorphosis of gunpowder cannot be represented by a chemical equation.

One might suppose that, perhaps, the pressure developed during explosion had an influence on the quantities of the products of combustion. From a comparison of the analytical results and the corresponding pressures, published by NOBLE and ABEL, this, however, appears not to be the case.[*] (See p. 85 of their first memoir.)

According to NOBLE and ABEL, the chemical metamorphosis of gunpowder during explosion is a very complicated process, which cannot be explained with the data at their disposal. BERTHELOT[†] arrived at a different conclusion.

The composition of the powders of Waltham Abbey can, according to him, be represented by the symbols

$$2KNO_3+3C+S$$

which require for 100 parts of powder :

Saltpetre	74·8
Carbon	13·3
Sulphur	11·8

The analyses gave :

Saltpetre	73·55	to	75·04
Carbon	10·67		12·12
Sulphur	9·93		10·27

---

[*] An increase of pressure appears to diminish the amount of carbonic oxide. But this is not always the case, and when it does occur, it is not sufficient to explain the variations in the other products of combustion.

[†] Comptes Rendus, tom. lxxxii., p. 487.

The combined weights of potassic sulphocyanate, ammonic carbonate, hydrogen, marsh gas, and sulphuretted hydrogen amount, according to Table II., to about 1·5 per cent. They evidently originate from secondary reactions, and may, accordingly, be neglected in the following considerations.

A theory of the explosion of gunpowder ought to explain the formation of potassic carbonate, potassic sulphate, potassic sulphide, carbonic acid and carbonic oxide. Potassic hyposulphite is not a primary product, but is formed during the analysis of the powder residue.

If we select two from several experiments published by NOBLE and ABEL, viz. :—one in which the maximum amount of potassic carbonate and the minimum of sulphate were produced, and another which yielded the largest quantity of potassic sulphate and the smallest of carbonate, then, according to M. BERTHELOT, the explosion which produced the results of the first case, may be represented by three equations—

$\frac{1}{3}$ of the powder was transformed according to equation

$$2KNO_3+3C+S=K_2S+3CO_2+N_2, \quad \ldots \ldots \quad \text{I.}$$

$\frac{1}{2}$ according to

$$2KNO_3+3C+S=K_2CO_3+CO_2+CO+N_2+S, \quad \ldots \ldots \quad \text{II.}$$

$\frac{1}{6}$ according to

$$2KNO_3+3C+S=K_2CO_3+1{\cdot}5CO_2+0{\cdot}5C+S+N_2 \quad \ldots \ldots \quad \text{III.}$$

and in the second case, with a maximum of potassic sulphate,

$\frac{1}{3}$ of the powder was transformed according to equation I,
about $\frac{1}{2}$ according to III,
$\frac{1}{8}$ according to

$$2KNO_3+3C+S=K_2SO_4+2CO+C+N_2, \quad \ldots \ldots \quad \text{IV.}$$

and $\frac{1}{12}$ according to

$$2KNO_3+3C+S=K_2SO_4+CO_2+C_2+N_2 \quad \ldots \ldots \quad \text{V.}$$

Between the limits, marked by these two cases, are contained all the experimental results of NOBLE and ABEL. If, therefore, we assume that in a given experiment one portion of the powder used burnt according to the equations of the first, and the rest according to those of the second case, the calculated results will agree with the observations. And if the proportions of powder, which are transformed according to the one or other system of equations, be changed from experiment to experiment, the quantities of the products of combustion obtained in each experiment can be calculated in a satisfactory manner.

The assumption that during explosion one portion of the powder is transformed according to one and another portion according to another equation or system of equations is justified in the opinion of M. BERTHELOT by the further assumption, that the local conditions in a mass of burning powder are not the same in all parts, and that the cooling is too rapid to allow the products to assume a state of chemical equilibrium.

If the products were left in contact at a high temperature for a longer time, they would react upon each other, and the final result would be the same as that represented by equation V., to which corresponds the greatest evolution of heat.

This theory of M. Berthelot is very ingenious, but does not agree with experience. Considerable amounts of carbon ought to be left free at the end of each explosion. In 28 experiments of Noble and Abel no free carbon was left, and only in three cases small insignificant quantities had escaped combustion. The equations III., IV. and V. cannot be applied to the combustion of the powders of Waltham Abbey. But even if the theory were correct, it would possess no practical value, because the quantities of the powder which would burn according to each of the two systems of equations, could not be known *à priori*, but would have to be found by experiment.

Berthelot invented his theory in order to explain the remarkable result of Noble and Abel's experiments, that the same description of powder, or powders of similar composition, yield the products of explosion, in different experiments, in variable proportions.

We will now proceed to show that this result can be explained, without hypothesis or theory, in a very simple manner.

For this purpose it is desirable to express the analytical results of Noble and Abel in a manner different from the one adopted by these investigators.* If we divide the numbers of Table II. by the corresponding molecular weights, we obtain another table expressing the number of molecules of the products obtained in the different experiments by the explosion of 1 grm. of powder. For Experiment XIX. we have :—

K_2CO_3	0·2615	gramme or	0·00189	mol.
K_2SO_4	0·1268	,,	0·00072	,,
$K_2S_2O_3$	0·1666	,,	0·00087	,,
K_2S_2	0·0252	,,	0·00017	,,
S	0·0012	,,	0·00004	atom.
CO_2	0·2678	,,	0·00608	mol.
CO	0·0339	,,	0·00121	,,
N	0·1071	,,	0·00765	atom.
H_2S	0·0080	,,	0·00023	mol.
CH_4	0·0000	,,	0·00000	,,
H	0·0008	,,	0·00080	atom.
KCNS	0·0004	,,		
KNO_3	0·0005	,,		
$Am_4H_2(CO_3)_3$	0·0002	,,		

Potassic carbonate, sulphate, sulphide, and hyposulphite, carbonic acid and carbonic

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[*] Noble and Abel discuss only the percentage quantities of the products of explosion.

oxide, nitrogen and sulphuretted hydrogen together form more than 98 per cent. of the exploded powder; accordingly, hydrogen, marsh gas, ammonic carbonate, potassic sulphocyanate and undecomposed saltpetre may, as non-essential products, be left out of consideration.

Potassic carbonate, sulphate, hyposulphite, and sulphide contain very nearly the entire amount of potassium of the exploded powder. If, therefore, we add the number of molecules of these bodies and multiply the sum by two, we obtain the number of molecules of saltpetre in 1 grm. of powder. In order to compare the results of experiments made with the same or with different descriptions of gunpowder it is desirable to calculate these results, not, as is usually done, for the same quantity of powder, but for such quantities as contain equal amounts of saltpetre or oxygen.

The products of Experiment XIX. contain 0·00730 atom of potassium and are derived from a weight of powder containing 0·0073 mol. of saltpetre. The same products contain 0·00189 mol. of K_2CO_3, hence:

$$0{\cdot}00730 : 0{\cdot}00189 :: 16 : x \text{ and } x = 4{\cdot}14.$$

And if the same mode of calculation is extended to the other products, we obtain

$$\begin{aligned} &4{\cdot}14K_2CO_3 + 1{\cdot}58K_2SO_4 + 13{\cdot}3CO_2 + 8{\cdot}38N_2 \\ &\quad + 1{\cdot}90K_2S_2O_3 + 2{\cdot}65CO + 0{\cdot}08S \\ &\quad + 0{\cdot}37K_2S_2 \qquad\qquad + 0{\cdot}5H_2S \end{aligned}$$

and, from these numbers, the following composition of the F. G. powder,

$$\begin{aligned} &16KNO_3 + 20{\cdot}09C + 6{\cdot}70S \\ &+ 0{\cdot}76N + 1{\cdot}0H \\ &\qquad\quad + 5{\cdot}69O. \end{aligned}$$

By a similar calculation we obtain:

Experiment LXXV.

$$\left.\begin{array}{l} 5{\cdot}10K_2CO_3 + 13{\cdot}08CO_2 + 0{\cdot}56H_2S \\ 1{\cdot}50K_2SO_4 + 2{\cdot}65CO \\ 0{\cdot}89K_2S_2O_3 + 8{\cdot}5N_2 \\ 0{\cdot}48K_2S_2 + 1{\cdot}58S \end{array}\right\} = \left\{\begin{array}{l} 16KNO_3 + 20{\cdot}83C \\ 0{\cdot}5N_2 + 6{\cdot}38S \\ 4{\cdot}78O \\ 1{\cdot}12H \end{array}\right.$$

Experiment IV.

$$\left.\begin{array}{l} 4{\cdot}75K_2CO_3 + 12{\cdot}92CO_2 + 0{\cdot}80H_2S \\ 1{\cdot}74K_2SO_4 + 3{\cdot}03CO \\ 0{\cdot}85K_2S_2O_3 + 8{\cdot}6N_2 + 1{\cdot}11S \\ 0{\cdot}65K_2S_2 \end{array}\right\} = \left\{\begin{array}{l} 16KNO_3 + 20{\cdot}7C \\ 6{\cdot}65S + 0{\cdot}6N_2 \\ + 1{\cdot}60H \\ + 4{\cdot}63O \end{array}\right.$$

Experiment XXXIX.

$$\left.\begin{array}{l} 5{\cdot}70K_2CO_3+13{\cdot}18CO_2+0{\cdot}41H_2S \\ +0{\cdot}78K_2SO_4+\ 3{\cdot}64CO \\ +0{\cdot}41K_2S_2O_3+\ 8{\cdot}3N_2\quad +2{\cdot}14S \\ +1{\cdot}10K_2S_2 \end{array}\right\}=\left\{\begin{array}{l} 16KNO_3+22{\cdot}52C \\ +6{\cdot}35S+0{\cdot}3N_2 \\ +0{\cdot}82H \\ +3{\cdot}45O \end{array}\right.$$

Experiment XIV.

$$\left.\begin{array}{l} 4{\cdot}77K_2CO_3+13{\cdot}44CO_2+0{\cdot}51H_2S \\ +0{\cdot}79K_2SO_4+\ 3{\cdot}62CO \\ +0{\cdot}36K_2S_2O_3+\ 8{\cdot}67N_2\ +0{\cdot}21S \\ +2{\cdot}04K_2S_2 \end{array}\right\}=\left\{\begin{array}{l} 16KNO_3+21{\cdot}83C \\ +6{\cdot}31S+0{\cdot}67N_2 \\ +1{\cdot}02H \\ +1{\cdot}05O \end{array}\right.$$

Experiment LXXVII.

$$\left.\begin{array}{l} 5{\cdot}95K_2CO_3+13{\cdot}78CO_2+0{\cdot}55H_2S \\ +0{\cdot}67K_2SO_4+\ 2{\cdot}88CO \\ +0{\cdot}89K_2S_2O_3+\ 8{\cdot}7N_2\quad +2{\cdot}93S \\ +0{\cdot}44K_2S_2 \end{array}\right\}=\left\{\begin{array}{l} 16KNO_3+22{\cdot}61C \\ +6{\cdot}81S+\ 0{\cdot}7N_2 \\ +1{\cdot}10H \\ +5{\cdot}64O \end{array}\right.$$

These equations represent only the quantitative relations between the constituents of the powders and the products of explosion, and, accordingly, fractions of atoms and molecules are admissible.

Experiments XIX. and LXXV. were made with F. G., Experiments IV. and XXXIX. with R. L. G., and Experiments XIV. and LXXVII. with P. powder of Waltham Abbey.

It will be noticed by comparing two experiments, made with the same description of powder, that the composition of the powder deduced from one of the experiments exhibits considerable differences from the composition derived from the other experiment, and neither of them agrees with the composition found by direct analysis.

The composition of the R. L. G. powder may serve as an example.

According to Experiment IV. . . $16KNO_3+20{\cdot}7C+6{\cdot}65S+4{\cdot}63O$
,, ,, Experiment XXXIX. $16KNO_3+22{\cdot}52C+6{\cdot}35S+3{\cdot}45O$
,, ,, Analysis of powder . $16KNO_3+19{\cdot}51C+6{\cdot}92S$

The products of combustion of Experiment IV. contain 1·2, and of Experiment XXXIX. 3 atoms more carbon than the powder used in these experiments, or in other words: the products of explosion in Experiment XXXIX. were found to contain 1·67 per cent. more carbon than the R. L. G. powder which was exploded.

The results of other experiments, made by Noble and Abel, differ from each other in a similar manner.

One is forced to conclude either that the methods of analysis adopted by Noble and Abel do not yield exact results, or that the powders exploded did not possess the composition which was attributed to them.

Gunpowder is a mechanical mixture of saltpetre, charcoal, and sulphur. It can hardly be expected that such a mixture should, even if the greatest care has been taken by the manufacturer, be perfectly homogeneous. Moreover, the burning of wood into charcoal will not always yield a product containing the same percentage amount of carbon, and as gunpowder is a mixture of 75 parts of saltpetre, 10 of sulphur, and 15 of charcoal, it appears *à priori* probable that the same description of powder from the same manufacturer will not always possess the same percentage composition. The amount of carbon, more particularly, may be expected to vary more or less.

In order to test this conclusion by experiment, I requested the late Mr. Wills to analyse a sample of R. L. G. and one of pebble (P.) powder, both obtained from the Royal Arsenal, Woolwich. His results, placed side by side with those of Noble and Abel, are given below.

	R. L. G.		P.	
	Noble and Abel.	Wills.	Noble and Abel.	Wills.
Saltpetre	74·95	75·10	74·67	74·26
Sulphur	10·27	8·96	10·07	9·51
Charcoal—				
Carbon	10·86	12·09	12·12	11·58
Hydrogen	0·42	0·54	0·42	0·51
Oxygen	1·99	2·12	1·45	2·55
Ash	0·25	0·20	0·23	0·33
Water	1·11	0·85	0·95	0·76
	99·85	99·86	99·91	99·50

It will be noticed that the amounts of carbon and sulphur found by Wills differ considerably from those found by Noble and Abel. But the best proof that the same description of powder from the same works may vary much in composition has been furnished by Noble and Abel themselves. In their first memoir they assign to R. L. G. powder the above composition; in their second paper "On Fired Gunpowder" they publish the following analysis:—

Saltpetre	74·43
Sulphur	10·09
Charcoal—	
Carbon	12·40
Hydrogen	0·40
Oxygen	1·27
Ash	0·21
Water	1·05
	99·85

The sample used in the first analysis was taken from the top, the one employed in the second from the bottom of the same barrel. Two analyses of powder out of the same barrel, executed by the same chemists, gave amounts of carbon which differ from each other by no less than 1·54 per cent.! We will now consider the effect of such a difference in the percentage of carbon on the relative quantities of the products of combustion.

If in one experiment 100 grms. of powder containing 75 parts of saltpetre, 10 parts of sulphur, and 10·86 parts of carbon, and in a second experiment 100 grms. of powder with 74·43 parts of saltpetre, 9 parts of sulphur, and 12·40 parts of carbon had been exploded, then, *cæteris paribus*, in the second experiment more potassic carbonate, more sulphide and less sulphate must have formed than in the first, and the quantitative differences of the same products furnished by the two experiments would be almost as great as the greatest differences actually observed by NOBLE and ABEL for the same description of powder in the whole series of their experiments. We arrive in this manner at a very simple explanation of the experimental results upon which NOBLE and ABEL have based the conclusions mentioned under No. 1 and 2 on p. 525, without the necessity of having recourse to a theory like the one advanced by M. BERTHELOT, or renouncing all explanation like NOBLE and ABEL.

It follows from Table I. that the differences of composition between P., R. L. G., R. F. G., and F. G. powders, one compared with the other, are not greater than the differences between two samples of R. L. G. powder taken out of the same barrel. From this it appears probable that only one mixture of saltpetre, charcoal, and sulphur is prepared at Waltham Abbey, and that from this one mixture the powders P., R. L. G., R. F. G., and F. G., differing only in size of grain and, perhaps, density, are manufactured. We may take then for the composition of the said powders the mean of the numbers of all published analyses. Taking into consideration only the saltpetre, sulphur, and carbon, we obtain—

R. L. G. First analysis by Noble and Abel	$16KNO_3+19{\cdot}51C+6{\cdot}92S$
„ Second „ „	$16KNO_3+22{\cdot}40C+6{\cdot}83S$
F. G.	$16KNO_3+20{\cdot}8C+6{\cdot}8S$
P.	$16KNO_3+21{\cdot}86C+6{\cdot}79S$

and for the mean $16KNO_3+21{\cdot}14C+6{\cdot}83S$.

If also the analyses of the late Mr. Wills are taken into consideration, then the mean composition of the powders of Waltham Abbey would be represented by the symbols

$$16KNO_3+21{\cdot}18C+6{\cdot}63S$$

The differences of composition found by analysis for the same description of powder are also of great importance in the calculations of the analytical results of the products of explosion. Noble and Abel determine the potassium in x parts of the solid powder residue, and by means of the number so obtained calculate the weight of the total residue. The amount of potassium in the quantity of powder exploded is known from the analysis of the powder, and as the whole of this potassium must reappear in the solid residue, it is easy to find the total residue if the weight of potassium in x parts of it are known.

The total weight of the powder residue subtracted from the weight of the powder used in the experiment gives as difference the total weight of the gases produced by the explosion.

This mode of calculation requires that the exact composition of the powder used in each experiment should be known. Messrs. Noble and Abel assume the composition of the powders of Waltham Abbey, P., R. F. G., and F. G., to be constant; they also consider in their first memoir that of R. L. G. to be so, but in their second memoir they base the calculations of the later experiments on the second analysis of this description of powder. This assumption is, as a matter of fact, not correct; on the contrary, we may take it as highly probable, that in any two experiments made by Noble and Abel with the same class of powder, the powder used in the one experiment was not exactly of the same composition as the powder used in the other. Accordingly the weight of the solid residue as calculated by Noble and Abel will have been found in some experiments too high, in others too low, and as a consequence of this the total weight of the gaseous products cannot have been exact. The correctness of this is proved by the differences between the composition of the powders calculated from the products of combustion, and the composition deduced from direct analysis (see pp. 530, 531). These errors will, however, compensate each other if we take the mean of the analytical results of all the experiments.

Before we proceed to do so it will be desirable to consider another circumstance which is not without influence on the final results of Messrs. Noble and Abel.

They burnt their powders in steel cylinders in quantities from 100 to 750 grms., so that the solid products of combustion after explosion remained from 60 to 120 seconds in contact in a fluid condition at a high temperature. Amongst these products we have not, as is usually assumed, potassium monosulphide, but disulphide, free sulphur, and potassium carbonate. But a mixture of polysulphides of potassium and potassium carbonate at a bright red or a white heat has a most powerful corroding effect upon metals. It is well known that the celebrated STAHL was of opinion that MOSES had dissolved the golden calf of the Israelites by means of a polysulphide of potassium.

" And he took the calf which they had made, and burnt it in the fire, and ground it to powder and strewed it upon the water, and made the children of Israel drink of it." —(Exod. xxxii. 20.)

Sulphur and ash of wood were known to the Jews, and these, at a high temperature, form liver of sulphur, capable of dissolving gold. The affinity of iron for sulphur is very strong. We may expect that, according to temperature, pressure, time of cooling, and last but not least, the condition of the surface of the cylinder, more or less of the sulphur of the powder would unite with the iron of the apparatus. This would have the same effect upon the products as if a powder with less sulphur had been burnt. If 400 grms. of powder of the average composition had been exploded, and 10 grms. of its sulphur were to unite with the iron of the apparatus, the potassium carbonate produced would be about 0·20 time greater than the amount obtained by the combustion of the same quantity of powder without this removal of sulphur by the iron of the apparatus.

According to the description given by NOBLE and ABEL of the solid powder residue, considerable quantities of ferrous sulphide were contained in it. Hence the variations in the quantities of the products of combustion of powders exploded in NOBLE and ABEL's apparatus will partly be due to the chemical action of iron and sulphur at high temperatures.

It will be observed by an inspection of the equations on pages 529 and 530, that considerably more oxygen was found in the products of combustion than was contained in the saltpetre of the exploded powder. This excess of oxygen cannot have been derived from the charcoal, or the moisture of the powder, because if it had, equivalent quantities of hydrogen ought to have been liberated. Charcoal contains more hydrogen than is necessary to form water with its oxygen. It is this excess of hydrogen which is set free, or which unites with sulphur, carbon or nitrogen forming sulphuretted hydrogen, marsh gas, or ammonia.

Hence, the excess of oxygen in the products of explosion must originate from some other source. NOBLE and ABEL, like LINCK, KAROLYI and others, adopted the method proposed by BUNSEN and SCHISCHKOFF for the analysis of the solid powder residue. This method requires that the aqueous solution of the substance should be digested with cupric oxide in order to convert the potassic sulphide into hydrate. The question suggested itself whether or not oxygen from the cupric oxide had formed with potassic sulphide, sulphate or hyposulphite.

27·3 grms. pure potassic hydrate were dissolved in water, which previously had been boiled in order to expell the air, and the solution divided into two equal parts. One of these parts was saturated with sulphuretted hydrogen in an atmosphere of hydrogen and then mixed with the other, and the solution of potassic sulphide, so obtained, digested with powder of sulphur in sufficient quantity to produce pentasulphide. The analysis of this liquid gave the following results :—

10 cub. centims. diluted with previously boiled water were acidulated with hydric chloride and heated to the boiling point. A precipitate of 1·094 grms. of sulphur was obtained. The filtrate evaporated to dryness and ignited, the dry residue moistened with hydric chloride, and then raised to a red heat gave 1·387 grms. of potassic chloride, corresponding to 0·726 grm. of potassium. On the assumption that 1 atom of sulphur had escaped as sulphuretted hydrogen, the liquid, according to these determinations, contains sulphur and potassium in the proportion of 4·67 atoms of the former to 2 atoms of the latter.

10 cub. centims. of the sulphide mixed with a solution of zinc sulphate, and the filtrate tested with an iodine solution in presence of some starch required 0·2 cub. centim. of the iodine liquid in order to produce a blue colour. 1 cub. centim. of the iodine solution corresponded to 1 cub. centim. of a solution of sodium hyposulphite containing 24·8 grms. of the salt in 1 litre.

Therefore, 10 cub. centims. of the polysulphide of potassium contain :

0·0038 grm. potassic hyposulphite
2·119 grms. potassic polysulphide.

To check these numbers, Mr. Cowper at my request dissolved the zinc sulphide in hydric nitrate of 1·5 sp. grav., and determined the zinc and the sulphur in the solution according to the usual methods.

He obtained :

Sulphur	1·285 grms.
Zinc.	0·556 grm.

or for every atom of zinc 4·69 atoms of sulphur.

180 cub. centims. of the solution of potassic polysulphide were digested in a hermetically closed flask with pure and previously ignited cupric oxide at common temperatures. The liquid assumed a brown colour, which still could be observed after two days' digestion. The flask was now placed in water of 35° C., whereupon the colour rapidly disappeared. The contents of the flask were now placed upon a filter, the cupric oxide and sulphide well washed, and filtrate and wash water united and kept in a closed bottle.

I. 25 cub. centims. of the liquid so prepared, neutralised with acetic acid, mixed with some starch solution, required 24·6 cub. centims. of iodine solution for the production of the blue colour.

25 cub. centims. in another experiment required 24 cub. centims. iodine solution.

25 cub. centims. in a third experiment required 24·2 cub. centims. iodine solution.

The mean of these experiments is 24·26 cub. centims. of iodine solution for 25 cub. centims. of the filtrate, hence, 100 cub. centims. of the united filtrate and wash water contain 1·844 grms. of potassic hyposulphite.

II. 100 cub. centims. of the same filtrate evaporated with pure hydric sulphate gave 3·653 grms. of potassic sulphate, which dissolved in water to a clear and neutral liquid. A similar quantity of hydric sulphate to that which had been used in this experiment, and out of the same bottle, left no residue after evaporation. 3·653 grms. of K_2SO_4 contain 1·6375 grms. of potassium, 1·844 grms. of $K_2S_2O_3$ contain 0·757 grm. of potassium; therefore, more than one-third, nearly one-half, of the potassium of the $K_2S_{4·67}$ in the original solution appears after treatment with cupric oxide as potassic hyposulphite.

The presence of potassic hyposulphite is assumed on account of the behaviour of the liquid with iodine solution. A direct proof of its presence appeared to be desirable.

Reactions of the filtrate of the cupric oxide.

a. Hydric chloride caused turbidity after some time, probably from sulphur.

b. Barium chloride, a white precipitate, only partly soluble in hydric chloride.

c. Cupric sulphate gave after neutralisation with acetic acid a blue precipitate, which turned dark brown at 70–80° C.

A mixture of sodic hyposulphite, potassic acetate and cupric sulphate behaved in a similar manner.

d. Lead acetate and silver nitrate, respectively, gave the same reactions as they do with a solution of sodic hyposulphite.

122 cub. centims. of the strongly alkaline filtrate were neutralised with acetic acid and allowed to evaporate over hydric sulphate under the receiver of an air-pump. After a few days a great number of prismatic crystals were observed. These crystals warmed with alcohol fused into an oily liquid, which recrystallised on cooling and did not dissolve in alcohol.

Alcohol added to the mother liquor of the crystals produced a crystalline precipitate.

The original crystals, and the crystalline precipitate united weighed 2·634 grms.

According to the determination with iodine solution 122 cub. centims. of the filtrate from the cupric oxide, contain 2·249 grms. anhydrous, or 2·604 grms. hydrated salt of the formula $3K_2S_2O_3, 5H_2O$. The crystals dissolved easily in 3 cub. centims. of water with absorption of heat; from this solution 2·457 grms. of salt were reobtained.

0·497 grm. of the same, dissolved in water and mixed with a solution of strontium chloride, gave, after two days' standing, a very small precipitate.

0·903 grm. of the salt, dissolved in 50 cub. centims. of water, gave, on addition of 1 grm. of barium chloride a white precipitate, which left, after treatment with boiling water, 0·017 grm. of barium sulphate. The filtrates of the barium sulphate yielded a fine crop of barium hyposulphite.

As this salt is, according to my experience, easily obtained in a pure form, the whole of the potassium hyposulphite was converted into the barium compound for the following analytical determinations.

0·882 grm. baric hyposulphite boiled with hydric nitrate gave 0·763 grm. of baric sulphate; the filtrate of this gave, on addition of barium chloride, another quantity of the same salt, which, after ignition and treatment with hydric chloride, weighed 0·753 grm.

Therefore in 100 parts:

	Experiment.	Theory.	
		BaS_2O_3	BaS_2O_3, H_2O.
Barium	50·86	51·70	48·41
Sulphur	23·60	24·15	22·61

The potassium salt from which the barium hyposulphite had been prepared gave the following reactions:—

a. Hydric chloride produced, a few moments after its addition, a slightly yellow turbidity.

b. Cupric sulphate gave a slight turbidity at common temperatures, on boiling, a copious brown precipitate.

c. Lead acetate, a white precipitate, which did not change its colour at 100° C.

d. Ferric chloride gave the usual violet colour.

e. Silver nitrate, a white precipitate, which turned black at higher temperatures.

According to these experiments there can be no doubt that, by the treatment of a solution of a mixture of potassic penta- and tetra-sulphides with cupric oxide, large quantities of potassic hyposulphite are formed.

The mother liquor of the potassic hyposulphite, from which this salt had been removed by means of alcohol, was left to evaporate in vacuo. The crystalline residue, heated in a platinum crucible, produced an oily liquid, which crystallised on cooling, like potassic acetate, and weighed 2·616 grms.

It was converted by means of hydric sulphate into 2·404 grms. of potassic sulphate.

2·616 grms. of potassic acetate should give, according to theory, 2·320 grms. of potassic sulphate. If we assume that, in the solution obtained by the treatment of potassic polysulphides with cupric oxide, only potassic hyposulphite and potassic hydrate are present, then, according to the numbers given on page 536, 122 cub. centims. of the filtrate from the cupric oxide should have given 2·697 grms. potassic acetate, instead of 2·616 as found by experiment.

Hence, we may conclude that 100 cub. centims. of the filtrate from the cupric oxide contain:

1·844 grms. potassic hyposulphite,
1·263 ,, ,, hydrate,

and a very small amount of potassic sulphate.

At my request, Mr. Cowper digested solutions of the other sulphides of potassium with cupric oxide and examined the products for potassium hyposulphite. He found that in a solution of potassic tersulphide (K_2S_3) nearly $\frac{1}{2}$, in one of potassic disulphide about $\frac{1}{5}$, and in one of potassic sulph-hydrate about $\frac{1}{12}$ of the potassium appears after treatment with cupric oxide as hyposulphite. All these experiments lead to the conclusion that the potassic hyposulphite found in solid powder residues by Bunsen and Schischkoff's method had been formed during the analysis of the said residues, and was not one of the original products of explosion. This conclusion is supported by the observation of Pape,* according to which potassic hyposulphite is decomposed at 225° C. into sulphate and pentasulphide of potassium.

At the conclusion of my experiments in July 1879, I communicated the results to Mr Abel, and he has since then confirmed my observations.

Noble and Abel say at the end of their second memoir (Phil. Trans., 1880, p. 277), "that although it would seem that in certain cases and under certain exceptional circumstances potassium hyposulphite does exist as a secondary, it exists in no case as a primary product, and should not, therefore, be reckoned among the normal constituents of powder residues."

Wishing to obtain a clear conception of the mode of action of cupric oxide in the analysis of a powder residue, I instituted the following experiments :—

6·157 grms. potassic sulphate, and 8·541 grms. of potassic carbonate were dissolved in 100 cub. centims. of water; the solution filled the space of 103·5 cub. centims.

10 cub. centims. of this liquid were mixed with 5 cub. centims. of a solution containing :

Potassium	0·389 grm.
Sulphur	0·498 „

or 3·12 atoms of sulphur for every 2 atoms of potassium.

The solution so prepared was then digested for two days with previously ignited cupric oxide in a well-stoppered bottle, at ordinary temperatures.

The mixture appeared brown, but became decolorised at 35° C.

The contents of the bottle were placed upon a filter; the black oxide and sulphide were well washed with boiling water, and both, filtrate and wash water united, were kept in a stoppered bottle. They filled the space of 578 cub. centims.

111 cub. centims. of the filtrate required 3·7 cub. centims. of the iodine solution.

192 cub. centims. of the filtrate required 6·3 cub. centims. iodine liquid. Hence, the entire filtrate contained 0·3623 grm. of potassic hyposulphite.

91 cub. centims. of the filtrate acidulated with hydric chloride, boiled for some minutes, separated from precipitated sulphur by filtration, and mixed with baric

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* Pogg. Ann., Bd. cxxii., p. 408.

chloride, gave a precipitate of baric sulphate weighing 0·158 grm. The whole filtrate therefore contains 0·749 grm. of potassic sulphate.

91 cub. centims. of the filtrate, acidulated with hydric chloride and evaporated to dryness, gave, after treatment of the residue with hydric sulphate, 0·395 grm. potassic sulphate. From this number it follows that the entire filtrate contained 1·124 grms. of potassium.

91 cub. centims. of the filtrate gave, with manganous sulphate, a precipitate, which generated with dilute hydric sulphate 0·042 grm. of carbonic acid. 578 cub. centims. of the filtrate contain 0·266 grm. carbonic acid, corresponding to 0·834 grm. potassic carbonate.

From the solution of the precipitate caused by manganous sulphate, 0·094 grm. of Mn_3O_4 was obtained, corresponding to 0·597 grm. for the entire filtrate; 0·461 grm. of this is derived from manganous carbonate, the remainder, 0·136 grm. of Mn_3O_4, from the manganous hydrate precipitated by potassic hydrate. 0·136 grm. of Mn_3O_4 corresponds to 0·139 grm. of potassium or 0·310 grm. of K_2S_3. The black cupric oxide and sulphide was dissolved in concentrated hydric nitrate and the solution precipitated by baric chloride; the weight of the baric sulphate was found to be 2·754 grms.

The following table contains the results of these experiments:—

	Composition of the original solution.	Found by analysis.
Potassic carbonate	0·829	0·834
,, sulphate	0·597	0·749
,, tersulphide	0·887	0·310
,, hyposulphite . . .	0·000	0·362
Potassium	1·124	1·124
Sulphur in CuO	0·000	0·378
,, in $K_2S_2O_3$	0·000	0·122

A considerable error attaches to the determination of the sulphur. The cupric oxide had been ignited in a Hessian crucible. From this, it appears, it became contaminated with silica and alumina. The baric sulphate, precipitated from the solution of the cupric oxide in hydric nitrate by baric chloride, contained a copper compound which I regard as a silicate, since it could not be got rid of even after long continued boiling with hydric chloride.

The analysis gives quite a wrong idea of the composition of the original solution. Not only is a portion of the potassic tersulphide converted into hyposulphite, it even appears that some has been oxidised into sulphate.

The following experiments prove the absence of sulphates in the reagents used for the above determinations.

15 cub. centims. of the solution of potassic tersulphide, acidulated with hydric

chloride, boiled, filtered and precipitated with baric chloride gave only 0·0005 grm. baric sulphate. 80 grms. of cupric oxide were dissolved in hydric chloride and the solution mixed with 1 grm. of baric chloride; even after two days' standing no precipitate could be observed.

0·854 grm. potassic carbonate, examined in a similar manner, was found to contain only 0·0007 grm. of sulphate.

1 grm. of sodic hyposulphite in 100 cub. centims. of water gave with barium chloride a crystalline precipitate which proved to be completely soluble in boiling water. Baric chloride, added to the water used in the experiments, and hydric chloride caused no precipitate.

1 grm. of sodium hyposulphite was dissolved in 200 cub. centims. of water, hydric chloride added, and the solution boiled during three-quarters of an hour.

Baric chloride precipitated 0·022 grm. of baric sulphate. Another similar experiment yielded 0·028 grm. baric sulphate.

From these experiments it is clear that in a solution containing hyposulphites and sulphates, the latter cannot accurately be determined by the ordinary method. The error from this source is, however, not sufficiently great to account for the discrepancy between the sulphate taken and found in the experiment described on page 539, and exhibited in the table given on that page.

Accordingly, it appears probable that by the action of cupric oxide upon potassic tersulphide, in *presence of potassic carbonate*, not only potassic hyposulphite but also potassic sulphate is formed.

15 cub. centims. of a solution of potassic sulphide were acidulated by addition of hydric chloride, and by boiling and filtration were separated from the precipitated sulphur. The potassic chloride left, after evaporation of the filtrate from the sulphur, was converted, by means of hydric sulphate, into potassic sulphate. The weight of this salt was found to be 2·554 grms.

5 cub. centims. of potassic sulphide solution, mixed with potassic hydrate and oxydised by chlorine, yielded 3·58 grms. of baric sulphate. 10 cub. centims. of potassic sulphide solution, precipitated by zinc sulphate, and the filtrate from the zinc sulphide examined by means of iodine solution for hyposulphites, gave numbers which indicated in 15 cub. centims. of the sulphide solution 0·114 grm. of potassic hyposulphite. Sulphuric acid was not found in the sulphide solution.

15 cub. centims. of potassic sulphide solution, out of the same bottle from which the quantities used in the above experiments had been taken, were mixed with solutions of 1·188 grms. of potassic sulphate, and of 1·650 grms. of potassic carbonate.

The mixture so prepared had the following composition :—

	Grms.		Grms.
Potassic carbonate . . .	1·650		
„ sulphate. . . .	1·188		
„ sulphide. . . .	2·534	Sulphur . .	1·4366
		Potassium .	1·0977
„ hyposulphite . .	0·114	Sulphur . .	0·0384
		Potassium .	0·0468

It was digested for a few days in a closed flask with 40 grms. of cupric oxide at common temperatures, and finally decolorised by the raising of its temperature for a short time to 35° C.

The contents of the flask were placed upon a filter and the oxide and sulphide washed with hot water. The volume of filtrate and wash water measured 1081 cub. centims.

The analysis of the liquid gave the following results:—

200 cub. centims. gave 0·394 grm. of baric sulphate; according to these numbers 1081 cub. centims. contain 1·589 grms. of potassic sulphate.

312 cub. centims. were precipitated with manganous sulphate. From the precipitate so obtained, 0·140 grm. of carbonic acid was developed by means of sulphuric acid, and 0·443 grm. of Mn_3O_4, prepared by the usual method. Hence, 1081 cub. centims. of filtrate contain 1·521 grms. of potassic carbonate and 0·708 grm. of potassium in the form of hydrate.

139 cub. centims. of the liquid required 6·4 cub. centims. of iodine solution. Therefore, 1081 cub. centims. contain 0·945 grm. of potassic hyposulphite.

200 cub. centims., evaporated with concentrated sulphuric acid, gave 1·069 grms. of potassic sulphate, containing 0·4793 grm. of potassium, or in 1081 cub. centims. 2·590 grms. of potassium.

From these data we find the following composition for the solution:—

	Composition of original solution.	Composition according to analysis.
Potassic carbonate	1·650	1·521
„ sulphate	1·188	1·589
„ sulphide	2·534 {S =1·4366 K=1·0977}	1·633 {S =0·925 K=0·708}
„ hyposulphite . . .	0·114 {S =0·0384 K=0·0468}	0·945 {S =0·318 K=0·387}
Potassium	2·609	2·590

The mixture of cupric oxide and sulphide, dissolved in concentrated nitric acid, yielded 8·16 grms. of baric sulphate, corresponding to 1·120 grms. of sulphur.

	Sulphur.	
	Contained in original solution.	Found by analysis.
Potassic sulphate	0·2180	0·291
„ sulphide	1·4366	. .
„ hyposulphite . . .	0·0384	0·318
Cupric oxide	0·0000	1·120
	1·693	1·729

From these experiments we conclude that by treatment of a solution containing potassic tersulphide, carbonate, and sulphate with cupric oxide, not only potassic hyposulphite but also potassic sulphate is formed.

Experiments with potassic monosulphide.

A solution of 5 grms. of pure potassic hydrate, in 100 cub. centims. of water, was divided into two equal parts, one of these was saturated with sulphuretted hydrogen and then mixed with the other.

20 cub. centims. of this solution gave, after evaporation with hydric chloride, 1·1015 grms. of neutral potassic chloride, corresponding to 0·812 grm. of potassic monosulphide (K_2S).

20 cub. centims. of the solution of the monosulphide were mixed with 2·219 grms. of potassic carbonate and 0·950 grm. of sulphate, and the mixture digested with pure cupric oxide at common temperatures, in a stoppered bottle, until all the potassic sulphide had been decomposed.

The contents of the bottle were treated in the same manner as in former experiments.

The filtrate of the cupric oxide measured 415 cub. centims.

160 cub. centims. required 2·1 cub. centims. of the iodine solution, therefore we have, in 415 cub. centims of the filtrate, 0·103 grm. of potassic hyposulphite.

It was ascertained, by special experiment, that the original potassic monosulphide did not contain hyposulphite.

60 cub. centims. of the liquid gave 0·185 grm. of baric sulphate, corresponding to 0·953 grm. of potassic sulphate in 415 cub. centims. of the filtrate.

100 cub. centims. gave with manganous sulphate a precipitate from which 0·174 grm. of carbonic acid and 0·428 grm. of Mn_3O_4 were obtained. It follows from these numbers that 415 cub. centims. of filtrate contain 0·758 grm. of monosulphide and 2·261 grms. of carbonate of potassium.

40 cub. centims. of the filtrate evaporated with hydric chloride, and the remaining potassic chloride converted by means of hydric sulphate into potassic sulphate, gave

0·482 grm. neutral potassic sulphate. Hence, 415 cub. centims. contain 2·238 grms. of potassium.

From the solution of the cupric oxide and sulphide in concentrated hydric nitrate, 1·684 grms. of baric sulphate were precipitated; 1·684 grms. of baric sulphate contain 0·231 grm. of sulphur.

	Composition of original substance.	Found by analysis.
Potassic carbonate	2·219	2·261
,, sulphate	0·950	0·953
,, monosulphide . .	$0{\cdot}812\left\{\begin{array}{l}K=0{\cdot}5765\\ S=0{\cdot}236\end{array}\right.$	$0{\cdot}758\left\{\begin{array}{l}K=0{\cdot}538\\ S=0{\cdot}220\end{array}\right.$
,, hyposulphite . .	0·000	0·103 S = 0·034
Potassium	2·255	2·238
Sulphur in cupric oxide . .	0·000	0·231

From these experiments it follows, as a general result, that if a solution containing potassic sulphate, carbonate and mono- ter- or pentasulphide of potassium is digested with pure cupric oxide, the determination of the potassic carbonate yields in all cases a nearly correct result, also for the sulphate an accurate value is obtained in a solution which contains the sulphide as monosulphide, but the numbers found for potassic sulphides are always incorrect. The potassium hyposulphite is formed by the action of cupric oxide upon pentasulphide of potassium in such quantities that a convenient method for the preparation of the salt might be based on the reaction.

Probably all the potassic hyposulphite, which was found in powder residues by Bunsen and Schischkoff's method of analysis, was formed during the analytical operations by the oxidation of potassic sulphide by the oxygen of the cupric oxide. At all events, we cannot assume that potassic hyposulphite is one of the products of the explosion of gunpowder, because at 225° C. it decomposes, according to Pape, into potassic sulphate and pentasulphide, and because in later experiments, in which zinc chloride was used instead of cupric oxide, Noble and Abel found very little potassic hyposulphite. We are, therefore, justified in replacing the hyposulphite of the analyses of Noble and Abel by its equivalent quantity of potassic sulphide.

The values found by means of Bunsen and Schischkoff's method for the potassic sulphate would be correct if the powder residues contained only potassic monosulphide. But as, according to Linck's and Noble and Abel's experiments, they contain one of the higher sulphides, it is not improbable that a portion of the sulphate observed has been formed during the treatment with cupric oxide from one of the said sulphides, or by the decomposition of potassic hyposulphite during the process of analysis.

The following considerations indicate how this possible error can be corrected:—

If powder is burnt in the apparatus of Noble and Abel, all the oxygen of the decomposed saltpetre is incorporated in the potassic carbonate and sulphate, the carbonic acid and oxide, and these, together with the potassic sulphide, nitrogen and free sulphur amount to about 96 per cent. of the exploded powder. The following calculation leads to the conclusion that the oxygen of the charcoal and of the moisture in the powder does not enter into the composition of the principal products of explosion, but is eliminated in union with hydrogen in the form of water.

According to analysis, the composition of the F. G. powder is represented by the symbols

$$16KNO_3+0{\cdot}044K_2SO_4+6{\cdot}88S+20{\cdot}81C+10{\cdot}78H+3{\cdot}52O$$

if ash and moisture of the charcoal are left out of consideration. In Experiment 17 of NOBLE and ABEL the hydrogen in ammonic carbonate, sulphuretted hydrogen, and in a free state, added together is equal to 4·1 atoms. This number deducted from 10·78 atoms, the hydrogen of the charcoal, leaves as difference 6·68 atoms. These 6·68 atoms of hydrogen must have formed water, for which operation 3·34 atoms of oxygen, or almost the exact quantity contained in the charcoal, is required.

Also the moisture of the powder cannot have contributed any oxygen, because if it had, an equivalent quantity of hydrogen ought to have been set free, or entered into combination with nitrogen, carbon, or sulphur. And as in all other experiments of NOBLE and ABEL, executed with F. G. powder from Waltham Abbey, the number of atoms of hydrogen, free or combined, which occur amongst the products of combustion, is less than 4·1, it follows, that in these other experiments also the oxygen of the charcoal or of the moisture of the powder has taken no part in the explosion. And finally, since several hundred grammes of powder were exploded in a *hermetically* closed steel cylinder, oxygen from the atmosphere cannot have entered into the composition of products of combustion of the powder.

If, then, we find in one of the experiments, after the potassic hyposulphite has been replaced by its equivalent of potassic sulphide, that the sum of the quantities of oxygen contained in the potassic carbonate and sulphate, carbonic acid, and oxide exceeds the oxygen derived from the decomposed saltpetre, we may assume that this excess of oxygen is owing to some sulphate which had been formed during the process of analysis, and, accordingly, we shall be justified in deducting this excess of sulphate from the total quantity found.

The errors which have been explained on p. 533, due to the mode of calculation, will compensate each other, if the *mean of all the experimental results* be taken.

If all these corrections are carried out we obtain in the form of a chemical equation an approximately correct expression of the metamorphosis of the powders of Waltham Abbey. This equation, deduced from the 31 experiments published by NOBLE and ABEL, is:

$$\left.\begin{array}{l} 4{\cdot}98K_2CO_3+13{\cdot}13CO_2+0{\cdot}84S \\ +0{\cdot}90K_2SO_4+3{\cdot}23CO \quad +0{\cdot}67H_2S \\ +2{\cdot}10K_2S_2 \quad +17{\cdot}34N \end{array}\right\} = \left\{\begin{array}{l} 16KNO_3 \ +1{\cdot}34N \\ +21{\cdot}35C+1{\cdot}34H \\ +6{\cdot}63S \end{array}\right\} \quad . \quad . \quad (I.)$$

The powder constituents on the right-hand side of the sign of equality have been calculated from the composition of the products of explosion.

The same constituents, as found by the direct analysis of the powders, are represented by the symbols:

$$16KNO_3+21{\cdot}18C+6{\cdot}63S$$

which are in close agreement with those deduced from the products of explosion. Powder of this composition, burnt according to the method of NOBLE and ABEL, will form the products of explosion in quantities as represented by equation (I.), if the small quantities of secondary products arising from the presence of hydrogen in charcoal, such as marsh gas, ammonia, and free hydrogen, are neglected.

The sulphuretted hydrogen is either the product of the direct union of hydrogen and sulphur at comparatively low temperatures, or of the action of carbonic acid and water upon potassic sulphide. In either case its formation has no direct connexion with the explosion, and it ought to be likewise omitted from an equation representing the metamorphosis of gunpowder.

0·84 atom of sulphur is represented as free, because there are no data to show how much sulphur has united with the iron of the apparatus.

It is usual to represent the potassic sulphide as monosulphide. Further on it will be shown that this is not correct, but that disulphide is really produced.

We may then replace equation (I.) by the more simple one

$$16KNO_3+21C+5S=\left.\begin{array}{l} 5K_2CO_3+13CO_2 \\ 1K_2SO_4+3CO \\ 2K_2S_2 \quad +8N_2 \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad (II.)$$

A portion of the sulphur of the powder has united with hydrogen and iron, hence the difference of sulphur in equations (I.) and (II.).

A powder consisting of

$$16KNO_3+21C+5S$$

exploded according to the method of NOBLE and ABEL in a vessel the substance of which is not attacked by the products of combustion, ought, *cæteris paribus*, to yield the products of explosion always in the proportions represented in equation (II.).

Under very great pressures the amount of carbonic oxide appears to diminish to a small extent; this variation of the carbonic oxide has, however, only a slight influence on the other products, amounting in the case of potassic carbonate, potassic sulphate, and potassic disulphide generally to less than 0·1 of a molecule. (See note, p. 526.)

Equation (II.) expresses only the quantitative relations between the powder constituents and the products of explosion; the reactions which occur during explosion, which of them are simultaneous, and the order in which they succeed each other have still to be determined.

The solid products of explosion possess the composition and the properties of liver of sulphur prepared with an insufficient quantity of sulphur. We can conceive that, at first, all the potassium of the saltpetre forms with carbon and oxygen potassic carbonate, and that, in another stage, sulphur acts on the potassic carbonate and produces the mixture known as the solid powder residue. Or we may conceive that potassic sulphate is the first product, and that this is afterwards reduced by carbon to potassic disulphide and carbonate. Both conceptions would lead to the same results.

The experiments of KAROLYI,* executed more than 17 years ago, contain the key to a chemical theory of gunpowder, and allow us to form an idea of the nature of the reactions, and the order in which they follow each other during the combustion of powder. He proposed to himself to decide by experiment whether or not the nature of the products and their relative quantities are dependent on the pressure which obtains during explosion. CRAIG† had asserted that under great pressure, such as would exist during the explosion in a piece of ordnance, much more potassic sulphide was formed than had been obtained by BUNSEN and SCHISCHKOFF under ordinary atmospheric pressure. KAROLYI took 36·836 grms. of Austrian cannon powder, which had, according to his opinion, a composition similar to that of the powder employed by BUNSEN and SCHISCHKOFF, and enclosed the same in a small metallic cylinder, suspended in the centre of a 60-pounder hermetically closed shell. The air was then pumped out of the shell by means of an air pump, and the powder exploded by an electric current. As soon as the pressure of the gases in the cylinder had attained a certain magnitude the cylinder burst, and its contents were scattered about the space of the exhausted shell.

The capacity of the shell and the amount of powder had been so adjusted that after explosion the gases in the shell should possess a tension of about 1·5 atmospheres; they were allowed to escape into tubes and sealed up for analysis. The solid products, which were removed from the shell by means of water, as well as the gases, were examined by the methods of BUNSEN.

A similar experiment was made with 34·153 grms. of rifle powder.

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* Pogg. Ann., Bd. cxviii., p. 544.

† SILIMAN'S Am. J., [2], vol. xxxi., p. 429; Chem. News, vol. iv., p. 18.

Composition of the powders.

	Cannon powder.	Rifle powder.
Saltpetre	73·78	77·15
Sulphur	12·80	8·63
Charcoal—		
Carbon	10·88	11·78
Hydrogen	0·38	0·42
Oxygen	1·82	1·79
Ash	0·31	0·28

Composition of the products of explosion.

	Cannon powder. Grms.	Rifle powder. Grms.
Potassic carbonate	7·14	7·096
,, sulphate	13·61	12·354
,, hyposulphite	1·04	0·605
,, sulphide	0·04	0·000
Carbonic acid	6·40	7·442
,, oxide	0·97	0·504
Nitrogen	3·60	3·432
Hydrogen	0·04	0·047
Marsh gas	0·15	0·167
Sulphuretted hydrogen	0·10	0·079
Carbon	0·94	0·887
Sulphur	1·73	0·397
Ammonic carbonate	0·99	0·908
Loss	0·08	0·235

Cannon powder gave 30·77 per cent. of gas and 69·25 per cent. solid residue. Rifle powder gave 34·86 per cent. of gas, and 65·14 per cent. solid residue.

Karolyi, comparing his results with those of Bunsen and Schischkoff, arrives at the conclusion that the nature and quantities of the products of explosion are not much influenced by the conditions under which the combustion takes place, but that the composition of the powder determines in a great measure the proportions in which the products of explosion are formed. Besides this, he deduces no other conclusions from his experiments. Karolyi's observations do not support Craig's assertion; the pressure in the metallic cylinder before explosion must have been great, yet very little or no potassic sulphide has been formed.

A few years later (1869) Fedorow* published the results of some experiments executed by him on the explosion of gunpowder. He, like Craig, concludes that under high pressure more potassic sulphide is produced than would be formed by the same powder under lower pressures.

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* Zeitschrift für Annalytische Chemie, Bd. ix., p. 127; Strecker, Jahresbericht, 1869, p. 1059.

The experiments of FEDOROW, however, do not prove anything of the kind. He fired powder from a pistol and from a 9-pounder cannon, and it appears that the solid products of the powder fired from the pistol were collected in a glass tube. A charge of 0·75 grm. of powder gave a residue with proportionally more potassic sulphate and less of potassic sulphide than one of 1·5 grms., and relatively still smaller was the potassic sulphate and greater the sulphide in the residue obtained by the firing of 3 lbs. of powder from the cannon.

M. FEDOROW concludes: "A smaller amount of powder remains unconsumed, less potassic sulphate, but more sulphide and carbonate are formed under higher than under lower pressures. Time acts like pressure. If the combustion of the powder is retarded the same effects follow as if the pressure had been increased. A charge of 1·5 grms. of a mixture consisting of 100 parts of meal powder, and 0·5 part of stearic acid, gave a residue with less potassic sulphate, but with more carbonate and hyposulphite than a similar charge of ordinary powder would have done."

Potassic sulphide is, as is well known, a substance endowed with great attraction for oxygen, not only at high, but also at ordinary temperatures. It appears that the air was not excluded from the glass tube into which M. FEDOROW fired his charges for the collection of the solid residues. A greater percentage of the potassium sulphide formed by a small charge, than of the sulphide of the products of a larger charge, will in this manner be oxidised.

It thus appears that M. FEDOROW's experiments can be explained without reference to pressure. But as far as the retardation of the combustion by stearic acid is concerned, we cannot ascribe to retardation an effect which is due to the stearic acid itself. Stearic acid at a red heat reduces potassic sulphate to sulphide and probably carbonate.

M. FEDOROW could not collect and examine the gases.

The experiments of FEDOROW do not establish a relation between pressure and the nature and quantities of the products of explosion, and, consequently, do not invalidate the conclusions of KAROLYI, as stated on page 547.

We will now proceed to explain, by means of the analytical results of KAROLYI, some of the reactions which take place during the combustion of gunpowder. For this purpose it will be desirable to express the composition of the Austrian powders, and the products of explosion, by chemical formulæ.

COMPOSITION of the powders.

	Cannon powder.	Rifle powder.
Saltpetre	16·00 mols.	16·00 mols.
Sulphur.	8·70 atoms.	5·66 atoms.
Carbon	19·80 ,,	20·57 ,,
Hydrogen	8·32 ,,	8·80 ,,
Oxygen.	2·50 ,,	2·34 ,,

COMPOSITION of the products of combustion of quantities of powder which contain 16 mols. of saltpetre.

	Cannon powder.	Rifle powder.
Potassic carbonate . . .	3·05 mols.	3·27 mols.
,, sulphate . . .	4·61 ,,	4·52 ,,
,, hyposulphite . .	0·31 ,,	0·20 ,,
,, sulphide . . .	0·01 ,,	0·00 ,,
Carbonic acid*	9·23 ,,	11·46 ,,
,, oxide	2·04 ,,	1·15 ,,
Nitrogen	7·55 ,,	8·06 ,,
Sulphuretted hydrogen .	0·17 ,,	0·14 ,,
Hydrogen.	2·36 atoms.	3·01 atoms.
Marsh gas	0·55 mol.	0·67 mol.
Ammonia	0·92 ,,	0·91 ,,
Carbon	4·61 atoms.	4·74 atoms.
Sulphur	3·16 ,,	0·79 ,,

Calculated from the products of combustion, we obtain for

Cannon powder . . . $16KNO_3+\underbrace{19{\cdot}48C+7{\cdot}66H}_{\text{Charcoal.}}+8{\cdot}58S$

Rifle powder . . . . $16KNO_3+\underbrace{21{\cdot}4C+8{\cdot}7H}_{\text{Charcoal.}\quad +1{\cdot}04N.}+5{\cdot}68S$

The potassic carbonate, potassic sulphate, potassic hyposulphite, and potassic sulphide, the nitrogen, carbonic acid, and carbonic oxide added together, amount in the case of the cannon powder to 89·6, and in that of the rifle powder to 92·0 per cent. of the exploded powder. 7·2 per cent. of carbon and sulphur of the cannon powder and 3·7 per cent. of the same elements of the rifle powder, remained free at the end of the explosion. The sum of all the other products, therefore, does not amount to more than 3 or 4 per cent., and as, with the exception of the nitrogen in the ammonia, no constituents of the saltpetre occur in them, they may be regarded as merely accessory products not directly concerned in the explosion.

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* Inclusive of the carbonic acid combined with ammonia.

As the composition of the Austrian service powders does not differ much from that of the powders of Waltham Abbey, it will be interesting to contrast the results of KAROLYI with those obtained by NOBLE and ABEL.

Remarkable differences will be observed if equation (I.), page 545 is compared with the composition of the products of combustion observed by KAROLYI and represented by means of chemical symbols on page 549. Equation (I.), as well as KAROLYI'S results, are calculated for quantities of powder containing 16 mols. KNO_3.

NOBLE and ABEL, according to equation (I.), found about a quarter of the potassium of the decomposed saltpetre as potassic sulphide, whereas in KAROLYI'S experiments the sum of the potassium in the hyposulphite, and in the sulphide of the products of combustion of cannon powder amounted to only $\frac{1}{26}$th, and in that of the rifle powder only to $\frac{1}{40}$th of the potassium of the saltpetre of the exploded powder. It seems to follow that the production of such small quantities of potassic hyposulphite and sulphide cannot be the direct result of the chief reactions of the explosion. KAROLYI obtained much more potassic sulphate and less potassic carbonate than the English chemists; a considerable quantity of the carbon of his powders remained unconsumed, whilst in NOBLE and ABEL'S experiments, as a rule, every trace of this element was burnt, although the English powders contain somewhat more carbon than the Austrian. Similar differences occur between the results obtained with the gaseous products. The gases obtained by KAROLYI were combustible, those of NOBLE and ABEL were not. The gases from the Austrian powders contained more hydrogen and marsh gas and less sulphuretted hydrogen than those from the mixtures of Waltham Abbey.

	Mean percentage by volume.	
	Austrian.	Waltham Abbey.
Hydrogen	6·41	2·50
Marsh gas	2·86	0·31
Sulphuretted hydrogen . . .	0·76	2·56

KAROLYI inclosed his powders in a thin brass cylinder hermetically closed, and ignited the charge by means of a galvanic current. As soon as the tension of the gases developed by the combustion had reached a certain magnitude the metal cylinder exploded, and its contents were scattered against the cold sides of the exhausted 60-pounder shell. Thus the combustion of the powder was interrupted before its completion. The correctness of this view is rendered apparent if the effect is considered which would have followed if the products of KAROLYI had remained in contact for some time at a red heat. The free carbon and hydrogen, and the constituents of the marsh gas would have been burnt at the expense of the oxygen of the potassic sulphate; the quantity of the latter would have been diminished and that of the sulphide increased, the free sulphur would have decomposed potassic carbonate with production of potassic sulphate and sulphide; in short, the products of combustion would have formed in similar proportions as we find them in NOBLE and ABEL's experiments. The products of several hundred grammes of powder remained in a fluid condition at a white heat in NOBLE and ABEL's steel cylinder from 60 to 100 seconds in contact, whereas the 36 grms. of burning powder were scattered after the explosion of KAROLYI's brass cylinders over the cold sides of a large iron shell, and their combustion occupied only a very small fraction of a second. The reactions between the powder constituents had time to complete themselves in NOBLE and ABEL's steel cylinder, but they had not in KAROLYI's small brass vessel. And although the time of the combustion in the experiments of the last-named chemist was very short, less than one second, we find all the saltpetre of the powder decomposed, and its constituents incorporated in the potassium salts, carbonic oxide and carbonic acid, or set free as nitrogen.

At the same time two of the potassium salts, the hyposulphite and sulphide, occur in such small quantities that we may regard them as secondary products, not connected with the chief reactions of the explosion, and accordingly neglect them.

Hence we have, as chief products formed in KAROLYI's experiments: potassic sulphate and potassic carbonate, carbonic acid and nitrogen, and perhaps carbonic oxide.

The combustion of gunpowder accordingly consists of two distinct stages; a process of oxidation, which is finished in a very short time, occupying only a very small fraction of a second, and causing the explosion, and during which potassium carbonate and sulphate, carbonic acid, some carbonic oxide and nitrogen are produced, and a process of reduction, which succeeds the process of oxidation and requires a comparatively long time for its completion. As the oxygen of the saltpetre is not sufficient to oxidise all the carbon to carbonic, and all the sulphur to sulphuric acid, a portion of the carbon and a portion of the sulphur are left free at the end of the process of oxidation. The carbon so left free reduces, during the second stage of the combustion, potassic sulphate, and the free sulphur decomposes potassic carbonate. Hydrogen and marsh gas, which are formed by the action of heat upon charcoal, likewise reduce potassic sulphate, and some hydrogen combines with sulphur, forming sulphuretted hydrogen.

Great variations of pressure appear to affect the proportions of the different products in a very slight degree, so that it may be regarded as doubtful whether pressure has any influence on them.

KAROLYI's experiments happen to be arranged in such a manner that the combustion of his powders could only proceed to the end of the first and the commencement of the second stage; in NOBLE and ABEL's explosions, the reactions of the second stage were also completed.

This view of the combustion of gunpowder explains not only the experiments of

Noble, Abel, and Karolyi, but is also in perfect accordance with the thermochemical relations of the products of explosion.

The heat of formation of a molecule of potassic sulphate is much greater than that of one of potassic sulphide, hence, the production of the former is to be expected during explosion. In short, the formation of the molecules of potassic carbonate, potassic sulphate, and carbonic acid is accompanied by the greatest evolution of heat.

Karolyi examined the products of explosion according to Bunsen and Schischkoff's method, which does not yield exact values for potassic sulphide and sulphate. The errors arising from this source will, however, be very small, if the potassic sulphide in the original powder residue is small. The small amount of potassic hyposulphite found by Karolyi proves that the products of combustion contained, in his experiments, very little potassic sulphide. As the potassic hyposulphite must be regarded as a product of oxidation of the potassic sulphide, it has been replaced in the following calculations by its equivalent of potassic sulphide. Nevertheless, the quantity of the latter does not exceed 0·32 of a molecule, hence the error caused by the method of analysis may be neglected.

We will now take the chief products of explosion observed by Karolyi, and calculate from their composition the quantities of powder constituents which took part in their formation, and arrange the results in form of equations.

Cannon powder.

$$\left.\begin{array}{rl} 3{\cdot}05 & K_2CO_3 \\ 4{\cdot}62 & K_2SO_4 \\ 0{\cdot}33 & K_2S_2 \\ 9{\cdot}23 & CO_2 \\ 2{\cdot}04 & CO \\ 7{\cdot}55 & N_2 \end{array}\right\} = \left\{\begin{array}{rl} 16 & KNO_3 \\ 14{\cdot}32 & C \\ 5{\cdot}28 & S \\ -0{\cdot}9 & N \\ 0{\cdot}13 & O \end{array}\right.$$

Rifle powder.

$$\left.\begin{array}{rl} 3{\cdot}30 & K_2CO_3 \\ 4{\cdot}49 & K_2SO_4 \\ 0{\cdot}20 & K_2S_2 \\ 11{\cdot}49 & CO_2 \\ 1{\cdot}15 & CO \\ 8{\cdot}00 & N_2 \end{array}\right\} = \left\{\begin{array}{rl} 16 & KNO_3 \\ 15{\cdot}94 & C \\ 4{\cdot}89 & S \\ 3{\cdot}99 & O \end{array}\right.$$

The entire quantity of saltpetre contained in the powders taken for these experiments was decomposed, and its constituents, with the exception of very small quantities of potassium and nitrogen, in potassic sulphocyanate and ammonia, reappear in the above chief products. The cannon powder contains for every 16 mols. of KNO_3 about 3 atoms of sulphur more than the rifle powder (p. 549). Nevertheless, during the first stage of the combustion the quantities of sulphur consumed in the formation of potassic sulphate are nearly the same in the experiments with both powders, and the potassic carbonate and disulphide are also almost identical.

The ratios of the oxygen in the potassic carbonate, sulphate, and carbonic acid are as follows :—

	Oxygen.		
	K_2CO_3	K_2SO_4	CO_2
Cannon powder	9·15	18·44	18·46
	1	2	2
Rifle powder	9·90	17·96	22·98
	1	1·81	2·33

The amount of oxygen in the products of explosion of the rifle powder has been found about 1·2 per cent. too high, consequently an error attaches to one or more of the analytical determinations.

The metamorphosis of the cannon powder during the first stage of the combustion can almost exactly, that of the rifle powder approximately, be represented by the equation

$$10(KNO_3)+8C+3S=2(K_2CO_3)+3(K_2SO_4)+6(CO_2)+5(N_2) \quad . \ . . \quad (III.)$$

It is worthy of notice that the ratios of the oxygen in the three principal products, potassic carbonate, potassic sulphate, and carbonic acid, are, according to equation (III.), of all possible ratios the most simple, if these products are to be formed by the combustion of a mixture of saltpetre, carbon, and sulphur.

From these considerations it appears to follow that during the explosion of gunpowder, or the first stage of its combustion, the constituents of powders which differ in their composition will act on each other in certain fixed stöchiometrical proportions.

It may be assumed as highly probable that of the infinite number of mixtures, which can be prepared from saltpetre, carbon, and sulphur, some will be more combustible than others, and, among the more combustible mixtures, one will be found containing the constituents in proportions most favourable for their transformation into the chief products of explosion. In this most combustible mixture the number of the molecules of saltpetre and of the atoms of carbon and sulphur will probably stand in simple arithmetical relations to each other, and if a mixture containing the constituents in other proportions be ignited, they will tend to react on each other in the stöchiometrical proportions of the most combustible mixture.

Equation (III.) can be transformed into:

$$16KNO_3+12{\cdot}8C+4{\cdot}8S=3{\cdot}2K_2CO_3+4{\cdot}8K_2SO_4+9{\cdot}6CO_2+8N_2$$

and as during the first stage of the combustion some carbonic oxide is probably also formed, we may write the equation instead

$$16KNO_3+13C+5S=3K_2CO_3+5K_2SO_4+9CO_2+CO+8N_2 \quad . \ . \ . \quad (IV.)$$

This equation explains the experiments of KAROLYI in a very satisfactory manner.

If the reactions went a little beyond the first stage of the combustion, and we add to the left of the sign of equality 1·32 atoms of carbon, which would reduce some potassic sulphate with formation of potassic sulphide and carbonate, carbonic acid and oxide, according to equations which will be given afterwards, numbers are obtained which are almost identical with those calculated from KAROLYI'S observations on the products of explosion of cannon powder (p. 552). The same remark applies to the products of the rifle powder, except the carbonic acid.

The excess of oxygen found in the products of rifle powder indicates that some error has occurred in the determinations of these products, and it seems to have influenced, almost exclusively, the carbonic acid. If the quantity of this substance is calculated according to the available oxygen of the decomposed saltpetre, a number is obtained which agrees very well with equation (IV.).

From the foregoing observations it follows that powders of the composition of the Austrian service powders—

$$16KNO_3+19{\cdot}8C\ +8{\cdot}7S$$
$$\text{and}\quad 16KNO_3+20{\cdot}57C+5{\cdot}66S$$

burn during the first stage of the metamorphosis according to the equation:

$$16KNO_3+13C+5S=3K_2CO_3+5K_2SO_4+9CO_2+CO+8N_2$$

and that the carbon and sulphur which the powders contain, beyond the quantities required by this equation, remain free.

The very combustible sporting powder of BUNSEN and SCHISCHKOFF contained

$$16KNO_3+13{\cdot}3C+6{\cdot}3S$$

Therefore, it will be seen that the constituents of the service powders react upon each other during the first stage of the explosion nearly in the same stöchiometrical quantities in which they are contained in the more highly combustible sporting powder.

If it be correct that equation (IV.) represents proportions of saltpetre, carbon, and sulphur, in which these substances will burn with greater facility than they do in the proportions of the service powders, then we can by means of equation (IV.) calculate the composition of a powder which shall be distinguished by its great combustibility.

Besides saltpetre, carbon and sulphur, hydrogen, oxygen, ash, and moisture are contained in gunpowder. The weight of these latter constituents is about 4 per cent. of the mixture. If we add 4 per cent. to the quantities represented by the symbols

$$16KNO_3+13C+5S$$

and consider hydrogen, oxygen, ash, and moisture united with carbon to charcoal, we obtain for the percentage composition of the most combustible mixture

Saltpetre	80·31
Sulphur	7·95
Charcoal	11·74

Sporting powders are required to burn quickly, and the composition of some of them approaches very closely to these theoretical values.

	Saltpetre.	Sulphur.	Charcoal.
English*	79·7	7·8	12·5
French	78·0	10·0	12·0
BUNSEN and SCHISCHKOFF's powder	78·99	9·84	11·17
Sporting powder in the year 1546	83·4	8·3	8·3

The heat relations of the products of explosion, as represented by equation

$$16KNO_3+13C+5S=3K_2CO_3+5K_2SO_4+9CO_2+CO+8N_2,$$

are of special interest.

If we assume for the heat of formation of potassic carbonate, potassic sulphate, and carbonic acid the following values—

$$K_2CO_3=279540 \text{ cal.}$$
$$K_2SO_4=344640 \text{ ,,}$$
$$CO_2=97000 \text{ ,,}$$

we obtain

$$3\times279540=838620$$
$$5\times344640=1723200$$
$$9\times97000=873000$$
$$838620:1723200:873000=1:2·05:1·04$$
$$\text{or nearly } 1:2:1$$

ratios which are very remarkable.

The total amount of heat produced by the combustion of $16KNO_3+13C+5S$ according to equation (IV.), is =1563189 cal. The maximum amount of heat is produced by a mixture which contains for every 16 mols. of saltpetre 8 atoms of carbon and 8 atoms of sulphur, according to the equation:

$$16KNO_3+8C+8S=8K_2SO_4+8CO_2+8N_2$$

and amounts to 1621450 cal.

Therefore, if the combustion of ordinary service powder takes place during the first stage according to equation (IV.), nearly the maximum quantity of heat is obtained which a mixture of saltpetre, sulphur, and carbon can produce. If the question were asked: in what proportions must the constituents of a given mixture of saltpetre, carbon, and sulphur react during the process of explosion, and what must be the ratios of the chief products of explosion, so that on one hand the total quantity of heat developed is as great as possible, and, on the other hand, the amounts of heat produced by the formation of the chief products shall stand to each other in a simple relation? the answer would be: the combustion must take place according to equation (IV.).

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* GRAHAM-OTTO, 'Lehrbuch der Chemie,' iv. edit., ii., p. 211.

But not only does this equation correspond to the most simple relations of the heat of formation of the principal products, it likewise requires the most simple distribution of the oxygen of the decomposed saltpetre. If the combustion of a mixture of saltpetre, carbon, and sulphur is to produce potassic carbonate and sulphate, carbonic acid and nitrogen, and if the oxygen of each of the first three products is to stand to the oxygen of the others in the most simple ratios possible, then the mixture must burn according to equation (III.), p. 553, and as the proportions expressed by equation (IV.) closely approach to those of equation (III.), it follows that equation (IV.) fulfils all the conditions and consequences explained in the foregoing lines. And, perhaps, these relations are the cause why mixtures of saltpetre, carbon, and sulphur of different composition burn during the first stage of the explosion according to equation (IV.), and if they contain more carbon and sulphur than is required by this equation, the excess of the two elements will remain free.

These interesting conclusions I deduce from the analytical data of KAROLYI and the corrected results of NOBLE and ABEL'S experiments. Their investigations, however, do not give any information about the reactions of the second stage of the combustion of gunpowder, the reduction of potassic sulphate by carbon, and the decomposition of potassic carbonate by sulphur. Hitherto it has been assumed that potassic monosulphide is formed; this is, however, a mistake.

According to BERZELIUS and MITSCHERLICH[*] the products of the decomposition of potassic carbonate by sulphur at a white heat are potassic sulphate and disulphide:

$$4K_2CO_3+7S=K_2SO_4+3K_2S_2+4CO_2$$

The question we have now to solve is: which of the sulphides of potassium is formed by the action of carbon upon potassic sulphate at a white heat?

BAUER[†] and WITTSTOCK[‡] obtained potassic carbonate and a polysulphide, but the amount of sulphur in the polysulphide was not determined.

A mixture of

26·5	grms. of potassic	sulphate
10·54	,, ,,	carbonate
2·5	,,	charcoal

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[*] GMELIN-KRAUT, 'Handbuch der Chemie,' Bd. ii., p. 39.

[†] Ibid., p. 33.

[‡] Ibid., p. 33.

was exposed in a porcelain crucible for half an hour to a temperature approaching white heat. The contents of the crucible dissolved completely in water, forming a deep yellow solution.

20 cub. centims. of this solution acidulated with hydric chloride, gave a copious precipitate of sulphur, and the filtrate of this precipitate left, after evaporation and treatment with hydric sulphate, 2·943 grms. of potassic sulphate. The potassium of

1·960	grms. potassic sulphate
0·779	,, ,, carbonate

of the salts originally taken, is, accordingly, present in 20 cub. centims. of the solution.

20 cub. centims. of the same solution gave by the usual method 1·038 grms. of baric sulphate, corresponding to 0·775 grm. of potassic sulphate.

19·28 cub. centims. of the same solution placed for some days in contact with 5 grms. of cupric oxide and the carbonic acid in the liquid determined according to Bunsen's method gave 0·37 grm. of this substance, which corresponds to 1·203 grms. of potassic carbonate in 20 cub. centims. of the solution.

1·96 grms. of potassic sulphate originally taken contain 0·36 grm. of sulphur.
0·775 ,, ,, ,, found in 20 cub. centims. contains 0·142 grm. of sulphur.
Sulphur in potassic sulphide = 0·218 grm.

20 cub. centims. of the solution contain 1·318 grms. of potassium.

1·203 grms. of potassic carbonate contain 0·680 grm. of potassium, and 0·775 grm. of potassic sulphate 0·347 grm. of potassium; the difference of 0·291 grm. represents the potassium in potassic sulphide.

Hence the potassic sulphide produced by the action of the carbon of the charcoal upon the mixture of potassic sulphate and carbonate contains:

	Grm.
Potassium	0·291
Sulphur	0·218
	0·509

and the composition of the salts in 20 cub. centims. of the solution of the fused mass is:

	Found.	Taken.
Potassic sulphate	0·775	1·960
,, carbonate	1·203	0·779
,, sulphide	0·509	0·000

The composition of the potassic sulphide can be represented by the symbols

$$K_2S_{1·82}$$

Therefore, potassic sulphate and the carbon of charcoal, react, under the conditions of the experiment, principally according to the equation

$$4K_2SO_4+7C=2K_2CO_3+2K_2S_2+5CO_2$$

Noble and Abel calculate their potassic sulphide as monosulphide, and in a special column give, as free sulphur, the sulphur not contained in the monosulphide, potassic sulphate and potassic hyposulphite. This so-called free sulphur was in reality contained in the residues partly in union with potassium as disulphide, partly with iron as ferrous sulphide.

If we imagine their free sulphur all combined with their potassic monosulphide, we obtain:

In 15 experiments united to 2 atoms of potassium, from 1·7 to 2·44 atoms of sulphur.

In seven experiments from 2·44 to 3·00 atoms of sulphur.

And in three experiments from 3·0 to 3·7 atoms of sulphur. The mean of 25 experiments would give us for 2 atoms of potassium 2·42 atoms of sulphur. But as a portion of the so-called free sulphur was in union with iron, it follows that in the powder residues 2 atoms of potassium were on an average combined with less than 2·42 atoms of sulphur. These numbers, however, apply only to their corrected results.

According to Berzelius and Mitscherlich the action of sulphur upon potassic carbonate at a white heat produces K_2S_2; the reduction of potassic sulphate in presence of carbonate, according to my own experiments, gives $K_2S_{1\cdot82}$; the mean of Noble and Abel's experiments for the composition of the potassic sulphide formed by the explosion of powder in their apparatus is less than 2·42 atoms of sulphur for every 2 atoms of potassium, hence we may conclude that the potassium sulphide formed during the second stage of the combustion of gunpowder is the disulphide, or at least contains the two elements in a proportion closely approaching the proportion in the disulphide. The following considerations confirm this conclusion.

The mean composition of the English service powder is:

$$16KNO_3+21\cdot18C+6\cdot63S$$

The Austrian rifle powder contains:

$$16KNO_3+20\cdot57C+5\cdot66S$$

The products of combustion of the former are, according to Noble and Abel,

$$4\cdot98K_2CO_3+0\cdot90K_2SO_4+2\cdot10K_2S+13\cdot13CO_2+3\cdot23CO+0\cdot84S$$

Those of the Austrian rifle powder, according to Karolyi, are

$$3\cdot27K_2CO_3+4\cdot52K_2SO_4+0\cdot2K_2S_2+11\cdot46CO_2+1\cdot15CO+4\cdot74C+0\cdot79S$$

Karolyi's products contain a considerable amount of free carbon, which element is completely burnt in Noble and Abel's experiments.

If we now assume that Karolyi's products had remained in contact at a high temperature, not a fraction of a second, but from one to two minutes, as was the case in Noble and Abel's experiments, the free carbon would have been oxidised by oxygen contained in potassic sulphate, and the free sulphur would have reacted upon the potassium carbonate, and the final result of these reactions would have been a quantitative relationship between the products similar to that found by Noble and Abel. The potassic carbonate, the potassic disulphide, the carbonic oxide, and carbonic acid would have increased, and the quantity of the potassic sulphate would have diminished. But the amount of potassic carbonate could not become greater if the reduction of the sulphate took place according to the equation :

$$K_2SO_4+2C=K_2S+2CO_2$$

hence, we must assume the formation of potassic disulphide. If we calculate from the analytical data of Karolyi the proportions of his products on the supposition that the carbon which had remained free had acted on potassic sulphate according to the equation :

$$4K_2SO_4+7C=2K_2CO_3+2K_2S_2+5CO_2$$

we obtain :

$$4{\cdot}65K_2CO_3+1{\cdot}12K_2SO_4+2{\cdot}22K_2S_2+14{\cdot}08CO_2+2{\cdot}65CO$$

quantities which are as near those found for the English powders as the composition of the Austrian rifle powder approaches those of Waltham Abbey.

If the potassic hyposulphite found by Bunsen and Schischkoff originated during the analysis of their powder residue, then the latter contained, for 16 mols. of decomposed saltpetre, 0·45 mol. of K_2S_2 and 0·33 mol. of K_2S.

Linck[*] found, amongst the products of the Würtemberg service powder, only potassic disulphide.

From Noble and Abel's[†] analysis of the products of English mining powder it would follow that for every 16 mols. of decomposed saltpetre, 4 mols. of K_2S are formed and 5 atoms of sulphur left free; this would give us for the composition of the potassic sulphide 2 atoms of potassium and 2·25 atoms of sulphur.

From these facts we conclude that the second stage of the combustion of gunpowder takes place according to the equations :

$$4K_2CO_3+7S=K_2SO_4+3K_2S_2+4CO_2 \quad \ldots\ldots \quad (V.)$$

$$4K_2SO_4+7C=2K_2CO_3+2K_2S_2+5CO_2 \quad \ldots\ldots \quad (VI.)$$

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[*] Ann. der Chemie und Pharm., Bd. cix (1859), p. 53.

[†] Phil. Trans. (1880), p. 207.

The possibility of dissociation requires the additional equation

$$K_2CO_3+K_2S_2+O_7=2K_2SO_4+CO_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad \text{(VII.)}$$

The final results of the reactions represented by equations (III.), (IV.), (V.), (VI.), and (VII.) can be expressed by one equation.

For this purpose let x, y, and z be positive numbers, and a denote how many molecules of carbonic oxide are formed by the combustion of a quantity of powder containing x molecules of saltpetre, y atoms of carbon, and z atoms of sulphur. The general equation representing the qualitative and quantitative relations between the constituents of a mixture of saltpetre, carbon, and sulphur on the one hand, and the products of *complete* combustion on the other, will then be :

$$\left.\begin{aligned} xKNO_3+yC+zS= & \ \tfrac{1}{28}[4x+8y-16z-4a](K_2CO_3) \\ & +\tfrac{1}{28}[20x-16y+4z+8a](K_2SO_4) \\ & +\tfrac{1}{28}[-10x+8y+12z-4a](K_2S_2) \\ & +\tfrac{1}{28}[-4x+20y+16z-24a](CO_2) \\ & +\tfrac{1}{2}xN_2 \\ & +aCO \end{aligned}\right\} \quad . \quad . \quad \text{(VIII.)}$$

As far as the application of this equation is concerned, the following remarks are, perhaps, not unnecessary.

The charcoal of gunpowder contains, besides carbon, also oxygen and hydrogen, ash and moisture. The oxygen of the charcoal is, as has already been proved before (p. 544), eliminated with some of the hydrogen as water. The rest of the hydrogen of the charcoal with nitrogen, carbon, and sulphur respectively, forms by-products, the total weight of which, as a rule, does not exceed 2 per cent. of the powder burnt.

The products of combustion, with the exception of those prepared by NOBLE and ABEL, contain always some unburnt carbon and sulphur, and frequently undecomposed saltpetre.

Hence we have :

a. Chief products: K_2CO_3, K_2SO_4, K_2S_2, CO_2, CO, and N_2.
b. By-products: H_2, H_2S, CH_4, NH_3, H_2O, and KCNS.
c. Constituents of powder, not burnt: KNO_3, C, and S.

Equation (VIII.) enables us to calculate, from that portion of the powder which produces the chief products (mentioned under *a*), the quantities of these products formed during complete combustion.

We will now proceed to prove, by examples, the correctness of this equation.

Bunsen and Schischkoff found in 1 grm. of sporting powder, and in the products of its combustion, the following quantities:—

1 grm. of powder—

Saltpetre	0·7899
Sulphur	0·0984
Charcoal—	
Carbon	0·0769
Hydrogen	0·0041
Oxygen	0·0307

Chief products of combustion—

Potassic sulphate	0·4227
,, carbonate	0·1264
,, hyposulphite	0·0327
,, sulphide	0·0213
Carbonic acid*	0·2159
,, oxide	0·0094
Nitrogen	0·0998
	0·9282

By-products—

Potassic sulphocyanate	0·0030
Hydrogen	0·0002
Sulphuretted hydrogen	0·0018
Oxygen	0·0014
Ammonia and water	0·0139
	0·0203

Powder constituents not decomposed or burnt—

Saltpetre	0·0372
Carbon	0·0073
Sulphur	0·0014
	0·0459

Hence—

Chief products	0·9282
By-products	0·0203
Unburnt powder	0·0459
	0·9944

---

* Inclusive of the carbonic acid of the ammonic carbonate.

If the potassic hyposulphite is replaced by its equivalent of potassic disulphide, the quantities of the chief products expressed in molecular weights, and calculated for 16 mols. of decomposed saltpetre, we obtain:

$$1{\cdot}97K_2CO_3+5{\cdot}25K_2SO_4+0{\cdot}45K_2S_2+10{\cdot}59CO_2+7{\cdot}7N_2+0{\cdot}33K_2S+0{\cdot}71CO.$$

From these symbols we calculate the following composition of the powder:

$$16KNO_3+13{\cdot}2C+6{\cdot}48S+0{\cdot}66O.$$

If now we substitute, in equation (VIII.), for x the number 16, for y 13·2, for z 6·5, and for a 0·71, we find for the chief products:

$$2{\cdot}25K_2CO_3+5{\cdot}01K_2SO_4+0{\cdot}74K_2S_2+10{\cdot}25CO_2+0{\cdot}70CO+8N_2$$

numbers which closely agree with those found by experiment.

The powder used by Bunsen and Schischkoff contains, according to analysis:

$$16KNO_3+13{\cdot}3C+6{\cdot}3S$$

a composition which is nearly the same as that deduced from the chief products of combustion.

The analytical method in this case causes no appreciable error in the determination of the potassic sulphate. Neither, according to my own analysis, does potassic monosulphide, nor, according to Noble and Abel's analysis of the residues of mining powder, does potassic disulphide produce potassic sulphate by treatment with cupric oxide. Bunsen and Schischkoff's residues contained a mixture of mono- and disulphide, and of both only a comparatively small quantity.

Linck[*] examined the products of combustion of the Würtemberg service powder according to the method employed by Bunsen and Schischkoff. He obtained the following results:—

Composition of the powder—

Saltpetre	0·7470
Sulphur	0·1245
Charcoal—	
Carbon	0·0905
Hydrogen	0·0041
Oxygen	0·0278
Water	0·0060
	0·9999

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[*] Ann. der Chemie und Pharm., Bd. cix., p. 53.

Chief products of combustion—

Potassic sulphate	0·2891
„ carbonate	0·1537
„ hyposulphite	0·0374
„ disulphide	0·0959
Carbonic acid*	0·2345
„ oxide	0·0118
Nitrogen	0·0952
	0·9176

By-products—

Potassic sulphocyanate	0·0116
Sulphuretted hydrogen	0·0238
Hydrogen	0·0003
Oxygen	0·0001
Ammonia and water	0·0098
	0·0456

Powder constituents not decomposed or burnt—

Saltpetre	0·0120
Carbon	0·0183
Sulphur	0·0031
	0·0334

Hence—

Chief products	0·9176
By-products	0·0456
Not burnt	0·0334
	0·9966

If we express the quantities of the chief products by means of molecular weights, and calculate how much of each would be formed by the combustion of a quantity of powder containing 16 mols. of saltpetre, we obtain :

$$2{\cdot}43K_2CO_3+3{\cdot}64K_2SO_4+0{\cdot}43K_2S_2O_3+1{\cdot}47K_2S_2+11{\cdot}71CO_2+0{\cdot}92CO+14{\cdot}94N$$

and expressed in powder constituents :

$$16KNO_3+15{\cdot}06C+7{\cdot}44S-1{\cdot}06N-0{\cdot}52O.$$

The powder contained, according to analysis :

$$16KNO_3+16{\cdot}3C+8{\cdot}4S.$$

---

* Inclusive of the carbonic acid in the ammonic carbonate.

The by-products contain no oxygen of the saltpetre, and with the exception of the potassium and nitrogen in the potassic sulphocyanate, and the nitrogen of the ammonia, their elements are derived from the charcoal and sulphur. The chief products should, accordingly, contain all the oxygen of the decomposed saltpetre. This, however, is not the case; 0·52 of an atom is wanting, and if the potassic hyposulphite, as we must assume, has been formed by the oxidising action of the cupric oxide upon the potassic disulphide, then no less than 1·81 atoms or nearly $\frac{1}{26}$th of the oxygen of the decomposed saltpetre have disappeared. LINCK himself finds in the products of combustion 1·75 per cent. or about $\frac{1}{22}$nd less oxygen than in the original powder, and this loss of oxygen would have appeared still greater if he had not assumed that the oxygen of the hyposulphite had been derived from the saltpetre. On the other hand, LINCK finds in the products of explosion 0·71 per cent. or $\frac{1}{12}$th to $\frac{1}{13}$th more of carbon, and 0·9 or $\frac{1}{13}$th to $\frac{1}{14}$th more of sulphur than in the original powder. Accordingly, it follows, either that LINCK's powder has not the composition which he ascribes to it, or that some considerable errors attach to the analytical data of the products of explosion. Hence, no near agreement can here be expected between theory and experiment.

If we replace the potassic hyposulphite found, by its equivalent of potassic disulphide, express the quantity of the chief products by molecular weights, and calculate the composition of that portion of the powder which was transformed into the chief products, we obtain

$$16KNO_3+15{\cdot}06C+7{\cdot}44S-1{\cdot}06N-1{\cdot}81O.$$

and if in equation (VIII.) we place,

$$x=16 \qquad y=15$$
$$z=7{\cdot}5 \qquad a=1{\cdot}0$$

we find the following theoretical values, which are placed by the side of the experimental numbers

		Theory.	Experiment.
$16KNO_3$ $15C$ $7{\cdot}5S$ =	K_2CO_3 . . .	2·14	2·43
	K_2SO_4 . . .	4·21	3·64
	K_2S_2	1·64	1·90
	CO_2	11·85	11·71
	CO	1·0	0·92

Some mistake appears to have occurred in the determination of the potassic sulphate and carbonate. The calculation could likewise be carried out in the following manner: The unburnt portion of the powder, and the quantities of the elements contained in the by-products, could be subtracted from the composition of the quantity of powder taken for experiment, and from the rest, the quantities of the products might be calculated according to equation (VIII.). This mode of calculation is in LINCK's case not applicable, because large quantities of charcoal remained unburnt, the composition of which is not known. This charcoal having been exposed to the high temperature developed by the combustion of the powder can no longer have had the original composition, but was probably nearly pure carbon.

The composition of the Austrian cannon powder can be represented by the symbols :

$$16KNO_3+19{\cdot}8C+8{\cdot}7S$$

if hydrogen, oxygen, ash, and moisture of the charcoal are neglected. All the potassium and the oxygen of the decomposed saltpetre reappeared in the chief products of explosion, but more than 4 atoms of carbon and 3 atoms of sulphur remained free. The amount of the sulphide formed by this powder was very small in KAROLYI's experiment.

The powder constituents which were transformed into the chief products are represented by the symbols

$$16KNO_3+14{\cdot}32C+5{\cdot}28S$$

and 2·04 mols. of CO had been formed (p. 549).

The values of these coefficients substituted for *x*, *y* and *z* in equation (VIII.) yield the following theoretical quantities of the chief products of combustion, to which are appended the quantities found by experiment :—

		Theory.	Experiments.
$16KNO_3$ $14{\cdot}32C$ $5{\cdot}24S$ =	K_2CO_3 . . .	3·06	3·05
	K_2SO_4 . . .	4·59	4·61
	K_2S_2	0·33	0·33
	CO_2	9·12	9·23
	CO.	2·04	2·04
	N_2.	8·0	7·55

a striking confirmation of theory is thus presented.

The analysis of the products of explosion of rifle powder carried out by KAROLYI is contaminated by a considerable error. He finds, correctly, no oxygen in the by-products ; but no less than 4 atoms or $\frac{1}{12}$th more oxygen appears in the chief products than in the decomposed saltpetre, and if from this the oxygen of the charcoal be subtracted there remain still 1·66 atoms of oxygen more than were contained in the entire powder. If these errors of experiment are corrected, by means of equation (VII.), a very good agreement of the numbers calculated according to equation (VIII.), and those derived from experiment, is obtained for rifle powder.

The mean composition of the powders of Waltham Abbey is expressed by the symbols

$$16KNO_3+21{\cdot}18C+6{\cdot}63S$$

and because, during their combustion, 0·67 mol. of sulphuretted hydrogen is formed, there remain

$$16KNO_3+21{\cdot}18C+5{\cdot}96S$$

for the formation of the principal products.

But as the portion of the sulphur which has united with the iron of the apparatus has not been determined by direct experiment, we are obliged to form an estimate of its amount from other considerations.

According to the remarks on pages 557–559, potassium disulphide is produced by the metamorphosis of gunpowder. The combustion of a quantity of powder containing 16 mols. of saltpetre produces, if we take the mean of all the experiments of NOBLE and ABEL, 0·90 mol. of K_2SO_4 and 2·1 mols. of K_2S_2, for which quantities 1 mol. of K_2SO_4 and 2 mols. of K_2S_2 have been placed in equation (II.). From this it follows that 5 atoms of sulphur have taken part in the metamorphosis, and that 0·96 atom of sulphur has remained free or has united with the iron of the explosion apparatus.

The experiments of KAROLYI support this conclusion. The Austrian cannon powder containing 8·7 atoms, and the rifle powder 5·66 atoms of sulphur for every 16 mols. of saltpetre, and in the principal products of combustion of the former 5·25 atoms, and in those of the latter 4·89 atoms of this element were found, the rest of the sulphur having remained free. In spite of the great difference of the amounts of sulphur in the two descriptions of powders, we find in their products of combustion, for 16 mols. of decomposed saltpetre, almost the same quantity of sulphur, 5 atoms, in both cases. In NOBLE and ABEL'S experiments the sum of the sulphur in the sulphate and disulphide is likewise equal to 5 atoms. From this equality we may conclude that in NOBLE and ABEL'S, as in KAROLYI'S experiments, during the first stage of combustion, or the stage of explosion, the powders were transformed according to equation (IV.), page 553, and that 5 of the 5·96 atoms of sulphur entered into combination. The remaining 0·96S should, during the second stage, have reacted with potassic carbonate according to equation (V.), page 559, but as this reaction appears not to have occurred, we may conclude, with great probability, that the 0·96 atom of sulphur united with the metal of the apparatus.

Since in NOBLE and ABEL'S experiments all the powder introduced into their apparatus was completely burnt, and as the sum of the weights of the secondary products, after deducting the sulphuretted hydrogen, is very small, and finally, because potassium, oxygen, and carbon in the principal products of combustion occur almost in the same proportions as in the saltpetre and charcoal of the original powder, we can substitute for x and y, in equation (VIII.), the values derived directly from the composition of the powder.

If then we put in equation (VIII.) $x=16$, $y=21·18$, $z=5$, and for a, the carbonic oxide, the number 3·23, found by experiment, we obtain for the principal products values which in the following table have been placed by the side of those found by experiment :—

		Theory.	Experiment.
$16KNO_3 + 21·18C + 5S =$	K_2CO_3	5·01	4·98
	K_2SO_4	0·96	0·90
	K_2S_2	2·01	2·10
	CO_2	12·93	13·13
	CO	3·23	3·23
	N_2	8·00	8·67

The theoretical numbers agree in a very satisfactory manner with those found by experiment.

According to what has been stated in these pages, we conceive the metamorphosis of gunpowder to take place in a shell, or in the bore of a gun, in the following manner. In the first moments after ignition, during the explosion, powders of different composition burn according to the equation

$$16KNO_3+13C+5S=3K_2CO_3+5K_2SO_4+9CO_2+CO+8N_2$$

and in case of a shell which will burst almost immediately and its contents be scattered about, no further changes take place.

In the bore of a gun the gases expand, move the shot, and by the performance of this work lose a portion of their energy; the products of the first stage of the metamorphosis, potassic carbonate and potassic sulphate, remain at a red heat, in a fluid condition, for a longer time in contact with free carbon and sulphur, and produce, according to equations (V.) and (VI.), an additional quantity of carbonic acid. This carbonic acid, which is generated during the movement of the shot in the bore, prevents the too rapid diminution of the tension of the gases; the heat of the solid products is, in part, transformed into *vis viva* of the gas molecules. If the gun were long enough and the quantities of carbon and sulphur not too large, every atom of the former might be oxidised by the oxygen of the potassic sulphate, and the entire amount of the sulphur be converted into potassic disulphide and sulphate by contact with potassic carbonate. But in reality this second stage of the metamorphosis is perhaps never complete; the shot will have left the gun before the termination of these comparatively slow reactions.

The mining powders, strictly considered, do not belong to the category of gunpowders; they contain a large excess of carbon and sulphur. But as their metamorphosis clearly shows the source of the by-products of the combustion of gunpowder, we will discuss here the analytical data furnished by NOBLE and ABEL of an experiment with a sample obtained from CURTIS and HARVEY.

COMPOSITION of the powder

	Saltpetre	61·66
	Sulphur	15·05
Charcoal	Carbon	17·93
	Hydrogen	0·66
	Oxygen	2·23
	Water	1·66
		99·19

which may be represented by the symbols:

$$16KNO_3+12{\cdot}3S+\underbrace{39{\cdot}18C+16{\cdot}16H+3{\cdot}65O}_{\text{charcoal.}}$$

This mixture contains sulphur and carbon nearly in the same proportions as they occur in the service powder of Waltham Abbey [S : C=1 : 3·21], and therefore differs by containing much less saltpetre.

The products of combustion, calculated for 16 mols. of decomposed saltpetre, yielded the following results:—

$$\begin{array}{llll} & 3{\cdot}66K_2CO_3+13{\cdot}47CO_2 & +2{\cdot}96H_2S & +2C \\ +0{\cdot}04K_2SO_4 & +14{\cdot}10CO & +1{\cdot}11CH_4 & +1{\cdot}27S \\ +4{\cdot}11K_2S_2 & +\ 7{\cdot}96N_2 & +2{\cdot}2H_2 & \\ +0{\cdot}36KCNS & & +0{\cdot}08[NH_4]_4H_2(CO_3)_3 & \end{array}$$

from which we calculate the composition of the powder as follows:—

$$16KNO_3+12{\cdot}85S+\underbrace{34{\cdot}94C+16{\cdot}2H+4{\cdot}42O}_{\text{charcoal.}}$$

If we contrast the above results with those obtained by the explosion of the service powders of Waltham Abbey, (I.), page 545, it is seen that the large excess of carbon and sulphur in the mining powder has not diminished the amount of carbonic acid, but greatly increased the quantity of carbonic oxide. In the service powders, the oxygen of the charcoal is eliminated, with hydrogen, as water; in the mining powder it is found at the end of the combustion in union with carbon as carbonic oxide. The hydrogen of the charcoal thus set free partly remains so, partly unites with carbon and nitrogen respectively, forming marsh gas and ammonia, and during the cooling of the products, at a lower temperature, gives rise to the generation of much sulphuretted hydrogen.

In consequence of the great excess of charcoal and sulphur, carbonic oxide, marsh gas and sulphuretted hydrogen are, calculated for 16 mols. of decomposed saltpetre, from four to five times greater in the products of the mining powder than in those of the service powder. The gases of the former are combustible, those of the latter are not.

The potassic sulphocyanate has been formed by the metamorphosis of CURTIS and HARVEY'S powder in quantities ten times as large as were observed amongst the products of the Waltham Abbey mixtures. It is well known that potassic carbonate, sulphur and charcoal, at a white heat, in an atmosphere containing nitrogen, will produce potassic sulphocyanate. The amount of potassic sulphate, as might be expected, present among the products of the mining powder is almost *nil*, and the source of the by-products of the combustion of gunpowder is laid bare.

If we discard the by-products, we obtain for the combustion of a powder with an excess of carbon the equation:

$$16KNO_3+28C+8S=4K_2CO_3+4K_2S_2+12CO_2+12CO+8N_2$$

or more simply:

$$4KNO_3+7C+2S=K_2CO_3+K_2S_2+3CO_2+3CO+2N_2$$

which represents with sufficient exactness the reactions between saltpetre, carbon, and sulphur, when an excess of carbon is present and is introduced in the form of charcoal. Carbonic acid and oxide have been found by experiment to be a little higher, in consequence of the action of the oxygen of the charcoal, potassic carbonate a little lower, in consequence of the formation of some potassic sulphocyanate, than is required by the foregoing equation.

It is also worthy of notice that in spite of the presence of free carbon, more than 13 mols. of CO_2 have remained undecomposed.

From the remarks of the preceding pages it follows that during the complete metamorphosis of powder, the reactions between the constituents of saltpetre, the carbon of the charcoal, and sulphur, take place according to equations (III.), (IV.), (V.), (VI.), and (VII.), and that by means of equation (VIII.) the products, namely, potassic carbonate, potassic sulphate, potassic disulphide, carbonic acid and nitrogen, which are formed during the combustion of a weight of powder containing x mols. of saltpetre, y atoms of carbon, and z atoms of sulphur, can be calculated in a satisfactory manner. It now remains to calculate, by means of the same equation, the quantities of heat, gas, and energy which powders of various composition are able to produce.

For this purpose we assume that we have to deal with mixtures of saltpetre, sulphur, and pure *carbon*, and that the combustion is complete, viz.: that it runs to the end of the second stage. If we conceive that during the transformation of the powder no carbonic oxide is formed, we should, as a consequence, have a considerable simplification of equation (VIII.) without influencing much the calculated amounts of gas and heat.

The conversion of the carbonic oxide to carbonic acid could only take place at the expense of the oxygen in the potassic sulphate; if it occurred according to the equation

$$K_2SO_4+4CO=K_2S+4CO_2$$

the volume of the gas would not be changed. But since potassic disulphide is formed, we have to base our calculations on equation

$$2K_2SO_4+7CO=K_2CO_3+K_2S_2+6CO_2$$

from which it follows that, if no carbonic oxide but only carbonic acid is produced, the volume of the entire gas will be diminished by $\frac{1}{7}$th of the volume of the carbonic oxide which in reality is formed.

The greatest amount of carbon in gunpowders generally, as far as I know, is contained in the mixtures of Waltham Abbey, and these also produce the largest quantity of carbonic oxide, 3 mols. or 6 vols. for every 16 mols. of decomposed saltpetre. In addition to 3 mols. of carbonic oxide, 13 mols. of carbonic acid and 8 mols. of nitrogen are generated, which together amount to 24 mols. or 48 vols. of gas.

Now, if in place of carbonic oxide, carbonic acid had been formed, the volume of the entire gas would have been 47·14 instead of 48 vols. In other words, if we frame our calculation on the assumption that only carbonic acid and no oxide has resulted from the combustion, we shall find for the English service powders 1·8 per cent. less gas than was actually obtained by experiment. And as other descriptions of powder contain less carbon than those of Waltham Abbey, in their case the error will be smaller than 1·8 per cent. If then we calculate the volumes of gas which mixtures of saltpetre, carbon, and sulphur in various proportions will produce, on the assumption that no carbonic oxide, but only carbonic acid is formed, we shall obtain numbers that will not differ much from the sum of the volumes of carbonic acid, carbonic oxide, and nitrogen produced by gunpowders containing corresponding quantities of saltpetre, carbon, and sulphur.

By adding the coefficients of carbonic acid and nitrogen of equation (VIII.), and putting $x=16$, and $a=0$, we obtain for the sum, G, of the molecules of carbonic acid and nitrogen, which a mixture of 16 mols. of saltpetre, y atoms of carbon, and z atoms of sulphur, by its complete combustion, can produce, the equation

$$G=\frac{160+20y+16z}{28}$$

and for the volume, V

$$V=\frac{160+20y+16z}{14} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (IX.)$$

In BUNSEN and SCHISCHKOFF's experiment 16 mols. of saltpetre, 13·3 atoms of carbon, and 6·3 atoms of sulphur were consumed in the formation of the chief products of combustion (page 562). The values $y=13·3$, $z=6·3$, placed in equation (IX.), give $V=37·62$. Now 16 mols. of saltpetre, 13·3 atoms of carbon, and 6·3 atoms of sulphur correspond to 1977·2 parts by weight, and if these parts are expressed in grammes, then

1 vol. of gas will be equal to 11·19 litres, and 37·62 vols.=420967·8 cub. centims. Hence, 1 grm. of the powder would produce 212·9 cub. centims. of gas.

But only 92·8 per cent. of the powder was transformed, according to equation (VIII.), therefore

$$\frac{212{\cdot}9 \times 92{\cdot}8}{100} = 197{\cdot}5 \text{ cub. centims.}$$

Bunsen and Schischkoff found 193·1 cub. centims. If we deduct from this number 4·5 cub. centims., the volume of the gaseous by-products, hydrogen, sulphuretted hydrogen, and oxygen, and add 7·4 cub. centims. for the carbonic acid of the ammonic carbonate, we obtain 196 cub. centims. for the gas found by experiment.

Hence:

Experiment.	Theory.
196 cub. centims.	197·5 cub. centims.

The chief products of the combustion of Linck's powder contain, according to his analysis, the powder constituents in the proportion

$$16KNO_3 + 15{\cdot}06C + 7{\cdot}44S.$$

If we substitute in equation (IX.) for y the number 15, and for z 7·5, we obtain $V=41{\cdot}42$. $16KNO_3+15C+7{\cdot}5S$ corresponds to 2036 parts by weight. Expressed in grammes 1 vol. of the gas will be equal to 11190 cub. centims., therefore 41·42 vols. =463489·8 cub. centims., and 1 grm. of the powder would yield 227·6 cub. centims.

As only 91·7 per cent. of the Würtemberg service powder was transformed, according to equation (VIII.), we have

$$\frac{227{\cdot}6 \times 91{\cdot}7}{100} = 208{\cdot}7 \text{ cub. centims. for the theoretical volume.}$$

Linck found 218·35 cub. centims.; but from this must be subtracted 15·67 cub. centims. for sulphuretted hydrogen, 3·56 cub. centims. for hydrogen, and 0·09 for oxygen, leaving 199·3 cub. centims. of gas as the product of combustion of 1 grm. of powder according to equation (VIII.). Adding 5·8 cub. centims. for carbonic acid in ammonic carbonate, we obtain

Experiment.	Theory.
205·1 cub. centims.	208·7 cub. centims.

If the difficulties which have to be overcome in order to obtain exact results in the determination of the products of explosion of gunpowder are considered, the differences between the theoretical and experimental numbers appear to come within the errors of observation.

In Noble and Abel's experiments 16 mols. of saltpetre, 21·35 atoms of carbon, and 5·1 atoms of sulphur were transformed according to equation (VIII.). If in equation (IX.) for y the number 21·35, and for z the number 5·1 is substituted, the volume of gas is found to be 47·73.

$16KNO_3+21{\cdot}35C+5{\cdot}1S$ are equal to 2035·4 parts by weight, and 47·73 vols.= 534098·7 cub. centims., if the weight is expressed in grammes, therefore: 1 grm. of the powder yields 262·4 cub. centims. of gas.

On an average, 93·75 per cent. of the weight of the powder was transformed, according to equation (VIII.), hence we have:

$$\frac{262{\cdot}4 \times 93{\cdot}75}{100}=246{\cdot}0 \text{ cub. centims.}$$

for the theoretical volume of gas formed by the combustion of 1 grm. of powder. The mean of Noble and Abel's observations is 268·7 cub. centims. From this number 14·5 cub. centims. must be subtracted for the volumes of sulphuretted hydrogen, marsh gas, and hydrogen, leaving 254·2 cub. centims. for the carbonic acid, carbonic oxide, and nitrogen. Therefore we obtain, for the volume of the gas produced by 1 grm. of service powder from Waltham Abbey,

Experiment.	Theory.
254·2 cub. centims.	246 cub. centims.

the difference between the two numbers is only 1·9 cub. centim. greater than the greatest difference between two observations made with R. L. G. powder.

The three descriptions of powder, Bunsen and Schischkoff's, Linck's, and Noble and Abel's, contain from 13 to 21 atoms of carbon, and from 6·3 to 8·4 atoms of sulphur for every 16 mols. of saltpetre, and are good representatives of gunpowder in general.

The values calculated for the volumes of the gases furnished by these powders are near enough to those found by experiment to show the correctness of the theoretical considerations on which equation (IX.) has been framed, and to justify the use of this equation for the determination of the volume of gas produced by mixtures which contain saltpetre, carbon, and sulphur in proportions different from those of the Waltham Abbey, the Würtemberg, or Bunsen and Schischkoff's powder.

The amount of heat generated by the combustion of a mixture of saltpetre, *pure* carbon, and sulphur can be found in the following manner:

If we assume, as in the case of the calculation of the gas, that no carbonic oxide is formed, that is to say put $a=0$, multiply the heat of formation of each product with its coefficient in equation (VIII.), add the products thus formed and subtract from the sum the heat of formation of saltpetre: the difference will be equal to the heat generated by the combustion of the mixture.

Heat of formation of 1 mol. $K_2CO_3 = 279530$*
,, ,, ,, $K_2SO_4 = 344640$†
,, ,, ,, $K_2S_2 = 108000$‡
,, ,, ,, $CO_2 = 97000$§
,, ,, ,, $KNO_3 = 119480$||

Hence we obtain for the heat of combustion, W

$$\left[\tfrac{4}{28}x+\tfrac{8}{28}y-\tfrac{16}{28}z\right]279530+\left[\tfrac{20}{28}x-\tfrac{16}{28}y+\tfrac{4}{28}z\right]344640$$

$$+\left[-\tfrac{10}{28}x+\tfrac{8}{28}y+\tfrac{12}{28}z\right]108000+\left[-\tfrac{4}{28}x+\tfrac{20}{28}y+\tfrac{16}{28}z\right]97000$$

$$-x\times 119480=\mathrm{W}\text{ ; or if } x=16,$$

$$\mathrm{W}=1000[1827{\cdot}154-16{\cdot}925y-8{\cdot}788z]\ .\ .\ .\ .\ .\ .\ .\ .\ \text{(X.)}$$

That is to say, a mixture of 16 mols. of saltpetre, y atoms of carbon, and z atoms of sulphur, will, by its complete transformation according to equation (VIII.), produce W units of heat.

An error attaches to W in consequence of the assumed non-formation of carbonic oxide.

The quantity of this substance produced by the Waltham Abbey powders is greater than that formed by other mixtures, but as the error attaching to W in the case of the English service powders does not amount to more than 2·6 per cent. of the total heat, an error smaller than the usual errors of observation, it may be neglected for the sake of the great simplification of the formula.

It is perhaps desirable again to call attention to the condition that the equations (IX.) and (X.) apply only to mixtures which contain their constituents in such proportions that they can completely transform themselves according to equation (VIII.). Mining powders are excluded.

The charcoal of gunpowder is, however, not pure carbon, but contains also hydrogen, oxygen, and water.

The high temperature generated by the explosion causes probably the dissociation of these elements, and if, as in NOBLE and ABEL's experiments, all the carbon is oxidised at the expense of the oxygen of the saltpetre, the oxygen of the charcoal will reunite with hydrogen and form water.

The heat absorbed by the decomposition of the charcoal is not known. A portion of the hydrogen unites with nitrogen, carbon, and sulphur respectively, forming ammonia, marsh gas, and sulphuretted hydrogen. The total heat which is either liberated or absorbed by all these secondary reactions, appears, however, to be a small

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* J. THOMSEN, 'Berichte der deutschen Chemischen Gesellschaft in Berlin,' Bd. xii., p. 2031.

† Ibid., p. 2032; Bd. xiii., p. 961.

‡ SABATIER, Comptes Rendus, tom. xc., 1557–1560; Chem. Soc. Journal, 1880, p. 689.

§ J. THOMSEN, 'Berichte der deutschen Chemischen Gesellschaft in Berlin,' Bd. xiii., p. 1329.

|| Ibid., p. 500.

quantity, when compared with the amount given off by the formation of potassic carbonate, potassic sulphate, potassic disulphide, and carbonic acid. The following condition has a greater influence on the heat of combustion of ordinary gunpowder.

The combustion ought to be complete; but in BUNSEN and SCHISCHKOFF'S as well as in LINCK'S experiments, a not inconsiderable portion of the powder remained unburnt. In every calorimetric determination all the products ought to be carefully examined, and this it seems was not done by those who have determined the heat of combustion of gunpowder. From the foregoing remarks we conclude that no close agreement can be expected between the heat of combustion calculated by means of equation (X.) for a mixture of 16 mols. of KNO_3, y atoms of C, and z atoms of S, and that generated by an ordinary gunpowder containing saltpetre, carbon, and sulphur in the same proportions.

In BUNSEN and SCHISCHKOFF'S powder we have for every 16 mols. of saltpetre, 13·3 atoms of C, and 6·3 atoms of S. If we substitute for y the number 13·3 and for z the number 6·3 in equation (X.), we obtain:

$$W=1546688 \text{ cal.}$$

$16KNO_3+13{\cdot}3C+6{\cdot}3S=1977{\cdot}2$ parts by weight, or one part of their powder would furnish 782 units of heat. BUNSEN and SCHISCHKOFF found 619·5. This number is, I believe, the result of one experiment made with 0·71 grm. of powder; it is evidently much too small. NOBLE and ABEL found, for the heat of combustion of the powders of Waltham Abbey, values which vary between 696 and 727 units. Their numerous calorimetric determinations were made by the combustion of the powders in the explosion apparatus, and several hundred grammes were used in each experiment. It is known that the combustion under such conditions is complete. But as the English powders contain much more carbon than BUNSEN and SCHISCHKOFF'S, or 4 per cent. less of saltpetre, they ought to have produced less heat. From these considerations it seems to follow that in BUNSEN and SCHISCHKOFF'S experiment a portion of the powder taken escaped combustion.

It has been shown that 16 mols. of saltpetre, 21 atoms of C, and 5 atoms of S take part in the metamorphosis of the powders of Waltham Abbey. If for y the number 21, for z the number 5, are substituted in equation (X.), the value of W is found to be equal to 1427789 cal.; but $16KNO_3+21C+5S=2028$ parts by weight, hence 1 grm. of powder would generate 704 units of heat.

NOBLE and ABEL found in the first series, comprising five experiments, numbers which give a mean of 702·34 units for 1 grm. of powder.

In another series of 19 experiments greater numbers were obtained than in the first. The mean of all 24 experiments is equal to 719·9 cal.

The theoretical number of 704 cal., however, corresponds to a mixture of saltpetre, sulphur, and pure carbon.

93·75 per cent. of the English service powders are transformed, according to equation

(VIII.), hence the calculated heat generated by the reactions between saltpetre, sulphur, and pure carbon of 1 grm. of English service powder is :

$$\frac{704 \times 93{\cdot}75}{100} = 660 \text{ cal.}$$

This theoretical quantity is 59·9 units, or 8·4 per cent. less than the amount found by experiment, a difference which would be much smaller if the amount of heat produced by the action of the sulphur upon the iron of the apparatus were known and could be subtracted from the experimental number. It is worthy of notice that the differences in the amounts of heat found in several experiments made with the same description of powder are nearly as great as the differences between the calculated and observed results.

A sample of powder manufactured by CURTIS and HARVEY, and marked No. 6, gave in four experiments

I.	II.	III.	IV.
732·9	744·9	755·7	784

units of heat, hence, between the first and last experiment a difference of 51·1 units.

Another heat determination may be here introduced. $16KNO_3+16{\cdot}4C+5{\cdot}5S$ of the Spanish pebble powder burnt according to the reactions on which equation (VIII.) is based, 16·4 for y, and 5·5 for z put in equation (X.), make $W=1501250$ cal.

As

$$16KNO_3+16{\cdot}4C+5{\cdot}5S$$

are equal to 1988·8 parts by weight, 1 grm. of the powder generates 754 cal.

If 5 per cent. are deducted for hydrogen, oxygen, and ash, we obtain :

$$\frac{754 \times 95}{100} = 716{\cdot}3 \text{ cal.}$$

for the calculated heat of 1 grm. of powder. Experiment gave 762·3, or 46 units more.

It follows as a general result from these considerations that the mean quantities of heat generated by the combustion of the English service and Spanish pebble powders are about 60 units greater than the theoretical values. If, however, the amount of heat generated by the action of the sulphur upon the iron of the explosion apparatus were known, and could be subtracted from the observed quantities of heat, the difference would become much smaller.

On the other hand, the theoretical numbers stand to each other nearly in the same ratios as do the corresponding experimental values.

The equations (VIII.), (IX.), and (X.) will now be used to determine the composition of an ideal powder, that is to say, of a powder composed of saltpetre, *pure* carbon, and sulphur, which shall, of all possible mixtures of this nature, possess the greatest energy; the results so obtained will be nearly correct for ordinary gunpowders. This would be the most general form of the problem of the explosion of gunpowder which could be proposed for solution to a chemist.

Equations (IX.) and (X.), viz.:—

$$V=\tfrac{160}{14}+\tfrac{20}{14}y+\tfrac{16}{14}z \quad \ldots\ldots\ldots\ldots\ldots \quad \text{(IX.)}$$

$$W=1000[1827{\cdot}154-16{\cdot}925y-8{\cdot}788z] \quad \ldots\ldots \quad \text{(X.)}$$

at once show that, if for a given weight of saltpetre, 16 mols., the carbon and sulphur of the mixture were allowed to increase, the volume of gas generated by combustion would likewise increase, but the quantity of heat will grow smaller, and if the carbon and sulphur be diminished the gas will also become less, but the heat of combustion will increase.

NOBLE and ABEL have called attention to the fact* that the products of heat and gas obtained by them in their various experiments with different descriptions of powder do not differ much from each other. The explanation of this interesting observation is to be found in equations (IX.) and (X.).

The work which can be performed by a given weight of powder will, *cæteris paribus*, be proportional to the volume of gas and amount of heat, respectively, which the powder can produce by its combustion, and hence, will be proportional to the product of both. This is, however, only approximately correct, because, if we have to compare the work which two powders of different composition can produce, the gases given off by the one will not contain the elements in the same proportion as those produced by the other; a portion of the energy developed will be consumed in the performance of interior work during the expansion of the carbonic acid. This portion is, however, very small.

The product of (IX.) and (X.) is:

$$10440{\cdot}88-12{\cdot}09y^2+1208{\cdot}39y-15{\cdot}95yz+993{\cdot}867z-5{\cdot}022z^2=\frac{W.V}{1000\times 2}=E\ . \qquad \text{(XI.)}$$

the factor 1000 in (X.) has been omitted, or W divided by 1000, and V by division by 2 has been converted from volumes into molecules.

The equations (IX.) and (X.) are based on the assumption that no carbonic oxide is formed during the combustion of a mixture of saltpetre, carbon, and sulphur. In consequence, the volume of gas calculated by formula (IX.) for a mixture of the composition like those of Waltham Abbey is 1·8 per cent. smaller, and the amount of heat according to equation (X.) 2·6 per cent. larger than it would have been if the carbonic oxide had been taken into consideration. These errors nearly compensate each other in the product E in the equation (XI.), so that the values of E are but *little* affected by putting a, the carbonic oxide, $=0$ in equation (VIII.).

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* Phil. Trans., 1880, p. 230.

Equation (XI.) can be used for the calculation of the relative energies of weights of powder containing 16 mols. of saltpetre, y atoms of carbon, and z atoms of sulphur.

The question now arises for what values of y and z will E in equation (XI.) assume a maximum value, provided that y and z render the coefficients of equation (VIII.) positive,—the condition which must be fulfilled in a chemical equation.

If we put $a=0$ in equation (VIII.), we obtain :

$$\left.\begin{array}{r} x\mathrm{KNO_3} \\ +y\mathrm{C} \\ +z\mathrm{S} \end{array}\right\} = \left\{\begin{array}{l} \tfrac{1}{28}[\quad 4x+\ 8y-16z](\mathrm{K_2CO_3}) \\ +\tfrac{1}{28}[\ \ 20x-16y+\ 4z](\mathrm{K_2SO_4}) \\ +\tfrac{1}{28}[-10x+\ 8y+12z](\mathrm{K_2S_2}) \\ +\tfrac{1}{28}[-\ 4x+20y+16z](\mathrm{CO_2}) \\ +\tfrac{1}{2}x\mathrm{N_2} \end{array}\right. \quad \ldots\ldots \quad \text{(XII.)}$$

in which, as in (VIII.), x, y, and z denote positive numbers. Let a rectangular coordinate system be given with its origin in point A, and the coordinates of a point P be represented by x, y, and z. The coefficients of potassic carbonate, sulphate, and disulphide in (XII.) will for certain values of x, y, and z be equal to 0. The equations :

$$\begin{array}{r} 4x+\ 8y-16z=0 \\ 20x-16y+\ 4z=0 \\ -10x+\ 8y+12z=0 \end{array}$$

satisfied by these values represent three planes which form a trihedral angle with its vertex in the origin and one edge in the x A y plane. The points within the trihedral angle have coordinates which will render all the coefficients of (XII.) positive, those situated outside give values for x, y, and z, which will make at least one of the three coefficients of the potassium salts negative. Hence, coordinates of the points within the trihedral angle denote quantities of saltpetre, carbon, and sulphur, which can transform themselves completely into potassic carbonate, sulphate, disulphide, carbonic acid, and nitrogen, whereas the points outside represent, by their coordinates, quantities of the powder constituents which cannot do so entirely, because one or the other of these constituents is in excess or defect.

The points on the faces of the trihedral angle correspond to mixtures which will burn with the production of two, those on the edges with only one of the three potassium salts.

But to show the connexion between the quantities of the constituents of a given powder and those of its products of combustion, we need only consider relative, and not absolute quantities.

If a straight line be drawn through the origin within the trihedral angle the ratios of the coordinates of every point upon it will be the same.

A plane at right angles to the x axis will cut the faces of the trihedral angle so as to form a triangle B D C (see fig. 1), and the coordinates of the points inside this triangle will represent all possible proportions of carbon and sulphur which can with a given weight of saltpetre transform themselves into the products of combustion indicated in equation (XII.). If then in (XII.) we attribute to x the constant value 16, we obtain :

$$\left.\begin{array}{l} 16KNO_3 \\ +yC \\ +zS \end{array}\right\} = \left\{\begin{array}{l} \tfrac{1}{28}[\quad 64+\ 8y-16z](K_2CO_3) \\ +\tfrac{1}{28}[\ 320-16y+\ 4z](K_2SO_4) \\ +\tfrac{1}{28}[-160+\ 8y+12z](K_2S_2) \\ +\tfrac{1}{28}[-\ 64+20y+16z](CO_2) \\ +8N_2 \end{array}\right. \quad \text{(XIII.)}$$

and from it the equations :

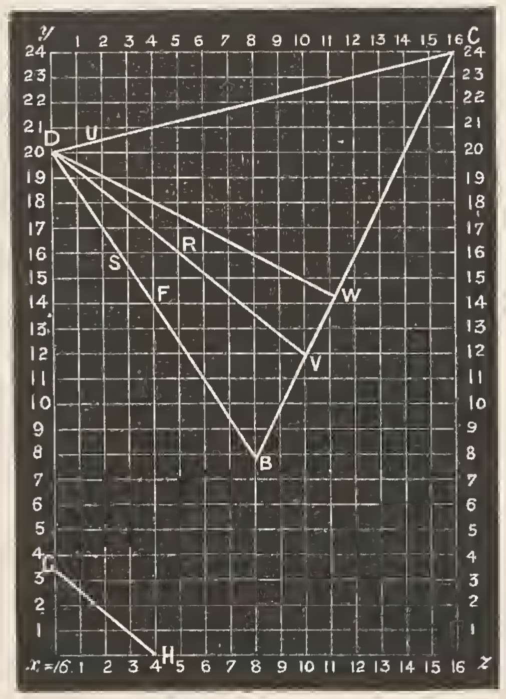
$$64+\ 8y-16z=0 \quad \text{(XIV.)}$$

$$320-16y+\ 4z=0 \quad \text{(XV.)}$$

$$-160+\ 8y+12z=0 \quad \text{(XVI.)}$$

of the lines of intersection of the plane at right angles to the x axis with the faces of the trihedral angle, in other words, the sides of the triangle B D C (fig. 1).

Fig. 1.



It will be observed that

(XIV.) is the equation of B C
(XV.) „ „ D C
(XVI.) „ „ B D.

The points of the side B C represent, by their coordinates, mixtures of carbon and sulphur with 16 mols. of saltpetre, which will by complete combustion produce no potassic carbonate; those of side D C such as will not form potassic sulphate, and finally, the coordinates of the points of side B D denote quantities of carbon and sulphur which will burn with 16 mols. of saltpetre without the formation of potassic disulphide. The coefficient of carbonic acid will never vanish, but be always positive, because if it is equated to zero it will represent the line G H in figure, which does not intersect the triangle B D C.

All points outside the triangle B D C have coordinates which render at least one of the three coefficients of the potassium salts in equation (XIII.) negative, and consequently have reference to mixtures of carbon and sulphur with 16 mols. of saltpetre, which contain either too much or too little of one or both of the two elements named.

The coordinates of the line B D represent mixtures of carbon and sulphur with 16 mols. of saltpetre, which will burn without the formation of potassic disulphide, and those of the line B C, such as will not produce potassic carbonate; hence, it may be concluded that the coordinates of B, the point of intersection of the two lines will correspond to a powder which will be transformed without formation of potassic carbonate and disulphide, and will only yield, as products of its combustion, potassic sulphate, carbonic acid, and nitrogen. The coordinates of point B are:

$$y=8$$
$$z=8$$

and these values substituted in equation (XIII.).

$$16KNO_3+8C+8S=8K_2SO_4+8CO_2+8N_2$$

or simplified:

$$2KNO_3+C+S=K_2SO_4+CO_2+N_2$$

Accordingly a mixture of 82·1 parts of saltpetre, 4·8 parts pure carbon, and 13 parts of sulphur may be expected to produce during complete combustion only potassic sulphate, carbonic acid, and nitrogen, and this conclusion is in perfect accord with the thermochemical relations of the reacting substances and with experimental results. And by a similar method of reasoning we arrive at the conclusion that the coordinates of the point D represent a mixture which will burn according to the equation:

$$16KNO_3+20C=8K_2CO_3+12CO_2+8N_2$$

or simplified :

$$4KNO_3+5C=2K_2CO_3+3CO_2+2N_2$$

and those of point C according to

$$16KNO_3+24C+16S=8K_2S_2+24CO_2+8N_2$$

or simplified :

$$2KNO_3+3C+2S=K_2S_2+3CO_2+N_2$$

if in this last equation the potassic disulphide be changed into monosulphide, then the equation would become identical with the old one, which for many years was supposed to represent the metamorphosis of all sorts of gunpowders.

As already observed, all points within the triangle represent, by their coordinates, mixtures of carbon and sulphur with 16 mols. of KNO_3, which besides carbonic acid and nitrogen, will yield during their combustion three potassium salts.

The geometrical construction of the coefficients of equation (XIII.), does not only offer the advantage of representing by the coordinates of the points within the triangle B D C all possible proportions of saltpetre, carbon, and sulphur which can transform themselves into potassic carbonate, potassic sulphate, potassic disulphide, carbonic acid and nitrogen, but it also enables us to deduce at once, geometrically, the quantities of these products of combustion.

If we desire to know the composition of all those mixtures which shall contain variable quantities of carbon and sulphur, but shall all produce by their combustion the same amount of potassic carbonate, we can deduce the answer from the following considerations :

For such mixtures the coefficient of the potassic carbonate in equation (XIII.) must assume a constant value. Hence,

$$64+8y-16z=c.$$

and

$$y=2z+\frac{c-64}{8}$$

the equation of a line parallel to the side B C of the triangle. The coordinates of the points of such a line indicate the composition of mixtures which will burn with production of the same amount of potassic carbonate. The amount of potassic carbonate is constant for each parallel line, but changes from one line to another. Now as they intersect the lines B D and D C, it is only necessary to ascertain the amounts of potassic carbonate corresponding to the points of one of these sides in order to know the amount of potassic carbonate formed by the combustion of a mixture represented by the coordinates of any point within the triangle.

Similar considerations lead to the equation :

$$y=\tfrac{1}{4}z+\frac{320-c}{16}$$

for mixtures which will burn with production of equal quantities of potassic sulphate.

This is the equation of a line parallel to the side D C. All these parallel lines intersect the line D B. If, then, we know the quantity of potassic sulphate corresponding to each point of D B, we shall likewise know the amount of this salt which any mixture, the composition of which is represented by the coordinates of one of the points of the triangle, can produce by its combustion. Now the powders, the composition of which is given by the coordinates of the points of B D produce by their combustion an amount of potassic sulphate which in molecules is directly expressed by the length of the corresponding abscissæ of the points.

Since x has been taken constant $=16$, the sum of the molecules of the potassium salts must always be $=8$, and as the points of the line B D represent only mixtures which burn with the production of two of these salts, potassic carbonate and sulphate, it is only necessary in order to know the respective quantities of each of these salts for a point F on B D, to subtract the value of the abscissa of F from 8 to obtain the molecules of potassic carbonate which would be produced by the combustion of a mixture the composition of which is given by the coordinates of F.

The coefficient of potassic disulphide in equation (XIII.) is $=-160+8y+12z$, from which we deduce the equation :

$$y=-\tfrac{3}{2}z+\frac{c+160}{8}$$

which is the equation of a line parallel to side B D. The points of such a line represent by their coordinates mixtures which will burn with the production of the same amount of potassic disulphide, which amount is constant for the same line, but changes from one to another. This amount is found for a mixture represented by the coordinates of a point P, if through P a line is drawn parallel to B D, and the abscissa of the point of intersection with the side D C is ascertained; half the length of this abscissa represents the number of molecules of potassic disulphide formed by the combustion of the mixture represented by point P.

For mixtures which are to burn with the evolution of the same quantity of carbonic acid we have :

$$-64+20y+16z=c$$

or

$$y=-\tfrac{4}{5}z+\frac{c+64}{20}$$

an equation which represents a line parallel to G H, on which for two points y, z and y', z', five times the difference of the ordinates is equal to four times the difference of the abscissæ.

The line D V in our figure is parallel to G H. In order to find the amount of carbonic acid which is developed by the combustion of a mixture the composition of which is represented by the coordinates of a point P, we have to draw through P a line parallel to D V or G H, and determine the length of the ordinate of the point of intersection with the side B C; this length is equal to the number of molecules of carbonic acid, because for all mixtures represented by the points of the side B C, the number of molecules of carbonic acid produced is equal to the number of atoms of carbon the mixtures contain.

We will now proceed to determine, by aid of the method just explained, the quantities of the products of combustion of a mixture the composition of which is represented by the coordinates of the point R on D V, $y=16$, $z=5$. A line drawn through R parallel to D C intersects D B in the point S; the abscissa of $S=3$, hence 3 mols. of potassic sulphate are produced.

A line drawn through R parallel to B C, cuts the side D B, in F; the abscissa of $F=4$; $8-4=4$; hence we obtain 4 mols. of potassic carbonate.

A line through R parallel to D B, intersects D C, in point U; the abscissa of $U=2$; $\frac{2}{2}=1$; hence 1 mol. of potassic disulphide is formed.

R is a point of D V, the ordinate of V, y, $=12$, hence we have 12 mols. of carbonic acid.

Nitrogen is for all mixtures a constant $=8N_2$, therefore the equation for the metamorphosis of a mixture, the composition of which is expressed by the coordinates of the point R is:

$$16KNO_3+16C+5S=4K_2CO_3+3K_2SO_4+K_2S_2+12CO_2+8N_2$$

The great advantage of the geometrical construction of the coefficients of equation (XIII.) consists in this, that we can at once ascertain by an inspection of figure B C D, the influence of all possible variations of the quantities of carbon and sulphur in given mixtures, upon the proportions of the corresponding products of combustion.

Similar considerations enable us to find the quantities of gas and heat.

If we add the constant 8 for nitrogen to the number of molecules of carbonic acid determined as previously explained, we obtain the total number of gas molecules produced by the combustion of a mixture represented by the coordinates of a given point.

The heat generated is found by equation (X.).

For powders which shall produce by their combustion the same amount of heat, we have:

$$1{\cdot}92y=-z+\frac{1827154-c}{8788}$$

for which we may adopt without serious error

$$2y=-z+\frac{1827154-c}{8788} \quad \ldots\ldots\ldots \quad \text{(XVII.)}$$

This is the equation of a line perpendicular to the side B C. For an appropriate value of c it becomes:

$$2y=-z+40$$

and then represents the line D W in the figure.

Mixtures, the composition of which can be represented by the points of such a line, will generate by their combustion very nearly the same amount of heat. A powder composed of $16KNO_3+20C$ corresponds to the point D, and one consisting of $16KNO_3+14{\cdot}4C+11{\cdot}25S$ to the point W. The first generates according to equation (X.) 1,488,654, and the second 1,484,569 units of heat, two numbers which differ only by 0·27 per cent., and may therefore be considered identical for practical purposes.

A line drawn through the point R, perpendicular to the side B C, intersects the latter in the point $y=13{\cdot}25$, $z=10{\cdot}6$; hence two powders composed of

$$16KNO_3+16C+5S$$
$$\text{and } 16KNO_3+13{\cdot}25C+10{\cdot}6S$$

will generate by their combustion the same, or more correctly, nearly the same amount of heat.

Consequently, if we know the heat of combustion of all the mixtures represented by the coordinates of the points of the line B C, then we know likewise the heat of combustion of all the mixtures the composition of which is represented by *any* point within the triangle. And we arrive at the same conclusion with regard to the amount of gas which a mixture can produce, the composition of which is represented by any point inside the triangle B C D. According to equation (X.) the heat of combustion reaches its maximum when y and z assume their smallest values, and, on the other hand, when y and z are greatest the heat of combustion will be a minimum. Therefore, an inspection of the triangle B D C teaches that of all the infinite number of mixtures of saltpetre, carbon, and sulphur which can be transformed according to equation (XIII.), the one which is composed of

$$16KNO_3+8C+8S$$

will produce by its combustion the greatest, and the one composed of

$$16KNO_3+24C+16S$$

the smallest quantity of heat: the first is represented by point B, the second by point C of the figure.

Further, it follows from equation (IX.) that the first of the above mixtures will form the smallest, and the second the largest quantity of gas.

If then we place ourselves at the point B of line B C, to which corresponds the generation of the greatest quantity of heat and that of the smallest quantity of gas, and move from B towards C, the amounts of heat produced by the mixtures represented by the coordinates of the several points will constantly decrease, and the volumes of gas increase, until the former reach in C their minimum, and the latter their maximum value.

We calculate for B and C

	Volume of gas.	Units of heat.
B . . .	32	1,621,450
C . . .	64	1,280,346

and between these numbers, 32 and 64 for the volume of the gas, and 1,280,346 and 1,621,450 for the units of heat, fluctuate the quantities of heat and gas which any possible mixture of 16 mols. of saltpetre with carbon and sulphur can produce, provided that these constituents, during combustion, transform themselves according to equation (XIII.). We will now show that the product, E, of the units of heat and the molecules of gas as given by equation (XI.), is greater for mixtures represented by points of line B C than for such as are represented by any other point within the triangle.

If we take, on the line D W, perpendicular to B C, the point $y=17$, $z=6$, then the mixture corresponding to this point and the one corresponding to point W will produce the same quantity of heat. The amount of gas generated by the mixture represented by point $y=17$, $z=6$, must be less than the quantity produced by the mixture corresponding to point W. Because, if we draw a line through $y=17$, $z=6$, parallel to D V, the point of intersection with B C will lie between W and V, but the further the point of intersection is away from W in the direction towards B the smaller the volumes of gas will be. Hence, the product of gas and heat for $y=17$, $z=6$ must be smaller than the one for point W, and the further we proceed from W towards D the smaller this product must be. But what holds good for the line D W also applies to every other perpendicular which can be drawn to B C.

Therefore the maximum value of the product of gas and heat must be produced by a mixture the composition of which is expressed by the coordinates of one of the points of the line B C.

If we represent the function on the right of the equation (XI.) by $F(y, z)$, and the equation of the line B C (XIV.) by $\phi(y, z)=0$, then the differential equation

$$\frac{dF}{dz}\cdot\frac{d\phi}{dy}-\frac{dF}{dy}\cdot\frac{d\phi}{dz}=0$$

together with $\phi(y, z)=0$ give the values of y and z, for which E in equation (XI.) becomes a maximum. We find $y=38{\cdot}02$ and $z=23{\cdot}0$. Hence, a powder composed of

$$16KNO_3+38C+23S$$

will by its complete combustion produce amounts of heat and gas the product of which will be the required maximum. But a mixture which shall transform itself according to equation (XIII.), can only contain per 16 mols. of saltpetre from 8 to 24 atoms of carbon, and from 8 to 16 atoms of sulphur. Hence, the product of the quantities of heat and gas will be a maximum for a powder composed of

$$16KNO_3+24C+16S$$

because according to the coefficients of equation (XI.) E will become greater and greater when y increases from 8 to 24, and z from 8 to 16, until it reaches its maximum value at a point $y=38$ and $z=23$, outside the triangle B C D.

For the purpose of calculating the values of E for different mixtures, we may simplify the coefficients and constant of equation (XI.), and write accordingly

$$10441-12{\cdot}1y^2+1208{\cdot}4y-16yz+994z-5z^2=E \quad . \quad . \quad . \quad \text{(XVIII.)}$$

E has been calculated by means of this equation for different values of y and z with the following results:—

$y=8,\quad z=8,\quad E=25941{\cdot}8$
$y=10,\quad z=9,\quad E=28416$
$y=12,\quad z=10,\quad E=30719{\cdot}4$
$y=14,\quad z=11,\quad E=32852$
$y=16,\quad z=12,\quad E=34813{\cdot}8$
$y=18,\quad z=13,\quad E=36604{\cdot}8$
$y=20,\quad z=14,\quad E=38225$
$y=22,\quad z=15,\quad E=39674{\cdot}4$
$y=24,\quad z=16,\quad E=40953{\cdot}0$

If, therefore, the carbon and the sulphur increase in different mixtures, the carbon by 2 atoms and the sulphur by 1 atom from

$$16KNO_3+8C+8S$$

to

$$16KNO_3+24C+16S$$

then parallel with this change of the carbon and sulphur, a regular increase of the product of heat and gas takes place, until for 24C and 16S it becomes a little more than one and-a-half times as great as for 8C and 8S.

If saltpetre and sulphur remain constant, and the carbon alone changes, then also an increase of the carbon is followed by one of E.

$y=8,\quad z=8,\quad E=25941{\cdot}8$
$y=16,\quad z=8,\quad E=32261{\cdot}8$
$y=18,\quad z=8,\quad E=33599{\cdot}8$
$y=22,\quad z=8,\quad E=35985{\cdot}8$

The energy of a powder of the composition:

$$16KNO_3+22C+8S$$

in about $\frac{2}{5}$ths greater than that of one containing:

$$16KNO_3+8C+8S.$$

If saltpetre and carbon are constant, but the sulphur changes, we obtain the following values for E:—

$y=14$, $z=4$, $E=27987{\cdot}0$
$y=14$, $z=11$, $E=32852{\cdot}0$

$y=16$, $z=6$, $E=30925{\cdot}8$
$y=16$, $z=8$, $E=32261{\cdot}8$
$y=16$, $z=12$, $E=34813{\cdot}8$

$y=18$, $z=8$, $E=33599{\cdot}8$
$y=18$, $z=13$, $E=36604{\cdot}8$

$y=20$, $z=0$, $E=29769$
$y=20$, $z=5$, $E=33014$
$y=20$, $z=9$, $E=35430$
$y=20$, $z=14$, $E=38225$

$y=22$, $z=8$, $E=35985{\cdot}4$
$y=22$, $z=15$, $E=39674{\cdot}4$

It follows from these examples that for a constant quantity of saltpetre, in varying mixtures of saltpetre, carbon, and sulphur, the relative energy of the mixtures increases with both the carbon and the sulphur, and reaches its maximum for 24 atoms of carbon and 16 atoms of sulphur, the highest amounts of these constituents which can exist in a powder according to equation (XIII.).

The difference of E for two mixtures of the same amount of saltpetre, but varying quantities of carbon and sulphur, becomes much smaller with equal weights of such mixtures. If, then, we multiply x, y, and z with their respective molecular or atomic weights, and divide E by the sum of the numbers so obtained, we find the relative energy, say E', of equal weights of various mixtures.

The following table gives the value of E' for mixtures which contain 16 mols. of KNO_3, y atoms of C, and z atoms of S.

$y=8$, $z=8$, $E'=13{\cdot}18$
$y=16$, $z=8$, $E'=15{\cdot}63$
$y=18$, $z=8$, $E'=16{\cdot}09$
$y=22$, $z=8$ $E'=16{\cdot}84$

$y=11,\qquad z=6,\qquad E'=13\cdot91$
$y=13,\qquad z=6,\qquad E'=14\cdot58$
$y=16,\qquad z=6,\qquad E'=15\cdot46$
$y=21,\qquad z=6,\qquad E'=16\cdot62$

$y=11,\qquad z=6,\qquad E'=13\cdot91$
$y=11,\qquad z=9\cdot5,\qquad E'=14\cdot41$

$y=14,\qquad z=4,\qquad E'=14\cdot63$
$y=14,\qquad z=8,\qquad E'=15\cdot11$
$y=14,\qquad z=11,\qquad E'=15\cdot38$

$y=16,\qquad z=2\cdot66,\qquad E'=15\cdot11$
$y=16,\qquad z=6,\qquad E'=15\cdot46$
$y=16,\qquad z=8,\qquad E'=15\cdot63$
$y=16,\qquad z=12,\qquad E'=15\cdot88$

$y=18,\qquad z=8,\qquad E'=16\cdot09$
$y=18,\qquad z=13,\qquad E'=16\cdot28$

$y=20,\qquad z=0,\qquad E'=16\cdot03$
$y=20,\qquad z=5,\qquad E'=16\cdot37$
$y=20,\qquad z=9,\qquad E'=16\cdot52$
$y=20,\qquad z=14,\qquad E'=16\cdot59$

$y=21,\qquad z=4,\qquad E'=16\cdot54$
$y=21,\qquad z=6,\qquad E'=16\cdot62$

$y=22,\qquad z=8,\qquad E'=16\cdot84$
$y=22,\qquad z=15,\qquad E'=16\cdot81$

$y=24,\qquad z=16,\qquad E'=16\cdot95$

It follows from these numbers that E' becomes greater when y or z, or both simultaneously, increase, but proportionately less so than is the case with weights of mixtures which contain equal weights of saltpetre, viz., 16 mols.

The smallest value of E' is 13·18, the highest 16·95; hence the latter is about 28 per cent. greater.

The highest value of E, on the other hand, is more than 50 per cent. greater than the lowest. Further, it is apparent that for mixtures for which y and z assume high values the differences of E' become very small.

The powder

$$16KNO_3+14C+4S$$

differs from

$$16KNO_3+14C+11S$$

by 7 atoms of sulphur.

The two mixtures

$$16KNO_3+22C+8S$$
$$\text{and } 16KNO_3+22C+15S$$

differ by the same amount of sulphur.

The two former show, for E′, the difference 0·75, the latter only 0·03; indeed, many of the various mixtures for which y and z have high values, give, when equal weights are considered, almost the same number for E′.

If we draw a line through the triangle B C D (see fig. 1), from the point $y=22$, $z=8$ to the point $y=8$, $z=8$, it will be observed that for mixtures represented by the coordinates of the points on the right-hand side of this line, the value of E′ only increases very little if the sulphur is increased beyond 8 atoms and the carbon kept constant. This circumstance is of great practical importance. The analyses of military and sporting powders known to me, all give for 16 mols. of saltpetre an amount of sulphur which varies between 5·5 and 8·7 atoms. There would be very little, if any, gain in energy if, for 16 mols. of saltpetre, more than about 8 atoms of sulphur were introduced into the powder; especially would this be the case with mixtures in which for 16 mols. of saltpetre more than 16 atoms of carbon are present.

E′ obtains its maximum value, 16·95, when the powder contains:

$$16KNO_3+24C+16S.$$

Such a large amount of sulphur does not, according to the foregoing remarks, contribute much to the value of E′, whereas, on the other hand, it must be very detrimental to the metal of the ordnance. For the mixture

$$16KNO_3+22C+8S$$

we have E′=16·84, hence, only 0·67 per cent. less than for $16KNO_3+24C+16S$.

If carbon and sulphur undergo a further diminution, the decrease of E′ becomes more rapid; for

$$16KNO_3+21C+4S$$

E′=16·54. If, therefore, we had to choose between the two mixtures

$$16KNO_3+24C+16S$$
$$\text{and } 16KNO_3+22C+8S$$

for the composition of a service powder the second would recommend itself as the more suitable.

We will now compare the composition and energy of the ordinary gunpowders with the results of the foregoing theoretical considerations.

The composition of the powders of Waltham Abbey can be represented by the symbols:

$$16KNO_3+21{\cdot}18C+6{\cdot}63S$$

which corresponds nearly to 75 parts of saltpetre, 10 parts of sulphur, and 15 parts of charcoal.

About these numbers fluctuate the compositions of the service powders of most nations.

Composition of gunpowders.

	Saltpetre.	Charcoal.	Sulphur.
England	75	15	10
Sweden	75	15	10
Russia	75	15	10
Prussia	74	16	10
Saxony	74	16	10
United States . . .	76	14	10
Austria	75·5	14·5	10

If, therefore, the composition of a gunpowder is required which shall possess nearly the greatest energy, and at the same time contain the smallest amount of sulphur compatible with this condition, an experience extending over 500 years has selected a mixture which contains saltpetre, carbon, and sulphur nearly in the theoretical proportions.

Composition of powders of Waltham Abbey. . $16KNO_3+21{\cdot}18C+6{\cdot}63S$
Theoretical composition. . . . . $16KNO_3+22C+8S$.

We concluded from Karolyi's experiments that the most inflammable and combustible mixture is represented by

$$16KNO_3+13C+5S$$

Bunsen and Schischkoff found in their sporting powder

$$16KNO_3+13{\cdot}3C+6{\cdot}3S$$

The value of E' for the proportions of saltpetre, carbon, and sulphur exhibited in the powders of Waltham Abbey is very nearly 16·62; for Bunsen and Schischkoff's sporting powder 14·58. Consequently 12·2 per cent. of the energy of the English service powder has been sacrificed in order to obtain the greater combustibility of the sporting powder.

According to composition, the service powders of France, Spain, Belgium, and Würtemberg are intermediate between the two powders just considered. They fluctuate about the proportions required by the symbols

$$16KNO_3+16C+8S$$

E' for these powders equals 15·63, or about 6 per cent. less than for the English; but they will, probably, be more inflammable and combustible than the latter.

It is worthy of notice that the points which represent, by their coordinates, the proportions of saltpetre, carbon, and sulphur in the gunpowders considered in this paper, are situated between two ordinates on our triangle B C D, for which respectively z assumes the values 5·5 and 8·7. The powders of Waltham Abbey, and BUNSEN and SCHISCHKOFF'S sporting powder contain per 16 mols. of saltpetre, nearly the same amount of sulphur; the former are represented by a point near the line D C, the latter by one near the line B D, of our figure B C D.

Summary of the main results.

1. The mean composition of the powders of Waltham Abbey can be represented by the symbols:

$$16KNO_3+21\cdot18C+6\cdot63S$$

A powder of this composition is transformed in NOBLE and ABEL'S apparatus according to the equation:

$$16KNO_3+21C+5S=5K_2CO_3+K_2SO_4+2K_2S_2+13CO_2+3CO+8N_2 \quad . \quad \text{(II.)}$$

The residue of the sulphur, 1·63 atoms, unites partly with hydrogen, partly with the iron of the apparatus.

2. The ordinary service and sporting powders contain for every 16 mols. of saltpetre from 13 to 22 atoms of carbon, and from 5·5 to 8·7 atoms of sulphur.

3. A powder composed of *pure* carbon, saltpetre, and sulphur furnishes by its *complete* combustion potassic carbonate, potassic sulphate, potassic *disulphide*, carbonic acid, carbonic oxide, and nitrogen, as chief products.

4. An increase of pressure appears, *cæteris paribus*, to diminish the amount of carbonic oxide, and, in consequence, according to equation (VIII.), to increase the quantities of potassic carbonate, potassic disulphide, and carbonic acid, and diminish that of potassic sulphate. These fluctuations depending on pressure are, however, *very small*. In NOBLE and ABEL'S Experiment No. 38, the pressure amounted to 18·6 tons, in Experiment No. 77 to 31·4 tons on the square inch, Experiment No. 38 gave for every 16 mols. of decomposed saltpetre 3·36 mols. of carbonic oxide, and Experiment No. 77, 2·9 mols. of this gas, or, for a difference of 12·8 tons in pressure, one of 0·46 mol. of carbonic oxide. A diminution of 0·5 mol. of carbonic oxide corresponds to one of 0·143 mol. in the amount of potassic sulphate, and an increase of 0·071 mol. in that of the potassic carbonate and disulphide, and 0·428 mol. in the quantity of carbonic acid. These fluctuations are probably not caused directly by the pressure, but by the differences in the rate of cooling after explosion.

5. The combustion of gunpowder takes place in two stages, one succeeding the other.

(*a*.) A process of oxidation during which potassic sulphate, carbonate, carbonic acid and nitrogen, and, *perhaps*, some carbonic oxide, but no potassic disulphide, are produced.

(*b*.) A process of reduction during which carbon and sulphur left free at the end of the first stage react with some of the products formed during that stage; the free carbon reducing potassic sulphate, with formation of potassic disulphide, potassic carbonate, and carbonic acid; the free sulphur decomposing potassic carbonate with the production of potassic disulphide, potassic sulphate, and carbonic acid [equations (V.) and (VI.)].

6. The first stage of the combustion, the explosion proper, takes place with powders of various composition according to equation:

$$10KNO_3+8C+3S=2K_2CO_3+3K_2SO_4+6CO_2+5N_2 \quad . \quad . \quad . \quad \text{(III.)}$$

But as some carbonic oxide is probably produced at the same time, the following will more correctly represent the reactions.

$$16KNO_3+13C+5S=3K_2CO_3+5K_2SO_4+9CO_2+CO+8N_2 \quad . \quad . \quad \text{(IV.)}$$

The constituents of the powder and the products of combustion are, according to (IV.), nearly in the same ratios as according to (III.).

7. The oxygen in the potassic carbonate stands to the oxygen in the potassic sulphate and carbonic acid, respectively, in equation (III.), in the most simple ratios which can exist, if these substances are to be produced by the combustion of a mixture of saltpetre, carbon, and sulphur. In other words, equation (III.) represents the most simple distribution of the oxygen of the decomposed saltpetre amongst the products of the first stage of the combustion. And because the products are, according to equation (IV.), nearly in the same proportions they assume to (III.), it follows that the distribution of the oxygen between potassic sulphate, carbonate, and carbonic acid, according to (IV.), nearly corresponds to the most simple possible distribution.

8. If the greatest possible amount of heat is to be evolved by the combustion of a mixture of saltpetre, carbon, and sulphur, and if at the same time potassic sulphate, carbonate, and carbonic acid are to be formed in such proportions that the heat of formation of one of them shall stand to the heat of formation of each of the others in the most simple ratio, then the combustion must take place according to equation (IV.).

The heat produced by the formation of 3 mols. of potassic carbonate stands to that produced by the formation of 5 mols. of potassic sulphate and 9 mols. of carbonic acid respectively, as

$$1 : 2{\cdot}05 : 1{\cdot}04$$

9. The ordinary gunpowders contain more carbon and sulphur than is required by equation (IV.).

This excess of carbon and sulphur is left free at the end of the first stage of the combustion.

The free carbon now acts according to equation:

$$4K_2SO_4+7C=2K_2CO_3+2K_2S_2+5CO_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad \text{(VI.)}$$

the free sulphur upon the potassic carbonate as follows :

$$4K_2CO_3+7S=K_2SO_4+3K_2S_2+4CO_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad \text{(V.)}$$

and both united form the second stage of the combustion. These reactions are endothermic ; heat is not evolved but consumed ; they are not of an explosive nature, and in practice are probably seldom complete.

The reactions of this second stage increase the volume of the gas formed during the first stage of the combustion and diminish the temperature of the products. A portion of the carbonic oxide is formed during the second stage by the action of free carbon or potassic disulphide upon carbonic acid.

10. The reactions represented by equations (III.), (IV.), (V.), and (VI.) can be expressed by one equation. If x, y, and z are positive numbers, and a indicates how many molecules of carbonic oxide are formed by the combustion of a weight of powder, containing x molecules of saltpetre, y atoms of carbon, and z atoms of sulphur, the following will be the general equation representing the complete chemical metamorphosis of powder.

$$\left.\begin{array}{l} xKNO_3 \\ +yC \\ +zS \end{array}\right\} = \left\{\begin{array}{l} \frac{1}{28}[4x+8y-16z-4a](K_2CO_3) \\ +\frac{1}{28}[20x-16y+4z+8a](K_2SO_4) \\ +\frac{1}{28}[-10x+8y+12z-4a](K_2S_2) \\ +\frac{1}{28}[-4x+20y+16z-24a](CO_2) \\ +aCO \\ +\frac{1}{2}xN_2 \end{array}\right.$$

11. If $x=16$, and $a=0$, the volume of the gas, (V.), generated by complete combustion is nearly

$$=\frac{160+20y+16z}{14} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad \text{(IX.)}$$

and the units of heat, W,

$$=1000[1827{\cdot}154-16{\cdot}925y-8{\cdot}788z] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad \text{(X.)}$$

y signifies in these equations the number of carbon and z that of the sulphur atoms in a weight of powder containing 16 mols. of saltpetre.

The volume of gas becomes greater and the heat of combustion diminishes with an increase of y and z, and *vice versâ*.

The mixture containing:

$$16KNO_3+8C+8S$$

produces the greatest amount of heat and the smallest quantity of gas, and the mixture represented by the symbols:

$$16KNO_3+24C+16S$$

the largest volume of gas and the smallest quantity of heat.

12. The product E obtained by the multiplication of V and W (equations IX. and X.) will approximately represent the relative energies of mixtures of various composition.

The mixture represented by the symbols

$$16KNO_3+24C+16S$$

is of all the infinite number of mixtures which can transform themselves according to equation (XIII.) the one for which E assumes the greatest value. Hence, a powder of this composition possesses the greatest energy.

13. If a mixture of saltpetre, carbon, and sulphur were required which shall possess nearly the greatest energy, and at the same time contain the smallest amounts of carbon and sulphur compatible with this condition, theory would point to the mixture:

$$16KNO_3+22C+8S$$

The service powders of most nations fluctuate about:

$$16KNO_3+21{\cdot}2C+6{\cdot}6S$$

14. Gunpowder, however, does not contain pure carbon, but besides this element hydrogen and oxygen as constituents of the charcoal.

The oxygen is eliminated with a portion of the hydrogen in the form of water, the remainder of the hydrogen remains either free or unites with carbon, sulphur, and nitrogen respectively, producing sulphuretted hydrogen, ammonia, and marsh gas.

These secondary products only amount from 1 to 2 per cent. of the powders.

15. Mining powders contain much more carbon than is required according to equation (XIII.). In consequence, the oxygen of the charcoal is not eliminated during the combustion of these powders with hydrogen, as water, but in combination with carbon as carbonic oxide and carbonic acid.

The hydrogen thus left free causes the formation of a comparatively large proportion of sulphuretted hydrogen and marsh gas. The potassic sulphocyanate is also produced in quantities much larger than those formed by the service powders on account of the carbon left free at the end of the combustion. If we neglect these secondary products, then the combustion of mining powder may be represented by the simple equation:

$$16KNO_3+28C+8S=4K_2CO_3+4K_2S_2+12CO_2+12CO+8N_2$$

The reactions can also be represented as follows:

$$16KNO_3+28C+8S=16KNO_3+22C+8S+6C$$

16 mols. of saltpetre, 22 atoms of carbon, and 8 atoms of sulphur transform themselves according to (XIII.) as follows:

$$16KNO_3+22C+8S=4K_2CO_3+4K_2S_2+18CO_2+8N_2$$

If, now, 6 atoms of carbon act on 6 mols. of CO_2 we obtain

$$6CO_2+6C=12CO$$

In this manner we can conceive the combustion of a mining powder to take place according to the same equations which apply to ordinary gunpowder, and that the excess of carbon in the mining powder causes, subsequently, the reduction of a portion of the carbonic acid to carbonic oxide.

XI. *On the Refraction of Plane Polarized Light at the Surface of a Uniaxal Crystal.*

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Received October 27,—Read November 17, 1881.

The laws of the reflexion and refraction of polarized light at the surface of a crystal in accordance with the electro-magnetic theory of light have been discussed by Lorentz (Schlömilch Zeitschrift, vol. xxii.), Fitzgerald (Phil. Trans., Vol. 171, 1880), and myself (Proc. Camb. Phil. Society, 1881). When a plane wave of electro-magnetic disturbance falls on the surface of separation between two different dielectric media six equations of condition are obtained. Three of these express the conditions that the electric displacement perpendicular to the surface and the electromotive force along the surface should be the same in the two media, while the other three do the same for the magnetic force and displacement. In all cases the six equations reduce to only four.

Let us suppose we know the amount and direction of the electric displacement in the incident wave. If both media are isotropic, these four equations give us the amounts and directions of the electric displacements in the reflected and refracted waves.

If the second medium is crystalline the possible directions of vibration in a wave travelling in it are known when the position of the wave is known; two of the equations as before give the amount and direction of the electric displacement in the reflected wave, the other two give the amounts of the displacements in the two refracted waves; the directions of these displacements being known from the position of the waves with reference to the axes of the crystal.

In general we have two refracted waves, the ordinary and extraordinary. Now, according to the electro-magnetic theory, light obeys the same laws as to propagation, reflexion, and refraction as this electro-magnetic disturbance, and the direction of the light vibrations coincides with that of the electric displacement, while the intensity of the light is measured by the energy of the disturbance. Our equations then give us the intensities of the two refracted rays which arise in general when a wave of polarized light falls on a crystal.

Consider now such an incident wave. Experiment tells us that there are two positions for its plane of polarization, in either of which one or other of the refracted waves disappears. The same result follows from the theory, and if we know the position of the face of incidence with reference to the axes of the crystal and also the directions of the two refracted wave normals, the theory enables us to calculate the angle between these two positions of the plane of polarization of the incident light.

But this angle is capable of direct measurement by experiment, and so the truth of the theoretical formulæ can be tested.

The experiment in its simplest form is as follows: a plane polarized beam of sodium light falls on a crystal of Iceland spar cut in the form of a prism, the position of whose faces relatively to the optic axis can be determined. The angle of incidence is observed and also the deviations of the two emergent wave normals.

From these data we can calculate the positions of the refracted wave normals in the crystal, and also of course the positions of the planes of polarization of the light travelling in these two directions respectively.

Let us suppose the polarizer to be a NICOL's prism, mounted in such a manner that the position of the plane of polarization of light emerging from it can be determined by means of a graduated circle attached to it; turn the NICOL until the extraordinary refracted wave disappears, then observe the position of the plane of polarization. But, there being only one ray, the ordinary, traversing the crystal prism, we can obtain from theory the azimuth of the plane of polarization of the incident light. Let us call this measured from some fixed plane θ_0. If the position of this fixed plane with reference to the NICOL can be found with accuracy we have here a means of comparing theory and experiment. We can eliminate the uncertainty in our knowledge of the relative position of the two planes of reference by turning the NICOL until the ordinary ray disappears, and reading the circle again; the difference between the two circle readings is the angle through which the plane of polarization has been turned; but the theory gives us again in this case, when there is only one refracted ray, the extraordinary, the value θ_E of the azimuth, measured from the same plane as before, of the plane of polarization of the incident light; the difference $\theta_0 \backsim \theta_E$ should be equal to the difference between the circle readings. Or again, having obtained as above a value for θ_0, alter the position of the spar prism so as to change the angle of incidence, and proceed as before; we can thus get a series of values of θ_0, the position of the plane of polarization of the incident light, for different angles of incidence as given by theory when the ordinary wave only traverses the crystal. But the readings of the polarizer circle give an experimental series of values of this same quantity and a comparison of these two series affords us a test of the theory. Since, in general, these two series of angles are not measured from the same zero point, there will, even if theory and experiment agree, be a constant difference between the two series depending on the difference of zeros. If the difference between corresponding values in the two series is not constant, but varies as the angle of incidence changes, we must infer that theory and experiment do not agree. We can test in the same manner the formula for the case in which the extraordinary wave only is propagated.

In practice, mainly in consequence of two difficulties, the experiments were conducted in a somewhat different manner. Unless certain rather elaborate conditions are fulfilled, the angle through which a NICOL'S prism is turned is not the angle through which the plane of polarization of the emergent light moves, and the difference between the two may, as I have shown (Phil. Mag., Nov., 1880, "On NICOL'S Prism"), be very considerable. Besides, when I began the experiments the only circle available was one graduated to 3′ of arc, which was not sufficiently accurate for my purpose. I allowed, therefore, plane polarized light to fall on my spar prism, and by turning it varied the angle of incidence until only one ray emerged. I observed the angle of incidence and the deviation, and from them calculated the azimuth of the plane of polarization of the incident light on the electro-magnetic theory. I then took a small cell with plane parallel glass sides and filled it with a weak solution of sugar; this I placed in the path of the incident light and thus produced a change in the position of its plane of polarization which brought the extraordinary ray into view again. This, by adjusting the angle of incidence on the spar prism, can again be made to vanish, and the angle of incidence and deviation being observed we can obtain a second value for the azimuth of the plane of polarization of the incident light. The angle between these two azimuths is the angle on the electro-magnetic theory through which the plane of polarization has been turned by the sugar cell. But this can be directly observed and the theory thus tested. Then I removed the sugar cell, altered somewhat the original plane of polarization, and again made the same observations, thus obtaining a series of values for the rotation produced by the cell corresponding to different angles of incidence on the spar prism.

The same observations were made using only the extraordinary ray. Thus the uncertainty arising from the want of adjustment and bad graduations of the polarizer circle was avoided.

The second difficulty was perhaps more serious. It was impossible to estimate with anything like sufficient accuracy the position of the spar prism for which the light of either ordinary or extraordinary ray was just quenched. To obviate this the apparatus was arranged as follows. The spar prism was mounted on the table of a spectrometer, kindly lent me by Professor STOKES, with a circle on silver and verniers reading to 10″. The instrument, and the method of adjusting the prism and focussing the telescope and collimator, have been described at length in my paper "On Plane Waves in a Biaxal Crystal" (Phil. Trans, 1879, p. 293). The sodium light was replaced by a strong source of white light, a powerful paraffin lamp, or the oxyhydrogen lime-light. A biquartz with the line of separation horizontal was placed between the polarizer and the collimator slit, and carefully adjusted by set screws, so that the light fell on it normally. Between the biquartz and the slit was placed a convex lens of about 20 centims. focal length, arranged so as to form an image of the biquartz on the slit of the collimator. The light from the slit fell on the spar prism, and two spectra, an ordinary and extraordinary, were formed and viewed by the telescope. Each of these spectra was divided horizontally into two parts, corresponding to the two parts of the biquartz, and when everything was adjusted the line of separation was seen clearly and distinctly.

Let us consider the ordinary image. Owing to the dispersion of the planes of polarization produced by the biquartz, the light of different colours in the incident wave is polarized in different planes, and a position can be found for the polarizer such that the plane of polarization of light of a certain colour on emerging from one-half of the biquartz is so related to the angle of incidence on the spar prism that the light of that colour is absent from the refracted beam. A dark band will be seen across one-half the spectrum in the place of this colour. Light of this colour emerging from the other half of the biquartz is in general not polarized in the same plane, and therefore in general, though a dark band will be formed in both halves of the spectrum, it will occupy different positions in the two. By turning the polarizer these bands appear to move in opposite directions across the field, and for one position of the polarizer the one can be brought vertically below the other. This position can be determined with great accuracy.

When this is the case the wave length of the light destroyed is clearly such that it has been rotated through 90° in opposite directions by the two halves of the biquartz. It is light then of a definite wave length, and we are thus able to place our prism with great accuracy in a position such that no light of one certain definite wave length is present in the ordinary wave. If, then, we are able to observe the angle of incidence and the deviation of the light of the same wave length in the extraordinary spectrum we shall have enough data to determine the azimuth of the plane of polarization of the incident light according to the electro-magnetic theory. It is easy enough to observe the angle of incidence. To find the deviation of the corresponding wave in the extraordinary spectrum, rotate the polarizer through about 90°; the dark bands will move out of the ordinary into the extraordinary spectrum, and the polarizer can be adjusted till they are brought to coincidence in it. When this is the case we know that it is light of the same wave length as before (viz.: that whose plane of polarization is turned through 90° by the biquartz), which is absent, and we have in the extraordinary spectrum a well-marked dark band, whose centre can easily be determined, and to which the needle point or cross wires of the observing telescope can be set with all the accuracy required. If we observe then the deviation of this dark band, we obtain the deviation in the extraordinary spectrum of the light which in the first part of our observation was wanting from the ordinary spectrum.

To escape the difficulty of having continually, when making observations for the determination of the position of the plane of polarization, to turn the polarizer through about 90° in order to get the deviation of the light in the extraordinary spectrum when the ordinary was quenched, or *vice versâ*, I divided the operation into two parts.

In the first I set the spar prism at a known angle of incidence and turned the polarizer until the dark bands coincided in the ordinary spectrum and then observed the deviation. I then turned the polarizer until the bands coincided in the extraordinary spectrum, and again observed the deviations; each of these observations was repeated five or six times and the mean taken. I then altered the angle of incidence by about 6° and made similar measurements.

In this manner I obtained a series of observations of angles of incidence ranging from 30° to 85°, with the corresponding deviations for both ordinary and extraordinary waves.

Let us call ϕ the angle of incidence, ϕ' that of refraction for the ordinary wave, ϕ'' for the extraordinary. From the observations the values of ϕ' and ϕ'' are easily determined by means of formulæ given by Professor STOKES (Brit. Ass. Report, 1862) and used by me in the paper already referred to.

If i be the angle of the prism ψ, ψ' the angles of emergence from and incidence on the second face, and D the deviation, we have

$$\psi = \mathrm{D} + i - \phi$$

$$\tan\frac{\phi'-\psi'}{2} = \tan\frac{i}{2}\tan\frac{\phi-\psi}{2}\cot\frac{\phi+\psi}{2}$$

$$\phi' + \psi' = i$$

and these equations give us ϕ'.

ϕ'' of course is found in the same manner.

I obtained thus a series of values of ϕ, ϕ', and ϕ''; now, of course, since ϕ' refers to the ordinary wave, if μ be the ordinary refractive index of the light used we should have $\frac{\sin\phi}{\sin\phi'} = \mu$, a constant, and this was found to be the case within small limits of error, showing that I had succeeded in quenching the same light throughout.

The value of μ was 1·662.

The values of $\phi'' - \phi'$ were also tabulated, and, of course, varied very slowly.

When this table had once been constructed, it was sufficient for the future to observe the angle of incidence; for knowing ϕ and μ, ϕ' is at once given by the formulæ $\sin\phi' = \sin\phi./\mu$ and ϕ'' by interpolation from the table. Thus the observations with the sugar-cell reduced to determining the angles of incidence at which the dark bands in the spectra coincided.

Each of these was determined five or six times and the mean taken.

We must now return to the theoretical considerations which enable us to express the azimuth of the plane of polarization of the incident light in terms of the angles of incidence and refraction and the position of the plane of polarization in the crystal.

Let us consider a plane-wave incident at an angle ϕ, let ϕ', ϕ'' be the angles of refraction for the two refracted waves respectively. The incident, reflected and refracted waves cut the plane face of incidence in the same line, let θ, $\theta_{,}$, θ' and θ'' be the angles between this line and the directions of the electric displacement in the incident reflected and refracted waves respectively.

Let a, $a_{,}$, a' and a'' be the amplitudes of the electric displacements in these directions.

Let q be the angle between the extraordinary wave normal and the corresponding ray.

Then (LORENTZ, 'Schlömilch,' xxii.; GLAZEBROOK, Proc. Camb. Phil. Soc., 1881) we have the equations

$$(a\cos\theta+a_{,}\cos\theta_{,})\sin^2\phi=a'\cos\theta'\sin^2\phi'+a''\cos\theta''\sin^2\phi'' \quad . \; . \; . \; . \; . \; . \; . \tag{1}$$

$$(a\sin\theta+a_{,}\sin\theta_{,})\sin\phi=a'\sin\theta'\sin\phi'+a''\sin\theta''\sin\phi''. \quad . \; . \; . \; . \; . \; . \; . \tag{2}$$

$$(a\cos\theta-a_{,}\cos\theta_{,})\sin\phi\cos\phi=a'\cos\theta'\sin\phi'\cos\phi'+a''\cos\theta''\sin\phi''\cos\phi''. \tag{3}$$

$$\begin{aligned}(a\sin\theta-a_{,}\sin\theta_{,})\sin^2\phi\cos\phi=a'\sin\theta'\sin^2\phi'\cos\phi'&\\ +a''(\sin\theta''\cos\phi''+\tan q\sin\phi'')\sin^2\phi''& \quad . \; . \; . \; .\end{aligned} \tag{4}$$

These equations express the conditions referred to previously and enable us to find $a_{,}$ a' a'' and $\theta_{,}$; θ', θ'' are known from the position with reference to the axis of the wave front in the crystal.

We can solve them in the general case, but for our present purpose it is sufficient to find the conditions that only one wave should be propagated in the crystal. Let us first take the ordinary wave; we may put $a''=0$ in the equations, and we get the condition

$$\tan\theta=\tan\theta'\cos(\phi-\phi') \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \tag{5}$$

by eliminating $a_{,}$ a' and $\theta_{,}$.

This then is the condition which must hold between the position of the plane of polarization of the incident light and the angles of incidence and refraction in order that only the ordinary wave may traverse the crystal.

If we desire to have only the extraordinary wave, put $a'=0$ and we obtain

$$\tan\theta=\tan\theta''\cos(\phi-\phi'')+\frac{\sin^2\phi''\tan q}{\cos\theta''\sin(\phi+\phi'')} \quad . \; . \; . \; . \; . \; . \tag{6}$$

In order to apply these formulæ we must find θ' θ'' and q in terms of the angles of incidence and refraction and constants.

Let the intersection of the incident, reflected and refracted waves with the face of incidence meet a sphere, centre O, in B (fig. 1).

Let the inward drawn normal to the face of incidence meet the same sphere in C, while the face itself cuts it in A B.

Let B I be the trace of the incident wave. B R of the refracted.

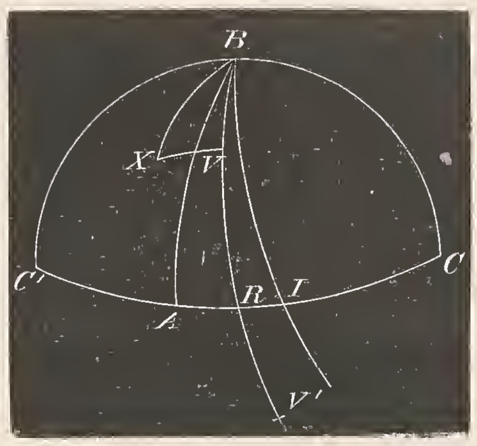
Let the optic axis cut the sphere in X. Join B X.

The prism used in the experiments was cut in such a way that X and R were on opposite sides of the arc B A, as in fig. 1.

Let B X $=\beta$ and let the angle A B X $=\lambda$. β and λ are known if the position of the face of incidence with reference to the optic axis is known.

Draw X V perpendicular to B R and take V V′ an arc of 90°. Then V V′ $=\frac{\pi}{2}$; also O V, O V′ are clearly the two possible directions of vibration in the wave front B R. O V is the direction of the extraordinary vibration O V′ of the ordinary.

Fig. 1.



Let us suppose that B R is the ordinary refracted wave corresponding to an incident wave B I. Then A B V$=\phi'$ and X B V$=\lambda+\phi'$; also B V$=\frac{\pi}{2}-\theta'$; and from the right angle and triangle X B V we have

$$\cos \mathrm{XBV}=\tan \mathrm{BV} \cot \mathrm{BX}$$

$$\cot \theta'=\tan \beta \cos (\lambda+\phi') \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

But we have from (5)

$$\cot \theta=\cot \theta' \sec (\phi-\phi')$$

Therefore

$$\cot \theta=\tan \beta \cos (\lambda+\phi') \sec (\phi-\phi') \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

If B R represent the extraordinary wave

$$\mathrm{BV}=\theta'', \quad \mathrm{ABV}=\phi'',$$

and we have

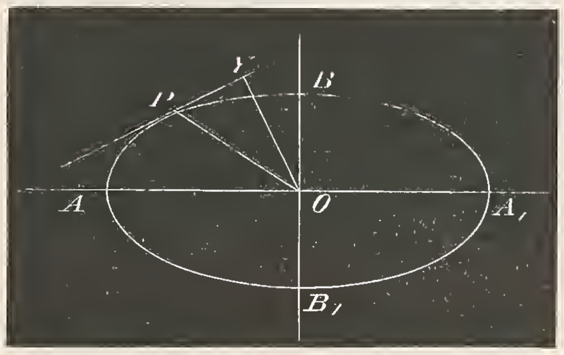
$$\tan \theta''=\tan \beta \cos (\lambda+\phi''). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

so that from (6) we obtain

$$\tan \theta=\tan \beta \cos (\lambda+\phi'') \cos (\phi-\phi'')+\sin^2 \phi'' \tan q / \cos \theta'' \sin (\phi+\phi'') \quad . \quad (10)$$

It remains now to find q, the angle between the extraordinary ray and the wave normal.

Fig. 2.



Let the figure (fig. 2) represent a section of the surface of wave slowness passing through the optic axis, O A, and the extraordinary wave normal, O P. Let O Y be perpendicular on the tangent plane at P, then O Y is, we know, the extraordinary ray. Let a and b be the principal refractive indices in direction O A and O B, and let $\mathrm{OP}=r$, $\mathrm{OY}=p$. Since OP and OY are respectively the directions of the wave normal and the ray, the angle $\mathrm{POY}=q$, and $p=r\cos q$; let the angle $\mathrm{POA}=\psi$.

Then

$$\frac{1}{p^2}=r^2\left\{\frac{\cos^2\psi}{a^4}+\frac{\sin^2\psi}{b^4}\right\}$$

Therefore

$$\sec^2 q=r^4\left\{\frac{\cos^2\psi}{a^4}+\frac{\sin^2\psi}{b^4}\right\}$$

Also

$$\frac{1}{r^2}=\frac{\cos^2\psi}{a^2}+\frac{\sin^2\psi}{b^2}$$

whence

$$\tan q=r^2\sin\psi\cos\psi\frac{a^2-b^2}{a^2b^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

But since r is a radius vector of the surface of wave slowness, $r=\sin\phi/\sin\phi''$, and we have

$$\sin^2\phi''\tan q=\frac{a^2-b^2}{a^2b^2}\sin^2\phi\sin\psi\cos\psi$$

Again X V (fig. 1) $=90^\circ-\psi$, and from the triangle X B V

$$\sin \mathrm{XV}=\sin \mathrm{BX}\sin \mathrm{XBV}, \quad \text{or} \quad \cos\psi=\sin\beta\sin(\lambda+\phi'')$$

Also

$$\cos \mathrm{BX}=\cos \mathrm{XV}\cos \mathrm{BV}, \quad \text{or} \quad \cos\beta=\sin\psi\cos\theta''$$

Thus

$$\sin^2\phi''\tan q=\frac{a^2-b^2}{2a^2b^2}\;\frac{\sin 2\beta\sin(\lambda+\phi'')\sin^2\phi}{\cos\theta''}$$

and equation (10) becomes

$$\tan\theta=\tan\beta\cos(\lambda+\phi'')\cos(\phi-\phi'')+\frac{a^2-b^2}{2a^2b^2}\;\frac{\sin 2\beta\sin(\lambda+\phi'')\sin^2\phi}{\sin(\phi+\phi'')\cos^2\theta''} \quad . \quad . \quad (12)$$

but θ'' can be found in terms of ϕ, ϕ'' and constants, from the formula

$$\tan\theta''=\tan\beta\cos(\lambda+\phi''),$$

and θ is thus expressed in terms of ϕ, ϕ'' and constants.

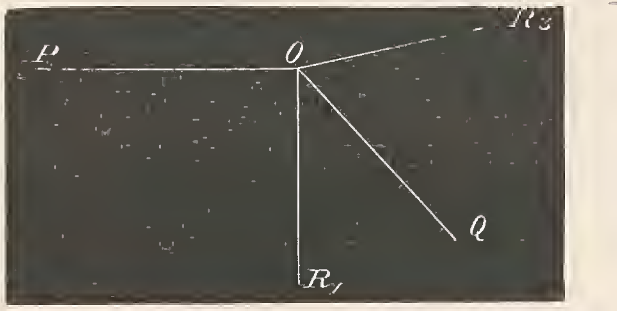
The value of a has been found already; it is the ordinary refractive index of the spar for the light used, and has been shown to be 1·662.

b is the extraordinary index, and may be taken with sufficient accuracy for the purpose from MASCART's or RUDBERG's determinations. Either of these give $b=1·488$ as the value of the extraordinary index, corresponding to the value 1·662 for the ordinary.

In the spar prism used one face coincided very closely with a cleavage plane. The other face was inclined to this plane at an angle of 39° 17′ 20″, and the edge of the prism—that is, the intersection of the two faces—was nearly coincident with that of two cleavage planes.

Let O R_1, O R_2, O R_3 (fig. 3), be normal to the three cleavage planes; O P, O Q to the faces of the prism. The incident light fell on the face normal to O P. Then experiment proved that P, Q, R_3, R_1 were very closely indeed in the same zone; for the present we shall treat them as if they were accurately so, and this zone will therefore be the principal plane of the prism.

Fig. 3.



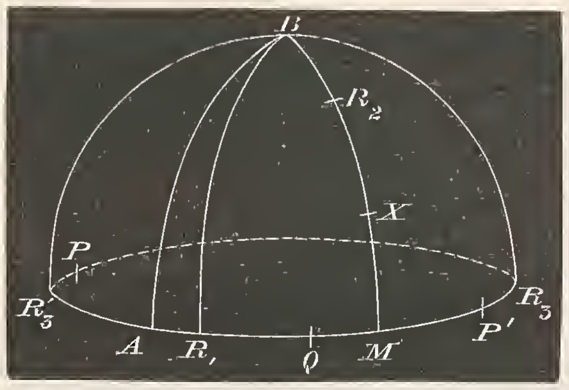
A series of observations on August 25, 1880, gave the values for the angles.

$$\text{P O R}_1 = 105^\circ\ 54'$$
$$\text{R}_1\text{ O Q} = 34^\circ\ 48'$$
$$\text{Q O R}_3 = 40^\circ\ \ 8'$$
$$\text{P O R}_3 = 180^\circ\ 50'$$

Each observation was the mean of four or five closely concordant ones. The prism was reset and relevelled in November and another series of measurements taken, which agreed with the above to within 1′.

Let the optic axis and the edge of the prism meet the sphere in X and B respectively (fig. 4). Then B R_2 X is a great circle which bisects the arc R_1 R_3 in M say.

Fig. 4.



Let P O produced backwards meet the sphere in P′. Then

$$R_1R_2 = R_2R_3 = R_3R_1 = 74^\circ\ 56'$$

and

$$XR_1 = XR_3 = XR_2$$

Also

$$\text{angle } R_1XM = 60^\circ$$

$$R_1M = \tfrac{1}{2}R_1R_3 = 37^\circ\ 28'$$

whence

$$R_1X = 44^\circ\ 37'$$

and

$$XM = 26^\circ\ 15'\ 30'$$

Thus

$$BX = 63^\circ\ 44'\ 30'' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

and BX is the angle denoted by β in the formulæ.

Again we have

$$R_3P' = 0^\circ\ 50'$$

Therefore

$$MP' = 36^\circ\ 38'$$

In the arc $P'R_1$ take $P'A = 90^\circ$. Then B A is the trace of the face of incidence, and

$$\lambda = ABX = AM = 90^\circ - MP' = 53^\circ\ 22' \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

These values of β and λ were used in reducing the experiments.

The error produced by assuming P, R_1, R_3, and Q to be in the same zone will be discussed later.

The results of the experiments are contained in the following tables.

Table I. is the interpolation table used as described above to find ϕ'' from the value of ϕ, and gives the series of corresponding values of ϕ, ϕ' and $\phi'' - \phi'$.

Table II. gives the values of ϕ, ϕ' and θ for the case in which the ordinary wave only traversed the crystal.

ϕ is directly observed, ϕ' and θ are found from the formula

$$\sin\phi' = \sin\phi/\mu$$

and

$$\cot\theta = \tan\beta\cos(\lambda + \phi')\sec(\phi - \phi')$$

where $\mu = 1{\cdot}662$, $\beta = 63^\circ\ 44'\ 30''$, $\lambda = 53^\circ\ 22'$.

The values are arranged in pairs. In the first experiment recorded in each pair there was no sugar cell in the path of the light. In the second experiment the sugar cell was interposed. The fifth column gives the differences between the two values of θ thus found, and this, if the formula given by theory were correct, ought to be the rotation of the plane of polarization produced by the sugar cell.

Each recorded value of ϕ is the mean of five observations. At high angles of incidence the difference between two observations of the same value of ϕ was sometimes as great as 10′. The error produced in the value of θ by an error $\delta\phi$ in ϕ is, for these values of ϕ, less than $\delta\phi/2$, so that the extreme difference between the values of θ calculated from each of the five values of ϕ, of which the mean is given in the table, may be as great as 5′. At high angles of incidence the mean error in the value of θ, as given in the table, is considerably less than 2′.

At lower angles of incidence, 45° to 30°, the differences in the observed values of ϕ were much less, and rarely amounted to 5′, but then the error produced in θ by an error $\delta\phi$ in ϕ is not far short of $\delta\phi$, so that the probable accuracy of the values of θ given in the table remains much the same as before.

TABLE I.

ϕ.	ϕ'.	$\phi''-\phi'$.
° ′ ″	° ′ ″	° ′ ″
83 11 0	36 41 10	1 2 40
78 10 30	36 4 50	1 0 50
68 11 30	33 57 40	56 20
65 34 35	33 13 10	54 40
56 49 0	30 14 10	50 20
52 28 40	28 30 10	48 0
48 32 30	26 48 10	46 10
38 17 50	21 53 40	41 10
31 5 15	18 5 50	37 20
30 36 0	17 50 10	37 0
24 42 0	14 33 40	33 0
21 3 10	12 29 0	30 0
17 45 0	10 34 10	26 50

TABLE II.

	ϕ.	ϕ'.	$\theta-90°$.	Rotation.
	° ′ ″	° ′ ″	° ′ ″	° ′ ″
1	80 47 0 70 59 0	36 26 10 34 40 10	0 33 20 4 55 30	4 22 10
2	78 14 0 69 29 0	36 5 20 34 18 0	1 29 20 5 54 10	4 24 50
3	73 43 0 66 6 20	35 16 40 33 22 30	3 30 10 7 47 50	4 17 40
4	70 57 40 63 57 50	34 39 50 32 43 0	4 56 30 9 11 50	4 15 20
5	66 6 20 59 51 50	33 22 30 31 21 30	7 47 50 11 58 10	4 10 20
6	64 12 30 58 9 30	32 48 10 30 44 20	9 2 0 13 11 50	4 9 50
7	59 45 0 54 8 30	31 19 0 29 11 10	12 3 20 16 9 30	4 6 10
8	57 54 20 52 24 30	30 38 40 28 28 30	13 22 50 17 28 10	4 5 20
9	52 16 40 47 1 40	28 25 10 26 7 10	17 34 10 21 35 50	4 1 40
10	46 50 40 41 30 50	26 2 10 23 30 10	21 44 10 25 50 0	4 5 50
11	41 41 30 36 4 10	23 35 30 20 45 0	25 41 40 29 54 20	4 12 40
12	36 5 40 30 18 50	20 45 40 17 40 50	29 53 20 34 0 10	4 6 50
13	30 36 0 24 25 0	17 50 10 14 24 10	33 48 10 37 54 20	4 6 10
14	24 42 0 17 45 0	14 33 50 10 34 10	37 43 20 41 54 30	4 11 10

The rotation produced by the cell was measured carefully by FIZEAU's method, and the mean of several observations in which the extreme difference was about 5′ gave the value 4° 6′ 15″.

Each of the numbers then in Table II., column 5, ought to be equal to 4° 6′ 15″, and this is evidently not the case. The differences begin by being too large, and allowing for the probable error of the experiment, they decrease fairly uniformly as the angle of incidence decreases until we reach an angle of incidence of about 52°.

As the angle of incidence is still further decreased there is a tendency to increase in the values of the rotation given by the table. The value 4° 12′ 40″, line 11, is pretty clearly too big, as also possibly is that in line 14. A very small displacement in the position of the sugar cell, so that the light traversed it somewhat obliquely, would give rise to an error of the kind here considered. The cell was usually adjusted by observing the beam of light reflected from its first face. This could be made with a little care to travel back through the biquartz, and in that case the light clearly fell normally on the cell.

The first twelve sets of observations recorded in Table II. were made on October 28, 1880, the last two a few days later.

Thus, unless there is some regular source of error in the experiments, we must conclude that the formula connecting the plane of polarization and the angle of incidence in the case in which only the ordinary ray traverses a crystal of Iceland spar, as given by the electro-magnetic theory, is only true approximately.

The method does not enable us to determine accurately by experiment the position of the plane of polarization of the incident light with reference to the face of the crystal on which it falls; we can however compare the rate of change of the position of this plane, as the angle of incidence varies, found from experiment with its value deduced from theory.

We arrive at the conclusion that this rate of change as given by theory is too rapid when the angles of incidence are large, that for angles of incidence of from 30° to 60° the theoretical and experimental rates agree, while for lower angles of incidence the theoretical rate is again possibly too great; the last inference however being a little uncertain.

Two other shorter sets of observations made about the same time confirm these statements exactly, and it does not seem worth while to print a table of the numbers actually arrived at.

TABLE III.

	ϕ.	ϕ''.	θ.	Rotation.
	° ′ ″	° ′ ″	° ′ ″	° ′ ″
1	82 55 0 69 21 30	37 42 10 35 13 10	0 48 30 4 12 40	3 24 10
2	77 24 20 67 0 0	36 58 10 34 33 40	1 37 0 5 18 0	3 41 0
3	73 4 40 64 57 30	36 16 0 33 56 20	2 35 10 6 21 40	3 46 30
4	69 3 30 61 37 30	35 8 20 32 50 30	4 21 0 8 17 10	3 56 10
5	66 42 0 59 57 0	34 28 10 32 15 20	5 27 10 9 19 30	3 52 20
6	64 46 50 58 17 0	33 52 50 31 38 20	6 27 40 10 26 0	3 58 20
7	61 29 40 55 37 0	32 47 50 30 35 50	8 22 0 12 19 0	3 57 0
8	55 43 0 50 27 30	30 38 10 28 25 50	12 14 50 16 14 0	3 59 10
9	50 18 0 45 15 0	28 21 50 26 2 30	16 21 0 20 27 20	4 6 20
10	45 17 0 40 17 0	26 3 20 23 35 50	20 26 0 24 34 40	4 8 40
11	40 9 20 35 4 30	23 32 0 20 53 10	24 41 0 28 51 20	4 10 20
12	35 7 30 29 50 30	20 54 0 18 1 20	28 48 50 32 58 30	4 9 40
13	29 40 0 24 4 0	17 58 10 14 44 50	33 3 20 37 12 30	4 9 10
14	24 15 0 17 45 0	14 51 10 11 1 0	37 4 50 41 23 0	4 18 10

To complete the investigation we should consider the effects of possible errors in the constants β and λ. This can be done more advantageously after we have tabulated the results of the experiments in which the extraordinary ray only was allowed to traverse the crystal.

Table III. contains these. θ as before is the azimuth of the plane of polarization of the incident light, which in this case is calculated from the formula (12)

$$\tan\theta = \tan\beta\cos(\lambda+\phi'')\cos(\phi-\phi'')$$

$$+\frac{a^2-b^2}{a^2b^2}\,\frac{\sin 2\beta\sin(\lambda+\phi'')\sin^2\phi}{\sin(\phi+\phi'')\cos^2\theta''}$$

where

$$\tan\theta'' = \tan\beta\cos(\lambda+\phi'')$$

$$a=1{\cdot}662 \qquad b=1{\cdot}488$$

$$\beta=63^\circ\ 44'\ 30'' \quad \lambda=53^\circ\ 22'$$

The observations recorded were made on November 16, 1880.

For the rotation of the sugar cell, measured by **Fizeau's** method, the value 4° 4′ 20″ was obtained as the mean of seven measures, the two extremes of which differed by 5′.

If our theory then were correct, each of the differences given in Table III., column 5, should be 4° 4′ 20″.

We see at once that this is very far from being the case. The values of the rotation commence by being much less than 4° 4′ 20″ and, with the exception of Experiment 4, increase fairly regularly as the angle of incidence decreases.

As was the case with the ordinary ray, the rotations agree with experiment for an angle of incidence of about 50°; from that point onwards the theoretical rotation is too great.

Thus the theoretical rate of change in the position of the plane of polarization is too small for high angles of incidence, but increases as the angle of incidence decreases, and finally becomes too great.

Several other series of experiments lead to the same conclusions.

It remains then to discuss the effect of an error in the values of λ or β, and this is all the more necessary, for we know that the values taken are only closely approximate.

Let us take the ordinary wave first for which we have

$$\cot\theta = \tan\beta\cos(\lambda+\phi')\sec(\phi-\phi')$$

and consider the effect of decreasing β by a given amount.

The logarithm of $\cot\theta$ is thus decreased throughout by the same quantity.

Now the change produced in θ by a given change in $\log\cot\theta$ is greatest when θ is nearest to 45°, thus by any change in β the values of θ will be more altered in Experiment 14 than in Experiment 1.

But we have to consider the change in the difference between two consecutive values of θ.

When θ is near 90° $\log\cot\theta$ changes more rapidly for a given change in θ than when it is near 45°.

If then in Experiment 1 we subtract from each of the values of $\log\cot\theta$ a certain quantity, it will alter the value of θ in the second line by an amount considerably greater than the alteration it produces in the first line, and thus the difference between the two values of θ will be reduced.

If on the other hand we consider Experiment 14, each of the values of θ there will be more affected by this change than in Experiment 1, but the effect produced on the value of θ in the first line will be nearly the same as that produced on the value of θ in the second, and thus the difference between the two values will not be so much altered.

Thus a decrease in β will decrease everywhere the theoretical value of the rotations, but it will affect the high angles of incidence considerably more than the low. It will thus tend to bring the theory more into accordance with experiments.

Let us see how it will affect the extraordinary ray.

The term in the formula with $\frac{a^2-b^2}{2a^2b^2}$ for a factor is practically a small and slowly varying correction except for the very highest angles of incidence; let us consider it as constant with regard to β and see how a decrease in β affects the values of θ supposed to depend only on the term

$$\tan\beta\cos(\lambda+\phi'')\cos(\phi-\phi'')$$

Exactly the same reasoning applies to this as in the case of the extraordinary wave.

The values of θ in Experiment 14 will be most decreased, but the alteration in the value of θ in the second line of Experiment 1 will be much greater than the alteration of the value of θ in the first line, while the change in the values of θ in Experiment 14 will be much the same for the two. Thus, by decreasing β the differences will be decreased throughout, but the changes will be greatest when the angles of incidence are large.

Thus a decrease in β will increase the differences between the theory and experiment.

Hence considering the ordinary and extraordinary rays together, a change in β will not reconcile the facts observed.

Calculation shows that decreasing β by 30′ decreases the differences in Experiments 4, 11, and 14 by 6′, 4′, and 1′ 20″ respectively for the extraordinary ray, and the amounts are about the same for the ordinary.

We must now consider alterations in λ; putting $\tan\beta=\mathrm{K}$ we have for the ordinary ray

$$\cot\theta=\mathrm{K}\sec(\phi-\phi')\cos(\lambda+\phi')$$

therefore

$$\delta\theta=\mathrm{K}\sin^2\theta\sin(\lambda+\phi')\sec(\phi-\phi')\delta\lambda.$$

Now when ϕ is large θ is nearly 90° and $\phi-\phi'$ is large.

Thus $\sin\theta\sin(\lambda+\phi')$ and $\sec(\phi-\phi')$ all increase with ϕ', and therefore $\delta\theta/\delta\lambda$ is greatest when ϕ is large.

Thus the alteration produced in θ is greater the greater the angle of incidence.

If then we decrease λ we shall reduce in every case the differences between the corresponding values of θ, but we shall reduce them most for high angles of incidence and thus tend to bring the theory more into accordance with experiment.

Turning now to the extraordinary wave, and again considering the first term only, we find

$$\delta\theta = -\mathrm{K}\cos^2\theta\cos(\phi-\phi'')\sin(\lambda+\phi'')\delta\lambda$$

Now as the angle of incidence decreases, $\cos^2\theta$ and $\sin(\lambda+\phi'')$ both decrease, but $\cos(\phi-\phi'')$ increases, and we must have recourse to numerical values most easily to consider the changes in the product.

Putting in the values for $\phi=82^\circ\ 55'$ and $\phi=17^\circ\ 45'$ we find $\delta\theta/\delta\lambda$ is numerically the greatest in the first case. Hence by decreasing λ we increase θ, and that most for the high angles of incidence.

Thus the decrease in λ increases the first value of θ in each of the experiments in Table III. by a greater quantity than the second. It therefore decreases the differences throughout, and does not tend to bring theory and experiment into agreement.

Hence taking both rays into account an alteration in λ will not produce the effect required.

Thus it is not possible to change the constants β and λ so as to produce greater agreement between theory and experiment.

But β and λ are both measured from the intersection of the incident wave and the face of incidence. In treating them as constants we have supposed that as the prism was turned this line of intersection remained parallel to itself, thus assuming that the prism was accurately levelled, so that the incident and refracted wave normals lie always exactly in a principal plane.

If this be not the case, β and λ become functions of the angle of incidence, and have not strictly the same values in the consecutive lines of Tables I. and II.

To test the effect of this error experimentally I set the spar prism so that the incident wave normal instead of lying in the principal plane was inclined to it at about 10′, and made a short series of measurements.

The results agreed very closely with those tabulated in Tables I. and II., and proved conclusively that the level error which I feel sure never amounted to as much as 2′ could not account for the difference between theory and experiment.

The same is fairly clear from the formulæ, for in Table II. we could reconcile theory and experiment by supposing that at high angles of incidence β is somewhat greater than 63° 44′ 30″, and that it decreases continually as the angle of incidence increases, but this assumption would just tend to increase the differences given in Table III.

Thus a small variable error in the value of β will not account for the observed facts.

One point, however, needs further investigation. An error has been confessedly committed in the value of β. Can we assign a limit to its value?

The fact that the four poles P, Q, R_1, R_3 are nearly but not quite in the same zone renders it impossible to determine accurately the position of P and Q. The prism was levelled by placing a mark across the slit, which when viewed directly appeared to coincide with the end of the needle point in the focus of the telescope. The telescope was then turned to view the images reflected from the faces of the prism, and the levelling screws of the prism adjusted until the same coincidence was established.

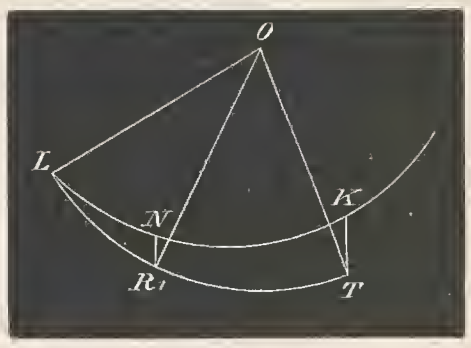
Now observation showed that when P and Q were level in this manner R_1 and R_3 were both a very little too low.

The vertical angular distance between the end of the needle and the mark across the slit, seen by reflection from R_1 or R_3, could be estimated by setting the needle on the slit, reading the vernier of the telescope and then moving the telescope with the needle until the horizontal distance between it and the slit appeared about equal to the vertical distance between the end of the needle and the mark.

This horizontal distance is found at once by again reading the vernier of the telescope.

I found as the mean of several closely concordant observations that when the angle of incidence of the light on the face R_1 was 20° 58′ the image formed by reflexion was 10′ too low, and the same exactly was the case with the image formed by reflexion at the same angle from R_3.

Fig. 5.



Let L O (fig. 5) be the direction of the incident light from the mark on the slit, O T the reflected ray, O R_1 the normal to R_1.

L R_1 T is a great circle, let L N K be the principal plane of the prism, T K and R_1 N being perpendicular to L N K.

$$LR_1=\phi=R_1T$$

Let the angle $TLK=\gamma$, $R_1N=x$.

The experiment has shown that $TK=10'$.

From the triangles T L K, R_1 L N

$$\sin x=\sin\gamma\sin\phi$$
$$\sin 10'=\sin\gamma\sin 2\phi$$

and

$$\phi=20°\ 58'$$

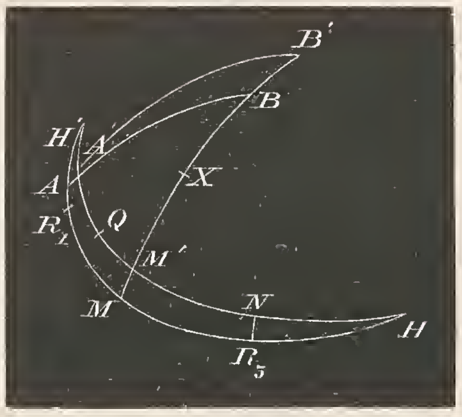
whence

$$x=6'$$

Thus R_1 is 6′ below the great circle L K, and so also is R_3.

Thus in reality the principal plane of the prism is inclined at a small angle to the plane $R_1 R_3$, the intersection of the two being the exterior bisector of the angle $R_1 O R_3$. Let H H′, fig. 6, be the line of intersection, B′, M′ the new position of B and M respectively, X the optic axis.

Fig. 6.



Then M, M′, X, B, B′, lie on a great circle, and BB′=MM′=MHM′

$$R_3H=90°-37° 28'$$
$$R_3N=6'$$
$$\sin MM'=\sin R_3N \operatorname{cosec} xR_3H$$

whence

$$MM'=8'$$

And if $\delta\beta$ be the change thus produced in β

$$\delta\beta=XB'-XB=BB'=8'$$

and if A′ is the new position of A

$$MA=\lambda \quad M'A'=\lambda+\delta\lambda$$

but

$$MA=M'A' \quad \therefore \delta\lambda=0$$

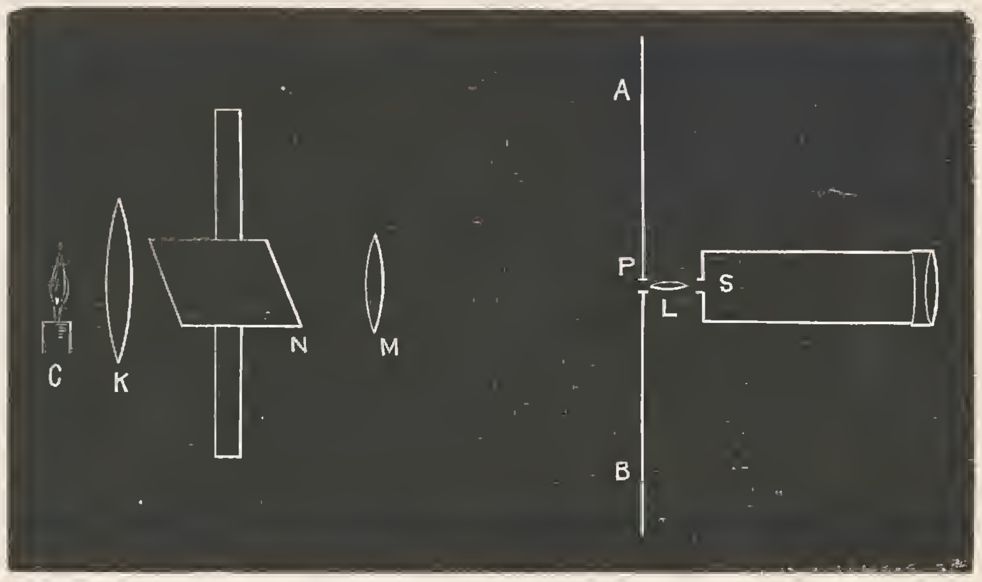
The value of β then, if these observations be true, is some 8′ too small. This would alter somewhat all the values of θ in Tables II. and III., but would produce little or no effect on the differences. The observations themselves, however, are only approximate, and can be used to give some idea of the limit of error made in giving to β the value 63° 44′ 30″, though hardly to determine its value more accurately. It is certain that β is rather too small; it is also fairly certain that it is not 10′ too small, and for our purpose at present, between these limits one value is as good as another. It therefore seemed that no real advantage would be gained by recalculating the theoretical values of θ for a value of β somewhat greater than 63° 44′ 30″.

The observations recorded above were made during the autumn of 1880, and at that time I was enabled by Lord RAYLEIGH's kindness to order, for the Cavendish Laboratory, two graduated circles, mounted so as to revolve about an axis at right angles to their planes, with tubes through their centres to carry a NICOL's prism or other such apparatus. The verniers attached read to 15″, and by means of various set screws all the adjustments requisite for accurate measurement of the position of the plane of polarization of the light emerging from the Nicol can be accomplished.

Before asking permission, therefore, to lay the preceding results before the Royal Society I waited until these circles were ready, and have repeated the experiments described above, using one of them instead of the sugar-cell to produce the rotation in the plane of polarization of the light.

The only additional adjustment necessary was to make sure that the axis of rotation of the polarizing Nicol was parallel to the incident light. The Nicol was placed at about a metre behind the slit of the collimator and adjusted by looking along the axis of the collimator until the centre of the face of the Nicol appeared to be in the axis. In the figure (7), which gives the apparatus in plan, S is the slit of the collimator, and

Fig. 7.



N the Nicol. Between the slit and the Nicol was placed a screen, A B, with another slit, P, adjusting it, so that P lay also in the axis of the collimator, and between P and S a lamp, L, so as to illuminate both slits. The lamp was so adjusted that the light from it fell along the axis of the collimator in one direction, while in the other after passing through the slit P, it fell on the face, N, of the Nicol and was there reflected. A convex lens was placed between P and N in such a position that the rays reflected from the face of N, formed an image of the slit on the screen A B. On rotating the Nicol round its axis this image moved over the screen, and if the axis of rotation is parallel to the axis of the collimator, as it should be, the centre of the image will describe a circle round P as centre. A horizontal scale was affixed to the screen along A B and the position of the Nicol adjusted until the image of the slit, as the Nicol was turned, cut this scale in points equidistant from P on either side of it.

This made it certain that the axis of the Nicol was in the same vertical plane as that of the collimator. But the axis of the collimator had been levelled with a spirit level, and by altering one of the feet on which the circle carrying the Nicol rested its axis was rendered horizontal. Thus the axis of the Nicol was rendered parallel to that of the collimator.

Behind the NICOL was placed a large lens, K, of somewhat short focal length, to act as a condenser, and at its focus the source of light, C, used in the experiment.

The lens M, the screen and lamp L, were of course removed, and when the narrow parallel beam of light from the lamp, C, after passing through the NICOL fell on the slit of the collimator, I knew that it traversed the NICOL in a direction parallel to the axis of rotation.

By means of a long handle and a HOOK'S joint the tangent screw of the circle of the polarizer could be turned by the observer when his eye was at the telescope.

The experiments were conducted in a somewhat different order to that adopted in the first set. The analyzing prism was placed so that the light fell on it at a known angle of incidence and the NICOL turned by hand until the two black bands appeared fairly close together in the ordinary spectrum. The polarizing circle was then clamped and turned by means of the handle until the bands were brought exactly into coincidence.

The reading N_0 of the NICOL circle was taken and also the deviation D_0 of the dark band in the ordinary spectrum. The NICOL circle was then turned, keeping the angle of incidence on the analyzing prism the same, until the dark bands were seen in the extraordinary spectrum and coincidence was established as before; the reading N_E of the NICOL circle was taken, and also the deviation D_E of the band in the extraordinary spectrum. The analyzing prism was then moved so as to alter the angle of incidence, and another set of observations taken, only in the reverse order. A series of values of the angles of incidence and the quantities N_0, N_E, D_0, D_E were thus observed. Each single observation was repeated five times and the mean taken and used in the calculations.

For a certain known angle of incidence, D_0 is the deviation of the light that is quenched in the ordinary spectrum. It is also however the deviation in the ordinary spectrum of the light that is missing from the extraordinary, when the coincidence between the black bands is established there.

D_0 is therefore the angle of deviation we must use in calculating the position of the plane of polarization of the incident light, when the ordinary ray only is transmitted. The experimental value of the angle determining this position, measured from some unknown zero, is N_E, for this gives us the position of the NICOL when the extraordinary wave is quenched.

Of course the theoretical values of the positions of the plane of polarization might have been determined as before from the values of the angle of incidence and ordinary refractive index, by the use of the interpolation Table I.

I wished, however, to make this series of observations entirely independent of the first, and so recalculated the interpolation table.

Since the position of the zero of the polarizer circle with reference to the analyzing prism is unknown, an unknown constant will, as before, come into our tables.

Let us determine it so that the theoretical and experimental values of the position of the plane of polarization in the first experiment on the ordinary wave agree.

Table IV. gives the values of ϕ, D_0+i, N_E, ϕ', θ_0 and the differences between θ_0 and N_E.

The last two columns give respectively the differences between the consecutive values of θ_0 and consecutive values of N_E.

The differences of θ_0 correspond to the rotation in the last column of Table II., and the differences of N_E to the constant experimental value of that rotation, viz., 4° 6′ 20″. The last two columns, therefore, enable us to compare these experiments with those recorded in Table II.

D_0+i is given, i being the angle of the prism because it occurred in this form in the formulæ, and is just as easily found at once as D_0.

Table V. gives the similar values for the case in which the extraordinary wave only is transmitted.

The zero from which N_0 is measured is of course that from which N_E has been measured.

As has already been said, each of the numbers in the columns $D+i$ and N is the mean of five observations.

The observations of the deviation rarely differed by as much as 20″, so that the mean is probably accurate to 5″. At the lower angles of incidence that is in the neighbourhood of the position of minimum deviation this would produce an error of about 1′ in the value of θ. When the angle of incidence is about 45° the error introduced into the value of θ by an error of 5″ in D is practically inappreciable.

Thus the theoretical values of θ are probably very accurate, the possible error being greatest when the angle of incidence is small, and then probably it is considerably less than 2′. The differences in the observed values of N were greater; except in the case of the first two sets recorded, when owing to the high angle of incidence very little light got through the prism and the field was very dark—the greatest difference between two observed experimental values of the same quantity was 5′. The mean error of the observations is rather under 1′ 30″, so that the recorded values of N_0 and N_E are probably accurate within this limit.

For the first two values of N_0 the extreme differences among the observations were as much as 10′, but a greater number of observations was taken. The mean error for them was 3′. In the case of the first two observations of N_E the difficulty was not so great; the extreme differences were 4′, and the mean error about 1′ 30″.

We proceed now to discuss the results of the table.

If the agreement between theory and experiment were complete, the differences recorded in the third column from the end in each table would be zero throughout.

In each case as there recorded, they increase for a time as the angle of incidence decreases and then decrease again. We of course must remember that our numbers do not give absolutely the difference between theory and experiment; there is an unknown constant to be considered which we have arbitrarily determined so as to make the difference in the first line of Table IV. zero. It may quite well be that we are

TABLE IV.—Ordinary wave only transmitted.

	ϕ.	D_0+i.	ϕ'.	θ_0-90.	$N_E-90°$.	θ_0-N_E.	Rotation theory.	Rotation experiment.
	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	′ ″	° ′ ″	° ′ ″
1	82 57 40	87 19 22	36 40 0	−0 5 50	−0 5 50	0 0	..	..
2	75 14 40	81 24 40	35 35 0	2 45 40	2 31 15	15 25	2 51 30	2 37 5
3	67 12 55	76 32 50	33 41 30	7 6 40	6 43 50	22 50	4 21 0	4 12 35
4	59 59 55	73 11 14	31 24 0	12 0 10	11 27 50	32 20	4 53 30	4 44 0
5	54 43 10	71 17 26	29 24 40	15 44 30	15 13 0	31 30	3 44 20	3 45 10
6	49 5 40	69 43 36	27 3 0	20 0 10	19 30 35	29 35	4 15 40	4 17 35
7	43 46 55	68 43 10	24 35 50	24 6 30	23 35 50	30 40	4 6 20	4 5 15
8	38 41 40	68 9 0	22 5 20	27 58 20	27 32 50	25 30	3 51 50	3 57 0
9	32 51 0	67 58 42	19 2 30	32 14 30	31 54 30	20 0	4 16 10	4 21 40
10	27 38 10	68 19 54	16 12 0	35 49 30	35 31 20	18 10	3 35 0	3 36 50
11	21 17 30	68 34 28	12 36 50	39 50 20	39 36 30	13 50	4 0 50	4 5 10

TABLE V.—Extraordinary wave only transmitted.

	ϕ.	D_E+i.	ϕ''.	θ_E.	N_0.	$N_0-\theta_E$.	Rotation theory.	Rotation experiment.
	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	′ ″	° ′ ″	° ′ ″
1	82 57 40	85 31 20	37 42 20	0 48 30	0 37 0	−11 30	..	..
2	75 14 40	79 38 20	36 33 20	2 10 15	2 8 50	− 1 25	1 21 45	1 31 50
3	67 12 55	74 47 28	34 37 40	5 11 15	5 24 50	13 35	3 1 0	3 16 0
4	59 59 55	71 25 8	32 16 30	9 15 15	9 34 50	19 35	4 4 0	4 10 0
5	54 43 10	69 28 57	30 14 40	12 58 0	13 17 45	19 45	3 42 45	3 42 55
6	49 5 40	67 51 28	27 50 50	17 17 0	17 38 6	21 6	4 19 0	4 20 21
7	43 46 55	66 43 56	25 21 10	21 38 30	31 58 20	19 30	4 21 30	4 20 14
8	38 41 40	65 59 43	22 46 0	25 55 30	26 12 10	16 40	4 17 0	4 13 50
9	32 51 0	65 33 18	19 41 0	30 39 0	30 49 50	10 50	4 43 30	4 37 40
10	27 38 10	65 33 18	16 47 20	34 38 15	34 45 30	7 15	3 59 15	3 55 40
11	21 17 30	66 9 22	13 7 20	39 6 30	39 12 0	5 30	4 28 15	4 26 30

wrong in so doing. The table, however, shows us conclusively that we cannot so determine this constant as to bring theory and experiment into exact agreement.

The last two columns in each table enable us to compare these results with those of our former experiments.

The one gives us the rotation of the plane of polarization when only one wave traversed the crystal according to the electro-magnetic theory, the other the measured value of the same rotation.

Consider Table IV. first. At high angles of incidence the theoretical value of rotation is greater than the experimental. As the angle of incidence decreases the two become more nearly equal and agree within the limits of the error of experiment between angles of incidence ranging from 55° to 45°. As the angle of incidence still further decreases the theoretical value of the rotation becomes decidedly less than the experimental.

In the main this agrees with Table II. At high angles of incidence the rotation given by theory is greatly in excess of that given by experiment, the two agree between 55° and 40°, but for the lower angles of incidence Table II. shows a slight increase in the theoretical value as compared with the experimental.

Turning now to the last two columns in Table V., we see that the theoretical rotation is at high angles of incidence less than the experimental, that as the angle of incidence decreases the two tend to become equal and agree very closely for angles of incidence between 55° and 45°, while from that point onwards the theoretical rotation is bigger than the experimental. The change in the relative values of the two is very regular, while the actual change of sign in their difference occurs between the values 49° 5′ 40″ and 43° 46′ 45″ of the angle of incidence.

Referring to Table III. we see that this is exactly the state of affairs there indicated. The theoretical rotation is at first less than the experimental and increases gradually as the angle of incidence decreases, becoming the greater between the values 50° 27′ 30″ and 45° 15′, almost the same limits as above, of the angle of incidence, and continuing so throughout the rest of the arc examined.

With the exception then of one or two observations recorded at the end of Table II., the results of the two series of experiments, the one made during the autumn of 1880, the other during the summer of 1881, agree and lead us to the conclusion that the laws arrived at by the electro-magnetic theory connecting the planes of polarization of the incident and refracted rays in the case of refraction at the surface of a uniaxal crystal are not exact but are probably close approximations to the truth.

These same laws have been arrived at by NEUMANN,[*] MACCULLAGH,[†] KIRCHKOFF[‡] and others on various assumptions as to the nature of the ether; and in the case of an isotropic refracting media agree with FRESNEL'S formulæ for the relative positions of the planes of polarization of the incident and reflected rays, though not for the refracted.

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[*] Abhand. der Akad. der Berlin, 1835.

[†] Irish Trans., 1837.

[‡] Abhand. der Akad. der Berlin, 1876.

The experiments having all been conducted with the same piece of spar do not afford data for deciding whether the differences observed are functions only of the angle of incidence, or whether, as is more likely, they depend partly on it and partly on the position of the face of incidence with reference to the axis of the crystal. Though in no case large, they are certainly considerably greater than the possible errors of experiment.

The experiments have been conducted by Lord RAYLEIGH'S kind permission in one of the rooms of the Cavendish Laboratory, Cambridge, and the apparatus used is chiefly the property of the Laboratory.

In conclusion, I would refer to a paper on the same subject by F. E. NEUMANN ("Beobactungen über den Einfluss der Krystaleflächen auf das reflectirite Licht." POGG. Ann., xlii.).

He polarized light by passing it through a tourmaline, and then allowed it to fall on a prism of Iceland spar at a known angle of incidence, observing the position of the plane of polarization when only one ray passed through the crystal. The light of a lamp was used and the crystal prism achromatised approximately for the ordinary ray. The prism was capable of rotation about a normal to the face on which the light fell so as to alter relatively to the axis of the crystal the position of the plane of incidence. The theoretical values of the position of the plane of polarization were calculated from formulæ given by NEUMANN (Abhandlungen der Akad. der Berlin, 1835), and MACCULLAGH (Irish Transactions, 1837), which are identical with those deduced from the electro-magnetic theory. In the eight observations recorded, the differences between the calculated and observed values of the azimuth of the plane of polarization range from $-8'$ to $+4'$.

Each observed value is the mean of 40 observations, but it is not stated how far the observations differ among themselves; observations were only made for a small number of values of ϕ, the angle of incidence, two at most for each position of the plane of incidence relatively to the axis of the crystal, and the values of ϕ chosen were 40°, 45°, 50° and 55°.

Now, it will be seen on referring to my tables, that for these values of ϕ the differences there recorded are small. Nothing is said in NEUMANN'S paper as to the method adopted to determine the values of ϕ' and ϕ'', the angles of refraction of the ordinary and extraordinary rays. In fact, they are both indeterminate, seeing that the experiment was made with white light. These are the only other experiments made to test the theory so far as it concerns the refraction of light.

The same paper of NEUMANN'S contains a series of experiments on the position of the plane of polarization of light reflected from the surface of the crystal under various circumstances, the results of which are compared with theory.

The comparison is also extended to some earlier experiments of SEEBECK'S (POGG. Ann. xxi., xxii., and xxxviii.). As far as one can judge from the details given, the differences are hardly greater than the possible errors of the experiment, for in all cases lamp light was used and the position of the plane of polarization determined by the quenching of one of the rays in a double image prism, and both these circumstances preclude great accuracy. On the other hand, the number of individual observations was very large, so that the mean deserves considerable weight.

XII. *On the Results of Recent Explorations of Erect Trees containing Animal Remains in the Coal-formation of Nova Scotia.*

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Received October 11, 1881,—Read January 12, 1882.

[Plates 39–47.]

The occurrence of remains of land animals in the interior of erect fossil trees is, so far as yet known, confined to certain horizons in the coal-field of the South Joggins in Nova Scotia. These remains were first discovered by Sir Charles Lyell and the writer in the summer of 1851. They were found in fragments of the sandstone filling an erect *Sigillaria* which had fallen from the cliff near Coal Mine Point. As other erect trees occurred in the beds from which this was supposed to have fallen, search was made by the writer in subsequent visits for additional trees; but up to 1876 only three of those which became accessible by the wasting of the beds were found to yield animal fossils. These, however, afforded many additional specimens, and several new species of Batrachians and Millipedes. The results of these explorations were published at various times in the Journal of the Geological Society of London,* in a work entitled 'Air-breathers of the Coal Period,' and in 'Acadian Geology;' and Dr. Scudder described the new species of Millipedes in the Memoirs of the Boston Society of Natural History.

The beds containing the productive trees being thus well known, and being exposed in a cliff and in a reef extending into the sea, it seemed probable that many others might be obtained by quarrying operations of no great difficulty. In 1878 the subject was brought under the notice of the Council of the Royal Society, and a grant of £50 was made from the Government Fund to aid in the extraction of these trees and the collection of their contents. With the aid of this grant, a thorough survey and examination has been made of the cliff and reef by Mr. Albert T. Hill, C.E., by Mr. W. B. Dawson, C.E., and by myself, with the kind aid of B. B. Barnhill, Esq., Superintendent of the Joggins Coal Mines. By these means, along with the removal of fallen débris and sand from the outcrop of the beds, twenty additional trees were discovered and were extracted by cutting and blasting; affording many additional specimens and much information respecting the conditions of accumulation of the beds and the manner of entombment of the animal remains.

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* Vols. x., xvi., xviii., xix.

Three annual reports have been made to the Society, detailing the plan and progress of the work. I may here merely state that, after preliminary clearing and exposure of the accessible trees, more especially in the reef extending from the shore and uncovered at low tide, the precise position of each was marked on the plan and section. The trees were then carefully taken out and their contents were examined. The portions containing animal remains were preserved in as large pieces as possible, and were boxed on the spot, the material of each tree being kept by itself. On being taken to Montreal, the whole of the material was cleaned and examined and carefully split up, each surface being scrutinised with the lens under a strong light. The fossils found were marked, keeping together the bones belonging to each skeleton, and were exposed as far as possible with fine chisels and needle points. As the work proceeded, drawings and photographs of the more important bones were made, more especially in the case of those which ran any risk of being damaged in the development of neighbouring or underlying fragments. The pieces belonging to each animal were then attached to cards or placed in separate drawers for study. This preliminary labour necessarily required much time, and though the accessible trees were exhausted in 1879, the final revision of the specimens and the microscopic examination of the bones and teeth have been completed only in the present year.

I may add that, unless additional specimens are exposed by falls of the cliff, further material of this kind can be obtained only by mining in the 6-inch coal supporting the trees, and its roof.

In the following pages I propose to notice as shortly as is consistent with clearness, the new facts obtained from the study of these interesting fossils, under the following heads :—

1. Geological relations of the beds containing the fossiliferous trees.
2. Character and contents of the trees.
3. Description of the included animal remains.

1. Geological Relations of the Beds.

The beds in question belong to Group XV. of Division 4 of the section of the South Joggins, tabulated by Sir W. E. Logan and the writer.* The detailed sequence of the beds more immediately connected with the fossiliferous trees, as noted in the course of the recent explorations, is as follows, in descending order; the dip of the beds being S. 30° W., at an angle of 32°.

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* 'Acadian Geology,' pp. 156 to 192.

	Ft.	ins.
1. *Gray sandstone*, massive, constituting the front of Coal Mine Point, which rises to a height of 93 feet, the top being composed of hard boulder-clay capping the sandstone (see section attached to plan)	30	0
2. *Shale and sandstone*	5	0
3. *Sandstone*, gray	4	0
4. *Coal*, two thin layers and shaly parting	0	8
5. *Underclay and gray shale*	10	0
6. *Coal*	0	3
7. *Underclay* and gray shale	3	0
8. *Sandstone*, gray and irregularly laminated, with erect *Calamites* and *Stigmaria in situ*, also surfaces with vegetable débris. Seems to have been deposited by currents having the direction of E. and W., or nearly so	6	6
9. *Sandstone*, gray argillaceous, graduating into the preceding, with small concretions of ironstone. Erect *Calamites* and *Stigmaria in situ*, also fronds of *Alethopteris lonchitica*	1	6
10. *Shale*, gray, with a few small concretions of ironstone, the texture becoming finer downward (soft and crumbling when exposed to the weather). Many prostrate plants, including *Sigillariæ*, *Stigmaria*, *Calamites Suckovii* and *C. Cistii*, *Lepidophloyos Acadianus*, *Cordaites borassifolia*, *Trigonocarpa*, *Sphenophyllum Schlotheimii*, *Pinnularia*, *Alethopteris lonchitica*, *Sphenopteris*, sp., *Lepidodendron Pictoense*. Also shells of *Naiadites carbonarius* and *N. elongatus*, scales and coprolites of fishes, and numerous valves of *Carbonia fabulina* and *C. bairdioides*	2	4
11. *Shale*, black and coaly. Fossils as in beds above, but more abundant and giving a carbonaceous character to the whole	1	1
[NOTE.—*Erect* Sigillariæ, *containing amphibian remains, stand on the next bed, and penetrate beds* 11, 10, *and* 9 *above, and some of them extend into bed* 8.]		
12. *Coal*, supporting erect *Sigillariæ*. This coal is laminated, and shows numerous impressions of the bark of flattened *Sigillariæ*	0	6
13. *Underclay*, gray, soft above, becoming harder below. Many *Stigmaria* roots with long rootlets	2	8
14. *Sandstone*, gray, with rootlets and some prostrate trunks of trees in the upper part	11	0
15. *Shale*, gray	0	7
16. Sandstone, gray	0	7
17. *Shale*, gray. Two erect trees, standing on bed below, penetrate this bed	3	6
18. *Coal*, and coaly shale, fern stipes, &c.	0	3
19. *Underclay*, with *Stigmaria* roots and rootlets	3	0
Total	86	5

Below this section is a thick series of sandstones and shales, intervening between the beds above and coal-group, No. XVI., of the general section. The sandstones contain numerous drift trunks of trees.

The great sandstone, No. 1, and the beds immediately below it appear in the face of Coal Mine Point, which has been produced by the resistance of the outcrop of this bed to the encroachments of the sea. East of this the coast recedes, still presenting a high cliff in which the beds crop out one by one to the surface. In front of the point the great sandstone, No. 1, and the minor bed, No. 8, extend in reefs seaward, and owing to the great rise and fall of the tide in Cumberland Bay a length of about 330 yards of these reefs is exposed at low tide (see map, section, and view of the coast, Plates 46 and 47).

From information obtained by HENRY POOLE, Esq., it would appear that the cliff recedes at the rate of about 25 feet in 40 years, so that trees are rarely exposed in the bank, and those existing on the reef, and which are the most accessible, represent the effects of denudation extending over at least five centuries.

The relative positions of all the trees observed are seen in the plan and section, from which it would appear that they occur in groups. There is reason to believe, however, that these erect trees are only survivors of a much more dense forest, of which the weaker and more perishable trees had been overthrown. The positions of the trees extracted before 1878 could only be indicated approximately.

The manner of accumulation of the beds enclosing the erect trees is rendered evident by their character and contents, and has been noticed in my 'Acadian Geology,' p. 190, *et seq.* The details ascertained by the recent excavations may be stated as follows:—

The underclay, No. 13, represents a loamy soil on which *Sigillariæ* must have flourished for a long time, filling it with their roots and rootlets. The result was the accumulation of the coal, No. 12, which is filled with flattened and carbonised bark of these trees, as is the black shale constituting its roof. Bark of *Lepidofloyos*[*] is however largely associated with that of *Sigillaria* in these beds. Leaves of *Cordaites* also occur in this coal, and its mineral charcoal shows under the microscope bast fibres of the inner bark of *Sigillaria*, with scalariform, uniporous, and reticulated tissues, probably belonging to the wood of *Sigillaria*, *Lepidofloyos*, *Calamites*, and *Cordaites.*[†] Leaves of the latter genus, as I have shown (*op. cit.*), constitute a large part of some thin coals at the Joggins, and some portion of the ligneous matter, which in former papers I have referred to other genera, may, since the discoveries of GRAND-'EURY, possibly be referred to *Cordaites*. The more compact portion of the coal when sliced shows shreds of epidermal tissue with a few rounded bodies, probably spores of ferns or lycopods.[‡]

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[*] *Ulodendron* of some English palæobotanists.

[†] 'Acadian Geology,' p. 168; and paper on "Coal Accumulation," Jour. Geo. Soc.

[‡] I cannot admit that the large trunks of silicified and calcified wood of the genus *Dadoxylon* (*Araucarioxylon*), so abundant in the coal-formation of Nova Scotia, belonged to *Cordaites*. Their foliage is more probably represented by the leafy twigs of *Walchia* or *Araucarites* found with them. It is also true that some ribbed trees with the markings of *Sigillaria* have wood of the structure attributed by GRAND-'EURY and RENAULT to *Cordaites*. Where the dividing lines between *Sigillaria*, *Cordaites*, and *Dadoxylon* will finally be fixed remains somewhat uncertain. As I have elsewhere argued, however, it is evident that under the names *Sigillaria* and *Cordaites* are included cryptogamous and gymnospermous trees of very different grades.

After a quiet accumulation of vegetable matter, sufficient to give six inches of pure coal, the area must have subsided or been overflowed with water, probably brackish, in which *Naiadites* and Entomostracans established themselves, but in which for a time very little sediment was deposited, the dead plants remaining on the surface of the submerged swamp, and possibly others drifted to the locality, forming, with a little fine argillaceous matter, the material of carbonaceous shales. In the meantime the stronger and larger trunks of the *Sigillaria* forest remained erect, and around their bases there gradually accumulated layers of mud, constituting the shale, No. 10, including drifted plants, while it is not unlikely that the abundant remains of *Sphenophyllum* and *Pinnulariæ* represent aquatic vegetation growing on the surfaces of the accumulating mud. At this time the greater number of the trees had either not yet become hollow or were too tall to receive any sediment. A few, to be noticed in the sequel, were however either wholly or in part filled with clay.

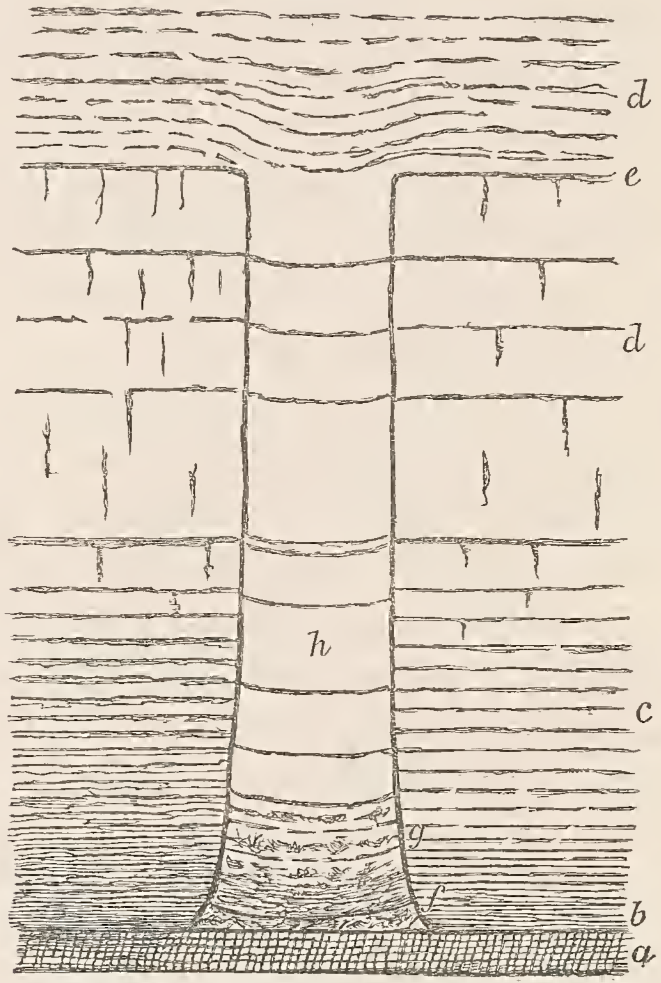
By gradual silting up, possibly aided by a slight elevation, the area again became capable of supporting land plants, as evidenced by the erect *Calamites* and *Calamodendra* which rise from the surface of the shale, and by the *Stigmariæ* in the succeeding sandy deposit. The surface however now became subject to periodical or occasional inundations bearing sand, at first fine and argillaceous, but afterwards coarser. In the intervals of these inundations the thickets of *Calamites* and ferns which occupied the ground were tenanted by Batrachians, Millipedes, and Land Snails.

In the meantime the larger and stronger *Sigillariæ*, which had remained erect, while sand was accumulating around their trunks, became hollow through decay, the strong outer rind alone remaining, while the inner bark and woody axis fell to pieces and dropped into the bases of the hollow cylinders. In this condition these hollow trees would constitute deep wells and pits in the soil, their openings more or less masked with herbage or with shreds of bark projecting above the surface. They no doubt served as places of retreat to Millipedes and Land-Snails; but to the small reptiles heedlessly passing over the surface they were pitfalls into which they fell, and being unable to escape, perished.

In connexion with this, it should be observed that the conditions of the case excluded all animals unable to creep or walk on land. Hence the assemblage of species in these trunks is of a special character, and includes none of those more aquatic forms of Batrachians which have been discovered in the aqueous deposits of the period. Further, only the smaller animals of the locality would be entrapped, larger species being little likely to fall into openings so narrow.

How long any of the hollow trees remained open it is impossible to say; but there is, as might be expected, evidence of successive stages and different modes in their filling. Some short bases of trees, resting on the coal and not extending into the shale, are mere disks of mineral charcoal, and may represent trees which had gone to decay even before the first submergence of the coal. Others seem to have been broken off before the close of the deposition of the shale, and are filled with that material. Others remained open till the deposition of the overlying sandstone, and must have been still about 9 feet high, when finally filled. These are the most productive of animal remains. The normal character of the deposits in such taller trees is represented in fig. 1. At the base is a layer of mineral charcoal representing the débris of the trunk itself, and sometimes containing a sandstone cast of the pith. Above this is a variable quantity of more or less indurated sandy matter, in layers, which, owing to pressure, are often concave above. This is blackened with organic matter, and contains fragments of bark and comminuted plants, with occasional leaves of *Cordaites* or fragments of stems of *Calamites*. It contains the greater part of the animal remains, though a few of these occur in the sandstone above. This deposit, which is the productive layer of the several trees, evidently consists of matter which fell into the hollow trunks or was washed in by rains, while the trees were still open at the surface of the soil. Above this productive layer are layers or sometimes a continuous deposit of sand, by which the trees were finally filled, and which contains no organic remains except fragments of drifted plants. Thus the productive portion of each tree is found near its base. After the final filling and the deposition of additional sediment, the greater compressibility of the matter contained in the trees

Fig. 1.



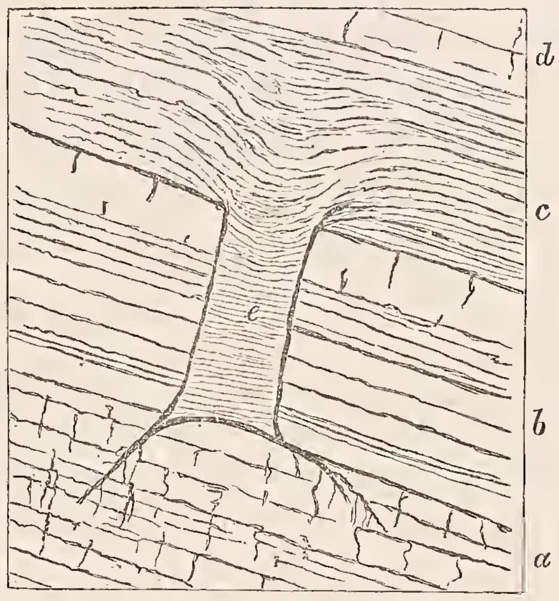
Section of erect tree.

(*a*) Coal. (*b*) Coaly shale. (*c*) Gray shale. (*d*) Sandstone. (*e*) Land surface.
(*d*) Argillaceous sandstone. (*f*) Mineral charcoal. (*g*) Carbonaceous deposit, with animal remains.
(*h*) Sandstone, filling interior of trunk.

caused this to settle somewhat, so that a funnel-shaped depression occurs over some of the taller trunks (figs. 1 and 2).

As opportunity occurred in the progress of the work, erect trees were extracted from other beds in the section, in the hope of discovering another fossiliferous forest; but with little positive result. In a trunk, about 15 inches in diameter, and standing in Section XXVI. of Division 4, Mr. A. T. Hill had found in 1876 several shells of *Pupa vetusta.* This tree, which was uniformly filled with compact argillo-arenaceous matter, was taken out, and a few additional *Pupæ* obtained, but no other remains; and no other trees could be seen in the bed (fig. 2). Another tree standing in Section XXIV., and 2 feet in diameter, was interesting in consequence of its well-marked Stigmarian roots, and of its standing on the sloping edge of a partially denuded shale (fig. 3). It contained no fossils.

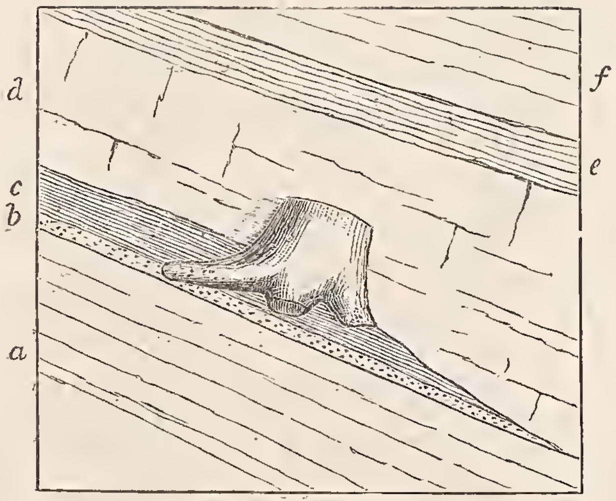
Fig. 2.



Erect tree in Section XXVI., containing shells of *Pupæ* and showing funnel-shaped depression.

(*a*) Underclay with roots and rootlets. (*b*) Sandstone and shale. (*c*) Shale. (*d*) Sandstone.
(*e*) Shale filling tree and containing shells of *Pupa* in lower part.

Fig. 3.



Erect tree in Section XXIV.

(*a*) Sandstone and shale. (*b*) Sandstone. (*c*) Shale. (*d*) Sandstone. (*e*) Shale. (*f*) Sandstone.
This tree had probably originally extended to the shale (*e*), or nearly so.

2. Characters and Contents of the Erect Trees.

It may be well in the first instance to present in a summary form the characteristics and contents of each tree extracted, taking them in order, from the shore cliff to the end of the reef, 1,063 feet in all, and including those extracted prior to 1878 with the others. The positions of the several trees may be seen on the plan and section.

No. 1. Lyell's tree of 1851, which comes first both in order of time and position. Diameter, about 17 inches; height, unknown; distance from crop of coal, No. 12, in bank, about 99 feet. Contained remains of *Dendrerpeton Acadianum*, *Hylonomus Wymani* or *Lyelli*, and *Pupa vetusta*.

No. 2. Extracted in 1860. Diameter, 1 foot 8 inches; height, 9 feet; distance from No. 1 about 73 feet; surface obscurely ribbed. Contained *Dendrerpeton Acadianum* and *Pupa vetusta*.

No. 3. Extracted in 1879. Diameter, 1 foot 8 inches; height, 2 feet 6 inches; distance about 20 feet from No. 2. Filled with argillaceous matter, and a little mineral charcoal at base. No animal remains observed.

No. 4. Extracted in 1879. A crushed coaly bark covering a mound of mineral charcoal. No animal remains observed. Distant about 20 feet from No. 3.

No. 5. Extracted in 1877. Diameter, 2 feet 6 inches; height remaining, about 4 feet. Surface with flat ribs. Distance from No. 4, 93 feet, and from the face of the cliff in 1879, 58 feet. This tree was extracted under unfavourable circumstances, owing to the interference of débris from the foot of the cliff, and high tides. It afforded portions of a skeleton of *Dendrerpeton Acadianum*, bones of *Hylonomus Lyelli*, *Pupæ*, and fragments of Millipedes. On one surface was a trail running round the circumference, indicating the efforts of an imprisoned Batrachian to escape. A few much decayed bones on this surface appeared to belong to *Dendrerpeton*.

No. 6. Extracted in 1879. Diameter, 2 feet; height remaining, less than a foot. This was probably the base of a tree, of which the upper part had been removed by the sea, leaving only the lowest layer. Afforded some much decayed bones, apparently of *Hylerpeton*. This tree was distant only a few feet from No. 5.

No. 7. Extracted in 1879. Distant from No. 5, 66 feet. Merely the base remaining. Consists of mineral charcoal without fossils.

No. 8. Extracted in 1879. Diameter, 1 foot 6 inches; remaining height, 1 foot. Consisted entirely of black carbonaceous matter with bones. It afforded several skeletons of *Hylonomus* and *Dendrerpeton*, the first observed remains of *Hylerpeton*, and many *Pupæ* and Millipedes. The skeletons in this tree were less disturbed than in any other I have examined; but owing to the long exposure of the base of the tree, after removal of the upper part, the material was very soft and much mud-stained, and the bones in a crumbling condition. This tree and Nos. 7, 9, 10 and 11 constitute a group near to each other, and of which owing to recession of the cliff only the bases remained.

No. 9. Extracted in 1878. Short coaly base of a tree with a few bones of *Hylerpeton*, much decayed.

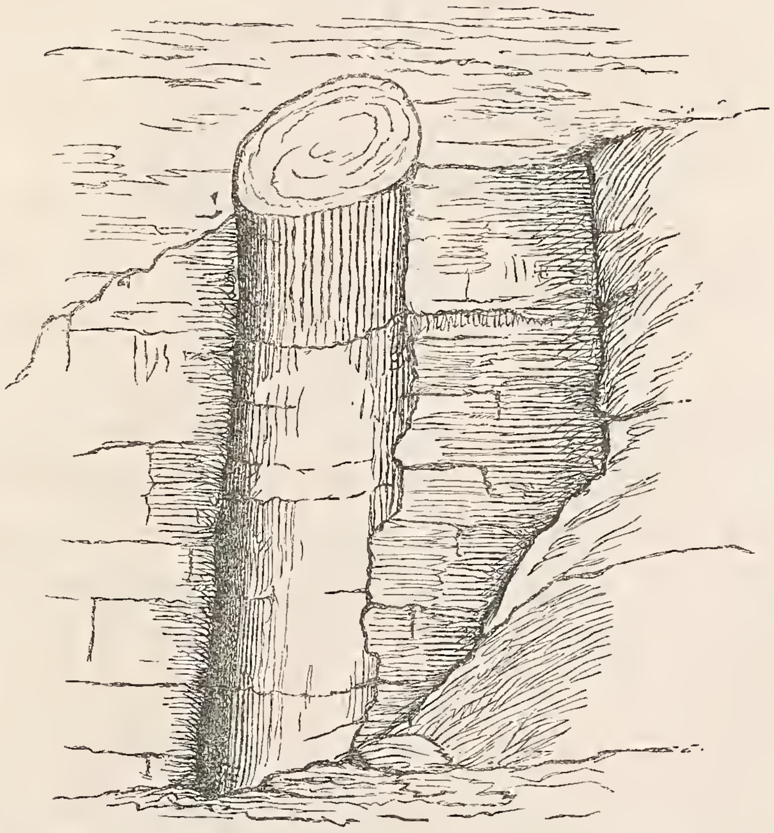
No. 10. Extracted in 1879. Diameter, 1 foot 6 inches. A short coaly base with much mineral charcoal. Detached bones of *Hylerpeton;* teeth and scales of *Sparodus;* remains of larvæ of insects (?).

No. 11. Extracted in 1879. Diameter, 15 inches, much crushed; height, about 2 feet; filled with argillaceous matter; unproductive. Distant 8 feet from No. 7, and exactly opposite No. 10.

No. 12. Extracted in 1878. Diameter, 2 feet 4 inches; height remaining, 3 feet 6 inches; distant 20 feet from No. 11. Surface marked with broad ribs and rows of leaf-scars. In upper part gray sandstone, in bottom 4 inches of dark-coloured material which afforded bones of *Dendrerpeton* and *Hylonomus,* also Millipedes and *Pupæ.* Some of the cuticle of *Dendrerpeton* is preserved. This tree had been inclined to the south-west, and was consequently somewhat flattened, so that its diameter in one direction was double that in the other, and the lower side had been crushed or bent inwards. In the base there were two inches of mineral charcoal, and over this about 2 inches of hard and dark-coloured laminated matter, which afforded many *Trigonocarpa* and fragments of plants. It also contained the whole of the animal remains above referred to.

No. 13. Extracted in 1878 (fig. 4). Diameter, 1 foot 6 inches; height, 9 feet;

Fig. 4.



Tree No. 13, as it appeared when partially exposed in the reef.

distant 100 feet from No. 12; surface roughly ribbed. From the position and great height of this tree, we found it most convenient to undermine it and to extract its basal part, leaving the remainder intact in the reef (Plate 45, fig. 141). The base contained about a foot of dark-coloured matter, with remains of *Hylerpeton, Hylonomus, Smilerpeton, Pupæ,* and Millipedes. This tree must have been a pit at least 8 feet in depth when the reptiles fell into it.

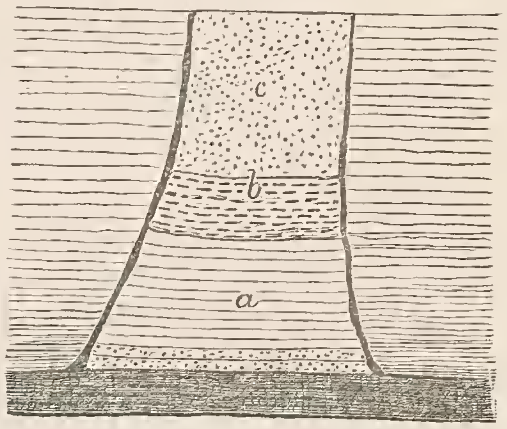
No. 14. Extracted in 1879. Diameter, 2 feet; height, 3 feet 6 inches; distant 13 feet from No. 13. Though there were in the base of this tree 3 inches of carbonaceous matter and mineral charcoal, no animal remains were found in it.

No. 15. Extracted in 1878. Diameter, 1 foot 6 inches; height remaining, 3 feet; distant 111 feet from No. 14. Consisted throughout of uniform gray sandstone without animal remains.

No. 16. Extracted in 1878. Diameter, 15 inches; height remaining, 4 feet 6 inches; distant 94 feet from No. 15. At the base had 6 inches of productive matter containing remains of seven individuals, of genera, *Smilerpeton, Dendrerpeton, Hylerpeton, Hylonomus,* and many *Pupæ* and Millipedes.

No. 17. Extracted in 1878. Diameter, 2 feet; height remaining, 2 feet. Close to No. 16. This tree had been filled in a somewhat peculiar manner (fig. 5). In the bottom were 2 feet of gray shale without animal remains, as if at the time when shale was in process of deposition around the trunk some fissure or opening had admitted a quantity of this material. Subsequently, the usual thin deposit of productive carbonaceous matter occurred, and then the sandstone filling. This tree contained bones of *Hylerpeton* and *Hylonomus,* with remains of Millipedes.

Fig. 5.



Tree No. 17, section showing material filling the interior.

(*a*) Gray shale filling lower part. (*b*) Carbonaceous matter containing animal remains. (*c*) Sandstone.

No. 18. Extracted in 1878. Diameter, 2 feet 5 inches; height, 7 feet; distant 44 feet from No. 17. Obscure ribs and leaf-scars. Filled with argillaceous matter, and destitute of animal remains. In the base was a layer of mineral charcoal, holding a sandstone cast of the medullary cylinder, 2 inches in diameter. The trunk spreads at base to 3 feet, and extends its *Stigmaria* roots on the roof of the coal. A prostrate and flattened trunk of *Lepidofloyos* was found immediately below it.

No. 19. Extracted in 1876. Diameter, 1 foot 6 inches; basal part, about a foot thick and partially compressed, alone seen; the upper part having been removed by the sea. The productive matter in its base afforded 13 skeletons of *Dendrerpeton*, *Hylonomus*, *Hylerpeton*, and *Fritschia*, besides Millipedes and *Pupæ*.

No. 20. Extracted in 1878. Diameter, 1 foot; distance from No. 18, 126 feet; Filled with shaly matter not productive.

No. 21. Extracted in 1878. Diameter, 1 foot 8 inches; height remaining, 4 feet; distant from No. 20 26 feet. About 5 inches of carbonaceous matter in the base, containing remains of *Hylonomus*, *Hylerpeton* and *Smilerpeton*. This tree is much crushed by lateral pressure at the base, and the bones obtained from it were in a fragmentary condition.

No. 22. Extracted in 1878. Diameter, 2 feet 3 inches; height remaining, 3 feet. Filled with sandstone to base, without fossils.

No. 23. Extracted in 1878. Diameter, 1 foot 6 inches; base only preserved. Close to No. 22. Some carbonaceous matter in the base believed to be productive, but accidentally mixed with the contents of No. 16.

No. 24. Extracted in 1878. Diameter, 15 inches; height remaining, 3 feet; distant 8 feet from No. 23. A little carbonaceous matter in hard laminæ of sandstone in base. Contained a few scattered scales and bones of *Dendrerpeton*, and some jointed objects supposed to be insect larvæ.

No. 25. Not extracted. About 18 inches in diameter, and several feet in length. Distant 136 feet from No. 24. This trunk is prostrate and partially flattened. It is visible only at very low tides, and I had no opportunity to extract it. Being prostrate, and so far distant from No. 24, it may possibly mark the limit of the erect trees in this direction.

It will be observed that of the erect trees catalogued above, ten were altogether unproductive. Of these, some had evidently been broken down and filled while the area was still submerged. Others, on the contrary, had remained inaccessible to animals till suddenly filled by the final irruption of sand. The intermediate conditions were those favourable to the entombment of land animals. Fifteen trees were more or less productive: a remarkable proportion when the combination of circumstances necessary to this result is considered. The greater part of the remains have however been obtained from nine or ten of the trees catalogued; but some of the others were only bases of trunks from which more productive portions may have been removed by the sea. The more productive trees are intermixed with the others, and there seems no local restriction in this matter, unless the scarcity of the erect trees toward the end of the reef may indicate a margin of the forest in that direction.

The trees range in diameter from a foot to nearly 3 feet, the ordinary diameter being about 18 inches. The measurements of diameter were made at a short distance above the base, where the trunks became approximately cylindrical. The extreme diameter of the largest tree at the base was 3 feet. The measurements of height refer to the actual condition of the trees. The maximum height of trees which had lost nothing by denudation is 9 feet; but several of those now of less height may, previously to the modern denudation of the beds, have been as tall as any of the others.

So far as can be ascertained, all the trees affording amphibian remains as well as the others standing erect with them, belong to the genus *Sigillaria* in its wider sense, or to the family *Sigillariæ.* Those retaining markings have the characters of the broad-ribbed species of *Sigillaria* proper, like *S. reniformis* and *S. Brownii,* which seem to have been among the largest trees of the coal-period, and those best fitted, by the density and indestructibility of their outer bark, for retaining an erect position. Trunks and branches referrible to the subdivisions *Rhytidolepis* and *Favularia,** and also trunks of *Lepidofloyos,* occur in a flattened condition in the black shale; and these as well as *Cordaites* must have constituted portions of the same forest, though none of them endured long enough to become repositories of amphibian remains. All the erect trees, so far as observed, had roots of the Stigmaria type.

It is deserving of remark in this connexion, that the circumstances of the growth and entombment of this forest entirely contradict those theories as to *Sigillaria* and *Stigmaria* which suppose that these plants grew in water or on submerged areas. In the present instance the surface on which the trees grew, now represented by the 6-inch coal, must have been underlain by several feet of peaty matter with prostrate trunks of trees, and no remains of aquatic animals. The forest contained not only *Sigillariæ* of different species but *Lepidofloyos, Cordaites,* and *Ferns.* When the area was submerged this vegetation was killed, while plants like *Sphenophyllum,* which could grow in water, apparently took its place, and were in turn replaced by *Sigillariæ* and *Calamites* when the area again became land. Further, when the reptiles fell into the erect trees the surface on which they walked, though subject to inundations, must ordinarily have been several feet above the drainage level, otherwise the hollow trees would have remained full of water.

No remains of any aquatic animals were found in the trees holding amphibian remains, unless certain minute annulated bodies, doubtfully compared by Dr. SCUDDER to leeches, can be regarded as of this character. The vegetable matters present consisted, first of the decayed material of the interior of the trunk itself, and, secondly, of fragments which had fallen into or been drifted into the trees after they became hollow. Most of these were pieces of bark and wood, but leaves of *Cordaites* were not uncommon, and there were numerous specimens of the little nutlets which I have elsewhere named *Trigonocarpum sigillariæ*,* and which must be fruits either of *Sigillaria* or of *Cordaites*. Pinnules of *Alethopteris lonchitica* were also occasionally met with, and fragments of *Calamites*. In one tree there occurred so large a mass of aerial roots of the type of *Psaronius*, as to render it probable that these roots may have grown in the interior of the decaying tree.

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* See 'Acadian Geology,' p. 432–3.

With reference to the mode of occurrence of the animal remains, it is to be observed that from the manner of their entombment all the bones of each specimen must be present in the matrix. On the other hand, having dropped asunder on decay of the soft parts, and having been liable to disturbance by water dripping into the trunks, they are often much displaced. In some instances, also, they have evidently fallen into the crevices of loose vegetable matter, afterwards consolidated. Besides this, the original inequality of the surfaces of deposit has been increased by the effect of pressure in depressing the centre, so as to give a basin-form to the layers and to produce a certain amount of displacement at the sides. The effect of this is increased by the circumstance that some of the amphibia seem to have crept close to the sides of the cavity in which they were imprisoned, and to have died in that position, so that their bones lie close to the bark, and in the portion of the deposit most bent by pressure. It is thus difficult to collect the whole of the bones belonging to a skeleton, and impossible to expose some of them without destroying others. I have been obliged, in consequence, to endeavour to secure and expose the more important portions of the skeleton in as perfect condition as possible, and in many instances it has proved possible to recover only a portion of the bones. The state of preservation of the remains is very different in different layers. In some cases considerable portions of the cuticle remain in a carbonised state ; this has occurred when carcases have been quickly covered with moist débris, or permanently water-soaked, so that the soft parts could become carbonised instead of decaying away. These portions of cuticle are unfortunately not laid out flat, so as to show the form of the animal, but much folded and crumpled ; and it would seem that where the cuticle has been thus preserved the bones have been specially liable to decay. The bones are often much softer than the matrix and of a brownish colour, but in some instances they are quite white and in excellent preservation. On a few of the surfaces they have been partially removed by the percolation of water. Their cavities are usually occupied with calcite, but sometimes with pyrite. When sliced they generally show their microscopic structure in a good state of preservation, though the presence of hard grains of pyrite in the softer bone often makes it difficult to prepare satisfactory slices. In a few cases bones of small amphibia and shells of *Pupa vetusta* have been found included in the mineral charcoal which forms the lowest layers in the trees, as if these animals had crept into the interstices of the fragments of decaying wood and bark.

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* 'Acadian Geology,' p. 477.

3. Description of the Animal Remains.

Batrachians.

As already stated, the circumstances attending the entombment of vertebrate animals in erect trees were of such a nature as to exclude the more aquatic forms. The species found in these repositories, therefore, constitute a special and peculiar assemblage, representing the more terrestrial types of the batrachian or amphibian life of the period.

The whole of the amphibian animals found in the twenty-four trees extracted may be included in twelve species, of which two are of doubtful character, owing to the imperfection of their remains. Of the remaining ten, eight belong to the family which I have separated* under the name *Microsauria*, and two are referable to the *Labyrinthodontia*, though perhaps to a special subdivision of that group. The whole may be included in the order *Stegocephala* of Cope.

The *Microsauria* are characterised by somewhat narrow crania, smooth cranial bones, simple or non-plicated teeth, well-developed limb bones and ribs, elongated biconcave vertebræ, bony plates and scales on the thorax and abdomen, and horny scales on the back and sides.

Though probably to be included in the group *Stegocephala*, as defined, and predominantly Batrachian in their affinities, they presented in form, clothing, and probably in habits of life, a close approximation to the lacertians. Their predominating terrestrial habits are evidenced by the circumstance that in nearly all the species the length of the femur coincides very nearly with that of the mandible, while their lacertian form is indicated by the fact that in death their crania and skeletons lie on one side and not on the back or front.

The following genera of this group are represented in the erect trees of the South Joggins :—

1. *Hylonomus.*—Form lizard-like, with the posterior limbs somewhat large in proportion to the anterior. Size, small. Mandibular and maxillary teeth numerous, small, conical, pointed. Palatal teeth minute. Abdominal scales oval. Four species, namely, *H. Lyelli*, *H. Wymani*, *H. multidens*, *H. latidens.*

2. *Smilerpeton.*—Form somewhat elongated and limbs short. Mandibular and maxillary teeth wedge-shaped, with cutting edges. Palatal teeth numerous, some of them large. Abdominal scales oval. One species, *S. aciedentatum.*

3. *Hylerpeton.*—Body stout with strong limbs. Mandibular and maxillary teeth strong, not numerous, grooved at apex. Palatal teeth numerous and some of them large. Thoracic plate broad. Abdominal scales pointed or oat-shaped. Two species, *H. Dawsoni* and *H. longidentatus.*

4. *Fritschia.*—Body lizard-like. Limbs large and well ossified. Mandibular

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* 'Air-breathers of the Coal Period.'

and maxillary teeth conical, grooved at apex. Abdominal scales slender and rod-like. One species, *Fritschia curtidentata.*

5. *Amblyodon.*—A genus characterised by stout cylindrical teeth, blunt at the apices; but otherwise imperfectly known.

The *Labyrinthodontia* are represented in these trees by the genus *Dendrerpeton* alone.

Dendrerpeton may be characterised as having a lizard-like form of body, with the anterior and posterior extremities nearly equal; the skull somewhat elongate with small orbits, and the nostrils placed at the front. The cranial bones sculptured. The teeth plicated at the base, more especially on their inner sides. A series of large teeth on the palate. The body was covered above with imbricated horny scales and had lappets or pendants at the sides. The abdomen was protected by thin bony scales semi-elliptical or oat-shaped in form, and arranged in a chevron pattern. There was probably also a thoracic plate. Two species, *D. Acadianum* and *D. Oweni.*

Of the above species six were more or less perfectly known previous to the recent explorations, but additional material has been obtained illustrating some of their characters. In the following notes these new facts will be stated, with more full descriptions of the new species.

1. *Hylonomus Lyelli,* Dawson (Plate 39, figs. 1 to 14, and fig. 27; also Plate 45, fig. 140).

[Journal of Geological Society of London, vol. xvi., 1859, p. 268. 'Air-breathers of the Coal Period,' 1868, p. 40. 'Acadian Geology,' 3rd edition, 1880, p. 370.]

This species is by much the most abundant in the erect trees examined. It is the type of the genus *Hylonomus* and of the family *Microsauria.* Its characters may now be given somewhat completely, as follows:—

General form lizard-like, with the hind limbs rather larger than the fore-limbs. Length when mature, 5 to 6 inches.

Head somewhat elongate; bones of skull smooth or with microscopic striæ, perfectly united, except at parietal foramen. Occipital condyle double, and apparently bony. Teeth simple, conical, numerous, about forty in each mandible, and nearly equal, except that a few of the anterior ones are rather larger than the others. The teeth are anchylosed to the jaw in a furrow protected by an external bony plate.

Vertebræ with cylindrical bodies, slightly concave at the ends. When partly exfoliated they appear hourglass-shaped, in consequence of the internal cartilage having the form of two cones attached by their apices. Zygapophyses conspicuous above; neural arches united to the bodies of the vertebræ, and with broad neural spines. Dorsal vertebræ with strong lateral processes. Caudal vertebræ apparently simple and cylindrical. Number of vertebræ in neck and trunk about thirty.

Ribs long and curved, with capitulum and tubercle, cartilaginous within.

Anterior limb slender, humerus with distinct keel; radius and ulna separate; toes, four or five.

Posterior limb with well-developed femur, with a trochanter and bifid articulatory surface at distal end; tibia and fibula shorter, separate; toes five, somewhat long and slender.

Pelvis large, principally composed of two large broad triangular ilio-ischiac bones.

Tail probably shorter than the body; thirteen vertebræ seen in one specimen.

Lower surface protected with a thoracic plate and numerous oval bony scales, with delicate lines of growth externally, and concave internally with thickened edges. Upper surface protected with imbricated horny scales. In front two rows of horny tubercles and plates, with epaulettes composed of bristle-like fibres projecting from the skin (Plate 39, fig. 27; Plate 45, fig. 140).

The dimensions of the largest individual found are as follows:—

Length of	head	about	2	centimetres.
,,	neck	,,	1·3	,,
,,	trunk	,,	7	,,
,,	posterior limb to heel		3	,,
,,	mandible		1·8	,,
,,	rib		1·3	,,
,,	humerus		1·4	,,
,,	femur		1·8	,,
,,	tibia		1·2	,,
,,	body of vertebra		2·5	millimetres.

Teeth, five in 1 millimetre.

A smaller individual gave the following measurements:—

Length of	mandible	7	millimetres.
,,	rib	6	,,
,,	humerus	5	,,
,,	femur	7	,,

Under the microscope the bone of *Hylonomus Lyelli* shows well-marked cells and canaliculi, the cells being oval in the cranial bones and more elongated in the limb bones. The dentine of the teeth presents very fine tubes radiating from the pulp cavity, and covered with a thin plate of structureless enamel at the outer surface.

In such specimens of this animal as are preserved in sufficient perfection to show its attitude, the head and trunk lie on one side, showing that the body was compressed laterally; a form fitting it to pass among dense vegetation. Coprolitic matter associated with its remains contains fragments of insects.

2. *Hylonomus Wymani*, Dawson (Plate 39, figs. 15 to 17).

[Journal of Geological Society, *l.c.* 'Air-breathers of the Coal Period,' p. 52.
'Acadian Geology,' p. 378.]

As compared with the last species this was smaller in size, more elongated in form, had the teeth less numerous (about twenty-two in the mandible), and shorter and more obtuse in form. There are six to seven in 1 millimetre.

Length of skull	8	millimetres.
,, mandible	5	,,
,, rib	5·5	,,
,, femur	6	,,
,, humerus	5	,,

This species is much more rare than the former; but quantities of minute bones, probably belonging to it, occur in the coprolitic matter.

Other characters of this species will be found in the memoir and work above cited.

3. *Hylonomus multidens*, s.n. (Plate 39, figs. 23 to 26).

This animal is known only by portions of bones of the head and a few other fragments. The scattered bones of the extremities are inseparable from those of *H. Lyelli* occurring with it. As compared with that species, the bones of this are smoother and more delicate. The teeth are more numerous and slender. The crushed distal end of a femur or humerus found near the skull indicates that the limbs were well developed.

Length of mandible		11	millimetres.
,, skull	about	15	,,
,, femur	,,	9	,,

Teeth, five to six in 1 millimetre.

4. *Hylonomus latidens*, s.n. (Plate 39, figs. 18 to 22).

Of this species fragments of three specimens were found in three distinct trees. It seems to have been somewhat stouter built than *H. Lyelli*, but with the limbs shorter in proportion. Its generic affinities are perhaps somewhat doubtful, as it presents in some respects characters intermediate between *Hylonomus* and *Hylerpeton*.

Mandibular and maxillary teeth broadly conical, about twenty in each mandible—three in 1 millimetre; anterior mandibular teeth somewhat larger than the others, and bent or hooked. Vomer or palate with minute teeth. Thoracic plate large. Scales of abdomen oval, but somewhat narrow, and tending to be oat-shaped.

Length of mandible (imperfect)	9	millimetres.
,, humerus	7	,,
,, vertebra	2	,,
,, tibia (?)	5	,,
,, thoracic plate	1·5	centimetre.
,, six caudal vertebræ	8	millimetres.

5. *Smilerpeton aciedentatum*, DAWSON (Plate 40, figs. 28 to 45).

[*Hylonomus aciedentatus*, DN., Journal of Geological Society, *l.c.* 'Air-breathers of the Coal Period,' p. 49. 'Acadian Geology,' p. 376.]

This animal was originally referred by me to the genus *Hylonomus;* but additional specimens and more careful comparison oblige me to place it in a new genus.

Its most remarkable distinction is the form of the mandibular and maxillary teeth, which are of a peculiar wedge-shape, being broad and oval at base and narrowed to a longitudinal edge at top. Thus, when viewed from the side they appear narrow and blunt, but when the jaw is broken across, and they are viewed from the rear or front, they appear broad and sharp-edged. The effect of this arrangement is that the jaw is armed with a closely-placed series of chisels or wedges, giving an almost continuous edge. At the end of the mandible some of the teeth are longer and more conical.

Another important character is that the palatal and vomerine bones seem to have bristled with teeth, mostly of very small size; but there are also some larger palatal teeth, of which some are sharply pointed and others blunt with furrowed points.

The vertebræ are of the same type with those of *Hylonomus;* but some which appear to be caudal have a pointed spine above, indicating perhaps a flattened tail. The ribs are short and stout.

Only a few fragments of the limbs have been found; they would seem to have been less developed than in *Hylonomus,* and perhaps this animal was less gifted with powers of walking on land than any of the other *Microsauria.* This would accord with the fact that its bones are much scattered, and occur chiefly in sandy layers, as if introduced in times of rain or of partial inundation.

Some fragments indicate thoracic plates, and there are numerous abdominal scales of oval form. Above, the body appears to have been clothed with small tubercles and horny scales, and to have had cuticular pendants like those of *Dendrerpeton.*

In a large specimen the mandible measures 2·2 centimetres, the femur 2 centimetres, and a rib the same. The following are measurements from a smaller specimen:—

Length of mandible	1·5 centimetre.
,, femur	1·5 ,,
,, humerus (?).	1·3 ,,
,, vertebra	3·5 millimetres.
,, rib	1 centimetre.

There are five teeth in 2 millimetres.

Under the microscope the bone shows elongated cells and a somewhat fibrous structure, and there is much cartilage associated with it. The teeth are composed of radiating tubes of ivory of remarkable coarseness, and with distinct lateral canaliculi; but the coarse tubes do not reach the surface to which only their terminal canaliculi extend, and there appears to be a thin superficial layer of dense and brilliant enamel having externally a microscopic vertical striation.

The above characters are taken from two specimens approaching each other in dimensions. A third specimen was of much smaller size, and had longer and more slender palatal teeth. It was originally set apart as a second species, under the name *S. acutidentatum;* but as the bones secured are few, and it may possibly be a young individual of the present species, I have thought it best to leave it undescribed in the meantime.

6. *Hylerpeton Dawsoni*, Owen (Plate 41, figs. 62 to 85).

[Owen, Journal of Geological Society, vol. xviii., p. 241. Dawson, 'Air-breathers of the Coal Period,' p. 55. 'Acadian Geology,' p. 380.]

The specimen on which Owen founded this genus and species was obtained by me in 1861. Unfortunately it consisted of only a few fragments of bone, the principal of which was a mandible, with some of the teeth remaining, and a fragment of a maxilla. The trees subsequently examined have afforded bones belonging to four additional individuals, and enable the following description to be given.

Bones of skull slightly striated, but not sculptured as in *Dendrerpeton.* Lower jaw with distinct ascending ramus or coronoid process, a feature not known in any other of these amphibia, but observed by Cope in his genus *Brachydectes*, which may be allied to *Hylerpeton*, but is known only by the jaws and teeth. Teeth, twelve in each ramus of the mandible, bluntly conical, slightly striated at the apex. Pulp-cavities large and longitudinally striated at the sides, though the teeth are not folded. Maxilla furnished with similar teeth, one of which near the front is larger than the others. Palatal teeth numerous, small and conical, with a few large teeth at the sides.

Vertebræ short, cylindrical, well ossified, with well developed zygapophyses and neural spines; ribs strong and much curved, with well developed division of the proximal ends; pelvis imperfect, but apparently large, with broad ilium.

Humerus half the length of the mandible; radius half as long as humerus; femur very large and stout, nearly as long as the mandible; leg bones and phalanges correspondingly stout.

The thoracic plate is indicated only by some fragments. The abdominal scales are narrow and pointed (oat-shaped), smooth externally and with a ridge at one side within.

The following are dimensions of the largest specimen:—

Length of	mandible	4·4 centimetres.
,,	largest tooth	5 millimetres.
,,	femur	3·5 centimetres.
,,	tibia	2 ,,
,,	humerus	2 ,,
,,	radius	1·5 ,,
,,	vertebra	6 millimetres.
,,	rib	3 centimetres or more.
,,	scales	5 to 7 millimetres.

Under the microscope the bone presents a coarser structure than that of *Hylonomus*, the bone-cells being large and of elongated form. The dentine of the teeth has coarse tubes with canaliculi, the appearance being very similar to that in *Smilerpeton*, though the tubes are scarcely so large.

It seems evident that *Hylerpeton Dawsoni* was an animal somewhat stout and broad in form, with a large and pointed head and massive limbs. It seems to have fed in part at least on Millipedes, as remains of these are found in coprolite associated with its bones. For some reason also, the specimens of this species seem to have been among the earliest introduced into the erect trees. Perhaps they sought their myriapodous food near these hollow trunks, or were in the habit of breaking up decayed wood in search of Myriapods.

7. *Hylerpeton longidentatum*, s.n. (Plate 42, figs. 86 to 109).

[Preliminary Notice, American Journal of Science, December, 1876.]

Two specimens of the species occur in the collections, neither of them perfect. Their characters warrant us in placing the animal in the genus *Hylerpeton*, but it is very distinct from the previous species, more especially in the length and slenderness of its teeth, and in the breadth of its thoracic plate.

Head much elongated, with the bones minutely pitted, and with delicate microscopic striæ, but not sculptured. Mandibular and maxillary teeth long and acute, pointing backwards, with the apex of their inner sides finely striated; twenty or more in each ramus of the lower jaw; palatal bones with several long slender teeth and many minute teeth. The mandibles found are not complete, but there are indications that there was an ascending process as in *H. Dawsoni*, but less developed. The narrowness of the dentary bone in Plate 42, fig. 86, is caused in part by the lower posterior edge being bent inward at (*a*), and by the posterior end being broken off above.

Vertebræ short and stout, and apparently well ossified. Ribs long, with double head and much curved.

Humerus longer than the femur, which is short and stout, if the bone taken for it is rightly determined.

Abdominal scales narrow, oat-shaped; thoracic plate large, broadly oval.

Length of mandible	4	centimetres.
,, vertebra	5	millimetres.
,, rib	2·5	centimetres.
,, humerus	1·5	,,
,, femur (?)	1·2	,,
,, tibia	8	millimetres.
,, mandibular teeth	3	,,

Six to seven teeth in 1 centimetre.

Under the microscope, the bone shows somewhat elongated cells similar to those of *H. Dawsoni*, but smaller. The teeth in cross-section near the base, present a somewhat complex structure, though the dentine is not folded. Next the pulp cavity there are straight radiating cells or tubes, coarse and with lateral canaliculi. This structure, which resembles that of *H. Dawsoni*, extends about half way to the surface, when the large tubes cease and the canaliculi form a dense network. On the outside of this is a thick layer of enamel with fine straight canaliculi. A section nearer the apex of the tooth would probably show a less thickness of the second or intermediate layer.

8. *Fritschia curtidentata*, s.n. (Plate 43, figs. 110 to 128).

[*Hylerpeton curtidentatum*, Preliminary Notice, American Journal of Science, *l.c.*]

This species is represented in the collections by two specimens—one well preserved and probably adult, the other smaller and less perfect. In 1876, when I possessed but one specimen, I referred it to *Hylerpeton;* but it differs from that genus in the absence of palatal teeth and in the abdominal armour, which consists of long slender rods instead of scales. This kind of protective structure, as is well known, occurs in HUXLEY's genus *Ophiderpeton*—a creature otherwise very different—and in COPE's *Sauropleura*, which in the parts known has much resemblance to the present genus.*

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* Certain species of *Urocordylus* described by HUXLEY and FRITSCH would seem to present forms of scales intermediate between the oat-shaped and rod-like types.

I have dedicated the genus to Professor ANTON FRITSCH, who has so ably illustrated the carboniferous and permian Batrachians of Bohemia.

Bones of the head very smooth, having only a few microscopic punctures. Teeth conical, somewhat obtuse, striated at the inner side of the apices; there are about thirty in each ramus of the mandible, and about twenty-seven in the maxillary bone. As in the other *Microsauria,* they are implanted in a furrow.

Vertebræ short and well ossified, three in a centimetre. Ribs strong, curved, about 1 centimetre in length.

Limbs robust, the bones better ossified than in any of the other species of *Microsauria.* Humerus round, and with a distinct keel on the shaft. Femur with well-formed articulating surfaces. Toes of hind foot probably five, central ones long and slender.

Thoracic plate of moderate size and somewhat rounded. Abdomen protected by needle-like rods, which are very numerous, and were probably arranged *en chevron.*

Length of mandible (not quite perfect) . .	2·1	centimetres.
,, maxilla	2	,,
,, rib	1	,,
,, humerus	2	,,
,, femur	2·4	,,
,, radius and tibia . . . . about	1	,,
,, toe of hind foot	7	millimetres.

Eight teeth in 5 millimetres.

Under the microscope the bone shows small rounded cells with numerous canaliculi. The cells in the limb bones are a little longer than the others. The teeth are simple, and consist of dentine traversed by moderately fine and somewhat tortuous tubes; these are much coarser than in *Hylonomus,* but finer than in *Hylerpeton.*

9. *Dendrerpeton Acadianum,* OWEN (Plate 40, figs. 46 to 51; and Plate 44, figs. 129 to 137, except fig. 131).

[OWEN, Journal of Geological Society, vol. ix. DAWSON, 'Air-breathers of the Coal Period,' p. 17; 'Acadian Geology,' p. 362.]

Portions of several specimens of this species are in the new material, but do not add much to the knowledge of its characters, which have been fully given by OWEN from the somewhat complete specimens obtained by me in 1859.

Some interesting fragments of cuticle were found in association with one of the skeletons, which show very well the scaly covering of the back and the lappets and edge-scales of the sides. These last probably formed a border or margin to the bony scales covering the abdomen (Plate 40, figs. 46 to 51).

A specimen of the skull was also obtained showing its internal surface, and exhibiting clearly the sutures of the several bones. This I have submitted to Professor Cope, and give a diagrammatic representation of it (Plate 44, fig. 132), showing his interpretation of the several bones, which apparently accord in number and arrangement with those of *Labyrinthodontia*, but differ from those of the *Microsauria*, in so far as the crushed condition of these latter enable an opinion to be formed.

Under the microscope, the bone of *Dendrerpeton* presents large and somewhat rounded or oval cells with numerous canaliculi filling the intermediate spaces. The teeth have the dentine folded at the base, but sometimes only on the inner side. The tubes of the dentine are simple and fine, in this resembling those of *Baphetes* and other Labyrinthodonts.

The abdomen of *Dendrerpeton* was protected by numerous broadly sub-oval scales, arranged *en chevron*, but I have not been able to detect with certainty any thoracic plate, though I have one specimen which I think may be a part of such a plate.

Other characters of this species will be found in the memoir and works above cited.

10. *Dendrerpeton Oweni*, Dawson (Plate 44, figs. 131, 138, 139).

[Journal of Geological Society, vol. xviii., p. 469. 'Air-breathers of Coal Period,' p. 32. 'Acadian Geology,' p. 368.]

This species is represented by four specimens in the new material, and these serve to establish its distinctness from the last mentioned, in its smaller size, its more delicate cranial sculpture, and its longer and more curved teeth. Its abdominal scales are also narrower and more pointed, approaching in this to the oat-like form of those of *Hylerpeton*. So far as known, the scaly covering of the back of this species was similar to that of *D. Acadianum*.

Other characters of the species will be found in the memoir and works above cited.

11. *Sparodus*, sp. (?) (Plate 40, figs. 52 to 56).

In the coaly matter or mineral charcoal at the base of tree No. 10, appeared a few fragments of an animal which may possibly belong to the above-named genus of Fritsch, though I am by no means certain of this identification or of the real nature of the animal.

The skull is represented by a fragment of a maxillary or intermaxillary bone, with blunt conical teeth. It is smooth or marked merely with microscopic dots. There is also a fragment which may be a palatal bone studded with minute teeth.

A few vertebræ associated with the above bones are long and narrow, with large zygapophyses and long neural spines. Length of body about 3 millimetres.

With these remains are a few bony scales different from those of any other species found in these trees, and more resembling scales of Ganoid Fishes. They are somewhat rectangular in form, enamelled on the surface and beautifully sculptured with waving lines.

In the same trunk were found some teeth and bones referable to *Hylerpeton Dawsoni*, and it is not impossible that the remains above referred to may have belonged to some creature devoured by that animal, and which would not otherwise have obtained admission to the interior of an erect tree. The tree itself had been removed by the sea, all but a little of the base, and this was in a very unsatisfactory state, so that doubt might even exist as to the limit between the deposit in the interior of the tree and that under its base.

12. *Amblyodon*, gen. nov. (Plate 40, figs. 57 to 61).

In tree No. 16 were found a few teeth and bones which do not seem referable to any of the genera above named, though pretty certainly belonging to a member of the group of *Microsauria*.

A fragment of a jaw 1 centimetre in length has ten cylindrical teeth, simple and smooth, with large pulp cavities and rounded regularly at the apices. With these are four vertebræ of the usual type, measuring together 1 centimetre. Fragments of cranial bones also occur and are obscurely pitted. There is also what seems to be the shaft of a limb bone and a few oval scales. A flat, somewhat rhombic bone with a style at one side may possibly be a thoracic plate or possibly a parasphenoid.

The material is too scanty for any satisfactory description of this animal, but I have named it provisionally *Amblyodon problematicum*.

13. *Coprolitic Matter.*

This occurs in several of the trees, not in masses of regular form but in indefinite patches. It is of a gray or buff colour, and usually highly calcareous. It is often filled with comminuted bones not determinable; but evidently of small Batrachians and probably of *Hylonomus*. Fragments of chitinous matter also abound in some of the coprolitic masses. In most cases they seem to belong to Millipedes, but in a few examples insect remains occur. They are not determinable; but in one specimen was a well-preserved fragment of a head apparently of a small neuropterous insect showing one of the compound eyes. Fragments of shells of *Pupa* are found in and near some of the coprolitic masses, and I think it probable that these pulmonates formed a part of the food of some of the Batrachian species.

Some doubt must of course exist as to whether the substances contained in the coprolite represent the ordinary food of the amphibia or only that to which they had access while imprisoned in the erect trees. The facts so far as they go would indicate that the larger amphibian species preyed upon the smaller, and the latter on Insects, Millipedes and *Pupæ.*

14. *Land Snails.*

The additional facts obtained in relation to these I have detailed in a paper communicated to the American Journal of Science.[*] *Pupa vetusta* proves to be by far the most abundant species. A very few crushed specimens of *Zonites priscus* were detected. The only new species found is a minute *Pupa* of different type from *P. vetusta,* and which I have named *P. Bigsbii.* These three species, with two described by BRADLEY from the coal formation of Illinois (*Pupa Vermilionensis* and *Dawsonella Meeki*) and a third from Ohio, recently described by Professor WHITFIELD (*Anthracopupa Ohioensis*) are so far the only known carboniferous pulmonates. The shell which I have named *Strophites grandaæa*,[†] from the Devonian of New Brunswick, is probably a still older representative of this group. Details with respect to them will be found in the paper in the American Journal of Science above referred to, and in that of Professor WHITFIELD in the same Journal for February, 1881.

15. *Millipedes, &c.*

Much additional material of this nature was obtained, but mostly in a very fragmental condition. On careful examination, I was able to refer nearly all the specimens to the species of *Xylobius* and *Archiulus,* already distinguished by Dr. SCUDDER in the material from the trees previously excavated. A number of other fragments, which seemed to indicate additional forms and species of arachnids, have been placed in Dr. SCUDDER's hands, and also some jointed objects which may be remains of larvæ of insects.

CONCLUSION.

It may be interesting to note the whole number of individual animals represented by the amphibian remains found, and the trees in which they were contained, as an indication of their distribution and relative abundance.

Microsauria.

1. *Hylonomus Lyelli*	12	specimens.	Trees No.	5, 8, 12, 13, 16, 19.
2. *H. Wymani*	7	,,	,,	1, 8, 17, 19, 21.
3. *H. multidens*	1	,,	,,	17.
4. *H. latidens*	3	,,	,,	12, 13, 21.
5. *Smilerpeton aciedentatum* .	3	,,	,,	8, 13, 21.
6. *Hylerpeton Dawsoni* . . .	6	,,	,,	6, 8, 9, 10, 16, 19, 21.
7. *H. longidentatum*	3	,,	,,	16, 17, 19.
8. *Fritschia curtidentata* . .	3	,,	,,	19, 21.
9. *Amblyodon problematicum* .	1	,,	,,	16.

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[*] November, 1880.

[†] Ibid.

Labyrinthodontia.

10. *Dendrerpeton Acadianum* .	9 specimens.	Trees No. 1, 2, 5, 8, 12, 13, 19, 24.
11. *D. Oweni*	4 ,,	,, 8, 16, 19.

Incertæ Sedis.

12. *Sparodus*, sp. (?)	1 specimen.	Tree No. 10.

Total individuals, 53.

The negative result that, under the exceptionally favourable conditions presented by these erect trees, no remains of any animals of higher rank than the *Microsauria* and *Labyrinthodontia* have been found deserves notice here. It seems to show that no small quadrupeds of higher grade inhabited the forests of Nova Scotia at the period in question. This is perhaps confirmed by the remarkably Lacertilian characters assumed by the *Microsauria* of the period, which seem to have occupied the place now taken by the smaller true reptiles. That there were larger Labyrinthodonts than those found in the erect trees we know from the *Baphetes planiceps* of the Pictou coal,* and from the remarkable footprints of *Sauropus Sydnensis*† and *Sauropus unguifer.*‡ It is to be observed also that as some of the amphibian animals found in the erect trees are represented only by single specimens, there may have been still rarer species, which may be discovered should other trees be exposed. Nor must we forget that the fauna of those swamps and low-lying plains of the carboniferous period, to which our knowledge is at present limited, may not fully represent that of the uplands of the period.

With reference to the probability of the discovery of additional remains in the beds to which this paper relates, I may state that new trees will no doubt be exposed from time to time by the gradual wasting of the cliff. Otherwise additional specimens can be procured only by regular mining operations carried on in the 6-inch coal and its roof. These would of course be costly, and the small amount of coal afforded by the 6-inch seam would contribute very little towards defraying the expense.

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* Journal of Geological Society, vols. x. and xi.; 'Air-breathers of the Coal Period;' 'Acad. Geology,' p. 359.

† 'Acadian Geology,' p. 358.

‡ 'Geological Magazine,' vol. ix.

(Received March 8, 1882.)

Note I.—On Horny Scales and other Appendages of Carboniferous Amphibians.

I have discussed these at some length in my 'Air-breathers of the Coal Period,' and as few new facts occurred in the more recent explorations, I have merely adverted to them in the text. It has however been suggested to me that some more detailed reference to them would be desirable.

I have referred to this kind of cuticular covering as being found in connexion with the bones of *Hylonomus Lyelli* and *Dendrerpeton Acadianum* and *D. Oweni.* It may have been present in other species, but of course was likely to be preserved only in rare instances. The examples figured in Plates 40 and 45 may serve to give an idea of the perfect manner in which it has sometimes retained its characters, though unfortunately, from the uneven and irregular surface of the deposits in the interior of erect trees, it is always folded and crushed, so that it does not retain its original form.

It is to be observed that in the species referred to, the thorax and abdomen were protected with bony plates and scales. The horny scales and plates seem to have been confined to the upper parts.

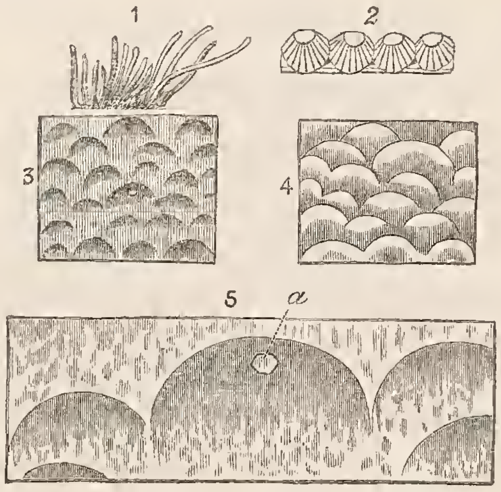
The horny scales and appendages are entirely different in appearance from the bony plates and scales. The latter are usually white or gray in colour, and present under the microscope true bony structures. The former are black, shining, and coaly in appearance, and are inseparable from the cuticle along with which they are preserved. The ordinary horny scales are semicircular, imbricated, or scattered sparsely over the surface of the skin. They are of different sizes, and the larger often show minute round pores, probably mucous or perspiratory pores. These scales are most conspicuous toward the upper and anterior parts of the body. Elsewhere they often degenerate into microscopic tubercles implanted in the skin. Along the sides, and perhaps near the margin of the upper scaly portion of the skin, there are in some specimens larger angular scales, apparently free at the margins, and forming a sort of Vandyke edging. In front the skin projects into long pendant lappets, terminated by similar angular points, and covered with oval scales, not imbricated, and each having a pore in its centre. These appear to have been present both in *Dendrerpeton* and *Hylonomus.* In the latter, as stated in the text, there were on the back and shoulders thick ridge-like and conical tubercles, having their surfaces sculptured with furrows, and *H. Lyelli* has two rows of flat horny bristle-like processes forming frills or epaulettes.

Portions of the scaly cuticle when carefully separated from the stone and mounted in balsam, are sometimes sufficiently thin to be studied as transparent objects. Viewed in this way, under a moderate power, the skin appears of a rich brown colour, and presents an areolar or cellular aspect, the scales appear dark brown, becoming black at their distal edges, and their pores appear as round transparent spots; viewed as an opaque object, the scaly skin appears black and shining, and the edges of the scales seem to be very thin and to bend upward as if free from the skin. The bristle-like appendages of *Hylonomus* are also translucent, when mounted in balsam; but even under a high power show only a faint indication of longitudinal fibrous structure (see woodcut, figs. 1 to 5). The thicker plates, when sliced, show, near the base, a few curved canals, probably vascular, but the upper part appears quite compact, and under a high power merely shows faint indications of tortuous fibres or tubes. Even the thickest show no bony structures whatever, and have throughout a carbonaceous or bituminous appearance.

When burned, the cuticle and horny scales give a strong flame, and emit a bituminous and ammoniacal odour, their chemical characters being those of highly bituminous coal or jet.

These portions of cuticle and horny scales are of rare occurrence, and appear to have owed their preservation to being embedded in wet fragments of bark and other vegetable matter, perhaps possessing a tanning quality; small loose bodies of similar character have been found which may have been horny plates detached from the skin by decay, but it is impossible to say to what they belonged. One of these is a semi-circular plate, about half an inch in diameter, and studded with conical tubercles. It may have been the armature of the snout of one of the *Microsauria*.

For additional descriptions and illustrations, see 'Air-breathers of the Coal Period,' p. 34, and Plates 1, 4 and 5; and 'Acadian Geology,' pp. 369–373.



Scales and Appendages of *Hylonomus* and *Dendrerpeton*.

Fig. 1. Ornamental horny processes of *Hylonomus Lyelli*, × 10.
,, 2. Horny tubercles of the same, × 10.
,, 3. Cuticle and scales of *Dendrerpeton* (transparent), × 10.
,, 4. The same (opaque), × 10.
,, 5. Portion of the same, × 50, showing cutaneous pore at (*a*).

(Received April 10, 1882.)

I have much pleasure in appending the following note, which I consider a most important addition to my paper, showing that two species of Scorpions have been entombed with the other tenants of the erect trees.

Note II.—On Additional Remains of Articulates obtained by Dr. Dawson *from Sigillarian Stumps in the Coal-field of Nova Scotia.** *By* Dr. Samuel H. Scudder.

The fragments sent to me for study, like those formerly received, consist in great part of myriapodal remains, often of single segments, and generally in a more or less crushed and flattened condition. In this respect they are not so well preserved as some of those previously studied, and obtained from erect trees in the same locality. Although all the species formerly separated occur in this collection, very little can be added to the statements then made. Two specimens occur of *Xylobius sigillariæ*, five of *X. similis;* three are somewhat doubtfully referred to *X. fractus*, eight to *X. Dawsoni*, and ten to *Archiulus xylobioides.* A single specimen of *X. Dawsoni*, showing four or five continuous segments, seems to prove that the elevated transverse ridge on each segment in this species was crowned by a single series of minute warts or raised points, not very closely set. A few specimens of different species exhibit the marks which were formerly interpreted as *foramina repugnatoria*, but are now presumed to be the casts of bases of spines,† thus bringing these species into more definite and probable relations to the carboniferous myriapods of Mazon Creek, though they plainly belong to a distinct group. Whatever spines they had must have been very small, slight, and wholly insignificant in comparison with those of the bristling *Archipolypoda* of the Morris beds. Careful search has been made for any other of those special features which distinguish the *Archipolypoda* from recent *Diplopoda*, but in vain, beyond the single but not unimportant point that the ventral plates, in *Archiulus* at least, are very broad and probably almost equally extensive in lateral expansion with the dorsal plates, a feature found nowhere in modern *Diplopoda.*

This is, perhaps, most clearly shown in two new species of *Archiulus*, discovered among these remains, and to which are referred a dozen or more specimens. One of these species is of about the same size with *A. xylobioides*, but has perfectly flat segments showing only a very slight and narrow transverse ridge at the anterior margin, occupying not more than one-fourth of the segment. The other is a smaller species, and has shorter and more simple segments, made slightly concave by the gentle elevation of both front and hind margins, but with no anterior ridge. In neither of these species could any trace of spines be found.

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* For descriptions of the remains previously discovered, see Mem. Bost. Soc. Nat. Hist., vol. ii., pp. 231–239, 561–562 (1873, 1876).

† Mem. Bost. Soc. Nat. Hist., vol. iii., pp. 148, 149 (1882).

Besides the myriopodal remains, there are a dozen fragments that must probably be referred to Scorpions. Of some of them there can be no doubt. The remainder are mere bits of integument showing the surface sculpture, but often with no natural borders whatever. In the character of the surface there is such difference as to indicate more than a single species. For though we should certainly expect to find considerable differences between the various parts of one and the same individual, the diversity here is too great, both in amount and nature, to render it at all probable that the difference may fairly be explained in such a manner. The better fragments exhibit a considerable portion of the stouter part of the body, enough to show its general form at least, and these point also to the probable existence of two species, of nearly the same size, but differing in form and sculpture ; the more fusiform-shaped species having a less roughened surface than is found in the more parallel-sided form. The latter agrees tolerably with the carboniferous genus found near Mazon, Ill., called *Mazonia* by MEEK and WORTHEN, and certainly belongs to the same group of Scorpions ; but in view of the remarkable addition to our knowledge of the carboniferous Scorpions in the promised publication of the researches of Mr. PEACH of the geological survey of Scotland, further study of these remains will best be postponed. In the meantime, they add another form of strictly land life to those already found in these remarkable repositories of fossils, and perhaps illustrate the utility of the bony and horny armour of the smaller Batrachians of the period, which may have had to contend with these active and venomous Arachnidans.

The occurrence of seven species of Millipedes in a few decayed trees in one locality, in connexion with similar discoveries in other parts of the world, tends to strengthen the probability of the suggestion, already made by Dr. DAWSON, that the animals of this type may have culminated in the Palæozoic period.

(Received September 4, 1882.)

Note III.—On the Footprints of Batrachians observed in the Carboniferous Rocks of Nova Scotia.

Though it is impossible to identify with certainty the footprints of particular species of the Batrachians of the coal-formation, and though most of the animals which have left impressions of this kind are much larger than those found in the erect trees, yet to give completeness to this memoir, it may be well to notice the indications of this kind which have been observed; more especially as they have not previously been brought together into one view.

For convenience, I shall refer the larger footprints, probably those of Labyrinthodonts of considerable dimensions, to the genus *Sauropus*, already established by LEA, and the smaller impressions, due perhaps to smaller Labyrinthodonts or to Microsaurians, and usually showing a longer stride and more slender toes, to the genus *Hylopus.* Besides these, there are the curious trails named *Diplichnites*,* and which may have been made by large unknown serpentiform batrachians.

Impressions of both types first appear in the lowest carboniferous or HORTON series, corresponding to the "Tweedian series" of TATE, the "calciferous sandstones" of McLAREN, and the carboniferous slate and Coomhala grit of JUKES. No bones of Batrachians have as yet been found in these beds, but the footprints indicate the presence at the beginning of the Carboniferous Period, and before the deposition of the lower carboniferous limestones, of both large and small species similar to those of the coal-formation. At the other extremity of the carboniferous system, footprints have been found on the sandstones of the upper coal-formation or permo-carboniferous series.

1. *Sauropus unguifer*, DAWSON.

[Geological Magazine, vol. ix. 'Acadian Geology,' 3rd edition, supplement, p. 62.]

This is the largest footprint yet found in the coal-formation of Nova Scotia. The length of the hind-foot, on one of the slabs having the largest and most distinct impressions,† is six inches, and its greatest breadth about five inches; the stride is from 12 to 13 inches, the distance of the two rows of tracks being about seven inches, and the hind-foot covering the impression of the fore-foot, which from other slabs would appear to have been smaller and shorter. The impressions show four ordinary toes, and a fifth outer toe armed with a long claw or spur, which sometimes trailed on the ground, and, when the foot was planted, was plunged into the mud. It was no doubt an aid to the animal in ascending inclined surfaces of mud.

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* Am. Journal of Science, 1873.

† Collected by SANDFORD FLEMING, Esq., C.E., and now in the Museum of the Geological Survey.

Very fine series of footprints of several individuals of different sizes, and showing different gaits, and in one instance the act of wallowing in soft mud, have been found in sandstone at Fillmore's Quarry, River Philip, Nova Scotia, and are now in the Museum of the Geological Survey. In some of the series of tracks there is no mark of the belly or tail. In others the belly has left impressions marked with longitudinal grooves, as if there were abdominal furrows or rows of scales. Descriptions of the principal specimens will be found in the publications above referred to.

2. *Sauropus Sydnensis*, DAWSON.

['Acadian Geology,' 3rd edition, p. 358, and woodcut.]

This animal had shorter and broader feet, with five toes and no indication of the peculiar claw of the previous species. The breadth of the foot was about three inches, the width of the body about six inches, and the stride about eight inches. The original specimen was obtained by R. BROWN, Esq., F.G.S., in the coal formations at Sydney, Cape Breton, and is now in the Museum of the MCGILL University.

Sauropus antiquior, DAWSON.

This species is based upon a series of footprints found by Mr. F. M. JONES, of Halifax, at Parrsboro, and now I believe in the Provincial Museum at Halifax, where I have seen the specimen. The horizon is probably that of the HORTON series. The footprint is about three and a-half inches wide, and scarcely half as much in apparent length. It shows four subequal toes, and an outer toe diverging from the others, and showing indications of a short claw. The shortness of the impressions in this species and in *S. Sydnensis* gives them a digitigrade aspect, while those of *S. unguifer* have a plantigrade appearance, varying however in different impressions.

Hylopus Logani, DAWSON.

['Air-breathers of the Coal Period,' p. 5, fig. 1. 'Acadian Geology, 3rd edition, p. 353.]

The original specimen of this footprint was found by Sir W. E. LOGAN at Horton Bluff in 1841, and was the first evidence of the existence of Batrachians in the Carboniferous Period. The specimen obtained by LOGAN is fully described in the works above cited. The impression has been made on a firm surface, and shows merely the marks of four claws or narrow toes. Each impression is about one inch in length; the distance between the right and left footmarks is about three inches, and the stride about four inches. There is no mark of the belly or tail. In 1881 a somewhat larger series of impressions, which should, however, probably be referred to the same species, was found in the same beds by Mr. PINEO, of Hantsport, and is now in the Museum of the McGill University. It shows indications of a fifth toe; and while the length of the foot is less than two inches, the stride is about eight inches, or more than four times the length of the foot.

5. *Hylopus Hardingi*, Dawson.

['Air-breathers of the Coal Period,' fig. 2, 'Acadian Geology,' 3rd edition, p. 356, fig. 139.]

This specimen, discovered by the late Dr. Harding, of Windsor, in the lower carboniferous shales of Parrsboro, indicates an animal of about the same size with *H. Logani*, and possibly nearly allied to it, but with five distinct and subequal toes which are long and slender. The footprints are about an inch in length, and those of the fore and hind feet are separate and of about equal size and similar form. The most remarkable feature of this series is the great length of the stride, which is nearly five times the length of the foot, and twice as much as the distance between the rows of tracks, apparently indicating that the animal stood as high on its legs as an ordinary Mammal.

6. *Hylopus Caudifer*, Dawson.

['Air-breathers of the Coal Period,' fig. 3.]

This is a slab with a series of footprints less than an inch in length and five-toed. The rows are distant from each other three inches and a-quarter, and the stride is three inches. There are at intervals marks of a tail trailed behind. This impression is in my own collection, and is from the middle coal-formation of the South Joggins, on gray ripple-marked sandstone.

In addition to the above, many obscure impressions have been found, which no doubt indicate several additional species. It is observable that in all the members of the carboniferous series, these footprints have been found most plentifully in the vicinity of those old ridges of land based on the older formations, which as I have shown in my 'Acadian Geology,' traversed the areas of deposition in Nova Scotia in the Carboniferous Period. On these isolated patches of land the Batrachians may have continued to exist throughout the period, undisturbed by the oscillations of elevation and depression which affected the lower levels.

It is evident that the smaller footprints to which I have referred under the generic name *Hylopus*, may have been produced by animals akin to those whose remains are found in the erect trees, though of somewhat larger dimensions; and it is instructive to observe that at the beginning of the Carboniferous Period there must have existed animals of this kind comparable in development of limb with the most highly endowed in this way of the *Microsauria*, and of greater bulk than those whose bones are found in the erect trees.

If we inquire as to the footprints of larger size included in the genus *Sauropus*, we know by the evidence of osseous remains that some large Batrachians inhabited the areas of coal deposition in Nova Scotia. More especially *Baphetes planiceps* (OWEN) must have been an animal of sufficient dimensions to have produced footprints like those of *Sauropus unguifer*. A second species of *Baphetes* is probably indicated by a jaw-bone found at the South Joggins, and which I have named provisionally *B. minor*. The *Eosaurus Acadianus* of MARSH must have been a large animal; but we do not yet know if it was capable of walking on the land.

On the whole, the evidence of footprints serves to indicate that both *Labyrinthodontia* and *Microsauria* existed in Nova Scotia throughout the Carboniferous Period, and that very many of the larger and important species still remain to be discovered. It seems remarkable that, while remains of Fishes are so abundant in the carboniferous shales of Nova Scotia and New Brunswick, nothing is as yet known as to the more aquatic types of Batrachians found so abundantly in the carboniferous strata elsewhere. This may either indicate a local paucity of these creatures, or may be a mere accident of preservation or discovery.

EXPLANATION OF THE PLATES.

(Figures not otherwise designated are of the natural size.)

PLATE 39.

Hylonomus Lyelli (figs. 1 to 14).

- Fig. 1*. Skull and portion of skeleton.
- Fig. 2*. Skull of the same. × 2.
- Fig. 3*. Skeleton. (*a*) Fore limb. (*b*) Vertebræ and ribs, the former with bony coating stripped off and showing hourglass-shaped casts of interior. (*c*) Femur. (*d*) Pelvis. (*e*) Remains of femur and tibia.
- Fig. 4. Section of tooth. × 500.
- Fig. 5. Cells of bone. × 500.
- Fig. 6. Portion of maxilla with teeth.
- Fig. 7. Teeth. × 25.
- Fig. 8. Mandible and teeth.
- Fig. 9. Maxilla. × 5.
- Fig. 10. Bones of foot.

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* Figures marked with an asterisk are from drawings by J. H. EMERTON. The remainder are from photographs by INGLIS and camera drawings by the author.

Fig. 11. Rib.
Fig. 12. Skull, crushed laterally. (*o*) Orbit. (*m*) Mandible. (*n*) Nasal bones.
Fig. 13. Skeleton, showing portions of limbs, vertebræ and ribs.
Fig. 14. Femur and thoracic plate.

Hylonomus Wymani (figs. 15 to 17).
Fig. 15. Mandible. × 8.
Fig. 16. Teeth. × 25.
Fig. 17. Vertebra. × 5.

Hylonomus latidens (figs. 18 to 22).
Fig. 18. Teeth. × 25.
Fig. 19. Section of jaw and teeth. × 5.
Fig. 20. Vertebra. × 2.
Fig. 21. Fragment of humerus. × 2.
Fig. 22. Mandible with teeth. × 5.

Hylonomus multidens (figs. 23 to 26).
Fig. 23. Mandible, maxilla, and cranial bones. × 3.
Fig. 24. Outline of another skull. × 2.
Fig. 25. Teeth. × 25.
Fig. 26. End of femur. × 2.

Hylonomus Lyelli (fig. 27).
Fig. 27. Diagram showing probable arrangement of ornamental horny scales (enlarged). (*a*) Horny scales enlarged. (*b*) Bony scales, natural size and magnified.

PLATE 40.

Smilerpeton aciedentatum (figs. 28 to 45).
Fig. 28. Section of tooth. × 500.
Fig. 29. Cells of bone. × 500.
Fig. 30. Teeth, lateral aspect. × 25.
Fig. 31. Teeth, longitudinal aspect. × 25.
Fig. 32. Section of tooth. × 50.
Fig. 33. Shaft of femur? × 2.
Fig. 34. Intermaxillary and teeth. × 25.
Fig. 35. Sections of teeth. × 25.
Figs. 36, 37. Palatal teeth. × 25.
Fig. 38. Mandible. × 2.
Fig. 39. Large palatal teeth. × 25.
Fig. 40. Fragment of femur. × 2.
Fig. 41. Rib. × 2.

Fig. 42. Palate. × 2.
Fig. 43. Caudal vertebra.
Fig. 44. Long palatal tooth. × 25.
Fig. 45. Bony scale. Natural size and magnified.

Dendrerpeton Acadianum (figs. 46 to 51).

Fig. 46*. Portion of cuticle, wrinkled and scaly. Fig. 46A. Lateral points. Fig. 47B. Lappets and pendants.
Fig. 47*. Ends of smaller pendants. Magnified.
Fig. 48*. Pendant. × 2.
Fig. 49*. Two points and scaly surface. × 4.
Fig. 50. Scaly surface. × 5.
Fig. 51. Pendant. × 2.

Sparodus, sp. (figs. 52 to 56).

Fig. 52. Tooth. × 25.
Fig. 53. Four of the smaller teeth. × 25.
Fig. 54. Three bony scales. × 5.
Fig. 55. Fragment of limb bone. × 2.
Fig. 56. Vertebra. × 2.

Amblyodon, sp. (figs. 57 to 61).

Fig. 57. Tooth. × 25.
Fig. 58. Section of tooth. × 5. Fig. 58A. Same. × 25.
Fig. 59. Fragment of thoracic plate.
Fig. 60. Shaft of limb bone.
Fig. 61. Rib.

PLATE 41.

Hylerpeton Dawsoni (figs. 62 to 85).

Fig. 62*. Section of palate with teeth. Fig. 62A. Same. × 2.
Fig. 63*. Section of large tusk. Fig. 63A. Same. × 2.
Fig. 64*. Mandible and teeth. Fig. 64A. Same. × 2.
Fig. 65*. Vertebra. End view.
Fig. 66*. Fragment of rib ?
Fig. 67*. Femur.
Fig. 68*. Humerus and vertebra.
Fig. 69*. Head of humerus.
Fig. 70*. Rib.
Fig. 71. Scales.
Fig. 72. Large scale.
Fig. 73. Bones of foot.
Fig. 74. Tooth. × 25.

Fig. 75. Dentine of tooth. × 500.
Fig. 76. Cell of bone. × 500.
Fig. 77. Scales. × 2.
Fig. 78. Head of rib.
Fig. 79. Rib.
Fig. 80. Tooth, showing ribbed interior and smooth surface.
Fig. 81. Fragment of vertebra. × 2.
Fig. 82. Section of body of vertebra. × 2.
Fig. 83. Section of tooth. × 50.
Fig. 84. Fragment of humerus—ribs and scales.
Fig. 85. Mandible of a small specimen. × 5.

PLATE 42.

Hylerpeton longidentatum (figs. 86 to 109).

Fig. 86*, 87*. Mandible and teeth. 86A. Same. × 2.
Fig. 88*. Fragment of palate with teeth. 88A. Same. × 2.
Fig. 89*. Thoracic armour.
Fig. 90*. Fragments of limb bones (these possibly belong to next species).
Fig. 91*. Limb bone and rib.
Figs. 92*, 93*, 94*. Vertebræ.
Fig. 95*. Fragment of scale.
Fig. 96. Section of tooth. × 500.
Fig. 97. Tooth. × 25.
Figs. 98, 99. Teeth showing grooved points. × 25.
Fig. 100. Cross section of tooth. × 25.
Fig. 101. Scale. × 2.
Fig. 102. Head of rib. × 2.
Figs. 103, 104. Palatal teeth. × 25.
Fig. 105. Limb bones. × 2.
Fig. 106. Cells of bone. × 500.
Fig. 107. Rib.
Fig. 108. Teeth and mandible. × 8.
Fig. 109. Mandible and thoracic plate. × 2.

PLATE 43.

Fritschia curtidentata (figs. 110 to 128).

Fig. 110*. Portion of mandible. Fig. 110A. Same. × 2.
Fig. 111*. Maxilla and teeth. Fig. 111A. Same. × 2.

Fig. 112*. Humerus, ulna, and tarsal bone.
Fig. 113*. Femur, distal end.
Fig. 114*. Humerus?
Fig. 115*. Limb bones.
Fig. 116*. Rib and limb bones.
Fig. 117*. Vertebræ.
Fig. 118*. Portion of palate.
Fig. 119. Tooth, showing grooved point. × 25.
Fig. 120. Tooth. × 25.
Fig. 121. Section of tooth. × 500.
Fig. 122. Cells of bone. × 500.
Fig. 123. Bones of anterior limb.
Fig. 124. Ribs, vertebræ, and bony rods.
Fig. 125. Various bones. (*a*) Maxilla. (*b*) Humerus. (*c*) Bony rods. (*d*) Femur. (*e*) Rib.
Fig. 126. Vertebræ.
Fig. 127. Mandible, curved by pressure.
Fig. 128. Various bones. (*a*) Maxillary. (*b*) Humerus and limb bones.

PLATE 44.

Dendrerpeton Acadianum (figs. 129 to 130 and 132 to 137).

Fig. 129. Interior of skull and mandible.
Fig. 130. Maxilla. × 2.
Fig. 132. Plan of constituent bones of skull represented in fig. 129. (*N*) Nasals. (*F*) Frontals. (*SOr*) Super-orbitals. (*P*) Parietals. (*PP*) Post-parietals. (*Pt*) Pterotics. (*SOc*) Super-occipitals. (*E*) Epiotic.
Fig. 133. Section of tooth. × 250.
Fig. 134. Cells of bone. × 500.
Fig. 135. Tooth. × 25. Showing plaited surface.
Fig. 136. Fragments of super-orbital and parietal, showing sculpture. × 5.
Fig. 137. Five teeth. × 5.

Dendrerpeton Oweni (figs. 131, 138, 139).

Fig. 131. Maxillary, showing sculpture and teeth. × 2.
Fig. 138. Teeth. × 25.
Fig. 139. Foot. × 2.

PLATE 45.

Hylonomus Lyelli.

Fig. 140. Skin and horny plates, tubercles, and scales of this species; from a micro-photograph. × 5. The specimen represents a fragment from the shoulder.

Erect Trees.

Fig 141. Tree No. 13, as standing in the reef; from a photograph by T. C. Weston, Esq., of the Geological Survey of Canada.

PLATE 46.

View of Coal-mine Point, South Joggins, Nova Scotia; from a photograph by J. C. Weston, Esq.

PLATE 47.

Plan of Low-tide Beach off Coal-mine Point, and section to illustrate the position of the trees containing remains of land animals, by W. B. Dawson, M.A., Assoc. Mem. Inst. C.E.

XIII. *Experiments to determine the Value of the British Association Unit of Resistance in Absolute Measure.*

By Lord RAYLEIGH, *F.R.S., Professor of Experimental Physics in the University of Cambridge.*

Received February 15,—Read March 9, 1882.

[PLATE 48.]

THE present paper relates to the same subject as that entitled "On the Determination of the Ohm in Absolute Measure," communicated to the Society by Dr. SCHUSTER and myself, and published in the Proceedings for April 12, 1881—referred to in the sequel as the former paper. The title has been altered to bring it into agreement with the resolutions of the Paris Electrical Congress, who decided that the ohm was to mean in future the absolute unit (10^9 C.G.S.), and not, as has usually been the intention, the unit issued by the Committee of the British Association, called for brevity the B.A. unit. Much that was said in the former paper applies equally to the present experiments, and will not in general be repeated, except for correction or additional emphasis.

The new apparatus (Plate 48) was constructed by Messrs. ELLIOTT on the same general plan as that employed by the original Committee, the principal difference being an enlargement of the linear dimensions in the ratio of about 3 : 2. The frame by which the revolving parts are supported is provided with insulating pieces to prevent the formation of induced electric currents, and more space is allowed than before between the frame and those parts of the ring which most nearly approach it during the revolution. It is supported on three levelling screws, and is clamped by bolts and nuts to the stone table upon which it stands. The ring is firmly fastened by nuts to two gun-metal pieces which penetrate it at the ends of the vertical diameter, and which form the shaft on which it rotates. The lower end of the bottom piece is rounded, and bears upon a plate of agate, on which the weight of the revolving parts is taken. A little above this comes the driving pulley (9 inches in diameter), and above this again the screw and nut by which the divided card is held. The top piece is hollow, forming a tube with an aperture of $1\frac{1}{4}$ inches, and is held by a well-fitting brass collar attached to the upper part of the frame. On this bearing the force is very small, so that the considerable relative velocity of the sliding surfaces has no ill effect. Notwithstanding its great weight, the ring ran very lightly, and the principal resistance to be overcome was that due to setting air in motion.

In the original apparatus the ring is very light, in fact scarcely strong enough to stand the forces to which it is subjected in winding on the wire. In order to avoid this defect, and also on account of its larger size, the new ring was made very massive. Cast solid, with lugs at the ends of what was to be in use the horizontal diameter, it was cut into two equal parts along a horizontal plane. The two parts were then insulated from one another by a layer of ebonite, and firmly joined together again at the lugs by bolts and nuts, after which the grooves, &c., were carefully turned. As it was intended to use two coils of wire in perpendicular planes, two rings were prepared. The smaller ring fitted into the larger, the end pieces passing through holes along the vertical diameters of both. But for a reason that will presently be given, only the larger ring was used in the present experiments.

In the spring of 1881 the larger ring was wound in Messrs. ELLIOTTS' shop under the superintendence of Dr. SCHUSTER and myself, and the necessary measurements were taken. On mounting the apparatus a few days later in the magnetical room of the Cavendish Laboratory, and making preliminary trials, we were annoyed by finding a very perceptible effect upon the suspended magnet even when the wire circuit was open. The currents thus indicated might have been due to a short circuit in the wire, or more probably (considering that the wire was triple covered, and that the winding had been carefully done) had their seat in the ring itself. Experiment showed that the insulation between the two parts of the ring, as well as between the wire and each part, was very good, so that no currents could travel round the entire circumference; but on consideration it appeared not unlikely that currents of sufficient intensity might be generated in those parts of the ring which lie nearest to the ebonite layer. The width of the ring (in the direction of its axis) was 4 inches, and the least thickness—that at the bottom of the grooves—about $\frac{3}{8}$ inch, so that the operative parts may be compared to four vertical plates $\frac{3}{8}$ inch thick, 4 inches broad, and (say) 6 inches high. In these plates currents will be developed during the rotation, whose plane is perpendicular to that of the currents in the wire.

The unwished for currents could doubtless have been much diminished by saw cuts in a vertical plane extending a few inches upwards and downwards from the insulating layer, but it appeared scarcely safe to assume that the ring would retain its shape under such treatment. It would have been wiser to have tried the effect of spinning the ring alone before winding on the wire, but we were off our guard from the fact that the old ring gave no perceptible disturbance.

Theory having shown that these currents, if really formed in the manner supposed, could be satisfactorily allowed for, we decided to proceed with the experiment. At the worst, the differential effect between wire circuit closed and wire circuit open could only be in error by a quantity depending upon the square of the speed, and therefore capable of elimination upon the evidence of the spinnings themselves; while if the view were correct that the disturbing currents were principally in a plane perpendicular to that of the wire, even the correction for induction would not be much affected. A special experiment, in which the ring (with wire circuit open) was oscillated backwards and forwards through a small angle in time with the natural vibrations of the magnet, allowed us to verify the plane of the currents. A marked effect was produced when the plane of the ring was east and west, but nothing could be detected with certainty when it was north and south—the opposite of what would happen with the wire circuit closed. After this, no doubt could remain but that most of the disturbance was due to currents in the ring, and subsequent spinnings after the removal of the wire have proved that no sensible part of it was caused by leakage through the silk insulation. The existence of this disturbance, however, so far modified our original plan as to induce us to omit the second ring as giving rise to too great a complication.

The suspended magnet was made of four pieces of steel attached to the edges of a cube of pith and of such length (about $\frac{1}{2}$ inch) as to be equivalent in their action to an infinitely small magnet at the centre of the cube. Before the pieces were put together the approximate equality of their magnetic moments was ascertained. The resultant moment was between six and seven times as great as that used in our former experiments. In virtue of the greater radius of the coil, this important advantage was obtained without undue increase of the correction for magnetic moment, which amounted to about ·004, only twice as great as before. The effect of mechanical disturbances, such as air currents, was still further reduced by diminishing the size of the mirror, particularly in its horizontal dimension. On both accounts the influence of air currents was probably lessened about 15 times, and, in fact, no marked disturbance was now caused by the proximity of a lamp to the magnet box.* In consequence of these changes, however, it was found necessary to introduce an inertia ring in order to bring the time of vibration up to the amount (about $5\frac{1}{2}$ seconds from rest to rest) necessary for convenient observation. The diameter of the ring was about $\frac{3}{4}$ inch, and the whole weight of the suspended parts was not too great to be borne easily by a single fibre of silk. A brass wire passing between the spokes of the ring prevented the needle from making a complete revolution.

The enlarged scale of the apparatus allowed us to introduce a great improvement into the arrangement of the case necessary for screening the suspended parts from the mechanical disturbance of the air caused by the revolution of the coil. A brass tube of an inch in diameter was not too large to pass freely through the hollow axis. At its lower extremity (fig. 1) it was provided with an outside screw, to which the magnet box was attached air-tight. By unscrewing the box, whose aperture was large enough to allow the inertia ring to pass, the suspended parts could be exposed to view, and by drawing up the brass tube they could be removed altogether, so as to allow the coil to be dismounted, without breaking the fibre. The upper end of the fibre was attached to a brass rod sliding in a socket at the upper end of the tube, by which the height of the magnet could readily be adjusted. The whole was supported on three screws

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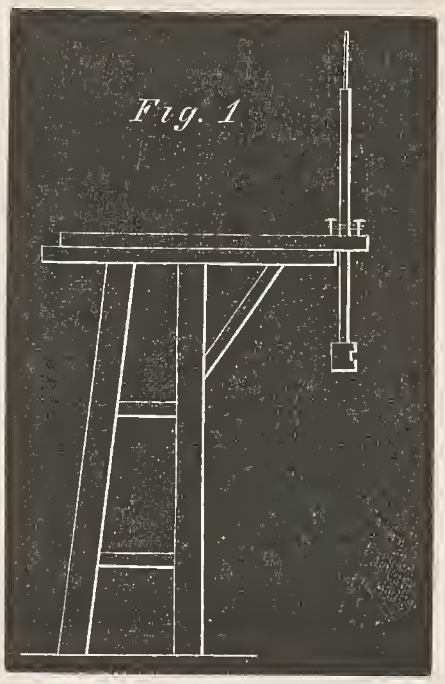
* See pp. 115–132 of the former paper.



passing through the corners of a brass triangle attached to the tube not far above the place where it emerged from the hollow axis. The points of the screws rested upon the same overhanging stand as in the former experiments (p. 113).*

The larger diameter of the tube made the system so rigid that no mechanical disturbance of the kind formerly met with was to be detected at the highest speed to

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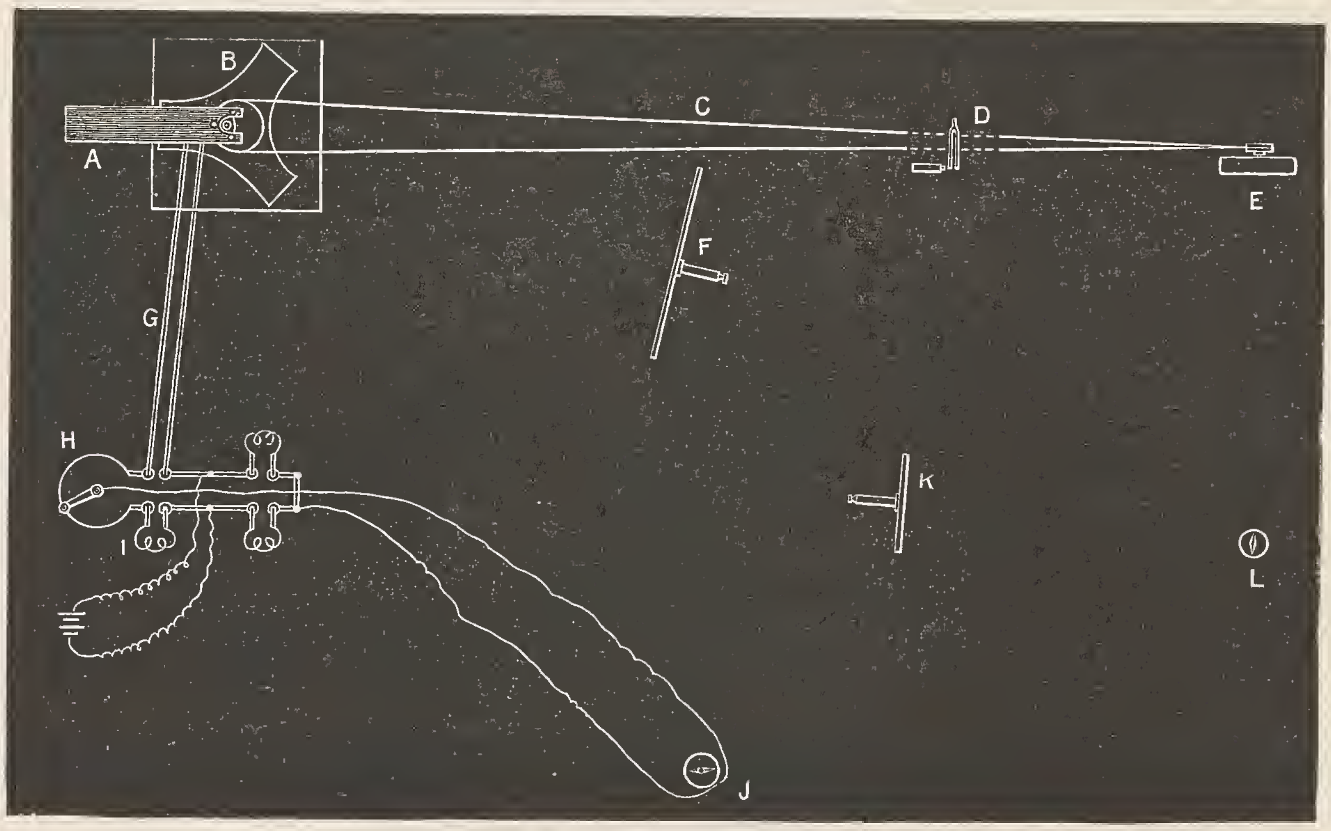
* June, 1882. The general disposition of the apparatus is shown in fig. 2.



A. Stand for suspended parts.
B. Frame of revolving coil.
C. Driving cord.
D. Electro-magnetic fork and telescope.
E. Water engine.
F. Principal telescope and scale.
G. Copper connecting bars.
H. Fleming's bridge.
I. Platinum-silver standard.
J. Bridge galvanometer.
K. Telescope and scale of auxiliary magnetometer.
L. Auxiliary magnetometer needle and mirror.

which we could drive the coil. Even a tap with the finger-nail upon the magnet-box produced but a small disturbance.

No change was required in the arrangements for regulating and determining the speed of the coil, which worked, if possible, more perfectly than before, in consequence of the greater inertia of the revolving parts. The divided card was, however, on an enlarged scale, and the numbers of the teeth in the various circles were so arranged that each circle was available for a distinct pair of speeds according as it was observed through the slits in the plates carried by the electric fork or over the top of the upper plate. The speeds actually used corresponded to 80, 60, 45, 35, and 30 teeth, seen through the slits, *i.e.*, about 127 times per second.

The greater resistance of the copper coil (23 instead of 4·6) rendered necessary a modification in the method of making the comparisons with the standard. The whole value of the divided platinum-iridium wire on FLEMING's bridge being only $\frac{1}{20}$ ohm, a change of temperature in the copper of not much more than a degree would exhaust the range of the instrument. To meet this difficulty it was only necessary to add resistances to the copper circuit so as to compensate approximately the temperature variations, for it is evident that it can make no difference whether the change of resistance of the entire revolving circuit is due to a rise of temperature, or to the insertion of an additional piece. The platinum-silver standard was therefore prepared so as to have a resistance (about 24 ohms) greater than any which we were likely to meet with in the copper, and the additional pieces were relied upon to bring the total within distance. As at first arranged, the additional resistance was inserted at the mercury cups, instead of a contact piece of no appreciable resistance. During the comparison with the standard it was transferred to another part of the circuit.

In the course of May, 1881, a complete series of spinnings were taken, the arrangements and adjustments being (except as above-mentioned) in all respects the same as with the old apparatus. Five different speeds were used, and each of them on three different evenings. The work of observing was also distributed as before, Dr. SCHUSTER taking the readings of the principal magnetometer, and Mrs. SIDGWICK the simultaneous readings of the auxiliary magnetometer, while I observed the divided card and regulated the speed. At each speed on each evening four readings were taken with wire circuit closed, two with positive and two with negative rotation, and in like manner four readings were taken with the wire circuit open. Observations on the zero with the coil at rest were for the most part dispensed with, as it was thought that the time could be better employed otherwise; in fact, the mean of the two not very different positions of equilibrium obtained with positive and negative rotation when the wire circuit was open, gives all that is wanted in this respect. In the actual reductions we only require the *difference* of readings with positive and negative rotations.

It was hoped that these observations would have been sufficient, but on the introduction by Dr. SCHUSTER of the various corrections for temperature, for the beats between the two forks, and for the outstanding bridge-wire divisions, the necessity for which disguises the significance of the numbers first obtained, it was found that the agreement of the results corresponding to a given speed was by no means so good as we had expected in view of the precautions taken and the accuracy of the readings. What was worse, there was evidence of a decided progression, as if the absolute resistance of the standard had gradually diminished during the time occupied by the spinnings.

It is not impossible that there really was some change in the standard which had then been newly prepared; but the discrepancies were not, as according to this view they ought to have been, proportional to the speeds of rotation. I am inclined rather to attribute them to shiftings of the paper scales. The principal magnetometer scale was composed of three lengths of 50 centims. each, cemented with indiarubber to a strip of deal. The compound scale thus formed was examined by Dr. SCHUSTER in March, 1881. Between the graduations of the first and of the middle piece there was a gap of about $\frac{1}{4}$ millim., and another of nearly the same magnitude between the middle and the third piece. When I re-examined the scale in July, the gap at 500 divisions had increased to $\frac{9}{10}$ millim., and that at 1000 to $\frac{1}{2}$ millim. Curiously enough, there were no observable errors in the equality of the divisions of the three parts taken separately; but the changes above-mentioned are sufficient to throw considerable doubts upon the value of the first series of spinnings. They have, however, been reduced by Dr. SCHUSTER, and the result is given below for the sake of comparison.

To be free for the future from uncertainties of this kind, I replaced the paper scale by a long glass thermometer tube by CASELLA, graduated into millimetres. The divisions were fine and accurately placed, but the imperfect straightness of the tube has rendered necessary certain small corrections in the final results. Probably a straight strip of flat opal would have been an improvement.

The second series of spinnings was made in August, 1881, and this, it was fondly hoped, would be final. To guard against possible change in the platinum-silver coil a careful comparison with the standard units was previously instituted by Mrs. SIDGWICK, of which the details are given later. As we had unfortunately lost the advantage of Dr. SCHUSTER's assistance, the observations at the principal magnetometer devolved upon Mrs. SIDGWICK. The much easier post at the auxiliary magnetometer was usually occupied by Lady RAYLEIGH; occasional assistance has been rendered by Mr. A. MALLOCK and by Mr. J. J. THOMSON.

In the conduct of the second series one or two minor changes were introduced. In order to know the temperature of the standard tuning-fork more accurately, a thermometer was placed between its prongs and read at the same time as the number of beats was taken. The insertion of the small resistances necessary to bring the copper coil within range of the standard was also arranged in a different manner. Some trouble had been experienced in getting a sufficiently good fit between the contact pieces used in the first series and the mercury cups. It is necessary that the stout copper terminals should press down closely upon the bottoms of the cups, and also that the mercury should not be liable to escape at high speeds from the effect of centrifugal force. Bits of indiarubber tubing were placed round the copper legs, by which a fair fit with the sides of the cups was effected; but I thought that it would be an improvement to revert to a single contact piece for the mercury cups of no sensible resistance, whose fit could be carefully adjusted, and to insert the extra resistances at the connexion of the other (outer) ends of the component coils. For this purpose binding screws were employed, pressing firmly together the flat copper terminals of the copper wire and of the German-silver resistance pieces. It is almost unnecessary to say that these short lengths of German-silver wire were doubled upon themselves before being coiled, and that the pieces were not touched between a spinning and the associated resistance comparisons. Used in this way the screwed up contacts seemed unobjectionable, even though the surfaces were not amalgamated.

On each night and for each speed a set of twelve spinnings was made, six with wire circuit open, and six with wire circuit closed. It was usual to take, first, two of the former (one with positive and one with negative rotation); secondly, to compare the resistances of the revolving circuit and the standard; thirdly, after inserting the contact piece and adjusting the indiarubber strap by which it was held down, to make the six closed contact spinnings; fourthly, to compare the resistances again; and lastly, to complete the open contact readings. Each spinning, it will be understood, involved the reading of several elongations (about six for the open contact and ten for the closed), from which the position of equilibrium was deduced.

Table II. (p. 691) gives all the results of the second series, except one for 35 teeth on August 27th, which was rejected on the ground that it exhibited such large *internal* discrepancies, as to force us to the conclusion that the contact piece had been inserted improperly. It will be seen that the agreement is good except on August 29th, in which case the deflections are as much as four or five tenths of a millimetre too small. These discrepancies, though not very important in themselves, gave me a good deal of anxiety, as they were much too large to be attributed to mere errors of reading, and seemed to indicate a source of disturbance against which we were not on our guard.

The least unlikely explanations seemed to be (1) a change in the distance of the mirror from the scale, which unfortunately had not been remeasured at the close of the spinnings, though this would require to reach 3 millims.; (2) imperfect action of the contact piece from displacement of mercury or otherwise; (3) a change of level in the axis of rotation. The anomalous result of August 27th seemed to favour (2), while on behalf of (1) it must be said that the stand of the telescope and scale as well as the support for the suspended parts of the principal magnetometer were of wood. It was just conceivable that under the influence of heat or moisture some bending might have occurred.

On my return to Cambridge in October we proceeded to investigate these questions with the closest attention. As repeated direct measurements of the distance of the mirror and scale were inconvenient, measuring rods (like beam compasses) were provided to check the relative positions of the telescope stand and of the upper end of the suspending fibre with regard to fixed points on the walls of the room. But no changes comparable with 3 millims. were detected, even under much greater provocation than could have existed during the August spinnings. The next step was to examine the action of the contact piece. For this purpose the coil was balanced against the standard as usual, except that the contact piece was inserted and connexion with the bridge made at the other ends of the double coil. It was presently found that the resistance *did* depend upon the manner in which the contact piece was pressed, and that to an extent sufficient to account for the August discrepancies. Eventually it was discovered that one of the legs of the contact piece, which by a mistake had been merely rivetted and not soldered in, was shaky.

After this there could be no reasonable doubt that the faulty contact piece was the cause of our troubles. In all probability the leg became loose on the 27th, in which case the earlier results would be correct. Moreover, the final means are not very different, whether the spinnings of August 29th are retained or not. This being the case, we might perhaps have been content to let the matter rest here; but in view of the importance of the determination, and the desirability as far as possible of convincing others as well as ourselves, we thought that it would be more satisfactory to make a third and completely independent series of spinnings.

In this series the faulty composite contact piece was replaced by a horse shoe of continuous copper, and a check was instituted upon the distance between mirror and scale. The opportunity was also taken to make a minor improvement in connexion with the auxiliary magnetometer. The somewhat unsteady table on which the telescope and scale had stood was replaced by one of stone, and the arrangements for illumination were improved by throwing an image of a gas flame on the part of the scale under observation. The same number of readings were made as in the second series, but we found it more expeditious to take the six open contact spinnings together. At the beginning of the evening it was desirable to commence with these open contact spinnings in order to give more time for the coil to acquire the temperature of the room, which always rose somewhat, although the lamps and gas were lit a couple of hours beforehand. Later in the evening we sometimes took the closed contact readings for two speeds consecutively, in order that the intermediate resistance comparison might serve for both. In other respects the arrangements were unaltered.

Full details of the observations and reductions are given below. It will be sufficient here to mention that the maximum discrepancy between any two deflections at the same speed amounts only to $\frac{7}{100}$ of a millimetre, so that the agreement on different nights is more perfect than could have reasonably been expected. At the lowest speed the above-mentioned discrepancy is less than one part in 3000, and at the highest speed less than one part in 6000. No spinnings in the third series were rejected, except on one or two occasions when it appeared *at the time of observation*, from the behaviour of the auxiliary magnetometer, that there was too much earth disturbance. The spinnings were then suspended, and the observations already obtained were not reduced.

At the close of the spinnings, Mrs. SIDGWICK made a further comparison of our platinum-silver coil with the standard units.

The value arrived at for the B.A. unit (·9865 ohm) differs nearly three parts in a thousand from that which we obtained with the original apparatus. This difference is not very great, and may possibly be accounted for by errors in the measurement of the coil (see p. 114 of former paper). If a coil be imperfectly wound, the mean radius, as determined by a tape, is liable to be too great. At any rate, this discrepancy sinks into insignificance in comparison with that which exists between either of these determinations and that of Professor KOHLRAUSCH,* according to whom the B.A. unit would be as much as 1·0196 ohms. With respect to the method employed by KOHLRAUSCH, I agree with ROWLAND† in thinking it difficult, and unlikely to give the highest accuracy; but how in the hands of a skilful experimenter it could lead to a result 3 per cent. in error, is difficult to understand. The only suggestion I have to make is that possibly sufficient care was not taken in levelling the earth-inductor. Although estimates are given of the probable errors due to uncertainties in the various data, nothing is said upon this subject. In consequence, however, of the occurrence of the horizontal intensity as a square in the final formula, in conjunction with the largeness of the angle of dip, the method is especially sensitive to a maladjustment of this kind. I calculate that a deviation of the axis of rotation from the vertical through 21′ in the plane of the meridian, would alter the final result by 3 per cent.‡

According to ROWLAND'S determination, the value of the B.A. unit is ·9912 ohm. The method consists essentially in comparing the integral current in a secondary circuit, due to the reversal of the battery in a primary circuit, with the magnitude of the primary current itself. The determination of the secondary current involves the use of a ballistic galvanometer, whose damping is small, and whose time of vibration can be ascertained with full accuracy; and it is here, I think, that the weakest point in the method is to be found. The logarithmic decrement is obtained by observation of a long series of vibrations, and it is assumed that the value so arrived at is applicable to the correction of the observed throw. I am not aware whether the origin of damping in galvanometers has ever been fully investigated, but the effect is usually supposed to be represented by a term in the differential equation of motion proportional to the momentary velocity. This mode of representation is no doubt applicable to that part of the damping which depends upon the induction of currents in the galvanometer coil, under the influence of the swinging magnet. If this were all, a correction for damping would be accurately effected on the basis of a determination of the logarithmic decrement, made with the galvanometer circuit closed in the same manner as when the throw is taken. In all galvanometers, however, a very sensible damping remains in operation even when the circuit is open, of which the greatest part is doubtless due to aerial viscosity; and it is certain that the retarding force arising from viscosity is not simply proportional to the velocity at the moment, without regard to the state of things immediately preceding.

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* POGG. Ann., Ergänzungband VI. Phil. Mag., April, 1874.

† American Journal, April, 1878.

‡ See p. 684.

In particular, the force acting upon the suspended parts as they start suddenly from rest in the observation of the throw, must be immensely greater than in subsequent passages through the position of equilibrium, when the vibrations have assumed their ultimate character. I calculate that in the first quarter vibration (*i.e.*, from the position of equilibrium to the first elongation) of a disc vibrating in its own plane and started impulsively from rest, the loss of energy from aerial viscosity would be 1·373 times that undergone in subsequent motion between the same phases. From this it might at first appear that in this ideal case the logarithmic decrement observed in the usual manner would need to be increased by more than a third part in order to make it applicable to the correction of a throw from rest; but in order to carry out this view consistently we should have to employ in the formula the time in which the needle would vibrate if the aerial forces were non-existent instead of the actually observed time of vibration. Now since the action of viscosity is to increase the time of vibration, the second effect is antagonistic to the first, so that probably the error arising from the complete neglect of these considerations is very small.

There is another point in which it appears to me that the theory of the ballistic galvanometer is incomplete. It is assumed that the magnetism of the needle in the direction of its axis is the same at the moment of the impulse as during regular vibrations. Can we be sure of this? The impulse is due to a momentary but very intense magnetic force in the perpendicular direction, and it seems not impossible that there may be in consequence a temporary loss of magnetism along the axis. If this were so, the actual impulse and subsequent elongation would be less than is supposed in the calculation, and too high a value would be obtained of the resistance of the secondary circuit in absolute measure. In making these remarks I desire merely to elicit discussion, and not to imply that ROWLAND's value is certainly four parts in a thousand too high.

Determinations of the absolute unit have been made also by H. WEBER,[*] whose results indicate that the B.A. unit is substantially correct. In the absence of sufficient detail it is difficult to compare this determination with others, so as to assign their relative weights.

The value of the B.A. unit in absolute measure is involved in the two series of experiments executed by JOULE on the mechanical equivalent of heat.[†] The result from the agitation of water is 24868, while that derived from the passage of a known absolute current through a resistance compared with the B.A. unit was 25187. The

[*] Phil. Mag., Jan., Feb., March, 1878.

[†] Phil. Trans., Part II., 1878. Brit. Ass. Rep., 1867; Reprint, p. 175.

latter result is on the supposition that the B.A. unit is really 10^9 C.G.S. If we inquire what value of the B.A. unit will reconcile the two results, we find—

$$1 \text{ B.A. unit} = \cdot 9873 \text{ ohm},$$

in very close agreement with the measurement described in the present paper. It should be remarked that in the comparison of the two thermal results some of the principal causes of error are eliminated; and it is not improbable that an experiment in which heat should be simultaneously developed in one calorimeter by friction, and in a second similar calorimeter by electric currents, would lead to a very accurate determination of resistance, more especially if care were taken so to adjust matters that the rise of temperature in the two vessels was nearly the same, and a watch were kept upon the resistance of the wire while the development of heat was in progress.

[*June*, 1882.—Since this paper was sent to the Society, Mr. GLAZEBROOK has worked out the results of a determination of the B.A. unit in absolute measure by a method not essentially different from that adopted by ROWLAND. The final number is practically identical with that of the present paper; and the agreement tends to show that the difference between ourselves and ROWLAND is not to be attributed to the use of a ballistic galvanometer.

Reference should have been made to the results of LORENTZ.* He finds as the value of the mercury unit *defined* by SIEMENS

$$1 \text{ mercury unit} = \cdot 9337 \, \frac{\text{earth quadrant}}{\text{second}}$$

The corresponding number calculated from the results of the present paper with use of the value of the specific resistance of mercury lately found (Proc. Roy. Soc., May 4, 1882) is ·9413. If we invert the calculation, we find that according to LORENTZ the value of the B.A. unit would be ·9786 absolute measure. The method of LORENTZ is ingenious, and apparently capable with good apparatus of giving a result to much within 1 per cent. Mrs. SIDGWICK and myself are at present making a trial of it.]

It will be desirable here to consider briefly some of the criticisms of KOHLRAUSCH and ROWLAND upon the method of the original British Association Committee, which has been adopted in the present investigation without fundamental alteration. The difficulty, remarked upon by KOHLRAUSCH, of obtaining a rapid and uniform rotation, has not been found serious, and I believe that no appreciable error can be due either to irregularity of rotation or to faulty determination of its rapidity. It has also been brought as an objection to the method that a correction is necessary on account of the magnetic influence of the suspended magnet upon the revolving circuit. The theory of this action is, however, perfectly simple, and the application of the correction requires only a knowledge of the ratio of the magnetic moment to the earth's horizontal force. If the magnetic moment is very small, the correction is unimportant; if larger, it can on that very account be determined with the greater ease and accuracy. It is probable that in the original experiments too feeble a magnetic moment was used, and that in consequence the suspended parts were too easily disturbed by non-magnetic causes; but this might have been remedied without increasing objectionably the correction in question. At any rate the larger coil of the new apparatus allows the use of any reasonable magnetic moment.

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* POGG. Ann., 1873.

Perhaps the least advantageous feature in the method is the necessity for creating a violent aerial disturbance in the immediate neighbourhood of a delicately suspended magnet and mirror. If, however, any deflection occurs in this way, very little error can remain when the open contact effect is subtracted from the closed contact effect. The difficulty of avoiding a sensible deflection, due to currents in the ring, when the wire circuit is open, is connected with a special advantage—*i.e.*, the possibility of assuring ourselves that there is no leakage from turn to turn of the coil. In the method followed by ROWLAND, for instance, such a leakage would lead to error, and could not be submitted to any direct test.

The correction for self-induction cannot be made very small without a disadvantageous reduction of the whole angular deflection; but so far as the wire is concerned it can be calculated *à priori*, or determined by independent experiment, with the necessary accuracy. There is reason, however, to think that the best method of treatment is to determine this correction from the spinnings themselves, combining the results of widely different speeds so as to obtain what would have been observed at a small speed. At small speeds it is certain that all effects of self-induction and of mutual induction between the wire circuit and other circuits in the ring will disappear.

Measurements of coil.

The mean radius of the coil, being the fundamental linear measurement of the investigation, must be found with full accuracy. There has been some difference of opinion as to the best method of effecting this. The greatest accuracy is probably attained by the use of the cathetometer. The measurement of the circumference of every layer by a steel tape has the advantage that the subject of measurement is three times as large, and is much less troublesome. The disadvantage is that if a layer be not quite even, there is danger of measuring the maximum rather than the mean outside circumference. In the present investigation the coil was so large that the tape could be employed without fear.*

Each of the component coils marked A and B had $18 \times 16 = 288$ windings, but in consequence of variations in the thickness of the triple silk covering, there was a difficulty in getting exactly 18 turns into each layer. In the eleventh layer of A it was necessary to be content with 17 turns, and to place an extra turn on the outside, so as to form the commencement of a seventeenth layer—a circumstance which of course was taken into account in calculating the mean. The number thus arrived at, after correction for the thickness of tape, is the mean *outside* circumference. What we require is the mean circumference of the axis of the wire; it may be derived from the first by subtraction of half the difference between the tape readings for the first layer, and for the bottom of the gun-metal groove.

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* The original Committee also employed the tape method. Their measurement of the length of the wire when unwound was not in order to find the mean radius, as SIEMENS and KOHLRAUSCH suppose, but to verify the number of turns.

The results obtained by Dr. SCHUSTER and myself when the coils were wound are:

	Coil A.	Coil B.
Mean of readings in millims.	1489·3	1487·5
Correction for tape	·6	·6
Mean outside circumference	1488·7	1486·9
Correction for thickness of wire	3·4	3·4
Mean circumference	1485·3	1483·5
Mean radius	236·39	236·11
Mean circumference of A and B	1484·4	
Mean radius of A and B (a)	236·25	
Axial dimension of section in millims.	19·9	19·9
Radial	15·9	15·4
Distance of mean planes ($2b'$)	65·95	

Two or three readings were taken of the circumference of every layer, and to prevent mistakes in the number of turns, the plan described by MAXWELL,* of simultaneously winding string on wooden rods, was followed. Without some such device, there is great risk of confusion.

In estimating the degree of accuracy obtainable, we must remember that the circumference of each layer is measured before the outer layers are wound on; any change produced by the pressure of these outer layers is a source of error. We had already observed a tendency in the measurements to be less during the unwinding of a coil than during the winding, and we fully intended to remeasure the coil after the spinnings were completed. This was done on December 6, 1881, by Mrs. SIDGWICK and myself. As we expected, somewhat smaller readings (by about $\frac{3}{4}$ millim.) were obtained for the circumference of the middle layers. The results were:

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* 'Electricity and Magnetism,' II., § 708.

	Coil A.	Coil B.
Mean radius	236·31	236·02
Mean of both	236·16	

or nearly one part in 2000 less than before. Of the two values, it would appear that the latter is more likely to represent the actual condition of the coil during the spinnings, and is therefore entitled to greater weight. If we give weights in the proportion of two to one, we get

Mean radius=23·619 centims.*

Calculation of GK.

We have

$$\mathrm{GK}=2\pi^2n^2a\sin^3\alpha\left\{1+\tfrac{1}{6}\frac{c^2}{a^2}+\tfrac{5}{8}\frac{b^2-c^2}{a^2}\sin^2\alpha\cos^2\alpha-\tfrac{1}{8}\frac{b^2}{a^2}\sin^2\alpha\right\}$$

in which

a =mean radius =23·625 (1st measurement)
b =axial dimension of section = 1·990
c =radial dimension of section = 1·565
n =total number of turns = 576
$2b'$=distance of mean planes = 6·595

$$\sin\alpha=a\div\sqrt{(a^2+b'^2)}$$

From these data we find

$$\begin{aligned}\log 2\pi^2n^2&=6{\cdot}81617\\ \log a&=1{\cdot}37337\\ \log\sin^3\alpha&=\bar{1}{\cdot}98744\\ \log\{\ldots\}&=\bar{1}{\cdot}99995\\ \hline \log\mathrm{GK}&=8{\cdot}17693\end{aligned}$$

But if we substitute the adopted value of a, *i.e.*, 23·619 centims., we have by subtraction of ·00011

$$\log\mathrm{GK}=8{\cdot}17682$$

Calculation of L.

We may write

$$\mathrm{L}=16^2\times18^2(\mathrm{L}_1+\mathrm{L}_2+2\mathrm{M}),$$

where L_1, L_2 are the coefficients of self-induction of the two parts, and M the coefficient of mutual induction without regard to the number of turns. L_1 and L_2 may be calculated from the formula

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* [August, 1882. At the time of use the tape was compared with a measuring rod, which again has been compared with a standard metre verified by the Standards Department of the Board of Trade. For the purposes of this investigation the differences observed are altogether negligible. I may add that the clock with which the standard tuning-fork was compared (see p. 137 of former paper) was rated from astronomical observations.]

$$L=4\pi a\left[\log_e\frac{8a}{r}+\tfrac{1}{12}-\tfrac{4}{3}(\theta-\tfrac{1}{4}\pi)\cot 2\theta-\tfrac{1}{3}\pi\operatorname{cosec}2\theta-\tfrac{1}{6}\cot^2\theta\log_e\cos\theta-\tfrac{1}{6}\tan^2\theta\log_e\sin\theta\right]$$

in which r is the diagonal of the section, and θ the angle between it and the plane of the coil. With this formula and with the dimensions as measured when the coil was wound, we get

$$L_1 \text{ (for A) } = 1029{\cdot}3 \text{ centims.}$$
$$L_2 \text{ (for B) } = 1031{\cdot}9 \text{ centims.}$$

It would not be difficult to calculate an approximate correction for the curvature of the coil, but this is scarcely necessary. (See p. 119 of former paper.) Adding the above, we have

$$L_1+L_2=2061{\cdot}2 \text{ centims.}$$

The value of M was found from the tables given as Appendix I. to § 706 of the new edition of MAXWELL's 'Electricity.' If we suppose each coil condensed into the centre of its section, we find $M=4\pi\times 33{\cdot}061$. A more exact calculation by the formula of interpolation explained in Appendix II. gives $M=4\pi\times 33{\cdot}140$, so that

$$2M=832{\cdot}88 \text{ centims.}$$

The final result is accordingly

$$L=16^2\times 18^2\times 2894{\cdot}1=2{\cdot}4004\times 10^8 \text{ centims.}$$

These calculations of the coefficients of induction have been made independently by Mr. NIVEN and myself, and are so far reliable; but we must not forget that the accuracy of the result depends upon the accuracy of the data, and that in the present case the diagonal of the section (r) on which the most important part of L depends is an element subject to considerable relative uncertainty. It is probable that the effective axial dimensions of the section is somewhat less than the width of the groove, and therefore that the real value of L may be a little greater than would appear from the preceding calculation.

Theory of the ring currents.

If the circuits are conjugate, the currents in the wire and in the ring are formed in complete independence of one another, a circumstance which simplifies the theory very materially. In the same notation as was used in the former paper (p. 105), and with dashed letters for the ring circuit, we have as the equation determining the angle of deflection (ϕ) when the wire circuit is closed,

$$\tan\phi+\tau\frac{\phi}{\cos\phi}=\frac{\frac{1}{2}\mathrm{GK}\omega}{\mathrm{R}^2+\mathrm{L}^2\omega^2}\{\mathrm{R}+\mathrm{L}\omega\tan\phi+\mathrm{R}\tan\mu\sec\phi\}$$
$$+\frac{\frac{1}{2}\mathrm{G'K'}\omega}{\mathrm{R'}^2+\mathrm{L'}^2\omega^2}\{\mathrm{R'}+\mathrm{L'}\omega\tan\phi+\mathrm{R'}\tan\mu\sec\phi\}$$

When the wire circuit is open the equation determining the angle of deflection (ϕ_0) is

$$\tan\phi_0+\tau\frac{\phi_0}{\cos\phi_0}=\frac{\frac{1}{2}\mathrm{G'K'}\omega}{\mathrm{R'}^2+\mathrm{L'}^2\omega^2}\{\mathrm{R'}+\mathrm{L'}\omega\tan\phi_0+\mathrm{R'}\tan\mu\sec\phi_0\}$$

Since τ is an extremely small quantity it is unnecessary to keep up the distinction between $\tau\phi/\cos\phi$ and $\tau\tan\phi$. By subtraction

$$(1+\tau)(\tan\phi-\tan\phi_0)$$
$$=\frac{\frac{1}{2}\mathrm{GK}\omega}{\mathrm{R}^2+\mathrm{L}^2\omega^2}\{\mathrm{R}+\mathrm{L}\omega\tan\phi+\mathrm{R}\tan\mu\sec\phi\}$$
$$+\frac{\frac{1}{2}\mathrm{G'K'}\omega}{\mathrm{R'}^2+\mathrm{L'}^2\omega^2}\{\mathrm{L'}\omega(\tan\phi-\tan\phi_0)+\mathrm{R'}\tan\mu(\sec\phi-\sec\phi_0)\}$$

The last term is small, and we may neglect ($\sec\phi-\sec\phi_0$) in combination with $\mathrm{R'}\tan\mu$.

Moreover

$$\frac{\frac{1}{2}\mathrm{G'K'}\omega}{\mathrm{R'}^2+\mathrm{L'}^2\omega^2}=\frac{(1+\tau)\tan\phi_0}{\mathrm{R'}+\mathrm{L'}\omega\tan\phi_0}$$

so that

$$(1+\tau)(\tan\phi-\tan\phi_0)=\frac{\frac{1}{2}\mathrm{GK}\omega}{\mathrm{R}^2+\mathrm{L}^2\omega^2}\{\mathrm{R}+\mathrm{L}\omega\tan\phi+\mathrm{R}\tan\mu\sec\phi\}$$
$$+(1+\tau)(\tan\phi-\tan\phi_0)\frac{\mathrm{L'}\omega\tan\phi_0}{\mathrm{R'}+\mathrm{L'}\omega\tan\phi_0}$$

If now we write (GK) for GK/$(1+\tau)$, we get

$$\tan\phi-\tan\phi_0=\frac{\frac{1}{2}(\mathrm{GK})\omega}{\mathrm{R}^2+\mathrm{L}^2\omega^2}\{\mathrm{R}+\mathrm{L}\omega\tan\phi+\mathrm{R}\tan\mu\sec\phi\}\{1+\frac{\mathrm{L'}\omega}{\mathrm{R'}}\tan\phi_0\}$$

The effect of L′ would therefore be to *increase* disproportionately the deflections at high speeds, *i.e.*, contrary to the effect of L. It appears, however, that in these experiments it could not have been sensible. At the highest speed $\tan\phi_0$ was about $\frac{1}{670}$, and ω about 26 per second, so that $\omega\tan\phi_0$ would be about $\frac{1}{26}$. The value of L′/R′ is difficult to estimate with any accuracy. But the value of L/R for the wire circuit is about ·01 second, and that for the ring circuit must be much less, so that the terms involving L′ may safely be omitted.

The quadratic in R then becomes

$$\mathrm{R}^2-\mathrm{R}\frac{\frac{1}{2}(\mathrm{GK})\omega(1+\tan\mu\sec\phi)}{\tan\phi-\tan\phi_0}+\mathrm{L}^2\omega^2-\tfrac{1}{2}(\mathrm{GK})\mathrm{L}\omega^2\frac{\tan\phi}{\tan\phi-\tan\phi_0}=0$$

whence

$$R=\frac{\frac{1}{2}(GK)\omega}{\tan\phi-\tan\phi_0}\left[\tfrac{1}{2}(1+\tan\mu\sec\phi)+\sqrt{\{\tfrac{1}{4}(1+\tan\mu\sec\phi)^2-U(\tan\phi-\tan\phi_0)^2\}}\right]$$

where

$$U=\frac{2L}{(GK)}\left\{\frac{2L}{(GK)}-\frac{\tan\phi}{\tan\phi-\tan\phi_0}\right\}$$

L *by direct experiment.**

Although the calculated value of L was the result of two independent computations, I considered that it would be satisfactory still further to verify it by an experiment with WHEATSTONE's balance. The statement of this method and the final formula, as given on p. 116 of the former paper, being approximate only, it will be convenient here to repeat them with the necessary corrections.

The four resistances in the balance are two equal resistances (10 units each), that of the copper coil P, and a fourth resistance Q (nearly equal to P) taken from resistance boxes, of which P is the only one associated with sensible self-induction. When P and Q are equal, there is no permanent current through the galvanometer; but if the galvanometer circuit be first closed and then the battery current be made, broken, or reversed, the needle receives an impulse, whose magnitude depends upon L.

If x denote the change of current in the branch P, the action of self-induction is the same as that of an electromotive impulse in that branch of magnitude Lx, and the effect upon the galvanometer is that due to this electromotive impulse acting independently of the electromotive force in the battery branch.

In order now to get a second quantity with which to compare the induction throw, the resistance balance is upset in a known manner. If while Q remains unaltered, P be increased to $P+\delta P$, there is a steady current through the galvanometer, which we may regard as due to an electromotive force $\delta P.x'$ in the branch $P+\delta P$, x' being the current through the branch. If θ be the deflection of the needle under the action of the steady current, α the angular throw, and T the time of swing from rest to rest, we have by the theory of the ballistic galvanometer as the ratio of the instantaneous to the steady electromotive force

$$\frac{T}{\pi}\frac{2\sin\frac{1}{2}\alpha}{\tan\theta},$$

subject to a correction for damping; so that this expression represents the ratio of $Lx:\delta P.x'$. If the induction throw be due to the make or break of the battery circuit, x represents simply the current in the branch P. In the case where the battery current is reversed, we may write 2Lx for Lx, understanding by x the same as before. As this method was the one actually adopted, we will write the result in the appropriate form

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* In consequence of the necessity which ultimately appeared of introducing an arbitrary correction proportional to the square of the speed of rotation, the result of the present section does not influence the final number expressing the B.A. unit in absolute measure. The method, however, is of some interest, and (it is believed) has not been carried out before with the precautions necessary to secure a satisfactory result.

$$\frac{\mathrm{L}x}{\delta\mathrm{P}.x'}=\frac{\mathrm{T}}{\pi}\frac{\sin\frac{1}{2}\alpha}{\tan\theta}.$$

In the formula as originally given by MAXWELL, and as stated in the former paper, the distinction between x and x' (the currents before and after the resistance balance is upset) was neglected. This step is legitimate if δP be taken small enough, to which course however there are experimental objections. In order that tan θ might be of suitable magnitude, it was found necessary to make the ratio of δP : P equal to about $\frac{1}{300}$, a fraction too large to be neglected.

In carrying out the experiment it was found more convenient to insert the additional resistance in the branch Q, leaving P unaltered. By the symmetry of the arrangement it is evident that this alteration is immaterial, and that we may take the formula in the form

$$\frac{\mathrm{L}}{\mathrm{Q}}=\frac{\mathrm{L}}{\mathrm{P}}=\frac{\delta\mathrm{Q}.x'}{\mathrm{Q}.x}\frac{\mathrm{T}}{\pi}\frac{\sin\frac{1}{2}\alpha}{\tan\theta},$$

x being the current in the branch Q when the resistance balance is perfect, x' the diminished current when the additional resistance δQ is inserted.

The principal difficulties in carrying out the experiment arose from variation in the battery and in the resistance balance. From these causes the results of two days' experiments were rejected, as unlikely to repay the trouble of reduction. On the last day (December 3, 1881) the first difficulty was overcome by using three large DANIELL cells (charged with zinc sulphate) in multiple arc. As precautions against rapid change of temperature the copper coils were wrapped thickly round with strips of blanket and deposited in a closed box. The delicacy of our arrangements was such that about $\frac{1}{1000}$ of a degree centigrade would manifest itself, so that it was hopeless to try to maintain the resistance balance absolutely undisturbed. The mode of applying a suitable correction will presently be explained. On December 3, partly by good luck, the necessary correction remained small throughout. In order to avoid a direct action of the current upon the galvanometer needle, the coil was placed at a considerable distance, at the same level, and with its plane horizontal. Any outstanding effect of the kind would, however, be eliminated from the final result by the reversals practised.

The induction throws were always taken by reversal of the battery current. A reversal has two advantages over a simple make or break. In the first place the effect is doubled and is therefore more easily measured; and in the second the battery is more likely to work in a uniform manner, the circuit being always closed except for a fraction of a second at the moment of reversal. The key was of the usual rocker and mercury cup pattern.

The galvanometer was one belonging to the laboratory of about 80 ohms resistance.

It was set up by Mr. Glazebrook for his experiments by an allied method, and with its appurtenances was ready for use at the time that this determination of self-induction was undertaken. The scale was divided into millimetres, and was placed at a distance of 218 centims. from the galvanometer mirror. The instrument was adapted for ballistic work, as the vibrations were subject to a logarithmic decrement of only about ·0142.

The electric balance was provided for by a resistance box from Messrs. Elliotts. The battery current after leaving the reversing key divides itself on entering the box, each part traversing 10 ohms. At the ends of these resistances come the galvanometer electrodes. The first part of the current now traverses the copper coil, and the second part other resistances, after which the two parts reunite and pass back to the battery. In the use of the "other resistances," a special arrangement was adopted which I must now explain. The resistance of the copper coil being somewhat under 24 ohms, the most obvious way to obtain a balance was to add to it a piece of adjustable wire until the whole would balance 24 ohms from the box. The objection to this plan is that the smallest known disturbance which we can then introduce, *i.e.*, by the addition or subtraction of a single unit, is much too great for the purpose.

The difficulty thus arising is completely met by the use of high resistances, taken from a second box, in multiple arc with the 24 ohms.

In order to balance the copper coil and its leading wires at the actual temperature (about 14°), 753 ohms were required in multiple arc with the 24. To calculate the resultant resistance we have

$$\frac{1}{24}+\frac{1}{753}=\cdot041666667+\cdot001328021=\cdot042994688=\frac{1}{23\cdot25869},$$

so that the resistance of the copper coil in terms of the units of the box is 23·25869. A suitable deflection θ was obtained by the substitution of 853 for 753 in the auxiliary box. In this case

$$\frac{1}{24}+\frac{1}{853}=\cdot041666667+\cdot001172333=\cdot042839000=\frac{1}{23\cdot34322};$$

so that the additional resistance was

$$\delta Q=\cdot08453 \text{ unit.}$$

It may be remarked that if the copper coil had been about 1° warmer, its resistance would have been greater by $\frac{1}{300}$th part, and the balance would have required 853 instead of 753 in multiple arc with the 24.

On account of the progressive changes already mentioned, it was advisable to alternate the observations of α and θ as rapidly as possible, and to occupy no more time than was really necessary in taking the readings. A good deal of time may be saved by working the key suitably, and by opening and closing the galvanometer branch (at a mercury cup provided for the purpose) so as to avoid producing unnecessary swings, and to stop those due to induction when done with; but it is unnecessary to go into detail in this part of the subject. After a little practice two induction throws, starting with opposite directions of the current, and two observations of steady deflection, one in each position of the reversing key, could be made in about seven minutes. The vibrations of the galvanometer needle were damped by the operation of a current in a neighbouring coil, the current being excited by a LECLANCHÉ cell and controlled by a key within reach of the observer at the telescope. The readings were taken by Mrs. SIDGWICK, while I reversed the battery current, shifted the resistances, and recorded the results.

In the simple theory of the method the induction throw is supposed to be taken when the needle is at rest and when the resistance balance is perfect. Instead of waiting to reduce the free swing to insignificance, it was much better to observe its actual amount and to allow for it. The first step is, therefore, to read two successive elongations, and this should be taken as soon as the needle is fairly quiet. The battery current is then reversed, to a signal, as the needle passes the position of equilibrium, and a note made whether the free swing is in the same or in the opposite direction to the induction throw. We have also to bear in mind that the zero about which the vibrations take place is different after reversal from what it was before reversal, in consequence of imperfection in the resistance balance. At the moment after reversal we are therefore to regard the needle as displaced from its position of equilibrium, and as affected with a velocity due jointly to the induction impulse and to the free swing previously existing. If the arc of vibration (*i.e.*, the difference of successive elongations) be a_0 before reversal, the arc due to induction be a, and if b be the difference of zeros, the subsequent vibration is expressed by

$$\tfrac{1}{2}(a \pm a_0) \sin nt + b \cos nt,$$

in which t is measured from the moment of reversal, and the damping is for the present neglected. The actually observed arc of vibration is therefore

$$2\sqrt{\{\tfrac{1}{4}(a \pm a_0)^2 + b^2\}}$$

or with sufficient approximation

$$a \pm a_0 + \frac{2b^2}{a}$$

so that

$$a = \text{observed arc } \mp a_0 - \frac{2b^2}{a}$$

In most cases the correction depending upon b was very small, if not insensible. The "observed arc" was the difference of the readings at the two elongations immediately following the reversal. As a check against mistakes the two next elongations also were observed, but were not used further in the reductions. The needle was then brought nearly to rest, and two elongations observed in the now reversed position of the key, giving with the former ones the data for determining the imperfection of the resistance balance. As the needle next passed the position of equilibrium, it was acted upon by the induction impulse (in the opposite direction to that observed before), and the four following elongations were read.

These observations of the throw were followed as quickly as possible by observations of the effect of substituting 853 for 753 units in the auxiliary arc. As soon as the vibrations could be reduced to a moderate amplitude, readings of three or four consecutive elongations were taken. The galvanometer contact was then broken, and the battery key reversed. When the needle had swung over to the other side, the galvanometer contact was renewed, and four elongations were observed. The difference between the two positions of equilibrium represented the disturbance of the resistance balance.

The whole of this disturbance, however, was not due to the additional 100 introduced, but required correction for the corresponding effect observed even with 753 units in the auxiliary arc. For this purpose it was only necessary to add or subtract the difference between the equilibrium positions of the needle with the key in the two positions, as deduced from the observations immediately preceding the induction throws; and in order to eliminate the influence of the progressive change, the mean of these differences as found before and *after* the insertion of the extra 100 units was employed. This result was compared with the mean of the four induction throws contiguous to it, two preceding and two following, and in this way a ratio obtained which was independent of the gradual but unavoidable changes in the battery current and in the copper resistance. After about half the readings had been taken the galvanometer connexions were reversed.

A specimen set of observations will now be given.

$3^h\ 36^m$	[753]	L	264·4
		Induction	246·6
$3^h\ 38^m$		R	262·5
$3^h\ 38^m$		Induction	245·9
$3^h\ 40^m$	[853]	R	182·3
$3^h\ 41^m$		L	344·7
$3^h\ 44^m$	[753]	L	264·4
$3^h\ 44^m$		Induction	245·7
$3^h\ 45^m$		R	263·1
		Induction	245·6

At $3^h\ 36^m$ with 753 units in the auxiliary arc and with battery key to the left, the position of equilibrium, as deduced from two elongations, was 264·4 on the galvanometer scale. The arc of vibration due to induction consequent on shifting the key from left to right, corrected for the free swing, but uncorrected for damping, was 246·6. In like manner with key to the right, the equilibrium position at $3^h\ 38^m$ was

262·5 and the arc due to induction was 245·9. The difference 1·9 between 264·4 and 262·5 represented the defect of balance. In the second set of induction throws the corresponding difference is 1·3, showing that the changes of temperature in progress were (at this stage) improving the balance of resistances. The difference between the readings R and L with 853 units is 162·4, the reading L being the higher. Since the reading L is also higher with 753 units, we have to *subtract* from 162·4 the mean of 1·9 and 1·3, *i.e.*, 1·6. The corrected value is thus 160·8. With this we have to compare the mean of 246·6, 245·9, 245·7, 245·6, *i.e.*, 245·9, and we thus obtain as the ratio of the two effects

$$\frac{245{\cdot}9}{160{\cdot}8}=1{\cdot}529$$

The numbers obtained in this way were 1·535, 1·532, 1·529, 1·528, mean 1·5310; and with galvanometer reversed 1·534, 1·529, 1·530, 1·530, 1·532, mean 1·5310. The reversal of the galvanometer appears to have made no difference, and we have as the mean of all 1·5310. The comparison of the partial results shows that during the hour and a half over which the readings extended the battery current fell slowly about one part in 120, and that the resistance of the copper gradually increased, until the balance was perfect, and afterwards became too great, the whole change being about one part in 6000, which would correspond to about one-twentieth of a degree centigrade.

A small correction is required in identifying the above determined ratio with $2 \sin \frac{1}{2}\alpha/\tan\theta$. If A be the induction arc and B be difference of equilibrium positions with 853 units when the commutator is reversed,

$$\tan 2\alpha=\frac{\frac{1}{2}\mathrm{A}}{\mathrm{D}}, \qquad \tan 2\theta=\frac{\frac{1}{2}\mathrm{B}}{\mathrm{D}}$$

where

D= distance of mirror from scale=218 centims.

From these we get

$$\frac{2\sin\frac{1}{2}\alpha}{\tan\theta}=\frac{\mathrm{A}}{\mathrm{B}}\,\frac{1-\frac{11}{32}\,\frac{\mathrm{A}^2}{4\mathrm{D}^2}}{1-\frac{1}{4}\,\frac{\mathrm{B}^2}{4\mathrm{D}^2}}$$

or in the present case with A=24·5, B=16·0,

$$\frac{2\sin\frac{1}{2}\alpha}{\tan\theta}=\frac{\mathrm{A}}{\mathrm{B}}(\cdot 99925)$$

and

$$\frac{\mathrm{A}}{\mathrm{B}}=1{\cdot}5310$$

So far we have omitted to consider the effect of damping, which must necessarily cause the observed value of A to be too small. If λ be the logarithmic decrement, the correcting factor is $(1+\lambda)$. The throw from zero to the first elongation is diminished by the fraction $\frac{1}{2}\lambda$, and the distance from zero to the second elongation is too small by the fraction $\frac{3}{2}\lambda$. Observations made in the usual manner after the other readings were concluded gave with considerable accuracy

$$\lambda = {\cdot}0142$$

The time of vibration was taken simultaneously. It appeared that

$$T = 11{\cdot}693 \text{ seconds}$$

A sufficient approximation to the ratio of currents $x' : x$ can be obtained by neglecting in both cases the current through the galvanometer, whose resistance (80 units) was considerable in comparison with the other resistances. On account of the small resistance of the battery, the difference of potentials at the battery electrodes may be regarded as given. On these suppositions we get at once

$$\frac{x'}{x} = \frac{10 + 23{\cdot}25869}{10 + 23{\cdot}34322}$$

whence

$$\log \frac{x'}{x} = \bar{1}{\cdot}99891$$

A more elaborate calculation, in which the finite conductivity of the galvanometer was taken into account, gave a practically identical result

$$\log \frac{x'}{x} = \bar{1}{\cdot}99886$$

We may now enter the numbers in the formula

$$L = \delta Q \frac{x'}{x} \cdot \frac{T}{2\pi} \cdot \frac{A}{B} ({\cdot}99925)(1+\lambda)$$

in which we must remember that δQ is to be expressed in absolute measure. Now the value given before, viz.: $\delta Q = {\cdot}08453$, is expressed in B.A. units. What this would be in absolute units involves the entire question to whose solution this paper is directed. We will suppose that

$$1 \text{ B.A. unit} = {\cdot}987 \text{ ohm}$$

δQ	log ·08453 × 10^9	= 7·92701
Correction to absolute units . . .	log ·987	= $\bar{1}$·99432
A : B	log 1·5310	= ·18498
Correction for finite arcs	log ·99925	= $\bar{1}$·99967
Correction for damping	log 1·0142	= ·00612
Time of vibration	log 11·693	= 1·06793
Ratio of currents	log (x'/x)	= $\bar{1}$·99886
		9·17889
	log 2π =	·79818
	log L	= 8·38071

whence

$$L=2\cdot4028\times10^8 \text{ centims.}$$

The value by *à priori* calculation is

$$L=2\cdot400\times10^8 \text{ centims.}$$

about one part in a thousand lower.*

Correction for level.

If the axis of rotation deviate from the vertical in the plane of the meridian a corresponding correction is required. If I be the angle of dip, and β the deviation of the axis from the vertical towards the north, the electromotive forces are increased in the ratio $(1+\tan I\,\beta):1$, in which proportion we must suppose GK increased. (See pp. 106, 124 of former paper.) The angle of dip at Greenwich for 1881 is about 67° 30′, so that

$$\tan I=2\cdot414$$

The correction for an error in level is thus of the first order, and is magnified by the largeness of the angle of dip in these latitudes. If the experiments were made at the magnetic equator, we should not only reduce the correction for level to the second order, but also obtain the advantage of a nearly doubled horizontal force.

Observations on the level were made by Dr. SCHUSTER on June 1, by myself on August 30, and by Mrs. SIDGWICK on October 13, and on November 11 and 23. The August observations gave $\beta=\cdot26'$; the October observations gave $\beta=\cdot30'$; and the November observations gave $\beta=\cdot25'$. The position of the axis is necessarily to a slight extent indefinite, and the differences are probably accidental. The same level was used throughout, and the value of its graduations was tested. We may take

$$\beta=+\cdot27'=+\cdot000079 \text{ circular measure}$$

and

$$1+\tan I\,\beta=1\cdot00019$$

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* A further small correction is called for by the fact that at actual temperature of the room (about 14°) the resistances given by the boxes were not exactly multiples of the B.A. unit. The difference in the case of the principal box, which is marked as correct at 14°·2, may be neglected, but the resistances taken from the auxiliary box (marked 18°·3) must have been smaller than their nominal value, to the extent of a little over one part in a thousand. By the same fraction δQ, and consequently L, must be *greater* than is supposed in the above calculation. The corrected value of L will be

$$L=2\cdot4052\times10^8$$

It is about *two* parts in a thousand greater than that found from the measured dimensions, and is, in my opinion, quite as likely to be correct.

Correction for torsion.

To determine τ, about five complete turns in either direction were given to the upper end of the fibre. The difference of reading for one turn was found to be in June 2·58, and in August 2·45. If we take as the mean 2·51, we get

$$\tau=\frac{2{\cdot}51}{4\pi\times2670}={\cdot}000075$$

Value of GK *corrected for level and torsion.*

Calling the corrected value $\mathfrak{G}\mathfrak{K}$, we have

$$\mathfrak{G}\mathfrak{K}=\frac{\mathrm{GK}(1+\tan\mathrm{I}\beta)}{1+\tau}=\frac{1{\cdot}00019}{1{\cdot}000075}\,\mathrm{GK}$$

so that

$$\log\mathfrak{G}\mathfrak{K}=8{\cdot}17686$$

The corrections are in fact almost insensible.

Calculation of U.

In this we take for $\mathfrak{G}\mathfrak{K}$ the value just found. For L we take the mean of the values found by *à priori* calculation and by direct experiment, *i.e.*,

$$\mathrm{L}=2{\cdot}4026\times10^{8}.$$

Thus

$$\log\frac{2\mathrm{L}}{\mathfrak{G}\mathfrak{K}}={\cdot}50485,\qquad\frac{2\mathrm{L}}{\mathfrak{G}\mathfrak{K}}=3{\cdot}1978$$

For the values of $\tan\phi$ and $\tan\phi_0$ we must anticipate a little. The ratio is itself in some degree a function of the speed, but it will suffice to take the values applicable to the highest speed, for which $\tan\phi_0:\tan\phi=7{\cdot}81:439{\cdot}41$.

Thus

$$\frac{\tan\phi}{\tan\phi-\tan\phi_0}=1{\cdot}0181$$

and

$$\log\mathrm{U}={\cdot}84325$$

Measurement of tan μ.

This is the tangent of the angle through which a suspended magnetic needle would be turned when the principal magnet is presented to it at a distance $\sqrt{(a^2+b'^2)}$ to the east or west, the axis of the principal magnet lying east and west. Actual measurements with the aid of the auxiliary magnetometer were made in April, June, and November; and as a check upon the constancy of the magnetic moment frequent observations were taken of the time of vibration.

To explain the procedure it will be sufficient to take the data of the November measurement. Two positions were chosen for the principal magnet, nearly equidistant from the suspended magnet, to the east and west. The length of the line joining the two positions was 695 millims., and it passed horizontally about 36 millims. below the suspended magnet. In each position the magnet was reversed backwards and forwards several times and readings taken. When the principal magnet was to the east, the mean difference of readings due to reversal was 13·55 divisions on the millimetre scale. When the principal magnet was in the westerly position, the corresponding difference of readings was 14·61. We are to take the mean of these, *i.e.*, 14·08, as the difference of readings due to reversal at a distance of 347·5 millims. The half of this, or 7·04, corresponds to the simple presentation or removal of the magnet. The distance from mirror to scale was 2670 millims, so that the tangent of the angle of deflection was $\frac{7\cdot04}{2\times2670}$. This result has to be adjusted to correspond with the distance $\sqrt{(a^2+b'^2)}$, in place of 347·5. Hence

$$\tan\mu=\frac{(347\cdot5)^3\times7\cdot04}{(238\cdot5)^3\times2\times2670}=\cdot00408$$

In this calculation the error due to the principal magnet having been necessarily placed at a different level from that of the suspended magnet is ignored. As a matter of fact a relatively considerable correction is required. If θ be the altitude of one magnet as seen from the other, the observed effect is too small in the ratio $(1-3\theta^2):1$. The above written value of tan μ requires to be increased about 3 per cent.; so that we take

$$\tan\mu=\cdot00420.$$

Measurement of D.

For the first and second series of spinnings the distance from mirror to scale was measured exactly as described by Dr. SCHUSTER (p. 126 of former paper). The value adopted for the second series, after correction for the thickness of the glass window in the magnet box, was

$$D=2669\cdot0 \text{ millims.}$$

The same method of measurement was applied at the beginning of the third set, and a watch was kept by means of the measuring rods already spoken of (p. 667). Slight movements were in fact observed, principally of the nature of a recovery of the telescope stand from the rather violent treatment to which it had been subjected as a test. Minute corrections are accordingly introduced into the tabular statement (p. 693), so as to make the results of different days comparable. At the close of the spinnings the direct measurement was repeated, when there appeared a slight discrepancy between the results obtained by Mrs. SIDGWICK and myself. It is in fact rather a difficult matter to say exactly when the pointer has advanced to the equilibrium position of the centre of a suspended mirror, which cannot be prevented from swinging. Although the amount in question was not important, I thought it might be more satisfactory to check the result by another method, and therefore arranged a travelling microscope focussed alternately upon the centre of the mirror and upon a scratch on the window of the magnet box, by which the distance between these two points was determined. The remaining distance between the scratch and the scale was easily measured with the rod. The result tended to confirm the smaller value previously found. The value adopted for the spinnings of the third series before November 5 is

2668·8 millims.

and for November 5 and subsequent nights

2669·4 millims.

From these numbers we have to subtract 1·1 millim., as a correction for the thickness of the glass window; so that

D before November 5 =2667·7 millims.
D November 5 and after =2668·3 millims.

These distances are expressed in terms of the divisions of the scale, whose exact agreement with millimetres is of no consequence.

Reduction of results.

In order to give a clear idea of the results and of the manner in which they have been reduced, it will be advisable to quote from the note book the details of one set of spinnings. I have chosen at random one of the third series made on October 31, 1881, with a speed of "45 teeth."

TABLE I.

		Time* P.M.	Readings of auxiliary magneto-meter in millims.	Readings of principal magneto-meter in millims.	Readings corrected by auxiliary magneto-meter.	Sum.	Difference.	Mean deflection.	Standard *minus* copper, in bridge-wire divisions.	Temperatures.			Beats per minute.	Correc-tion to middle of bridge.	Correc-tion to 13° of standard.	Correc-tion to 59 beats.	Correc-tion to 17° of fork.	Corrected deflection.
										Standard.	Air.	Fork.						
Contact open.	I.	8.16	81·3	593·38	593·38	1197·24	10·48	5·29										
	II.	8.18	81·6	604·16	603·86	1197·27	10·45											
	III.	8.20	82·2	594·31	593·41	1197·51	10·69											
	IV.	..	83·0	605·80	604·10	1197·55	10·65											
	V.	8.23	83·8	595·95	593·45	1197·50	10·60											
	VI.	8.25	84·2	606·95	604·05													
		8.36	..	..	..	..	..	..	+212	9·95	12·2	12·5	56					
Contact closed.	VII.	8.45	83·1	903·38	901·58	1197·69	605·47	302·56	Mean −52	Mean 10·02	..	Mean 13·05	Mean 56·5	+·03	−·27	−·10	−·12	302·10
	VIII.	8.47	82·5	297·31	296·11	1197·65	605·43											
	IX.	8.50	81·4	901·64	901·54	1197·96	605·12											
	X.	8.52	81·0	296·12	296·42	1197·75	604·91											
	XI.	8.55	81·0	901·03	901·33	1197·89	604·77											
	XII.	8.58	81·1	296·36	296·56													
		9.3	..	..	..	..	..	..	−316·5	10·1	12·0	13·6	57					

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* To prevent misapprehension it may be mentioned that the times given in this column are approximate only and were not relied upon to secure simultaneous readings of the two magnetometers. A signal was passed from one observer to the other at the beginning and end of the readings.

The first column gives the number of the spinning, the first six being made with wire circuit open, and the last six with the wire circuit closed. In spinnings I., III., V., VIII., X., XII., the rotation was in the direction reckoned negative, and in the remaining ones positive. The second column gives the time, the third the reading of the auxiliary magnetometer, the fourth the reading of the principal magnetometer, the fifth the result of correcting the latter by the former, the sixth and seventh the approximately constant sums and differences of consecutive pairs of numbers in the fifth column, and the eighth gives the mean deflection from zero, *i.e.*, 5·29 for the open contacts, and 302·56 for the closed contacts.

The ninth column shows the results of the resistance comparisons between the platinum-silver standard and the revolving copper coil before and after the closed contact spinnings. The first number (+212) means that at $8^h\ 36^m$ the resistance of the standard exceeded that of the copper by 212 bridge-wire divisions, each of which represents $\frac{1}{20000}$ of an ohm. It will be seen that during the spinnings the resistance of the copper increased, which accounts for the gradual fall observable in the seventh column. The mean of the comparisons before and after spinning is taken to correspond with the mean deflection 302·56. The three following columns show respectively the temperatures of the water in which the standard was immersed, of the air in the neighbourhood of the copper coil, and of the standard tuning-fork, while the thirteenth column gives the number of beats per minute between the electrically maintained and the standard fork.

For the sake of more convenient comparison of the results obtained at the same speed on different nights, small corrections are calculated to reduce the actually observed deflections in the eighth column to what they would have been in a standard condition of the resistance and of the speed. Under each of these heads we have two corrections to consider. In the first place the copper circuit differed in resistance from the standard coil by the outstanding (−52) divisions of the bridge wire. The resistance of the whole being about 24 ohms, each division of the wire corresponds to one part in 480,000, so that in the present case the correction is additive and equal to 52 parts in 480,000, *i.e.*, is equal to +·03 division of the scale. This is given in the fourteenth column. Secondly, the resistance of the standard itself depends upon a variable temperature. The mean temperature of the standard in this series was about 13°, to which all the observations are reduced. In the present case the temperature was below the normal, so that the resistances were too small and the deflections too large. Accordingly the correction is negative. To estimate its amount the change of resistance with temperature is taken at three parts in 10,000 per degree ; so that in the present case we are to subtract 2·8 parts in 3000 of the whole deflection, *i.e.*, ·27, as entered in the fifteenth column. With use of these corrections we obtain the deflection as it would have been observed had the resistance of the revolving circuit (together with the long copper bars by which it was connected with the bridge) been on every occasion exactly that of the standard at 13°.

In like manner two other very small corrections have to be introduced to make the results correspond exactly to a normal speed of rotation. The standard number of beats is taken at 59, and the standard temperature of the fork at 17°. In the specimen set the number of beats is $2\frac{1}{2}$ per *minute* too small, which means that the octave of the electrically maintained fork made (relatively to the other fork) $2\frac{1}{2}$ complete vibrations per minute *too many.* The whole number of vibrations per minute being 60×127, the speed was too great by $2\frac{1}{2}$ parts in 60×127, by which fraction the observed deflection must be reduced. The correction is thus $-{\cdot}10$. But besides this the standard fork at 13·05° vibrated faster than its normal rate at 17°, by about one part in 10,000 for each degree of difference. The correction for this is $-{\cdot}12$.

In addition to the corrections already mentioned the observations of November 5 and after were subjected to another small correction for the observed change in D.

The accompanying Table (II.) exhibits the results of the second series in a manner which after what has been said will not require much explanation. Column VIII. gives in each case what the deflection would have been if the revolving circuit and the copper connecting bars had exactly balanced the platinum-silver standard at 16°, the electric fork vibrating at such a speed as to give 59 beats per minute with the standard fork at 17°, and thus allows us to test the agreement or otherwise of the results obtained on various occasions at the same speed. From this point onwards the means only need be considered; but as there is reason (as already explained) to distrust the observations of August 29, I have added a second mean from which the distrusted elements are excluded. The deflection (d) thus arrived at is equal to $\mathrm{D}\tan 2\phi$, whereas what we require is $2\mathrm{D}\tan\phi$. The connexion between the two quantities is obtained in a moment from the formula

$$2\tan\phi=\tan 2\phi\,(1-\tan^2\phi)$$

by successive approximation. Thus

$$2\tan\phi=\tan 2\phi\,\{1-\tfrac{1}{4}\tan^2 2\phi+\tfrac{1}{8}\tan^4 2\phi\},$$

or

$$2\mathrm{D}\tan\phi=d-\tfrac{1}{4}\frac{d^3}{\mathrm{D}^2}+\tfrac{1}{8}\frac{d^5}{\mathrm{D}^4}.$$

Column X. gives the value of $2\mathrm{D}\tan\phi$, XI. that of $2\mathrm{D}\,(\tan\phi-\tan\phi_0)$ in the notation of p. 675, and XIII. that of $\log\,(\tan\phi-\tan\phi_0)$.

For the further calculation we require the value of ω. If f be the frequency of vibration of the electrically maintained fork, F that of the standard at 17°, N the number of teeth,

$$\omega=\frac{4\pi f}{\mathrm{N}},$$

and when the number of beats is 59 per minute,

$$f=\tfrac{1}{2}(\mathrm{F}-\tfrac{59}{60}).$$

TABLE II.—Second series.

Date, August, 1881.	Open contact deflections.	Closed contact deflections.	Correction to middle of bridge wire.	Correction to 16° of standard.	Correction to 59 beats.	Correction to 17° of fork.	Closed contact deflections, corrected to standard resistance and speed.	Correction for scale reading $-d^3/4D^2$ $+d^5/8D^4$	Closed contact deflections corrected for scale reading.	X. *minus* II.		Logarithm of $\tan\phi - \tan\phi_0$.
	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.	XI.	XII.	XIII.
						60 TEETH.						
15	3·97	228·60	−·15	+·01	·00	·00	228·46	[− ·42			D=2669·0 millims.	
23	3·87	228·68	−·33	−·01	·00	·00	228·34	+ ·00				
26	3·95	228·41	+·04	+·01	−·03	·00	228·43	=]				
Mean	3·93	. .	. .	. .	. .	. .	228·41	− ·42	227·99	224·06		$\bar{2}$·62298
						45 TEETH.						
19	5·25	302·22	−·33	·00	·00	−·01	301·88	[− ·97			D=2669·0 millims.	
24	5·27	302·31	−·34	+·01	−·02	−·01	301·95	+ ·01				
27	5·21	302·48	−·50	·00	−·04	−·01	301·93	=]				
Mean	5·24	. .	. .	. .	. .	. .	301·92	− ·96	300·96	295·72		$\bar{2}$·74350
						35 TEETH.						
15	6·68	383·41	+·22	+·03	·00	·00	383·66	[−1·98			D=2669·0 millims.	
24	6·82	383·90	−·18	+·02	+·02	−·01	383·75	+ ·02				
29	6·81	383·53	−·18	−·01	−·05	−·02	[383·27]	=]				
Mean	6·77	. .	. .	. .	. .	. .	383·56 or 383·70	−1·96 −1·96	381·60 or 381·74	374·83 or 374·97		$\bar{2}$·84645 or $\bar{2}$·84662
						30 TEETH.						
19	7·86	442·79	−·01	+·02	·00	·00	442·80	[−3·04			D=2669·0 millims.	
26	7·80	442·73	−·18	+·01	−·03	·00	442·53	+ ·04				
29	7·81	441·85	+·20	+·02	−·03	−·01	[442·03]	=]				
Mean	7·82	. .	. .	. .	. .	. .	442·45 or 442·66	−3·00 −3·00	439·45 or 439·66	431·63 or 431·84		$\bar{2}$·90773 or $\bar{2}$·90794

For F at 17° we take 128·130 (see p. 137 of former paper), so that

$$f=63\cdot574.$$

Thus

$$\log\,(2\pi . f . \mathfrak{GK})=10\cdot77832=\log\,(10^{10}\times 6\cdot0023),$$

and

$$R=\frac{10^{10}\times 6\cdot0023}{N(\tan\phi-\tan\phi_0)}\left[\tfrac{1}{2}(1+\cdot00422\sec\phi)+\sqrt{\{\tfrac{1}{4}(1+\cdot00422\sec\phi)^2-U(\tan\phi-\tan\phi_0)^2\}}\right]$$

in which

$$\log U=\cdot84325$$

TABLE III.—Second series.

	Number of teeth.			
	60.	45.	35.	30.
R by preceding formula in ohms. .	23·639	23·655	23·660 23·651	23·670 23·659
Resistance of standard at 16°. . .	23·642	23·658	23·663 23·654	23·673 23·662
Resistance of standard at 13°. . .	23·621	23·637	23·642 23·633	23·652 23·641

The immediate result of the formula is the resistance in absolute measure of the revolving circuit, on the supposition that with the connecting bars it exactly balances the standard at 16°. The resistance of the standard itself is therefore given by addition of the resistance of the bars, *i.e.*, ·003 ohm. In the last line the results are reduced to the temperature of 13° for comparison with the third series.

TABLE IV.—Third series.

Date, 1881.	Open contact deflections.	Closed contact deflections.	Correction to middle of bridge wire.	Correction to 13° of standard.	Correction to 59 beats.	Correction to 17° of fork.	Correction to D=2667·7.	Closed contact deflections corrected.	Correction for scale reading as in Series II.	Correction for curvature of scale.	Closed contact deflections corrected for scale.	XII. *minus* II.		Logarithm of tan ϕ − tan ϕ_0.
I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.	XI.	XII.	XIII.	XIV.	XV.
							60 TEETH.							
Oct. 22	3·92	228·47	+·19	−·06	−·06	−·07	..	228·47					D=2667·7 millims.	
Oct. 29	3·95	228·74	−·08	−·10	−·06	−·08	..	228·42						
Nov. 7	3·92	228·21	+·27	+·13	−·06	−·01	−·05	228·49						
Mean	3·93	..	..	..	..	..	..	228·46	−·42	+·05	228·09	224·16		$\bar{2}$·62339
							45 TEETH.							
Oct. 29	5·20	302·93	−·37	−·20	−·08	−·11	..	302·17					D=2667·7 millims.	
Oct. 31	5·29	302·56	+·03	−·27	−·10	−·12	..	302·10						
Nov. 5	5·22	301·84	+·31	+·17	−·11	−·02	−·07	302·12						
Mean	5·24	..	..	..	..	..	..	302·13	−·96	+·08	301·25	296·01		$\bar{2}$·74414
							35 TEETH.							
Oct. 22	6·77	383·49	+·64	−·16	−·05	−·12	..	383·80					D=2667·7 millims.	
Oct. 31	6·77	383·92	+·39	−·33	−·10	−·15	..	383·73						
Nov. 7	6·73	383·48	+·29	+·18	−·10	−·03	−·09	383·73						
Mean	6·76	..	..	..	..	..	..	383·75	−1·96	+·11	381·90	375·14		$\bar{2}$·84702
							30 TEETH.							
Oct. 29	7·92	443·64	−·25	−·26	−·12	−·17	..	442·84					D=2667·7 millims.	
Nov. 5	..	442·31	+·54	+·22	−·17	−·03	−·10	442·77						
Nov. 7	7·90	..	..	..	..	..	..	..						
Nov. 10	7·94	442·74	+·16	+·19	−·11	−·04	−·10	442·84						
Mean	7·92	..	..	..	..	..	..	442·82	−3·00	+·11	439·93	432·01		$\bar{2}$·90830

TABLE V.—Third series.

	Number of teeth.			
	60	45	35	30
R by formula	23·616	23·618	23·627	23·635
Resistance of standard at 13° .	23·619	23·621	23·630	23·638

If we compare the results of the second and third series at the same speed, we find the agreement satisfactory (with a partial exception at the speed corresponding to 45 teeth), especially if we take the means from which the observations of August 29 in the second series are excluded. Adding together all the results of each series we should obtain from the second series 23·638, or with exclusion of August 29, 23·633, and from the third series 23·627, between which the extreme difference is less than one part in 2000. When, however, we compare the values obtained from observations at different speeds, we see from both series, but more especially from the third, evident signs of a tendency to rise with the speed, as if the self-induction of the revolving circuit had been underestimated. In view of the remarkable concordance of the results obtained at the same speed on different nights, it is impossible to attribute these discrepancies to errors of observation, and it is important to consider what cause of systematic disturbance can have remained unallowed for. The first question which presents itself is whether it is possible to admit an error in the adopted value of L sufficient to explain the progression. The proportional correction for self-induction is approximately $-\mathrm{U}\tan^2\phi$, or for the speed of 30 teeth ·0457. For the speed of 60 teeth the correction will be only one-fourth of this. To bring the results for the two speeds into agreement it would be necessary to increase the value of U by nearly 3 per cent., which would correspond to an increase of about one per cent. in L. It is difficult to believe that the value of L adopted for the wire circuit can be in error to this extent.

Another direction in which an explanation might be looked for would be the influence of air disturbance, or from tremor. The accompanying table, however, shows such an extraordinary agreement of the open contact deflections, both among themselves and with numbers proportional to the speeds of rotation, as to prevent us from supposing that this cause of disturbance can have operated.

TABLE VI.—Deflections with wire circuit open.

	Number of teeth.			
	60	45	35	30
Mean of first series	3·89	5·22	6·74	7·85
„ second series . . .	3·93	5·24	6·77	7·82
„ third series	3·93	5·24	6·76	7·92
After the wire had been removed, December 7 . . .	. .	5·23	6·76	. .
Numbers proportional to speed	3·94	5·25	6·76	7·88

On the whole, it would appear to be the most probable explanation that there were currents in the ring flowing in circuits not conjugate to the wire circuit, and therefore influencing the induction phenomena. But whatever view we may take on this matter, there is no reason to doubt that the true value will be obtained by introducing such a correction proportional to the square of the speed as will harmonise the several results, a course equivalent to determining the coefficient of self-induction from the spinnings themselves. In this way the numbers corresponding to any two speeds may be made arbitrarily to agree, but the numbers for the two remaining speeds will afford a test of the admissibility of this procedure. The only hypothesis upon which the simple mean of the numbers already obtained for the various speeds should be taken as final would appear to be one that would attribute to the discrepancies an accidental character, and seems quite out of the question.

The simplest way to carry out the correction will be to determine the amount of the coefficient from the two extreme speeds. The squares of the speeds are as

$$1 : \tfrac{16}{9} : \tfrac{144}{49} : 4;$$

so that the difference of the numbers for the two extreme speeds, 23·638—23·619, *i.e.*, ·019, is three times the quantity by which the lowest is to be reduced. We are accordingly to subtract respectively

$$\cdot 0063,\ \tfrac{16}{9}\times \cdot 0063,\ \tfrac{144}{49}\times \cdot 0063,\ 4\times \cdot 0063$$

with the following results.

TABLE VII.—Third series.

	Number of teeth.				Mean.
	60.	45.	35.	30.	
Resistance of standard at 13°, uncorrected	23·619	23·621	23·630	23·638	23·627
Correction proportional to square of speed	·006	·011	·018	·025	. .
Resistance of standard at 13°, corrected .	23·613	23·610	23·612	23·613	23·612

It will be seen that the agreement is practically perfect, the coefficient given by the extreme speeds suiting also the requirements of the intermediate speeds. The maximum difference corresponds to about $\frac{3}{100}$ths of a millimetre only in the deflections of the principal magnetometer. The number $23{\cdot}612 \times 10^9$ is therefore to be regarded as the resistance in absolute C.G.S. measure of the platinum-silver standard at 13°. If, however, the correction be rejected, the result will be different by decidedly less than one part in a thousand.

Although the experiments of the second series will not bear comparison with those of the third, it may be well to mention that they lead to substantially the same conclusion. The simple mean (taken with exclusion of August 29) of all the values is 23·633, and after introduction of the correction proportional to the square of the speed, 23·618.

The results of the first series of spinnings are given in Table VIII. They have been reduced by Dr. SCHUSTER, so as to show the value of the platinum-silver standard in absolute measure from the observations of each night at each speed. The mean radius of the coil was taken from the first measurements, and a somewhat higher value of U was employed than the subsequent calculation of the ring currents seemed to justify.

TABLE VIII.—First series.

Teeth.	Resistance in absolute measure of standard at 13°.				
80	23·651,	23·632,	23·628	..	Mean 23·637
60	23·646,	23·629,	23·601	..	Mean 23·625
45	23·678,	23·691,	23·686	..	Mean 23·685
35	23·608,	23·615,	23·632,	23·665	Mean 23·630
30	23·644,	23·639,	23·628	..	Mean 23·637
					23·643

Comparison with the standard B.A. units.

Four distinct sets of comparisons between the platinum-silver standard and the ultimate B.A. units have been effected in the course of these investigations, and two distinct methods have been followed. In the first method two coils of about five units, called for brevity [5]'s, were compared separately with five standard units combined in series with mercury cups. Secondly, the two [5]'s in series were compared with a [10]. Thirdly, the [10], the two [5]'s, and four singles were combined in series and compared with the platinum-silver standard [24]. The differences in every case were expressed in divisions of the wire of FLEMING'S bridge, whose value in terms of the B.A. unit is known. This method is simple enough in principle, but the arrangement of so many mercury connexions is troublesome, and the calculation of the innumerable temperature corrections necessary is tedious. The labour would have been greater still had we not been able to avail ourselves of the previous work of Professor FLEMING, who had carefully compared the various standard units, and had drawn up a chart on which is exhibited the comparative resistances of the coils over a considerable range of temperature. The mean B.A. unit, in terms of which our results are expressed, was defined by him, but the differences between the single standards is scarcely of importance for our purpose. In calculating the temperature corrections for the two [5]'s, the [10], and the [24], which were all of platinum-silver wire, the co-efficient ·0003 per degree has been used. The temperatures were those of the water in which the coils were immersed. They never differed much from the temperature of the room, and were referred to a Kew standard. The results of three comparisons, executed by Mrs. SIDGWICK, are as follows:—

Resistance in mean B.A. units of platinum-silver standard at 13°.

July, 1881	23·9326
September, 1881	23·9341
November, 1881	23·9348

In February, 1882, a fourth determination was executed by myself, in which a different method was employed. Five coils approximately equal to each other and to five units were arranged in a closed case upon a tube of brass. The ten copper terminals emerged below from the ebonite bottom of the case, and rested in mercury cups upon an ebonite base-board, which was so arranged that by a single motion the terminals could be transferred from one set of cups which combined the coils in series to another set which combined them in multiple arc.* In this way resistances are obtained in the proportion of 25 : 1, independently of any exact equality between the single coils; for it is obvious that if the resistance in series is given, the resistance in multiple arc is a maximum in the case of equality, and therefore varies little, even if the equality be not exact. By the aid of this apparatus the [24] was compared with a standard unit, without the assistance of other coils. In the first place [24]+[1] was compared with the five coils in series, and in the second place the [1] was compared with the five coils in multiple arc. The only precaution necessary is to effect the second comparison so quickly after the first that the five coils have no time to change their temperature. Two determinations by this method on different days gave as the resistance of [24] at 13°

23·9350, 23·9358—mean, 23·9354.

It would seem not impossible that the resistance of [24] has gradually increased, but the changes are unimportant. We will take as the resistance with which the absolute measurement is to be combined, that found in November 23·9348; so that

$$23{\cdot}9348 \text{ B.A. units} = 23{\cdot}612 \times 10^9 \text{ C.G.S.} = 23{\cdot}612 \frac{\text{earth quadrant}}{\text{second}}$$

Hence, as the result of the investigation, we conclude that

$$1 \text{ B.A. unit} = {\cdot}98651 \frac{\text{earth quadrant}}{\text{second.}}$$

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* I believe that Professor ROWLAND has used a contrivance of this sort.

XIV. *On the Comparative Structure of the Brain in Rodents.*

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Received October 13,—Read November 17, 1881.

[Plates 49, 50.]

In a former memoir published in the Philosophical Transactions[*] I have detailed the result of researches into the minute anatomy of the cortex cerebri in the Pig as contrasted with that of the Sheep, the Cat, and other animals. In continuation of the same series of researches it is proposed to consider here the results obtained by a still more extended enquiry into the structure of the brain in the Rodentia, and with this object in view, I shall divide my subject into the two lines of investigation followed out:—first, *the histology of the entire cortical envelope*, and secondly, *the central projections of the olfactory organ.* It is necessary that such an account be preceded by a short sketch of the external conformation of the brain in Rodents, and that its topography should be mapped out under the more recent and accurate nomenclature introduced by the late Professor Broca, whose work upon the comparative anatomy of the convolutions in Mammalian brains[†] is by far the most philosophical and trustworthy treatise which has appeared on this subject since the publications of Turner,[‡] Bischoff,[§] and Ecker.[||] Since the cerebrum of the Rabbit affords the type for the brain of the Rodent, and since this animal, together with the Rat, was chiefly utilised in these investigations, my sketch will be limited to the brains of these two animals.

External Conformation of Brain in the Rat and Rabbit.

When investigating the structure of the brain in the Pig and Sheep I had to deal with a highly developed olfactory apparatus, associated with a richly convoluted cortex. In the brain of the small Rodents now under consideration there is also a notable development of the olfactory organ and its central connexions, but on the other hand the great extra limbic or parietal mass is wholly devoid of gyri. The Mammals possessing such smooth, non-convoluted brains have been grouped in an artificial class by OWEN, who terms them *Lissencéphales;* the animals possessing convoluted brains are termed *Gyrencéphales.* BROCA, on the other hand, has adopted the terms Osmatic* and Anosmatic Mammals,† as indicative of a highly-developed olfactory apparatus in the one, as contrasted with the defective, rudimentary, or wholly absent olfactory lobes of the other class. Both these terms are extremely useful in such investigations as we at present are concerned with, and I shall therefore freely adopt them, together with the nomenclature of the convolutions advocated by BROCA. From what has been stated it is obvious that Osmatic Mammals may have smooth or convoluted brains, but the Rat and Rabbit present us with typical brains of the Osmatic Lissencéphales. The cerebrum in these animals is mapped out into two grand divisions :—

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[*] "Researches on the Comparative Structure of the Cortex Cerebri," Phil. Trans., Part I., 1880.

[†] "Anatomie Comparée des Circonvolutions," par M. Paul Broca. Revue d'Anthropologie.

[‡] "Convolutions of the Human Cerebrum Topographically considered." W. Turner, Edin. Med. Journ., 1866.

[§] "Die Grosshirnwindungen des Menschen." Dr. Th. L. W. Bischoff. München, 1868.

[||] "The Convolutions of the Human Brain." Dr. Alex. Ecker. Translated by J. C. Galton.

1. A median portion, encircling the basal ganglia and peduncular mass as it leaves the cerebrum, forming the great limbic lobe.
2. The more exposed hemispheric mass forming the outer and upper aspect of the brain corresponding to the parietal lobe of higher animals, and which I shall term, after BROCA, the extra-limbic mass.

The former, or great limbic lobe, is divisible into three portions—

a. An upper arc corresponding to what has been called the convolution of the corpus callosum (gyrus fornicatus).
b. A lower arc, also called the gyrus hippocampi.
c. An anterior arc, formed by the olfactory lobe, which unites the upper and lower limbic arcs.

Looked at from above, the brain of the Rat is heart-shaped, with the apex directed forwards. The hemispheres diverging behind reveal a minute, semi-translucent, pale-grey body, ovoid in form, projecting betwixt them, the pineal body, posterior to which the corpora quadrigemina appear. The surface of the extra-limbic mass is almost perfectly smooth, being marked only by a few extremely delicate venules. There is no distinct frontal, occipital, or temporal lobe differentiated from this extra-limbic mass, which must be regarded as entirely constituting a parietal lobe. We therefore find no trace of a fissure of ROLANDO, as is distinctly seen in Osmatic Gyrencéphales (*e.g.*, Pig and Sheep).‡ We shall, however, retain the terms frontal and occipital poles, as indicative of the anterior and posterior extremity of the hemisphere, it being understood that these terms do not imply the existence of a frontal or occipital lobe. The inner margin of the hemispheres bordering upon the great longitudinal fissure may also be conveniently spoken of as the sagittal border. Along this border, in the hinder half of the hemisphere, there is seen a strip of cortex, much paler than the remaining portion of the hemisphere, about 2 mm. wide behind, but becoming gradually narrower as it extends forwards. This pale strip is not marked off by any linear depression, yet it forms an important region characterised by a peculiar type of cortex. In the Rabbit, on the other hand, the same sagittal region is mapped off from the extra-limbic mass outside it by a constant though shallow depression (Plate 49, fig. 1, K K), the representative of the primary parietal sulcus of higher animals, and which, as in the Pig and Sheep, separates the sagittal from the Sylvian gyri of the parietal lobe. Moreover, the upper aspect of the brain in the Rabbit is of more pyriform contour, and remarkably attenuated in front, so that the pointed frontal poles contrast strangely with the broader frontal extremities of certain animals included in the same category (fig. 8, Beaver, *op. cit.*, BROCA.). The hemispheres of a four-months' old Rabbit measured 28 mm. long, and weighed (together with corpora quadrigemina) just six grammes. The extra-limbic lobe is traversed by three or four delicate channels for blood-vessels, which course upwards and backwards from the limbic fissure, and very slightly indent its surface. At the widest portion the cerebrum has a diameter of 30 mm., whilst in the Rat it is but 15 mm. across. The brain of the Rat displays the same cordate outline at its base, the hemispheres widely separated behind by the descent of the cerebral peduncles. The lower limbic arc forms so prominent a feature as *wholly to conceal from view* the lateral portions of the extra-limbic lobe (Plate 49, fig. 2, N). The base of this cordate area is therefore formed by the gyrus hippocampi, in front of which extends the olfactory lobe. Along the course of each olfactory lobe the superficial olfactory medulla (external root) runs back, rapidly narrowing as it approaches the gyrus hippocampi, in which it loses itself (Plate 49, fig. 2, M). Betwixt the two superficial olfactory bands, and in front of the optic commissure, lie two pyriform grey areas, the olfactory field of GRATIOLET (Plate 49, fig. 2, P). They are bounded externally by the fasciculus just named, and lie in contact with one another on the inner side, the optic nerves being here placed superficial to them. The optic nerves terminate in a well-marked optic commissure (Plate 49, fig. 2, R), from which the optic tracts diverge, and are lost beneath the gyrus hippocampi. The tuber cinerum and infundibulum forms a prominent mass of grey matter behind the optic commissure (Plate 49, fig. 2, L), and are crossed behind the optic tracts by a distinct white band of fibres extending across the angular interval left betwixt the diverging tracts. Stellate pigment cells of a jet-black hue accumulate more or less thickly along the whole lower limbic arc and olfactory lobe. They are especially frequent over the basal aspect of the olfactory lobe and along the limbic fissure, back even as far as the posterior extremity of the latter.

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* Includes the large majority of Mammalian brains.

† The Cetacea, Amphibious Carnivora, and Primates.

‡ Vide Plate 9, fig. 2, *op. cit.*

Upon separating the olfactory lobe from the frontal end of the hemisphere and reflecting it backwards, we become aware of the fact that the frontal extremity is obliquely truncated, sweeping forwards and upwards to the frontal pole, and thus forming an oblique depression in which lies the opposed olfactory lobe. Into this part of the hemisphere runs the upper or fourth olfactory fasciculus. If we direct our attention to the base of the brain in the Rabbit, on the other hand, we observe that the great limbic lobe does not, as in the Rat, conceal by its extension outwards the extra-limbic portion of the hemisphere, a large portion of which projects on either side of the well-marked limbic fissure. The olfactory lobe runs directly forwards beneath the frontal extremity of the hemisphere, its external or superficial fasciculus coursing backwards parallel to, and 2 mm. distant from, the limbic fissure (Plate 49, fig. 1, M), until at the gyrus hippocampi it curves inwards and is lost to view. On the median aspect of the Rabbit's brain the upper limbic arc is deep anteriorly, but narrowed behind, and curving down behind the corpus callosum, forms a deep fossa, here overlapped by the occipital pole in which lies the quadrigeminal bodies. Below this it becomes continuous with the gyrus hippocampi or lower limbic arc (Plate 49, fig. 1, B). Along the upper arc two delicate furrows are seen, the representatives of the sub-frontal and sub-parietal segments of the limbic fissure (Plate 49, fig. 1, A). In the Rat, however, there is no trace of any such furrows, the upper arc being here directly continuous with the sagittal portion of the parietal lobe. In both animals the Sylvian fissure is nearly absent, being indicated by a very slight depression betwixt the gyrus hippocampi and the olfactory region (Plate 49, figs. 1, 2, S).

Section I.—The Cerebral Cortex.

The complete cortical envelope of the cerebral hemispheres in the animals under consideration (and this with special application to the Rabbit and the Rat) may be readily mapped out into six distinct regions characterised by a notable diversity in their laminar constitution (Plate 49, fig. 1). These regions, abruptly marked off from each other by their topographical relationships as well as their intimate structure, are the upper limbic arc, the lower and anterior limbic arcs, the extra-limbic lobe, the olfactory bulb, and the cornu ammonis; but whereas the two former of these regions are subject to further modifications of their textural laminations—modifications which are not simply those of *degree*, but of *kind*—it results that I must extend my classification of the varied cortical realms beyond the limits mapped out by former authorities. Thus Meynert distinguishes five *types* of cortical lamination as follows:—

1. A common type.
2. Occipital type.
3. Sylvian type.
4. Type of cornu ammonis.
5. Type of olfactory bulb.

He also casually refers to a peculiar constitution of the olfactory area, but enters into no detailed account of the same, stating that it still awaits a monographical exposition.*

Since, in accordance with the rule which I find it essential to adopt as to what is to be understood by a divergence in the *laminar type* of the cortex, I should exclude from his list the third or Sylvian, the diversities of structural type embraced in MEYNERT'S category will be four only. This, however, but very inadequately expresses the truth as regards the cortex of the Mammalian brain, for I find at least *eight characteristic cortical areas* distinguished by their laminar structure. These we may enumerate as follows :—

1. Type of upper limbic arc.
2. Modified upper limbic type.
3. Outer olfactory type.
4. Inner olfactory type. } Comprised within the limits of the lower and anterior limbic arcs.
5. Modified olfactory type. }
6. Extra-limbic type.
7. Type of cornu ammonis.
8. Type of olfactory bulb.

In enumerating these types I have distinguished them by terms having special reference to their regional distribution, not that I believe this to be the better method ultimately to adopt, but the simplest in our present state of defective knowledge upon these points. With advancing knowledge of the intimate structure and peculiarities of the various regions of the cerebral cortex it will be found advisable to term these varied types after some important and essential structural peculiarity. The same remark applies to the enumeration of the individual cortical layers which are usually denominated by their relative positions as first, second, or third layers, &c. This method cannot fail to mislead and introduce much confusion, since we are constantly meeting with intercalation of fresh layers or the absence of others, and as a result of this method the same denomination becomes applied to layers which are wholly different in constitution. It is, therefore, on this account necessary to term each layer after some characteristic structural feature, more especially one which finds its counterpart in all the various members of the Mammalian series, and to subordinate the numbering of layers to the expression of their relative depth.

Upper Limbic Arc (Plate 49, fig. 1).

Adopting the above considerations, I find it necessary to regard the cortex typical of the upper limbic arc as consisting of four well differentiated layers, which may be termed respectively—

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* "The Brain of Mammals," STRICKER'S 'Human and Comp. Histology,' Sydenham Soc. Trans., vol. 2, p. 414.

1. Peripheral cortical zone.
2. Small pyramidal layer.
3. Ganglionic layer.
4. Spindle cell layer.

1. *Peripheral cortical zone.*—As seen by the naked eye this is a uniformly light grey belt sharply defined from the darker layers beneath. Its structure, revealed by microscopic aid, does not differ from that of higher animals, being constituted of a fine neuroglia matrix derived from the delicate prolongations of connective cells supporting the meshwork formed by the repeated sub-divisions of the apical processes of nerve-cells in the underlying strata. We meet here, as in the Sheep, Pig, and other animals, with the DEITER cell. These cells are found immediately beneath the pia-mater, their processes extending downwards into this layer and *a larger process invariably being connected* with a nucleus along the sheath of a blood-vessel. As regards depth there is a very evident diminution of this layer upwards and outwards over the exposed aspect of the hemisphere as well as backwards towards the occipital pole. Thus in regions of this upper limbic arc anterior to the corpus callosum the depth is ·511 mm., whilst posterior to this commissure it scarcely attains to ·377 mm. A still more notable diminution in depth occurs outwards, viz.: in extra-limbic regions where this outer cortical zone measures but ·279 mm. in depth. In the cortex of the Pig's brain a similar fact was noted and is recorded in the table of depths of the cortical layers given in my former paper.*

2. *Small pyramidal layer.*—In the upper limbic arc of the brain of Man and of some of the higher Mammalia I have described the typical formation as that of a five-laminated cortex. In the same region, however, of the Rabbit's brain the cortex is four-laminated, and this arises from the absence of the small oval and angular elements which in Man constitute the second layer. In certain localities, more especially extra-limbic in position, a shadowing forth of this stratum is represented by the closer aggregation of elements superficially which become also appressed in small clumps at intervals and may possibly represent a rudimentary form of this layer. So little marked is this in the upper limbic arc that it may virtually be considered devoid of this layer. The second layer, then, is in this region constituted by a stratum of small pyramidal nerve-cells as well as a few scattered angular cells. In the size of these elements, in their mode of distribution, in the depth of the stratum and its relationship to subjacent layers a very close resemblance is borne to the upper half of the third layer of higher animals. In the pyramidal cells of the human cortex a notable increase in their dimensions occurs with their depth so that at the lowest level of this series in Man I find cells whose proportions, though not usually so great as the cells of the still lower ganglionic layer, still render them giants beside their pigmy representatives higher up. This increase in size is *gradually* attained.† Now this feature is wholly absent in the small

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* *Op. cit.*, p. 62.

† "The Cortical Lamination of the Motor Area of the Brain." Proceedings Roy. Soc., No. 185, 1878.

pyramidal layer of the Rabbit—the cells remain small to their lowest level. In form, these cells are usually pyramidal or pyriform, whilst the small irregular, angular cell is met with chiefly on the outskirts of this layer where it joins the outer cortical zone. The apex process of these cells divides and subdivides into a delicate network of fibrils, those which are deeper seated undergoing the primary division at some distance from the cell, those nearer the cortical zone undergoing instant division, so that instead of a single elongated apex process the apex of the cell itself is bifid or horned. A similar feature pertains to the subjacent layer, the apex process of which may be traced often into the outer cortical zone ere primary division occurs. It therefore appears probable that whilst the greater bulk of nerve-fibre element of the lower four or five layers is formed by the numerous secondary processes of the nerve-cell,* the outer cortical zone derives its nervous element chiefly from the apex process of cells of underlying strata. The fact that the bifurcate or horned cell is met with at the commencement of this layer in sparse detached clumps has been alluded to above as significant probably of a rudimentary form of the second layer of higher animals. Now it has been shown that this latter stratum is also extremely defective and in great part entirely absent in the cortex of the Sheep; in the Pig it is also but a shallow belt averaging ·093 mm. deep, increasing in certain regions to ·139 mm., or even ·186 mm., whilst in Man its greatest development is fully ·279 mm. in depth. It appears clear, therefore, that the development of this belt bears an important relation to the position of the animal in the scale of organisation. The average size of these small pyramids in the Rabbit is 13μ long by 9μ broad, with a nucleus of 8μ in length—the largest do not exceed $17\mu \times 12\mu$. This closely approximates to the upper series of the third layer in Man, which in the ascending frontal gyrus measure $12\mu \times 8\mu$, and in the ascending parietal gyrus $15\mu \times 11\mu$.

This second layer attains its greatest depth in that portion of the upper limbic arc lying in front of the corpus callosum, gradually diminishing in depth backwards, so that behind the corpus callosum it is but ·372 mm. as contrasted with ·883 mm., the depth of the anterior region. In the direction of the extra-limbic mass, *i.e.*, over the exposed aspect of the hemisphere, the depth of this layer is often increased to 1·023 mm., being double the depth in the limbic lobe on a corresponding plane.

3. *Ganglionic layer.*—Immediately beneath the small pyramids of the second layer at the frontal end of this upper arc we find the far larger cells of the ganglionic layer. These cells, whilst differing widely from those of the ganglionic layer in Man, approximate to them sufficiently in many features to justify us in regarding them as representatives of the same series. Thus they are the largest and most characteristic cells in the cortex; they lie immediately upon the spindle layer, with a slight pale poorly-celled zone extending above and below it as in Man; their processes are peculiarly elongated, extending right through the pyramidal series into the outer

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* By secondary processes are meant all extensions from the cell exclusive of the apex and basal or axis cylinder process.

cortical zone ; and lastly, with the increase from a four to a five-laminated cortex by the addition of a small angular layer, the latter lies immediately between the pyramidal layer and this stratum of large cells, which I therefore propose to consider as identical with the ganglionic series in Man. This intercalation of an angular layer between them and the pyramids suffices to distinguish them from the large pyramids of the lower half of the third layer in human brain. Another character common to this series and that of Man and higher animals is their distribution in confluent clusters, as a deep stratum on the one hand, or on the other, as the solitary or linear arrangement.* The confluent and linear arrangement of these elements is as marked a feature here as it is in the Pig and Sheep. It has been noted that the great development of the ganglionic series in Man was found associated with the five-laminated, and not the six-laminated, cortex, and was peculiarly distributed over the upper limbic arc in its anterior half—so in the Rabbit these cells vastly predominate in the four-laminated cortex, *i.e.*, before the intercalation of an angular layer occurs, and is richly distributed over the upper limbic arc and its immediate neighbourhood. To sum up the special features whereby I identify this layer with the common ganglionic series :—

a. The large size and characteristic form of the cells.
b. The histological structure of the stratum.
c. Peculiar distribution of apex processes.
d. Relationship to the angular layer.
e. Arrangement in dense clusters or solitary cells.
f. Special development in certain areas.

It now remains to consider how they differ histologically from the same series in higher animals, and this object will be best furthered by considering in detail the general structure of the ganglionic layer as regards—1, depth ; 2, form of cell ; 3, size of cell.

1. *Depth of ganglionic layer.*—Great variations in depth occur throughout this layer, corresponding to a more or less rich development of cells. Upon examining sections taken vertically through the hemisphere, as at B (Plate 49, fig. 3), one cannot fail to have the attention arrested by the rich development of these cells in the upper limbic arc, and their rapid diminution and thinning out in the outer regions of the extra limbic mass. Referring to our figure of the upper limbic arc in the Rabbit (Plate 49, fig. 3), let us briefly examine the depth of this layer along the length of this arc, as also along the upper angle where it unites with the extra limbic portion. The depth attained is as follows :—

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* "The Comparative Structure of the Cortex Cerebri." BEVAN LEWIS. Phil. Trans., Part I., 1880, page 38.

	Limbic lobe. mm.	Union with extra-limbic. mm.
At A. (Plate 49, fig. 3) . .	·376–·418	·743
B. „ . .	·511	·743
C. „ . .	·232	·511
D. „ . .	·232	·372
E. „ . .	·279	·376

Hence its depth is greatest at A and B, increasing upwards and outwards towards the extra-limbic portion, and thus forming a specially rich formation in Ferrier's region (7).* When these measurements are contrased with those of the same stratum in the Pig,† a great similarity is at once perceived. As before shown, the depth varies with the two kinds of arrangement observed by these cells, viz.: the clustered or the solitary. Thus in the Pig the clustered cells attained an average depth of ·595 mm., whilst the dense confluent series in the Rabbit average ·506 mm. The depth, however, affords but an imperfect guide to the relative richness of the stratum in cells—a better notion is gained by the fact that the quarter-inch field will reveal in the confluent series as many as 90 to 100 cells, whilst the laminar series will give to the same power not over 6 or 8 cells.

2. *Form of cell.*—The cell of this series is usually an elongate pyramid, although the oval and large fusiform cell is occasionally met with, as is also a bifurcate or horned cell; the latter, however, far less common than in the Sheep and Pig. It was stated as regards the form of these cells in the Pig, a striking uniformity in their contour was observable, "by far the greater proportion taking the form of an elongate pyramid, the few exceptions occurring being usually gigantic spindles." ‡ In this respect the ganglionic cells in the Rabbit's brain agree with those of the Pig and Sheep in not exhibiting the great irregularity in marginal conformation, which is a peculiar character of the same series of cells in Man. The group of ganglionic cells from the cortex of the Pig, figured in my last memoir, represents the typical form here.§

3. *Size of cell.*—The dimensions attained by these cells along this region from before backwards have a fair uniformity. The largest cells are found at C and D (Plate 49, fig. 3), where they measure 32μ long by 18μ broad, having a nucleus 13μ in length. Anterior to this they measure $24\mu \times 17\mu$, whilst posterior to it they diminish gradually in size to $26\mu \times 16\mu$ behind the corpus callosum. With the exception of these slight variations I may state that throughout the upper limbic arc *great uniformity in the size of the structural elements of this layer is maintained.* When examining the structure of the parietal or extra limbic portion of the hemisphere we shall find still larger cells,

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* Vide fig. 36, area 7, in brain of Rabbit. "The Functions of the Brain."

† *Op. cit.*, p. 62.

‡ *Op. cit.*, plate 7.

§ *Op. cit.*, page 43.

although they do not approach the dimensions of the same elements in the Pig. Thus in the upper limbic arc of the Pig the larger cells averaged $48\mu \times 17\mu$, the largest in the Rabbit being $32\mu \times 18\mu$, measurements suggestive not only of the greater size of these cells in the Pig, but of their far more *elongate* contour. The typically elongate cell however occurred in the Sheep, as exhibited by their measurements of $46\mu \times 11\mu$.

The above considerations lead us to conclude that whilst these cells undoubtedly represent the ganglionic series of Man and the higher Mammalia, yet in size, contour, and structural type these cells in the Rabbit lose the strong distinctive features which characterise the ganglion cells of Man, whilst on the other hand they acquire structural affinities to the *larger pyramids* in the human cortex approximating as closely to them as was the case in the Sheep and Pig; in short, the structural type of the ganglionic cell is modified, being less complex.

Regional distribution (Plate 49, fig. 3).—In describing the cortex of the Pig and Sheep it was shown that this ganglionic layer assumed the form of a nested or clustered and a laminar or linear arrangement of its cells, but that in the former case the clusters were either far apart or discrete, or tended to become confluent. In the Rabbit the true nested arrangement is not to be found, as the clusters all tend to become confluent. The characteristic formation therefore over the upper limbic arc is that of a deep and dense layer, the cells uniformly distributed throughout, and in this plan again they more closely resemble the large pyramidal cells of Man than the subjacent layer. In the portion of the upper limbic arc which bends down in front of the corpus callosum (Plate 49, fig. 3, A and B), there is a most notable development of this layer. The ganglionic belt is here deep, and consists of large cells densely congregated and commencing abruptly beneath the small pyramidal layer. Its depth and richness in cells increases upwards towards the *exposed aspect* of the hemisphere, the quarter-inch field usually showing 60 to 80 cells, so that the margin of the longitudinal fissure here has a most exceptionally rich development of this layer. This rich formation is continued outwards *over the whole of the areas numbered* 7 *and* 9 *by* FERRIER,[*] and terminates only at the limbic fissure. As in the corresponding series in higher animals, the deepest portion of this layer is pale, having very few small cells, although at wide intervals apart are found some of the largest size ganglionic cells. Again referring to the diagram, we find from C to D the decreasing depth of the layer shows a corresponding diminution in the richness of its cells, which, however, are still numerous as far as D, and increase in number toward the outer or exposed aspect of the hemisphere. Beyond the sagittal margin, however, we find that from being closely congregated, the poorest development of cells yet met with occurs, as they rapidly thin out into an almost linear series down the outer aspect of the hemisphere as far as the great limbic fissure. Betwixt them and the small pyramidal layer I met with a belt of small angular cells, so that the cortex becomes five-laminated. Beyond D, the layer becomes rapidly shallower and the cells distant and sparse, 18 to 20 cells alone being seen in the quarter-inch field in striking contrast to their abundance in the areas A and B.

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[*] "Functions of the Brain," fig. 36.

Continuing to diminish in depth and in the number of its cells, this layer becomes a very insignificant tract beyond E, being, however, always most richly developed along the sagittal angle of the hemisphere. The whole exposed or extra-limbic aspect of the hemisphere, from D to E, is distinctly five-laminated, and a typical linear arrangement of cells is maintained in the ganglionic layer—the cells being placed at wide distances apart, and all on the same plane or at a uniform depth from the surface of the cortex.

Modified Upper Limbic Type (Plate 50, fig. 4).*

It has been already inferred that the structural type of the *posterior extremity* of both upper and lower limbic arcs exhibit very important modifications. The change, as far as it affects the upper limbic arc, is as follows:—On the outer aspect of the hemisphere a shallow sulcus is seen (Plate 49, fig. 1, K) disposed parallel to the great longitudinal fissure and mapping off from the extra-limbic mass outside it, a portion of the hemisphere which in the convoluted brains of higher animals corresponds to the superior parietal or sagittal convolutions.

Now this sulcus forms an *abrupt boundary* betwixt two forms of lamination; external to it lies the cortex typical of the larger mass of the extra-limbic lobe, internal to it is a cortex distinguished by a remarkable granule formation. It has been noted that as we examine the upper limbic cortex towards the posterior extremity of the callosal commissure an intercalated series of granule and angular elements becomes interposed betwixt the small pyramidal and ganglionic layers (vide Plate 50, fig. 4). Now, these small angular cells increase rapidly in number, and approaching the surface wholly replace the second or small pyramidal layer. In lieu of the latter, therefore, we eventually find here a deep belt of densely congregated small granule-like bodies, the granule-like character being due to the large size of their nucleus compared with their investing protoplasm. They are very closely crowded at the upper limits of the layer, but less so at deeper levels, taking a position in horizontal serried rank which is an especial feature of this formation—a peculiarity due to their separation into rows by the passage betwixt them of arciform fasciculi or bundles of medullated fibres running parallel to the surface of the cortex. In this region, therefore, the cortex is still four-laminated, consisting of—

1. Outer cortical zone (Plate 50, fig. 4, A).
2. Deep belt of granule-like cells (Plate 50, fig. 4, B).
3. Ganglionic belt (Plate 50, fig. 4, C).
4. Spindle-cell layer (Plate 50, fig. 4, D).

This formation is not limited to the upper limbic arc, but stretching over on to the exposed aspect of the hemisphere, extends to the shallow parietal sulcus above described (Plate 49, fig. 1, K). If we follow this formation backwards it is found to occupy the whole remaining median aspect of the hemisphere as far back as the limbic sulcus.* From this point downwards its distribution is more and more restricted to the inner moiety of the limbic lobe until it finally dwindles into an insignificant tract to disappear at the posterior extremity of the gyrus hippocampi. The gradually narrowing tract as it comes downwards and inwards leaves an angular portion of cortex betwixt it and the limbic sulcus, whose apex corresponds to the extremity of the latter sulcus (Plate 49, fig. 1, C). In this region is included the formation alluded to as the modified lamination of the inferior limbic arc. Vertical sections through the hemisphere behind the callosal commissure when stained by aniline black exhibit to the naked eye a remarkably defined dark belt corresponding to the granule belt just described (Plate 50, fig. 4, B). This belt can be traced outwards through the whole limbic arc and is seen to *terminate abruptly at the parietal sulcus.* On the other hand, fresh unstained vertical sections exhibit clearly to hand-magnifiers two delicate white stripes, one lying in the lower half of the first layer and the other lying within the granule belt of the second layer. These are found invariably in this region of the upper limbic arc (modified limbic type), but disappear as they bend over the marginal or sagittal angle to reach the exposed aspect of the hemisphere. They are constituted by two stripes of medullated fibres and will receive further attention when we come to describe the central projections of the olfactory organ. The depth of the individual layers of this modified upper limbic type immediately behind the corpus callosum was as follows :—

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* The area covered by this formation is shaded by dots in Plate 49, figs. 1 and 3.

1. Peripheral cortical zone, ·325 mm. to ·372 mm.
2. Granule layer, ·372 mm.
3. Ganglionic layer, ·604 mm.
4. Spindle layer, ·604 mm. to ·744 mm.

The granule layer where deepest does not exceed ·418 mm., nor does this formation in any region attain the depth of the small pyramidal layer which it displaces and which latter usually measures ·604 to ·833 mm. deep. The substitution of this densely crowded, though shallow, layer for the deeper pyramidal series, together with the *horizontal* connexions rather than vertical, established by means of the various parallel arciform medullated belts, accounts for the notable shallowness of the whole cortex posteriorly as compared with anterior realms. Thus near the occipital pole the depth of the cortex is 1·737 mm., whilst in the neighbourhood of the frontal pole it is 2·577 mm. The granule cell is very small and averages $10\mu \times 8\mu$, having a nucleus measuring 4μ to 5μ. A large number, however, scarcely reach the dimensions of $9\mu \times 9\mu$. In form they are conical, the nucleus constituting their chief bulk, whilst the very limited and delicate protoplasm extends irregularly into several processes, the most notable being usually, although by no means universally, apical.

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* Represented by dotted area on medium aspect of brain. Plate 49, fig. 1.

For a short distance internal to the primary parietal sulcus the upper stratum still contains a limited number of small pyramidal cells freely interspersed with these granule-like bodies, and giving this upper boundary a more densely packed appearance; but the lower portions of this second layer are constituted purely of granule cells. The latter resemble in every respect the intercalated belt of angular cells of the five-laminated realms of this animal, and, except in size, the smaller granules of the inner olfactory type and those distributed in the deep medullated structures of the olfactory area.

To sum up the chief features of the modified upper limbic cortex—

1. It is a four-laminated cortex with a dense belt of granules for its second layer.
2. The granule cell formation ends abruptly at the parietal sulcus.
3. The upper layer possesses a deep arciform medullated band.
4. The granule cells are disposed in serried rank by a similar arciform stripe of medulla.

The lower and anterior limbic arcs which next claim our attention comprise three distinct types of cortex—the outer olfactory, the inner olfactory, and the posterior or modified lower limbic.

Outer Olfactory Type (Plate 50, fig. 5).*

Vertical sections carried through the hemisphere, which pass through the septum lucidum, present us with four well-marked because structurally differentiated regions. These are, on the inner side or median aspect, the upper limbic arc; on the upper and outer aspect of the hemisphere the extra-limbic cortex, extending down as far as the limbic sulcus; beyond the sulcus, and occupying the basal aspect of the hemisphere, the outer olfactory portion of the lower limbic arc extends, and internal to this is a region continuous with the basal portion or head of the corpus striatum—the inner olfactory cortex.† The five-laminated cortex extending over the greater portion of the extra-limbic segment terminates abruptly at the limbic sulcus (Plate 49, fig. 1, C), immediately beneath which a wholly distinct formation commences, that of the outer olfactory cortex. This cortex is constituted by three layers, possessing but two belts of nerve-cells:—

1. A peripheral cortical zone (Plate 50, fig. 5, A).
2. A densely-compressed but shallow layer of small and very irregular pyramidal cells (Plate 50, fig. 5, B).
3. A scanty series of large pyramidal cells (Plate 50, fig. 5, C).

The outer peripheral zone has superimposed upon it, and also imbedded in its structure, the medullated fibres of the superficial olfactory fasciculus, which is seen in vertical sections as an oval or oblong belt, its fibres cut across transversely, and forming a notable white prominence upon the exposed surface, whilst it is also deeply imbedded so as apparently to rest in a slight sulcus of this cortical layer. The second layer of nerve-cells bends inwards in an arched form beneath it. The fibres of this superficial olfactory fasciculus (Plate 49, fig. 2, M) may be traced not only backwards towards the gyrus hippocampi, but spreading downwards into the olfactory cortex and arching outwards over its whole extent. The apex processes of the nerve-cells of the second and third layers blend with this fasciculus, whose exact connexions will be considered further on. The depth attained by this outer cortical zone varies between ·186 mm. and ·232 mm. In intimate structure it is fundamentally a supporting matrix of neuroglia, as in other regions, the nervous elements imbedded in it consisting of a dense meshwork formed by the branching of the apex processes of the nerve-cells beneath (Plate 50, fig. 5, B), and the ramifications of medullated fibres from the superficial olfactory fasciculus which are found at all depths of this layer beneath the surface. Immediately beneath the peripheral zone lies the layer of small pyramidal cells, forming but a shallow belt of densely-crowded elements, very irregular in contour, but usually pyramidal, oval, or fusiform, their apex processes bifurcating early and approaching in character very closely the small pyramidal cells of the second layer in other regions. These cells have a peculiar arrangement in clumps or appressed groups, especially in close proximity to the superficial olfactory tract. The nerve-cells vary between $18\mu \times 10\mu$ and $19\mu \times 12\mu$ in size, and possess a nucleus whose diameter is 9μ. Scattered sparsely among these smaller cells are pyramidal cells of greater dimensions, which become more frequent below the first belt of nerve-cells, forming here the third layer, through which a few really large pyramids are scattered, often measuring $31\mu \times 13\mu$, with a nucleus 13μ in diameter. The depth of the second layer is ·139 mm. to ·186 mm., that of the third or large pyramidal layer, varying from ·106 mm. (olfactory region) to ·568 mm. (gyri hippocampi). Tracing the passage of the extra-limbic formation into that of the olfactory region at the limbic sulcus, we find the layers of nerve-cells assume the laminated aspect peculiar to a sulcus. Thus the cortex becomes much shallower by the rapidly decreasing depth of the second or small pyramidal layer, and the close approach towards the latter of the ganglionic series. The second layer at the inner side of the sulcus becomes shallower and more compressed; the larger pyramidal cells become sparse, form a less distinct layer, and are wholly absent from the innermost portion of the olfactory region, so that a clear space here intervenes betwixt the small pyramidal belt and the claustral formation beneath. Thus the second layer of the lower limbic arc is directly continuous with the small pyramidal layer of the cortex of the vault, whilst the large pyramids of its third layer are continuous with the ganglionic series of cells. The claustrum is seen beneath these layers descending from the parietal region as a deep stratum of spindle cells from ·511 mm. to 1·116 mm. in depth (Plate 50, fig. 5, D). Arching beneath the limbic sulcus, and separated by medullated fasciculi from the corpus striatum, they appear to terminate superficially near the inner border of the superficial olfactory band. Here, as will be mentioned further on, they seem to communicate with a superficial arciform system. The axis of these cells is arranged parallel to the surface of the cortex.

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* Area covered by this type of cortex is shown in Plate 49, fig. 1, by cross-hatching at base.

† The two lower of these regions are shown in diagram (Plate 49, fig. 8).

Gyrus hippocampi (Plate 49, fig. 1, B). The structure of the cortex of this region is in most respects identical with that of the anterior segment of the limbic arc just described. The layers are three in number and alike in constitution, but the development of the third or large pyramidal layer is a more marked feature here. Whereas the second or small pyramidal layer was the only fully developed formation in the outer olfactory cortex, in the gyrus hippocampi the third or large pyramidal cells form a well differentiated layer, increasing in size and number towards the posterior extremity of this gyrus. Here they are identical in appearance with the pyramidal cells of the third cortical layer of human brain, showing in vertical sections six or seven branches and a long apical process. The depth of this layer also increases from before backwards.

*Inner Olfactory Type.**

Near the inner margin of the superficial olfactory fasciculus we find the commencement of a thin and peculiar cortex, which spreads over the anterior perforated space to the inner margin of the hemisphere, where, with a slight bend upon itself, it becomes continuous with the septum lucidum (Plate 49, figs. 1, 2, P). This cortex covers an area the relationship of which will be more particularly described further on, and which we shall denominate, after GRATIOLET, the olfactory area, or in contradistinction to the remaining cortex extending from it to the limbic sulcus—the inner olfactory area. The coarse or naked-eye appearances of this cortex are peculiar, as is also its histological constitution. Vertical sections carried through the hemisphere in this region will exhibit a wavy disposition of the second layer, which assumes a folded or duplicate arrangement wholly peculiar to this realm of the brain. In these V or W-shaped duplicatures the superficial layer of the cortex does not participate, so that at one time the second layer is close to the surface and at other times quite remote. The layers constituting the cortex here are three :—

1. Superficial or peripheral zone.
2. Layer of granule cells.
3. Layer of spindle cells.

1. *Peripheral zone.*—The first layer has an average depth of ·232 mm., but from the duplicatures of the subjacent layer its depth will, of course, be subject to great variation. In structure it is similar to the same layer in other regions of the cortex, differing, however, in being perforated by numerous large vessels, which pass through

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* This region is seen in diagram (Plate 49, fig. 8), also by black area at base, Plate 49, fig. 1.

the cortex in straight or arched direction, within perivascular channels, which in the fresh state attain a diameter of ·110 mm. In the deeper region of this peripheral zone I find a shallow belt of arciform medullated fibres, which originating near the inner margin of the peripheral olfactory fasciculus (apparently continuous with the claustrum) is here continued inwards over the whole of the inner olfactory area.

2. *Granule cell layer.*—These cells forming the involuted layer characterising this region attain the average dimensions of 9μ long by 6μ broad, and contain a large oval or spheroidal nucleus 6μ in longest diameter. Amongst the cells of this layer we also find an abundance of very minute granules scarcely attaining a diameter of 5μ. These latter resemble the granules of the modified upper limbic cortex, but are smaller in size than these. They are common to the cortex here and to the peculiar medullated region which extends as far as the central olfactory fasciculus and anterior commissure, and may be regarded as specially the olfactory region of the nucleus caudatus. The depth of this region amounts to 3·27 mm., whilst the granule cell formation or second layer is ·116 mm. deep.

3. *Layer of spindle cells.*—Immediately beneath the wavy layer of small angular and granule cells lies a formation of fusiform cells, the elements of which attain a notable magnitude. Whilst the majority of these cells measure $32\mu \times 13\mu$, we frequently meet with far larger elements attaining the dimensions of $60\mu \times 9\mu$, with an oval nucleus 13μ in longest diameter. These spindle cells have their long axes parallel to the surface of the cortex, and lie along the course of similarly disposed medullated fibres which bend inwards to continue beneath the same layer of the septum lucidum. This cortical stratum is therefore essentially an arcuate fasciculus of medullated fibres connected with unusually large spindle cells, which bring the cortex of the inner olfactory area in connexion with that of the septum lucidum and upper limbic arc. Beneath the cortical layers above described are the densely aggregated olfactory fasciculi, which traverse the deep structure of the olfactory area, and from which vertical prolongations descend to the deeper layers of the cortex, probably fusing with the arciform medulla.*

Modified Lower Limbic Type (Plate 50, fig. 6).†

In the anterior portions of the lower limbic arc and the olfactory lobe, it was stated that the cortex was chiefly characterised by the development of the second layer as closely appressed clusters of small pyramidal cells, and with the exception of a scanty formation of large pyramids, no other nerve-cells are present. The further back we examine the lower limbic arc the greater is the development of the larger pyramidal cells, until eventually we reach a region where the large cells are much more densely grouped. Here, however, a remarkable change has occurred in the second layer, the cells of which are no longer pyramidal but swollen, inflated, globose or flask-shaped (Plate 50, fig. 6, B). They are also greatly increased in size, measuring $37\mu \times 32\mu$, or $37\mu \times 23\mu$; whilst others are more elongated, measuring $46\mu \times 27\mu$. Their nucleus has a diameter of 13μ. Their dimensions are thus *more than double* those attained by the elements of the second layer in other regions of the brain, whilst their irregular mode of branching as a bifurcate cell gives this formation the aspect of an unusually rich development of the ordinary second layer. Scattered amongst these larger cells, and descending as a distinct layer, are small pyramidal cells measuring but $23\mu \times 13\mu$, so that in this region *the series of larger cells is placed superficial to the smaller elements.* As regards either pyramidal series in other regions of the cortex, this latter feature is never observed—the upper series being invariably composed of smaller cells. Immediately beneath this third layer of pyramidal cells is a clear white belt devoid of nerve cells, and to this succeeds a belt of spindle cells (Plate 50, fig. 6, D, E). The cortex of this modified olfactory type is therefore composed of five layers :—

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* Diagramatically represented in Plate 49, fig. 8, by *f*.

† Area covered by this form of cortex is the light shaded portion T to W in figs. 1 and 3.

1. Peripheral cortical zone (depth ·418 mm.).
2. Layer of large inflated cells (depth ·279 mm.).
3. Small pyramidal cells (depth ·372 mm.).
4. Pale belt devoid of nerve cells (depth ·184 mm.).
5. Spindle series (depth ·372 mm.).

The region embraced by this formation is that portion of cortex at the extreme posterior extremity of the lower limbic arc where the upper limbic arc unites with the gyrus hippocampi (light-shaded area from T–W, Plate 49, fig. 1). It extends as far as the upper extremity of the limbic fissure seen on the median posterior aspect of the hemisphere, being bounded externally by the limbic sulcus (C) and internally by the gradually vanishing granule formation of the modified upper limbic cortex (dotted area, Plate 49, fig. 1). Sections carried *horizontally* through the occiput at this site will therefore exhibit from without inwards four types of lamination :—

1. Five-laminated type of extra-limbic cortex.
2. Modified olfactory type.
3. Modified upper limbic type.
4. Type of cornu ammonis.

Independent of the peculiar arrangement of cells and disposition of the layers in this region, a characteristic feature is acquired in the presence of two arciform stripes of medullated fibres, disposed as in the modified upper limbic cortex, one above the second layer and the other immediately beneath the same. They will be more minutely described in the section on the central projections of the olfactory lobe. For the present it will suffice to bear in mind the important fact that both these modified occipital regions of the great limbic lobe are characterised by the *double stripe of intra-cortical arciform medulla,* and by the *dense granule belt* of the one, as contrasted with the *great inflated cell formation* of the second layer in the other.

Extra-limbic mass.—Having now completed the examination of the great limbic lobe, I have next to consider the greater portion of the hemisphere which lies external to the constricting limbic arc, or that portion which in higher animals enters into the formation of a frontal, parietal, and tempo-spheroidal lobe. As in the detailed account of the various layers of the cortex in the upper four-laminated limbic arc I have anticipated many points bearing upon the structure of this extra-limbic region, it appears only necessary here to allude to the general points of divergence or similarity borne to other regions, and not to consider each individual layer minutely. The cortex, then, of this region* is five-laminated, consisting of similar layers of nerve-cells to those found in the upper limbic arc along with the interposition of a layer of small angular cells. The relative position of layers is as follows :—

1. Peripheral cortical zone.
2. Small pyramidal layer.
3. Belt of granule or angular cells.
4. Ganglionic series.
5. Spindle cells.

As regards diversity of cell formations, therefore, this is by far the richest region examined, and but a cursory glance at sections from near the occipital and frontal poles, as well as midway betwixt these points, will satisfy us that though this richly-laminated cortex has a most extensive distribution the varied layers differ much in their relative development in these different regions. Thus the small pyramidal layer is uniformly rich and deep, whilst the ganglionic series is especially developed in the anterior and median regions, and the granule and angular cells predominate in the posterior realms. Directing attention to the variations which occur in *mid-cortical regions* from within outwards, *i.e.*, from the great longitudinal fissure outwards and downwards to the lower boundary of this region, the limbic fissure, I find first, that the outer cortical zone, or first layer, *diminishes in depth*, attaining its greatest shallowness near the limbic fissure ; next, the small pyramidal layer *increases in depth and richness of cells* in the same direction ; thirdly, the ganglionic series tend to become distributed, after the laminar or solitary type, as we approach towards this region ;† whilst in the upper realms, near the median fissure, they retain the dense confluent aspect and depth of stratum which is likewise seen in the upper limbic arc. It will likewise be observed that the intercalated layer of angular elements commencing in the posterior half of the upper limbic arc extends over the far greater area of the extra-limbic mass, and progressively increases in richness of cells and distinctness of lamination outwards and especially backwards towards the occipital pole. As already stated, this type of lamination terminates abruptly at the limbic sulcus, the small pyramidal and ganglionic series alone being continued beyond into the lower limbic arc.

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[*] The five-laminated cortex of this region is the unshaded area at summit and sides, Plate 49, fig. 1.

[†] The solitary arrangement is seen well in section from sulcus of limbic lobe of Pig, *op. cit.*, plate 7.

Again, if our attention be directed to the variations observed from before backwards, *i.e.*, from frontal to occipital pole, over the extra-limbic realm, we first observe a progressive decrease in the depth of the outer cortical zone in the latter direction, with a decrease in size of the small pyramidal cells of the second layer. With this there is also associated a progressive increase of the granule or angular series which attains its greatest development posteriorly. On the other hand, the ganglionic series, from being rich in cells and deep as a stratum, at the frontal pole undergoes a progressive impoverishment along the regions midway betwixt the great longitudinal fissure and the limbic fissure, thinning out gradually to a laminar formation, whilst towards the extreme occipital cortex the ganglionic belt again becomes deep, with a layer of granules superimposed. From these particulars it may be gleaned that—

a. The cortex near the frontal pole in the extra-limbic realm approaches the type of the four-laminated upper limbic cortex.

b. The cortex midway betwixt frontal and occipital pole is a typical five-laminated formation, and represents, as does the six-laminated cortex of Man, the *common type*, because more extensively distributed.

c. The cortex of the extra-limbic realm at the occipital pole is a still further modified form of the five-laminated, but is by no means a distinct *kind* of cortex.

A notable diminution in size of the small pyramidal cells of the second layer occurs from the frontal to the occipital pole of the extra-limbic realm. Thus in front of the corpus callosum they average $22\mu \times 12\mu$, with a nucleus of 9μ, and progressively decrease in size until, on a level with the primary parietal sulcus, they measure $13\mu \times 11\mu$ (nucleus 8μ). Towards the occipital pole they again increase to $20\mu \times 11\mu$. The same features as regards size of nerve-cell were observed along the upper limbic arc ($17\mu \times 12\mu$ to $13\mu \times 9\mu$). On the other hand, the second layer in the *anterior and lower limbic arcs*, it will be remembered, increase in size towards the occipital pole, wholly lose their pyramidal contour, and from measuring $19\mu \times 12\mu$ become globose, and measure $37\mu \times 32\mu$.

It must be borne in mind that whilst the extra-limbic cortex appears to be developed out of the four-laminated cortex by the intercalation of a belt of granule cells, the modified upper limbic cortex is characterised by the granule cell-formation actually replacing the second layer of small pyramids, which is here either very defective or wholly absent.

Distribution of the Ganglionic Series.

Proceeding upon the same line of inquiry which was adopted in my former article upon the brain of the Pig and Sheep, it remains for me now to map out the area over which the ganglionic series is spread, and more especially that division of the series characterised by a *rich confluent and deep belt of cells* co-extensive with a four-laminated cortex. Such a formation extends over the anterior half of the upper limbic arc, and stretches upwards and outwards over the exposed aspect of the hemisphere. By far the richest formation lies in the latter position—viz. : on the exposed aspect of the vault, in a lengthened strip of territory lying parallel with the great longitudinal fissure, and represented by the shaded area in the diagram.* It extends forwards well into the frontal extremity, covers the whole outer aspect of the hemisphere here, and becoming much poorer in cells, spreads along the lower confines of the extra-limbic mass, skirting the limbic fissure as far as, and even behind, the slight depression which in this animal represents the Sylvian fissure. Between these two tracts the enclosed area is spanned by a *five-laminated cortex* whose ganglionic series is poor, and distributed in the linear or solitary arrangement. The region which may therefore be considered, *par excellence,* as that of a rich ganglionic formation, is the narrow strip of cortex along the sagittal margin of the hemisphere from A to D (Plate 49, fig. 3), extending over the outer aspect of the hemisphere betwixt A and B as far down as the limbic fissure, and next to this the upper limbic arc. The far less characteristic formation adjacent to the limbic fissure, however, is interesting chiefly from the fact that it foreshadows a richer structure, which appears along this course in higher animals. It will be convenient to name these formations the sagittal and Sylvian formations of large ganglionic cells. Thus it was shown that the five-laminated cortex, with its clustered cells, in the Pig not only extends over the anterior limbic arc and frontal lobe, but bends back over the first and second parietal gyri "from their union with the ascending parietal (post-Rolandique, BROCA) towards and around the Sylvian fissure."† In the Pig, therefore, as also in the Sheep, we have demonstrated the distribution of the rich ganglionic layer over the regions bordering upon the limbic fissure. Just as in these and other allied animals we have betwixt the two regions of the clustered ganglionic cells an intermediate district characterised by a laminar arrangement of this series and an intercalated series of angular elements, so in the Rabbit a similar cortex intervenes betwixt the inner and outer series of rich ganglionic areas. We thus conclude that the fundamental type of the structure of the cortex over the extra-limbic and upper limbic realms is identical in these various animals. The dense layer of ganglionic cells bordering upon the great longitudinal fissure maintain a fair uniformity of size throughout. A large number of measurements give the following average dimensions :—

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* Area shaded by small crosses, Plate 49, fig. 1.

† See fig. 3, p. 44, "On the Comparative Structure of the Cortex Cerebri." Phil. Trans., Part. I., 1880.

Plane of Section.	Length.	Width.	Nucleus.
	μ	μ	μ
Anterior to corpus callosum	32	16	13
On a plane with Sylvian depression	34	16	13
Posterior to Sylvian depression	34	17	13
Posterior border of corpus callosum	34	16	12
Through superior parietal sulcus	24	18	12
Posterior to latter	31	19	13

These nerve-cells are therefore larger than those of the corresponding series in the upper limbic arc (vide table of measurements), also larger than those of the lower limbic arc, which average for the olfactory gyrus $25\mu \times 13\mu$; for the gyrus hippocampi $31\mu \times 13\mu$; and for the modified olfactory type near the occipital pole $23\mu \times 13\mu$. Larger cells occur occasionally, scattered widely apart, the largest registered attaining the dimensions of $60\mu \times 18\mu$. If these average dimensions be contrasted with those of the *second cortical layer*, in the region of the modified olfactory type (Plate 50, fig. 6, B), the important fact is disclosed that this peculiar formation consists of cells which attain a much higher average than that of the ganglionic cells. These globose inflated cells usually average $37\mu \times 32\mu$, and frequently reach the dimensions of $46\mu \times 27\mu$. These are therefore the largest nervous elements to be found throughout the cortex cerebri. The lower limb of the rich ganglionic tract extending from the frontal pole along the limbic fissure beyond the Sylvian depression, possesses cells of a smaller size than those of the corresponding upper tract. Thus in front of the corpus callosum the nerve-cells near the limbic fissure measure $27\mu \times 17\mu$, as contrasted with $32\mu \times 16\mu$ for those of the upper tract; behind the Sylvian depression those of the lower tract measure $27\mu \times 17\mu$, whilst those of the upper tract measure $34\mu \times 17\mu$.

The rich aggregation of cells in the ganglionic layer along the median exposed portion of the vault results in a deep stratum, which near the frontal extremity measures ·558 mm., and posteriorly and internal to the superior parietal sulcus ·604 mm. If this be contrasted with the pale, solitary belt in five-laminated realms, measuring but ·186 mm., its wealth of structure will be apparent. Another fact to be noted is that this layer of ganglionic cells is both deep (·604 mm.) and rich in cells in that area of the granule formation bounded externally by the primary parietal sulcus, and covers the median surface of this posterior region of the hemisphere as far back as the summit of the occipital pole. We must therefore conclude that although the ganglionic cells (next to those near the occipital pole) are the largest throughout the cortex cerebri, the distinctive feature relative to this formation is not the large size attained by the cells, as is the case in the highest of the Mammalian series—for they exhibit in the Rabbit, as in the Pig and Sheep, a remarkable uniformity throughout; but, on the other hand, this formation is especially notable for its richness in nerve-cells—a wealth demonstrated both in depth of stratum and close aggregation of its constituents.

The Cornu Ammonis (Plate 49, fig. 7).

Peripheral cortical zone.—In the cornu ammonis this stratum may be readily seen to consist of a superficial and deeper region, which by MEYNERT and others have been regarded (in the human brain) as two distinct laminæ, representative of the first and second cortical layers elsewhere. But whereas the second layer of superficial angular cells is not developed in the Rabbit, the lower portion of this zone can scarcely be regarded as representative of a series of cells which do not appear in the most complex regions of the cortex in this animal. These two divisions have been named the nuclear and lacunar layers—the latter also termed by KUPFFER the *stratum reticulare.* On close examination of this inner or reticulated structure it becomes apparent that it clearly corresponds to the deeper portion of the peripheral zone of other cortical regions. In diverse realms of the cerebral cortex it was stated that this peripheral zone in various animals exhibited two clearly distinct portions—the outer or more superficial being often distinguished by a layer of horizontally disposed medullated fibres following the various involutions of the cortex and parallel to the surface, and extending often to a depth of one-third or even one-half that of the first layer. Now this peripheral stratum appears as the representative of the nuclear layer of the cornu ammonis. The inner or deeper portion of this zone elsewhere was seen to be composed of a dense network, formed by the apex processes of the subjacent cells dividing and subdividing in their course upwards. This was found to be the case in the Cat, Sheep, Pig, and Rabbit. In the cornu ammonis the same network is apparent, but being here entirely constituted by the apex processes of much larger cells—the ganglionic cells—it is a coarser and far more prominent formation. The whole depth of this peripheral zone is on an average ·836 mm., the outer medullated layer being about ·511 mm. The nuclear lamina, or as I prefer to term it, the medullated division of the peripheral zone, includes the following constituent elements:—

a. Band of medullated fibres derived from re-union of meshwork of the divided apical processes of ganglionic cells.

b. Numerous large blood-vessels.

c. A neuroglia matrix, containing a profusion of the ordinary nucleated connective cell; also the DEITER'S corpuscle and perivascular nuclei.

d. Spindle-form cells.

Of these constituent elements the blood-vessels are large, and dip through the medullated portion from the investing pia mater into the deeper division of the peripheral zone. Here they divide into a series of anastomosing branches, which, with the meshwork of nerve processes, gives this region its characteristic reticulated aspect.

The vessels throughout this course exhibit wide perivascular channeling, and are bordered by the usual perivascular nuclei. The connective element is similar in every respect to that found in this layer in other parts of the brain—the larger cells, measuring 9μ across, are usually nearly globular, and have a nucleus 6μ in diameter. They are very numerous, and throw off on all sides delicate fibrillar processes to constitute the neuroglia framework. The DEITER's cell is found immediately beneath the vascular investment of the cortex. Scattered through the medullated division of this cortical zone, and *especially near the confines of the reticular stratum*, are the nerve cells alluded to. They are elongate spindles, measuring $34\mu \times 12\mu$, with an oval nucleus 12μ in diameter. These cells lie parallel to the medullated fibres, throwing off branches from either pole in the direction of these fibres, but also almost invariably exhibiting a lateral process, which dips down into the cortex in the direction of the reticulated layer. These spindle cells are not numerous, and, as stated by MEYNERT, may readily be overlooked. In connexion with their presence here, it is interesting to recall similar spindle cells of great dimensions which have been pointed out as existing in certain portions of this peripheral zone in the olfactory cortex.

Striate layer.—Immediately beneath the vascular and nervous plexus constituting the reticulated portion of the peripheral zone lies the striate layer, so named after the striated aspect given by the numerous apex processes of the ganglionic layer, which radiate towards their site of common union—the nerve plexus. The depth attained by this layer averages ·558 mm., but near the commencement of the infolding of the true cornu ammonis it is much shallower, the branching of the apex processes occurring nearer to the cell. Radiating in the same direction outwards, and lying side by side with these nerve processes are numerous vessels, which rise from the ependyma of the lateral ventricles, and after supplying the medulla of the cornu, unite with the vessels of the peripheral zone (from the pia mater) through the medium of its vascular plexus. The region of these apical radiations undoubtedly corresponds to the small pyramidal layer in other realms of the cortex of the Rabbit.

Ganglionic layer.—This layer is formed by cells quite characteristic in their distribution of the cornu ammonis formation. It forms a shallow stratum varying in depth from $\cdot 093\mu$ to $\cdot 139\mu$, the average depth being $\cdot 112\mu$. The cells vary in contour, being frequently oval, elongated, or fusiform, rarely pyramidal, and much more commonly of a swollen pegtop-like contour. At the commencement of the inrolling of the cortex to form the cornu the ganglionic cells may be clearly traced into this formation, first, as a scattered series or deep belt, in which the cells approximate closer and closer until they form a belt two to five deep, in which the cells are so closely applied to each other as to be in actual contact, and form an almost unbroken file through the greater portion of the true cornu ammonis; beyond the second turn of this sigmoid fold, however, they again become more widely separated and scattered so as to form a deeper belt. In hardened sections the shrinking separates the cells widely apart, and they may be observed surrounded by a clearly defined pericellular space. In fresh sections, on the other hand, their close approximation renders their outlines obscure except in extremely fine and carefully prepared sections. Near the commencement of the first involution where the cells are more widely scattered, these elements are observed to be encircled usually by a number of nucleated protoplasts, as are also the spindle cells, which latter have frequently such numbers heaped upon their surface as quite to conceal their real contour. These pericellular elements also tend to render the ganglionic cells of the true cornu ammonis obscure in outline. The more elongated or fusiform cell is prevalent in the lower end of the cornu ammonis or that portion bounded externally by the lower limbic arc. That portion, however, lying internal to the upper limbic arc has the characteristic pegtop-shaped cell throughout its whole extent. Each ganglionic cell besides its apex process, gives off several secondary processes from its basal extremity, and especially a primary branch from the base, which becomes an axis cylinder process. The secondary branches are directed towards the medulla and rapidly divide and subdivide within the subjacent formation (stratum moleculare). The axis cylinder process passes into the medulla of the alveus. The involuted cornu ammonis is most fully developed just posterior to the corpus callosum, and sections taken across the hemisphere at this site will therefore present us with the most extensive tracts of the ganglionic and other layers, the reticulated portion of the peripheral zone being also best studied here. The nearer we approach the inferior extremity of the cornu the more contracted is the area of section, and consequently the less extended are its nerve-cell layers, whilst the individual cells of the ganglionic series not only become more elongated in contour, but exhibit less and less of the striate formation from early subdivision of their apex processes. In the latter site, therefore, in lieu of the straight, elongated apex process of the cells becoming more and more attenuated and thus forming a forest-like structure of delicate twigs (striate layer) they commence as coarse contorted branches, which at the distance of ·046 mm. from the cell sub-divide into brush-like groups of filaments. The dimensions attained by these cells are an average length of 23μ and a width of 18μ, the nucleus measuring 13μ. At the commencement of its involution, however, where the cells are larger, more scattered, and elongated in contour, the dimensions are $37\mu \times 18\mu$ (nucleus 13μ), the apex process becoming bifurcate at from 30μ to 60μ from the cell. A few spindle cells or even pyramidal cells are found occasionally here and there at wide distances apart within the striate layer separated from the main body of cells.

Spindle-cell layer.—Beneath the ganglionic layer we come to a belt in which the neuroglia basis supports the fine fibrillar network formed by the secondary processes of the super-imposed cells. The presence of this nervous network accounts for the uniform faint staining which is here seen in aniline black preparations—a staining common to all parts of the grey cortex possessed of a network of *non-medullated* nerve fibrils and contrasted strongly with the perfectly white unstained layer of *medullated* fibres beneath this belt. Passing up from the ventricular ependyma covering the medulla of the cornu through this stratum numerous large blood-vessels are seen on their way to the capillary plexus beyond the striate region. These vessels in hardened preparations show to a marked degree the wide perivascular channels around them. The stratum which we above described has from its fine granular aspect been termed by KUPFFER *stratum moleculare.* It is wholly absent in Man, where, according to MEYNERT, the axis processes of the ganglionic cells unite immediately with the medulla. In the Rabbit, however, the spindle cell is well represented here, not only scattered widely apart through the *stratum moleculare,* but arranged in some numbers along line of union of this stratum with the deeper medulla. Numbers of spindle cells of large size are also met with at all depths of the deep medulla of the cornu lying parallel with the direction of the medullated fibres. This spindle cell medulla communicates directly with the spindle series of the limbic lobe at the commencement of the inrolling of the cornu, and lies parallel throughout its course with the involutions of this organ. Occasionally an unusually large fusiform cell will be met with immediately beneath the ganglionic series measuring as much as $60\mu \times 11\mu$, and thus rival in size the similarly large spindle cells met with beneath the cortex of the inner olfactory region along the course of the arciform fasciculi. These fusiform elements lie embedded in and continuous with the medulla of the fornix, and usually exhibit three or more processes, one from each pole of the cell following the course of the medulla and the others lateral in origin and smaller in diameter. One of the larger processes may often be traced directly up into the layer of ganglionic cells.

Significance of the Fissures and Sulci of the Brain.

The foregoing considerations as regards the intimate structure of the cerebral cortex in the Rat and Rabbit, taken together with the results of my former examination of the cortex in the Pig, Sheep, and the Cat, lead to important conclusions relative to the significance of the fissures and sulci of the brain.

Hitherto it has been customary to regard the convolutionary arrangement as dependent in a great measure upon the progressive growth and development of the cortex, restricted and modified by the encircling cranial bones, the varied folds and complex fissures being really in this sense mere accidents of development, their uniformity of distribution in different classes of animals being due to the uniform action of the forces thus brought into play. Our investigations would lead us to a far different conclusion, nor can we, judging from our histological data, regard the divisions mapped off from each other by fissures and primary sulci as other than structurally distinct organs. In no case is this so palpable as in the various regions of the great limbic lobe, for here both a superficial survey, *i.e.,* of the structure of the cortex and an investigation of the deep medullary connexions of these regions conclusively confirm the statement now advanced. In the former memoir* I insisted strongly upon the fact that certain sulci or fissures formed sharply-defined boundary lines betwixt regions differing in the structure of their cortex: these were the crucial and infra-parietal sulci, together with the fissure of ROLANDO. My later investigations enable me still further to extend this list by others, viz.: the limbic fissure, olfactory sulcus, and primary parietal sulcus. Thus in the Sheep and Pig we find that the crucial and sub-parietal sulci separates the five-laminated cortex, with its clustered ganglionic series, from the six-laminated cortex, with its solitary ganglion cells, the former lying in front and the latter behind these fissures. In like manner the inner extremity of the crucial sulcus formed the boundary betwixt the five-laminated cortex of the upper limbic arc anteriorly, and the peculiar granule formation of this arc behind the crucial sulcus, which extends as far as the retro-limbic annectant. In the Rat and Rabbit it is again observed that the limbic fissure, in separating the lower limbic arc from the parietal or extra-limbic mass, sharply defines two entirely distinct types of cortical lamination. Above the fissure lies the *five-laminated cortex* of the extra-limbic mass, with its intercalated series of granule cells; beneath it lies the *three-laminated cortex* of the outer olfactory region and gyrus hippocampi, whilst still further back there is found internal to it the remarkable modified olfactory cortex which has been described near the occipital pole. Within the great limbic lobe we find a sulcus, dividing two important and morphologically distinct regions. These regions are the outer and inner olfactory areas whose boundary line is the olfactory sulcus, wherein lies imbedded the superficial olfactory fasciculus. It was moreover shown that the superior parietal sulcus forms a sharp line of demarcation betwixt the five-laminated type of the parietal mass and the peculiar dense granule formation of the posterior or modified upper limbic cortex. In the higher animals, as was indicated in the former paper, the fissure of ROLANDO, here lying at the frontal pole, separates a five from a six-laminated cortex. We surely have here a most important fact, and one which, if followed up in further researches, will enable us to map out the convolutionary surface of the brain after a less arbitrary plan than the one usually adopted. The more fully I investigate the minute structure of the cortex and its deep connexions, the more forcibly am I impressed by the belief that the various fissures and sulci are not mere accidental productions, but have a deep significance of their own, dividing off the cortical superficies into *morphologically* if not *physiologically distinct organs*. Hitherto the fissures and sulci which I have found to be the boundary lines of distinct cortical realms are the following:—

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* *Op. cit.,* p. 45.

1. The limbic fissure.
2. The infra-parietal sulcus.
3. The crucial sulcus.
4. The superior parietal sulcus.
5. The inter-parietal sulcus.
6. The olfactory sulcus.
7. The fissure of ROLANDO.

In the lowest of the animals examined, the Rat and Rabbit, we find the *anterior lower limbic arc* already well developed, distinct from surrounding regions, and forming the far greater bulk of the great limbic lobe. We find the limbic fissure deep and complete as far back as the region where in higher animals the retro-limbic annectant unites the limbic to the occipital lobe. Within this fissure lie the three distinct cortical regions of the outer, inner, and modified olfactory types. On the other hand the *upper* limbic arc in these animals is far from being so definitely separated from the neighbouring regions of the extra-limbic mass. The anterior or four-laminated segment of this arc is, in fact, directly continuous with the exposed cortex of the vault, no fissure, sulcus, or depression however small intervening. In connexion with this fact it is, however, important to bear in mind the identical nature of the laminated type of this portion of the limbic arc and the exposed margin of the hemisphere which bounds the longitudinal fissure (sagittal). The modified limbic type of the posterior segment of this arc, however, is sharply defined from the five-laminated cortex of the extra-limbic mass by the primary parietal sulcus. In the Pig and Sheep, on the other hand, whilst the bulk and differentiation of the lower limbic arc is maintained, there is a decided advance in this respect evident in the upper limbic arc also. The anterior portion of this arc, from its size, actually overlaps so as to appear upon the upper surface of the hemisphere, whilst a most characteristic and deep sulcus, the *crucial*, separates it sharply from the complex convoluted surface of the parietal lobe.* The oblique course of this sulcus carries it inwards to the median aspect of the hemisphere, midway betwixt frontal and occipital pole, where it meets the subparietal sulcus, regarded by BROCA as the continuation upwards of the limbic fissure. This latter sulcus, sweeping backwards, does not join the limbic fissure posteriorly, being separated by the retro-limbic annectant, but as the latter annectant corresponds to the region which in the Rat and Rabbit is characterised by the modified olfactory type of cortex, this sub-parietal sulcus forms a line of demarcation betwixt the latter type of cortex and the granule formation of the upper arc. The mapping out of distinct morphological areas exhibited in the lower limbic arc of the smooth brain of the Rat and Rabbit, is a feature which in the convoluted brain of the Pig and Sheep becomes extended to the upper limbic arc also through the medium of the crucial and sub-parietal sulci. Moreover in these convoluted brains the deep inter-parietal sulcus also divides two highly characteristic formations. Still higher, as in the Carnivora, the fissure of ROLANDO constitutes a similar limitary zone, the cortex of the ascending parietal (post-Rolandique of BROCA) being six-laminated; that of the ascending frontal five-laminated.

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* *Op. cit.*

Section 2.—CENTRAL PROJECTIONS OF THE OLFACTORY ORGAN.

The Olfactory Lobe and Bulb.

The olfactory lobe is constituted by a tubular prolongation of the cerebral cortex given off from the inferior aspect of the anterior pole of the hemisphere. Within the confines of the limbic fissure its cortex is directly continuous with that of both extremities of the limbic lobe along the lower and lateral aspects of the brain, whilst above and outside the boundaries of the limbic fissure it is overlapped by the projecting frontal extremity of the hemisphere, with the cortex of which its surface is also continuous. Its continuity with the anterior extremity of the upper and lower limbic arcs thus completes the circuit made by the great limbic lobe.

The summit of the olfactory lobe is capped by the olfactory bulb, which completely ensheaths its extremity and receives the olfactory nerve fibres.

The connexions of the olfactory cortex, inclusive of the bulb, are in this animal both complicated and extensive—complicated in that its medulla brings it into relation with most diverse regions of the brain, and extensive in so far that its fibres are projected back into the occipital pole of the hemisphere, and embrace in their circuit the whole extent of the limbic arc, thus associating the most distant realms of the brain in direct organic connexion. For convenience of description as well as upon anatomical grounds, we may consider the olfactory medulla as consisting of the following distinct systems :—

1. A central decussating and commissural fasciculus.
2. A connecting system with the striate body.
3. An arciform series.
4. A superficial medullated band.

1. *Central fasciculus.*—A rapid convergence of fibres from the bulbus olfactorius constitutes the origin of a central medullated core for the olfactory lobe, which soon becoming almost cylindrical in contour courses backwards through this lobe, and arching slightly outwards plunges into the anterior extremity of the caudate nucleus. Owing to its outward curve, vertical sections through the frontal extremity exhibit an oval contour of this fasciculus, such oblique sections measuring 1·3 mm. in its greater diameter, by ·78 mm. in its shorter diameter. It is, however, a uniformly cylindrical fasciculus, its usual diameter being ·87 mm. Receiving no fresh addition of fibres, the central olfactory fasciculi of both hemispheres attain their greatest distance apart within the corpus striatum (4·76 to 5·046 mm.), and thence converge towards the descending pillar of the fornix, passing in this course slightly upwards in relation to the cortex at the base, and lying betwixt the corpus striatum on the outer side and the thickened folia of the septum lucidum internally, in close relationship to the floor of the lateral ventricles. In the Rabbit, the central canal is seen lined with endothelium,

and surrounded by the central medullary fasciculus, forming a direct communication between the olfactory lobe and the lateral ventricle. Still converging, these two fasciculi decussate in the median line immediately in front of the pillars of the fornix, constituting by this convergence the anterior commissure of the cerebrum, being in this latter course imbedded in the septum lucidum. The resultant fibres of this decussation pass out from the commissure as a compact fasciculus, and are not projected directly backwards, but arch *suddenly outwards*, passing through the substance of the corpus striatum. In this course the fasciculi of either side diverge so rapidly that they lie very nearly in one and the same line, a single vertical section through the hemispheres presenting us with the commissural band and both posterior fasciculi as far as the outer border of the striate body. To follow the further course of these posterior projections from the olfactory bulb, horizontal sections of the cerebrum of the Rat should be examined, and it will then be found that each fasciculus passes through the striate body, and upon reaching its most external border each olfactory band sub-divides into several secondary fasciculi similar in appearance to those which result from the union of the fibres of the first link of the projection system at their connexion with the receiving surface of the corpus striatum (Plate 49, fig. 9, A).

These sub-divisions of the posterior olfactory fasciculus curve directly back into the occipital pole (Plate 49, fig. 9, B). If *vertical* sections of the hemisphere at this site be examined, the connexions of the cortex with the corpus striatum by the projection system of fibres will be found to be chiefly affected by the upper surface of the ganglion, with which a large bulk of coarse fasciculi are connected (Plate 49, fig. 10, A); yet the *whole outer aspect* of the ganglion in like manner receives numerous fasciculi, which form a complete medullated investment over its outer surface. This medullated investment being augmented by fewer fibres, and yet fewer at lower levels, eventually thins off into an insignificant tract (Plate 49, fig. 10, B). Now it is by no means improbable that a certain number of minute fasciculi in the Rat pass from the sub-divisions of the posterior olfactory fasciculus upwards along this medullated tract. I have repeatedly imagined that I could trace such a connexion, and if this turns out to be correct then we have here a distribution similar to although far more restricted than what I have assured myself is the case with the Rabbit's brain. The position of this posterior olfactory fasciculus from the anterior commissure in its course through the corpus striatum is parallel to the base of the lenticular wedge, lying in fact betwixt its two main divisions, as may be clearly seen in horizontal sections taken at the proper plane, in which will also be very evident the fact, that whilst these fibres run directly backwards, the majority of the projection fibres of the corpus striatum pass directly outwards or forwards to the cortex (Plate 49, fig. 9, A–C). The olfactory fasciculus, prior to the formation of the anterior commissure, measures in its shortest diameter ·78 mm. to ·87 mm.; the fasciculus emerging from the commissure, as it leaves the septum lucidum to enter the corpus striatum, measures but ·52 mm. This reduction in bulk is due to the absence of a series of fibres which run from one olfactory bulb to that of the opposite side, as a purely commissural system. This of course is in accordance with the fact indicated by GRATIOLET, MEYNERT, and others, that in the Rodentia and Carnivora, where the olfactory apparatus is largely developed, the great bulk of fibres proceeding from the anterior commissure communicate with the olfactory lobe; whilst in Man and Apes, where its development is small, the larger portion of the commissure is distributed backwards to the occipital and to the temporo-sphenoidal lobes. The central olfactory fasciculus behind the anterior commissure exhibits a notable difference in its distribution in the Rabbit and the Rat, a divergence which has an important bearing upon the functional relationships of the olfactory lobes. In the Rat, as has just been stated, the fasciculus passes outwards through the corpus striatum, and *immediately* upon reaching its outer border divides into a brush-like head of nerve bundles, some of which probably unite with the projection fasciculi passing through them up to the vault, but by far the most extensive portion turns back, and runs towards the occipital pole of the hemisphere (Plate 49, fig. 9). In the Rabbit, on the other hand, when the posterior olfactory fasciculus reaches the outer surface of the corpus striatum it does not, as in the Rat, break up into numerous diverging bundles, but bends upwards and ascends as a more or less compact fasciculus in contact with the outer surface of the ganglion (Plate 49, fig. 11, A). Upon reaching the level of the upper pole of the striate ganglion it meets with the accumulating mass of the projection and callosal systems (C, D), and at this spot it suddenly breaks up into an extensive brush-like head of fibres (B), which pierce through the mass of the projection system, and are thence directed towards the cortex of the inner margin of the hemisphere, *i.e.*, the margin bounding the great longitudinal fissure (E). In vertical sections which have been carried successfully through the plane of the posterior commissural fasciculus, we may readily trace it—first in its horizontal course through the striate ganglion, next curving round and up the outer aspect of that ganglion, and lastly on its arrival at the upper pole, grasping, as it were, by a mass of tentacular-like extensions, the base of the projection system, decussating freely with the callosal fibres, meeting them from an opposite direction, and with the projection fibres betwixt striate body and cortex, which in part traverse the same course.* What is the destination of this important system of fibres? It appears indeed to form that extensive arcuate system which lies beneath the cortex of the summit of the hemisphere at these planes. This arcuate system may be readily seen lying betwixt the cortex of the vault and the great mass of the projection and callosal systems above the corpus striatum (Plate 49, fig. 11, F). They therefore are directed nearly at right angles to the radiating callosal fibres for the cortex of the vertex, and running parallel to the surface of the cortex, terminate in the region of the large ganglionic cells which have been referred to as seen in the cortex along the inner margin of the vault. In short, then, the olfactory bulbs of the Rabbit are thus brought into direct and decussating connexion through an extensive course with the specialised cortex of the inner marginal aspect of the brain in its anterior regions—a region proved by FERRIER to have motor endowments. In the Rat, on the other hand, this fasciculus is brought into relationship not so much with the motor cortex as with the cortex of the occipital pole, with which it has most extensive connexions. No doubt need be entertained upon these facts after a careful examination of horizontal and vertical sections of the frozen brain of these animals, and by dissection and teasing out of these structures. In vertical sections I have repeatedly succeeded in tracing the posterior commissural fasciculus upwards towards the projection system above, and have teased out, by means of a dissecting needle, the whole of its course from the anterior commissure to the brush-like head, leaving it a compact band of fibres entirely separated from the surrounding structures.

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* The whole of this course is represented in Plate 49, fig. 11.

The olfactory lyre.—This is a structure which I have ventured to name by what appears a very appropriate term when its configuration is thoroughly understood—it has up to the present been entirely overlooked in descriptions of the olfactory apparatus. MEYNERT distinctly states, when describing the posterior extensions from the anterior commissure in his classical work on the brain of Mammals,* that the fasciculus passes "uninterruptedly through the corpus striatum." This statement is, however, undoubtedly an error, and it requires but a careful examination of vertical sections through this commissural tract in the Rabbit or a dissection of the fresh brain of the Rat or Rabbit to show not only that such is not the case, but that MEYNERT and others must have entirely overlooked here a structure which is most peculiar and important in its bearings. Vertical sections through the fresh brain of the Rabbit show very distinctly oval fasciculi of no mean size separating the strands of the posterior olfactory band, largest and most numerous near the anterior commissure (Plate 49, fig. 11, G). They consist of divided medullated fibres, which distinctly traverse the structure of the olfactory fasciculus from the portion of the striate body lying in front to that lying behind the commissure. In passing through the olfactory fasciculus a very apparent increase in the size of this tract is occasioned by this separation of its strands, so that a great oval enlargement characterises the earlier portion of this fasciculus in the Rabbit. A dissection of the brain of the Rat will indicate still more readily the nature of the structures here exhibited. Such a dissection will be described in detail further on—let it suffice here to state that by this method we expose a series of delicate fasciculi of medullated fibres passing from the posterior olfactory band forwards parallel to the central olfactory fasciculus. In part, these fibres traverse the structure of the olfactory band at right angles to its course, passing thus into the corpus striatum behind it, whilst others attached to the anterior margin of the same band curve inwards to accompany the rest of its fibres across the commissure. In passing forwards these fibres form a delicate floor or dissepiment betwixt the structure of the corpus striatum proper and that portion of the same ganglion which forms the olfactory area. This delicate sheet of medullated strands occupies a triangular space bounded internally by the central olfactory fasciculus, behind by the posterior olfactory fasciculus, and externally and deeply by the superficial olfactory medulla, and betwixt these boundaries the fibres pass forwards like the strands of a lyre in relation to its several sides—hence the term by which I have denominated this structure.

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* STRICKER'S 'Handbook,' Sydenham Soc. Trans.

Tœnia semicircularis.—This long arciform band of medullated fibres arising from the summit of the gyrus hippocampi follows the curved inner surface of the caudate nucleus through the whole of its course, and consists of superficial and deeper-seated fibres, the latter connected with the ganglionic structure of the caudate nucleus. It has been affirmed by numerous authorities that this arciform band terminates in the descending pillar of the fornix.* That the more superficial fibres so terminate I will not deny; but I have hitherto failed in *these animals* to assure myself of the fact. Of its deeper fibres I can speak with confidence. They not only *do not terminate* in the fornix, but end in two distinct directions, the deepest curving downwards to enter the anterior perforated space (Plate 49, fig. 12, 2); the fibres above this series arch inwards and forwards to enter the posterior commissure (Plate 49, fig. 12, 1). The latter connexion is a most important one to recognise, and of its existence I have repeatedly assured myself by horizontal and vertical sections. The fibres enter the commissure from behind in such a direction as to ensure their decussation within it. The cortex of the gyrus hippocampi and caudate nucleus are thus brought into crossed relationship with the olfactory bulbs.

2. *Connexions with striate ganglion.*—The anterior extremity or head of the caudate nucleus bends downwards to the base, and approaching the median line is separated from the ganglion of the opposite side by the mass of the septum lucidum. At the base the ganglion becomes almost superficial, being merely covered by a very thin layer of cortex, which is continuous with the general surface of the olfactory lobe. Medullated fibres from the cortex of the olfactory lobe and the granule layers of the bulb pass from before backwards into this basal extension of the caudate nucleus in numerous bundles, which in vertical sections appear mapped out into triangular wedge-shaped spaces by the large vessels traversing this region (pars perforata antica). This basal region of the corpus striatum, the olfactory field of GRATIOLET, is best studied in fresh preparations by horizontal sections carried across the hemispheres. The brain is placed on the section plate of a microtome with its base downwards, frozen to the height of a few millimetres, the unfrozen portion removed, and then successive sections should be made down as far as the cortex of the base. All these sections examined fresh will prove most instructive. A section carried in this way through the cortex shows this basal or olfactory realm of the brain to be clearly mapped out into two very distinct areas. One, the most internal, is of pyriform contour, its narrow end continuous with the olfactory lobe, its larger end directed backwards. This area constitutes the base of the corpus striatum and has a peculiarly characteristic structure. The other, or outermost area, is constituted by a continuation of the olfactory cortex with the lower

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* MECKEL, ARNOLD, JUNG, and LUYS.

limbic arc, and is sharply separated from the former area by the superficial olfactory fasciculus, whilst on its outer side it is of course bounded by the limbic sulcus. Following exactly the outer border of the pyriform area, the superficial olfactory band is naturally at first curved outwards, and then inwards towards the Sylvian depression. In sections taken across a slightly higher level the same regions are distinctly seen and their structure rendered clear. The innermost area is now seen to consist of numerous oblong and elliptic masses of grey matter enclosing nerve corpuscles; these grey nuclei vary much in size, but usually measure ·348 mm. to ·870 mm. in length by ·204 mm. to 300 mm. in width. They are closely embraced by bundles of medullated fibres, which form dense fasciculi betwixt them and are all directed from before backwards. They originate in the cortex of the olfactory lobe and pass through this ganglionic region. Mingled with these is a separate system of fibres which arise from the granule layers of the olfactory bulb, and coursing chiefly along the inner basal aspect of the olfactory lobe also enter this olfactory area, but do not form fasciculi. Their peculiarity consists in their running as separate fibres very irregularly, as bending, decussating and crossing each other in various directions so as to give the impression of their forming an open meshwork. These nerve fibres measure ·002 mm. to ·004 mm. in diameter. They penetrate the oval nuclei, and are found throughout their substance. As they do not extend beyond this region they terminate almost certainly in the nerve-cells of its grey masses. The outer surface of this olfactory region is covered, as was indicated by MEYNERT, by a thin layer of cortex, whose structure has already been fully described (p. 714). In passing through the olfactory area this deep mass of medulla is disposed in numerous longitudinal divisions or clumps well shown in vertical sections, from which smaller fasciculi are seen to descend to the cortex of the base, where they become apparently continuous with the arciform medulla superficial to and beneath the second cortical layer. Continuous with this deep olfactory medulla is a minor segment given off from the superficial olfactory fasciculus along its inner margin, and which can be best demonstrated by longitudinal sections through the hemisphere. The following scheme represents in a diagram the numerous systems of medullated fibres seen in vertical sections through this olfactory area (Plate 49, fig. 8). In the first place, we trace the cortex from the limbic fissure (*a*) downwards beneath the superficial olfactory fasciculus (*b*) prolonged over the basal aspect of the caudate nucleus. Its second layer is seen disposed in the peculiar duplicatures described, and beneath which lie the clumps of deep olfactory medulla seen in cross section (*m*) forming massive fasciculi contrasted strangely with the more distant fasciculi of the corpus striatum proper. Betwixt the deep medulla and the second layer is the deep arciform belt (*d*) with its great spindle cells apparently continuous with the claustral formation externally (*c*) and with the arciform medulla passing up the septum lucidum internally (*e*). Descending from the deep medulla into this arciform belt are numerous vertical fasciculi (*f*), which may also probably form connexions with the second layer of the cortex. The deep medulla gives off laterally a dense mass of fibres which ascend deeply within the substance of the septum lucidum (*g*), whilst the cortex of the septum is seen to possess a double arciform belt, one superficial to its second layer (*h*), and the other beneath these cells (*k*). These arciform stripes communicate by vertical intermediate fibres with each other in their course upwards to the corpus callosum.

From this olfactory area an important medullated fasciculus descends to enter the motor columns of the cord, hence affording a direct channel for the impulses propagated on excitation of the olfactory bulb. The double connexion with motor columns of the medulla and the cerebral cortex is the usual condition in Mammalia, although, as BROCA has indicated, the connexion with the *cortex* alone is the only constant feature.

3. *Arciform fasciculi.*—If the olfactory lobe be reflected downwards, away from the frontal pole, an upper medullated connexion betwixt the former and the hemisphere, may be traced lying invariably over the central canal of this lobe. But by far the more important arciform fasciculi form two series, which taking origin in the cortex of the olfactory lobe, are destined for the gyrus fornicatus and posterior regions of the brain. In vertical sections through the frontal end of the hemisphere, near the commencement of the caudate nucleus, the central olfactory fasciculus is seen in transverse section, imbedded in the head of this nucleus, which rises dome-like above it. Descending from the region of the central medulla of the hemisphere, are two medullary finger-like processes which appear to grasp the caudate nucleus betwixt them. The outer is constituted by fasciculi arising from the external borders of the caudate nucleus and disappears in the central medulla. The inner lies parallel with the central mass of medulla, but is wholly distinct from it, and constitutes the arciform fasciculi, which originate in the cortex of the olfactory lobe at the base and median aspect, and curving up the inner side of the hemisphere is closely connected with the cortex of the upper limbic arc. At the median aspect, where the upper limbic arc unites with the olfactory cortex, numerous medullated fasciculi arising from the latter follow the bend of the hemisphere, and thence arching strongly inwards come into relationship with the under surface of the corpus callosum, and turning backwards, course along its inferior aspect, whilst other fasciculi pierce its structure and join the arciform system of the gyrus fornicatus, which rests upon the upper surface of the corpus callosum. (Plate 49, fig. 12, ^{7}). At its origin this important fasciculus is seen in vertical sections, ascending in a sigmoid curve, with the convexity first inwards and then outwards towards the under surface of the callosal commissure. A little further back the septum lucidum is found to the inner side of the olfactory area. In this animal it appears as a largely-developed structure, heart-shaped in vertical section, and formed of two lateral folioles uniting centrally, and not enclosing a ventricle betwixt them as in Man. Through the substance of the septum a large number of medullated fibres ascend to the inferior aspect of the corpus callosum (Plate 49, fig. 8, G). The longitudinal band formed by the arciform fibres of the olfactory and upper limbic

cortex is distributed eventually to a peculiar region already described, near the occipital pole of the hemisphere, which is covered by cortex of the modified upper limbic type. This termination will be described when I refer to the central expansions of the olfactory medulla.

4. *Superficial olfactory fasciculus* ("External root").—This fasciculus appears, from a surface view, as a flattened white lamina covering the basal aspect of the olfactory lobe. Widest in front, it encircles the greater surface of this lobe, grasping its lower, outer, and portion of its upper aspect. Thence directed backwards, it sweeps round the olfactory area, forming, as already stated, its outer boundary, and becoming rapidly attenuated, runs as a delicate white streak inwards, in a direction which, if continued, would pass nearly at right angles across the origin of the optic tracts. In this, its latter course, it passes over that slight furrow which here indicates the Sylvian fissure, and is lost in the substance of the gyrus hippocampi. Vertical sections show that this fasciculus is not a mere flattened lamina, but a thickened oval or oblong band, which is sunk to a little depth in the peripheral zone of the cortex, covered externally by a delicate layer of DEITER's cells connected with its pia mater, from which large vessels dip vertically through its substance and enter the subjacent cortex.

In what manner does this medullated band terminate? It has long been known that in Man and the higher Mammals the external olfactory medulla ends in the temporal and occipital lobes, but in these animals it is seen by the naked eye that a rapid attenuation is undergone ere it reaches the Sylvian furrow (Plate 49, figs. 1, 2, M). To comprehend its distribution we must examine closely vertical and horizontal sections through this region anterior to the Sylvian depression. In vertical sections the superficial olfactory fasciculus forms the inner border of what we have termed the external olfactory region, limited outwards by the limbic sulcus. The cortex of this region is composed, as already stated, of the three layers—the peripheral zone; the irregular and angular cells; the large pyramidal layer. Now beneath the lowest layer of this cortex, at variable depths, is found an important formation of spindle-cells, which taking the reclinate position as regards the surface, descends from the extra-limbic portion of the hemisphere. This belt of spindle-cells is the representative of the claustral formation of higher Mammals, and is directed towards the surface of the cortex below, along the inner margin of the superficial olfactory fasciculus. It forms therefore a deep belt, dividing off the olfactory area from the external olfactory regions (Plate 49, fig. 8, *c*). Having noted this fact, a horizontal section should be taken through the hemisphere, close to its basal cortex, as was done for the examination of the connexion of the olfactory lobe with the olfactory area. If such a section pass through the deeper part of the superficial olfactory band we shall note that whilst its outer fibres are long and continued uninterruptedly backwards, the innermost fasciculi constantly tend to bend outwards and pass through and across the longitudinal band, arching in fan-like manner over the whole external olfactory region as far as the limbic sulcus. This passage of the inner marginal fibres of the band into the cortical substrata must eventually lead to its total disappearance. These fan-like radiations are continuous with the cells of the cortical layers; yet it does appear as though a connexion was also established betwixt its innermost fibres and the claustrum, which latter formation, however, is mainly continuous with the arciform spindle-celled medulla of the olfactory area (Plate 49, fig. 8, *d*). Prior to the arrival of this superficial medullated band at the Sylvian furrow a notable diminution in its width has occurred, apparent to the naked eye. This is due to the detachment from its margin of a large proportion of fibres which have already been alluded to as running continuous with the deep olfactory medulla within the olfactory area. It has been usual to speak of the central projections of the olfactory medulla under the terms olfactory roots, an *external, middle, and internal root* being usually those described in higher animals. A fourth also is described as a *superior root*, connecting the upper surface of the olfactory lobe to the hemisphere. By the external root is indicated what I have above described as the superficial olfactory fasciculus; the connecting system with the striate body is identical with the so-called middle root, whilst the arciform series includes the remaining two roots, the internal and superior.

Dissection of Basal Olfactory Regions.

The tracing of the coarser strands of medullated fibres by microscopic examination of sections, should invariably be supplemented, where possible, by frequent dissections which often enable us to fully confirm by this second method results arrived at by the former. As regards the olfactory regions at the base the following points may by this method be satisfactorily demonstrated in the Rabbit and the Rat:—

a. Uninterrupted course of the central olfactory fasciculus into anterior commissure.
b. Projection of posterior olfactory fasciculus into the occipital lobe.
c. Connexion of the latter fasciculus also with the cortex of the vertex.
d. Peculiar connexion of the same fasciculus with the corpus striatum.
e. Relationships of olfactory area at greatest depth below cortex.
f. Descent of the tænia semicircularis into olfactory area.

The above facts may be confirmed by placing the brain of the Rat in its natural position with the vertex upwards, and keeping it during dissection moistened occasionally by a few drops of a 2 per cent. solution of common salt. With a delicate pair of dissecting scissors the membranes stretching across between the hemispheres are divided through their whole length, and the latter sliced off horizontally outwards upon a level with the corpus callosum by a sharp and fine scalpel. This commissure is next divided along the median line from behind forwards, each segment being turned outwards. Such a section exposes the corpus striatum in its extra- and intra-ventricular portions placed in front of the cornu ammonis. In the median line separating the two striate bodies lie the double leaflets of the septum pellucidum. The callosal fibres are seen to meet those ascending from the internal capsule and upper pole of the ganglia. If the extra-ventricular nucleus be pressed gently outwards away from the caudate nucleus, we observe numerous white fasciculi beneath its surface arching upwards parallel to the latter to enter the projection mass at the upper pole, whilst running parallel with the converging tract of the fornix, and closely connected with the whole inner border of the caudate nucleus, is the fairly broad white band—the tænia semicircularis. By means of a fine dissecting needle and camel-hair brush the whole remaining dissection may be completed. First, the septum pellucidum is gently raised from behind and thrown forwards, the hemispheres pressed asunder so as to expose the descending pillar of the fornix, in front of which the anterior commissure crosses from one to the other hemisphere. By means of the needle the continuation of this commissure, with the olfactory lobe of one side, may be most readily traced, and upon cautiously teazing out the substance of the striate body on one side, the continuation from the commissure outwards through its substance may be quite as easily traced. We now see gleaming deeply through the basal structures of this region a pale band which may be regarded as the outer side of a remarkable triangle, the inner side being constituted by the central olfactory fasciculus, and the base by the anterior commissure and its posterior extension. Now this triangular area is the deeper portion of the pyriform region recognised at the base as GRATIOLET'S olfactory area. Closely examined by a hand magnifier we now see the very peculiar structure alluded to as the "olfactory lyre." Gently raise by the needle the posterior olfactory fasciculus and examine also with a lens. From its anterior border arise numerous delicate fibres constituting more or less fine fasciculi, the largest arising near the anterior commissure, whilst a few delicate offsets are given off from the commissure itself. Whilst these fasciculi curve inwards and appear *continuous with the olfactory tract into and through the commissure,* other fasciculi may be seen passing through the structure of the posterior olfactory band directly backwards, separating its strands and *emerging behind to enter this region of the corpus striatum.* Traced forwards these delicate fasciculi lie parallel to and on a plane with the central olfactory fasciculus, forming as it were a floor for this triangular space (the strands of the lyre), and in fact sharply defining the upper limits of the olfactory area from the structure of the caudate nucleus proper. With the scissors divide these fibres along their line of connexion to the posterior olfactory tract, and teaze out with the needle the structures lying immediately beneath. We now see by means of a low power, and even by the naked eye, the coarse fasciculi enclosing betwixt their strands the oval grey nuclei of the olfactory area. Next, by a delicate dissection with needle and moistened brush, the posterior olfactory fasciculus may be followed from the outer margin of the striate nucleus backwards; its brush-like division here sending a small detachment of fibres upwards along the outer aspect of the corpus striatum to reach the vertex. The posterior division of the brush may readily be traced into the inferior arc of the great limbic lobe, its fasciculi never rising above the level of the limbic sulcus. They are ultimately distributed to that peculiar region at the occiput characterised by the modified olfactory cortex.

It now remains for us to follow up the extensions of medullated fasciculi from the olfactory lobe, and to trace the expansions into the distant cortical areas of the central olfactory and arcuate olfactory systems. Prior to doing so, it will be advisable to consider the structure and cortical connexions of the corpus striatum and callosal commissure.

Corpus Striatum and its Cortical Connexions.

Corpus striatum.—This ganglionic body exhibits in these animals, as in higher members of the animal kingdom, a marked division into lenticular and caudate segments, or, in other words, extra- and intra-ventricular nuclei, which assume also a higher level as compared with the thalamus, since the largely developed cornu ammonis and the bulky fibrous fornix commences at higher and anterior planes, and not, as in Man, *behind* the thalamus. These structures therefore intervene betwixt the cortex of the vertex and thalamus, and displace the latter to a lower level, so that sections carried horizontally through the upper part of the hemisphere will exhibit the striate body and cornu ammonis cut through in one and the same plane. Anteriorly the corpus striatum exhibits a large rounded head, which bends downwards to the base of the hemisphere within the confines of the superficial olfactory fasciculus. Posteriorly it becomes rapidly attenuated and is continued as a small cylindrical belt of grey matter—the tail of the caudate nucleus, which curves backwards and downwards within the descending horn of the lateral ventricle to terminate in the inferior limbic arc, near the summit of the gyrus hippocampi (Plate 49, fig. 10, C). To study the form and relationship of the striate body, vertical sections should be examined passing through the septum pellucidum in front of the optic commissure. This ganglion is then seen as a large irregularly pyriform structure, its greater mass directed upwards, its upper and outer border nearly concentric with the surface of the hemisphere (Plate 49, fig. 12, 4),* whilst the inner or intra-ventricular border lies partly adjacent to the thick foliole of the septum lucidum (fig. 8, *b*), and at the angle of union with the latter the transverse section of a compact band of medullated fibres is seen, the central olfactory fasciculus (Plate 49, fig. 8, *h.*). The true structure of the corpus striatum ends upon a level with this olfactory fasciculus, and is mapped out by a succession of fasciculi, which are cut across and lie in a continuous line from the central olfactory fasciculus to the outer margin of the ganglion. These fasciculi are of greater size than those beneath them, and are constituted by the strands of the "lyre" already alluded to. Beneath this line the striate nucleus still extends as far

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* The diagram (fig. 8) exhibits in outline a part of the caudate nucleus and its full basal relationships.

as the base, forming the modified structure referred to as the olfactory area, and which is here covered by a thin layer of cortex.* The outer margin of the striate body is sharply differentiated from the outer olfactory realm (lower limbic arc), and terminates abruptly at the inner margin of the superficial olfactory band, to which point run the fibres of the claustral formation (Plate 49, fig. 8, *c*). On the inner side, at its base, we find this olfactory area of the corpus striatum bounded not only by the central olfactory fasciculus, but by sheaves of fibres arising from the cortex of the olfactory lobe (Plate 49, fig. 8, G, *g*), and running as the arciform fibres up the septum lucidum to reach the under surface of the corpus callosum. In these anterior realms the internal capsule which separates the two main divisions of the striate body is not as yet clearly differentiated, and hence the large oval mass is constituted by the blending together of the caudate and lenticular nuclei, and the fasciculi here being chiefly projections from the cortex of the frontal lobe are seen in transverse or oblique section. To obtain a good view of the fibres radiating from the frontal pole through the structure of the corpus striatum vertical sections must be taken across the hemispheres. At the summit or upper pole of the striate body (Plate 49, fig. 11, H) the fibres of the corpus callosum (D) meet those of the projection system from the cortex to the basal ganglia, and at their line of intersection mark off the intra-ventricular (K) from the extra-ventricular portion (L). We have therefore in such sections a large ganglionic mass formed by the blending of the two striate nuclei, enclosed in a concentric manner by the hemisphere, and exhibiting the following systems of fibres: the callosal fasciculi (D); the striate radiations to cortex (C); the claustral formation (Plate 49, fig. 8, *c*); the external (*b*); median or deep (*m*); internal and central olfactory fasciculi (G and *h*). In vertical sections carried through the hemisphere of the Rabbit on a plane with the anterior commissure the distinction betwixt the two nuclei of the corpus striatum is very obvious. Here the caudate nucleus lies reclinate upon the sloping oblique side of the internal capsule, measuring from 9 mm. to 10 mm. in its longer, and 2·5 mm. in its shorter diameter, its upper pole directed upwards and outwards, receiving the radiating fibres from the cortex; its inferior pole directed downwards and inwards as a thin prolonged margin, exhibiting on its ventricular aspect the transverse section of the stria cornea (Plate 49, fig. 12, 4). Beneath the stria cornea with the structure of this inferior pole are nerve fasciculi, which in part pass downwards into the anterior perforated space (Plate 49, fig. 12, 2) (olfactory area), whilst a few arch inwards and are directly continuous with the anterior commissure (Plate 49, fig. 12, 1). Beneath the sloping roof formed by the internal capsule lies the lenticular nucleus, which is distinctly wedge-shaped in form, its base directed outwards and curved concentrically to the surface of the hemisphere, its apex directed downwards and inwards terminates in the internal capsule (Plate 49, fig. 9, D). The lenticular nucleus appears to consist of outer and inner segment; the inner,

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* Vide Plate 49, fig. 8. All structures below *h*.

forming the peduncular portion and apex of the wedge (Plate 49, fig. 9, D), contains a far greater proportion of fibres than the outer, which are consequently much more closely approximated, consisting, as MEYNERT indicates, of fibres extending from the outer segment, as well as fibres originating afresh in itself. The further our examination of these nuclei by vertical sections extends, the more restricted becomes the dimensions of the lenticular nucleus, the mass of the thalamus intervening until on a plane with the origin of the pineal peduncles ; its extent is very insignificant,* whilst the tail of the caudate nucleus (Plate 49, fig. 10, D) and the stria cornea are seen as two small oval bodies, separated from it by the intervening cortical connexions of the ganglia. In these posterior planes, vertical sections passing through the tuber cinereum will exhibit within the substance of the inferior limbic arc two noteworthy structures : first, a compact cylindrical band, consisting of some thirty or more fasciculi of medullated fibres, which results from the convergence of fibres from the cortex of this region, and is really the temporal extension of the stria cornea ; and secondly, just external to it lies a somewhat oval nuclear grey body, the tail-like prolongation of the caudate nucleus (Plate 49, fig. 10, C). In sections through the hemispheres at that site where the retiring optic tracts are concealed betwixt the crus cerebri and hemisphere, the stria cornea will appear to a certain extent of their course cut lengthwise and entering the substance of the limbic lobe external to the optic tract (Plate 49, fig. 10, E).

Coronal projections to the motor ganglia and internal capsule.—Traced from the cortex downwards it is found that the whole marginal aspect of the hemisphere at the vertex and for a certain distance outside the median line, constitutes the area whence the coronal radiations arise which converge to the internal capsule and corpus striatum below. Upon arriving at the centric pole of the corpus striatum, the fibres form a dense projection mass, which in the form of small fasciculi spread out and almost completely enfold the upper and outer aspect of this ganglion, the innermost series entering the ganglion direct, the outer fasciculi proceeding along the arched surface of the lenticular nucleus to enter it at its outer aspect, whilst the intermediate fibres terminate at various points along the intervening surface (Plate 49, fig. 10, A). The receiving surface or centric pole of the striate body is therefore a very extensive one, being constituted by the whole of its upper and greater part of its outer aspect. The above description of its cortical connexions holds good only as regards that region where the centric pole of these ganglia attains its greatest height, viz.: anterior to the thalamus where the corpora striata lie in closer contiguity. To study its exact distribution a series of vertical sections of the brain of the Rat should be examined at different planes from before backwards. Thus, in regions of the septum lucidum, in front of the anterior commissure, we find the innermost fasciculi of the projection mass where they leave the ganglia only 2 mm. distant from the marginal angle of the hemisphere.

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* Vide Plate 49, fig. 10, between A and B.

In regions where the thalamus intervenes, as at the origin of the optic tracts, the lenticular nucleus retiring further outwards makes the distance between the points 5½ mm. In these regions the coronal fibres are distributed chiefly to the upper aspect of the hemisphere and not so exclusively to its marginal angle. Still further back on a plane with the optic radiations to the thalamus, the distance betwixt the same points is 6½ mm., and here the great mass of the projection system no longer runs upwards to the vertex, but immediately upon the emergence from the lenticular nucleus they bend directly backwards towards the occiput and are therefore exhibited as transverse sections of fasciculi (Plate 49, fig. 10, F). A limited supply can still be traced upwards to the cortex of the outer and upper aspect of the hemisphere (Plate 49, fig. 10, A). These facts warrant us in asserting—

1st. In regions anterior to the thalamus the coronal connexions betwixt cortex and motor ganglia with the internal capsule, arise *par excellence* from that marginal aspect of the vertex whose cortex is characterised by the great ganglionic cell formation, and as entering into the composition of FERRIER's motor realm.

2nd. That the median cortex (upper limbic arc) has no connexion with the same ganglia or capsule throughout any of its course.

3rd. That as these ganglia retire outwards before the intervening thalamus, their coronal connexions arise chiefly from the upper and outer aspect of the hemisphere and still further back from the occipital cortex at their own level.

From what has already been stated as regards the distribution of the callosal commissure, it is evident that in the anterior regions of the brain the marginal and exposed aspect adjacent to it will receive a much larger supply of medullated fibres than with the more external aspect, or the cortex of the median aspect, which receives callosal fibres only. In accordance with this a glance at vertical sections of fresh brain shows the dense sheaves of fasciculi which radiate to the marginal angle (Plate 49, fig. 11, E), and the comparative paucity of medullated supply to the limbic arc and outer aspect of hemisphere.

Junction of projection fasciculi with motor ganglia.—The uniform medullary radiations unite into delicate fasciculi which pass between the fibres of the callosal commissure, decussating with them at various angles, and just where they arrive at the surface of the striate ganglion unite into coarser fasciculi and pass into the outer segment of the ganglion, often bending inwards at right angles to their original course (Plate 49, fig. 11, C). The fasciculi thus formed by convergence of the coronal radiations have an average diameter of ·05 mm., but towards the upper pole of the lenticular nucleus large fasciculi are found with a diameter of ·093 mm., much more closely aggregated. At the lowest level of this body they form long delicate bundles (·025 mm. diameter), which pass uninterruptedly after a straight or curved course into its inner segment, whilst midway betwixt these points the fasciculi are far more widely scattered apart and are of medium size (Plate 49, fig. 13, A–B). The delicate bundles traversing the callosal fibres which by their convergence form the coarse fasciculi of the outer lenticular nucleus are usually one-third or one-fourth the diameter of the latter. Each coarse fasciculus is formed by the union of two or three, and occasionally a large number of such bundles, and during their passage through the outer grey segment frequently divide into two, and appear in their further course through the inner segment subject to still further subdivision. At the upper pole of the lenticular body fasciculi are observed passing from the first segment directly into the internal capsule without previously traversing the inner segment, whilst coronal fasciculi are here also distinctly seen to pass down the internal capsule without establishing any connexion with the ganglia. Upon a plane passing through the anterior or olfactory commissure in horizontal sections of the brain, other fasciculi of medium size, are directed directly backwards into the inner segment of the striate body from the frontal aspect of the caudate nucleus, and obliquely backwards and inwards from its marginal aspects, each fasciculus frequently subdividing on its way but occasionally passing uninterruptedly through the outer segment. In this course they are directed over, beneath, and through the posterior olfactory fasciculus (Plate 49, fig. 9, D). Within the inner segments we meet with two systems of fibres: the coarse fasciculi already dealt with, and a large proportion of minute medullated fibres running separately, and not in fascicles. These ultimate nerve fibres are peculiarly liable to varicosity—they take curved and spiral directions—cross each other frequently, are often much contorted, but are chiefly distributed longitudinally along the course of the large bundles, or in arches across the inner segments of the lenticular nucleus concentric to its base.

They attain a diameter of ·004 to ·005 mm. At a little lower level we find them still more abundantly, but here the orifices of large divided vessels and the peculiar oval nuclei betwixt the coarser bundles indicate that we are examining the anterior perforated space or olfactory area, and now we immediately identify these minute twisted fibres with those already described as peculiar to this region, and can readily trace their connexion with those surrounding the granule layers of the olfactory bulb. At higher levels passing through the anterior end of callosal commissure, coronal radiations pass betwixt cortex of the tip and inner margin of the frontal lobe to the head of the caudate nucleus—the fibres of the corpus callosum crossing their course in their radiations outwards.

To recapitulate—

1. The anterior end of the frontal lobe is connected chiefly with the head of the caudate nucleus.

2. The cortex of the marginal angle at the vertex in front of the thalamus and of a limited surface outside is connected with the centric pole of the lenticular nucleus.

3. The cortex of the outer aspect of the hemisphere behind the latter, and that of the outer aspect of the occipital lobe is connected with that portion of the wedge-shaped nucleus which bends backwards and downwards towards the base.

4. The cortex of the olfactory bulb and lobe is represented in the olfactory area of the nucleus caudatus.

5. The cortex of the inferior limbic lobe will also be strongly represented in the extension of the lenticular nucleus in this direction.

The callosal commissure.—According to the researches of FOVILLE the fibres of the corpus callosum are distributed to none of the convolutions of the brain, but consist of tegmental fibres which, having passed upwards through the thalamus and corpus striatum, unite over these ganglia as a true commissure of the crus cerebri. All my fresh preparations lead me to regard this conclusion as quite fallacious, and demonstrate in the clearest possible manner the passage of the callosal fibres from the convolutions of one hemisphere to those of the other. As maintained by ARNOLD, OELLACHER, and MEYNERT, it is *truly and exclusively a commissure of the hemispheres.* In vertical sections of the brain the following facts I have repeatedly and satisfactorily demonstrated. In sections thus taken through the region of the septum lucidem, the callosal commissure upon entering either hemisphere meets the mass of the projection system obliquely, and decussating and interweaving with this system proceeds along the curved centric pole of the striate body, becoming more and more attenuated as it extends along the lateral aspects of the ganglia, owing to the constant distribution of fibres from its mass to the cortex throughout the whole of this course. In the first part of its course, where it first blends with the projection fibres to the ganglia, a very large proportion of its mass (one-third to one-eighth of the whole) takes a curved course upwards and inwards so as to reach the *median and marginal cortex here* (Plate 49, fig. 12, 6). By far the greater part accompanies the projection fibres, being similarly distributed, viz.: to the cortex of the upper marginal border of the hemisphere and the region just external to it. At this curve of the corpus callosum, where it interweaves with the projection fasciculi prior to their dispersion as coronal radiations, an angular projection is formed which, taking the whole length of the commissure into consideration, really is a prolonged ridge astride which rests the longitudinal fasciculus of the olfactory arcuate system (Plate 49, fig. 12, 7). In vertical sections taken through more posterior planes, it is seen that the portion of the callosal commissure which thus supplies the inner marginal regions of the hemisphere is distinctly to be traced through the whole of its extent as separate from the lower segment which supplies the cortex further outwards. So notably is this the case that all prepared sections show, as a rule, a distinct separation between these two callosal segments, the inner as it were peeling away from the outer—a natural condition due to their opposed direction. It is therefore obvious, at a first glance, that these inner regions of the brain which are similar and identical are connected by the same series of fibres, and the same may readily be maintained for the outer regions of the hemispheres. In fact, the regular superposition of fibres above one another, and the great uniformity of their course in either hemisphere, conclusively show that they unite identical regions of both sides. The most inferior fibres of the callosal commissure are the lowest throughout their course, and are placed next to the ganglia in their course to the cortex, which for them is the most external part of the hemisphere, *i.e.*, the undermost portion of the corpus callosum terminates in the cortex just above the limbic sulcus externally. On the other hand, the uppermost series of fibres in this commissure connect the more internal aspects of the brain. A superficial section lengthwise through the corpus callosum would therefore deprive the median and inner regions of the brain of their commissural connexions, whilst the *deeper* the incision was carried the more *external* the regions of the hemispheres which would suffer likewise.

Expansion of olfactory arcuate system into occipital cortex.—An important distribution of fibres from the olfactory cortex has been already traced into the occipital lobe lying like a longitudinal fasciculus (of inverted V-shaped form in vertical section) upon the prominent ridge formed by the apposition of the callosal and projection fasciculi (Plate 49, fig. 11, F). The ultimate destination of these fibres may best be studied in vertical sections through the hemisphere behind the posterior commissure. In such sections we expose the central medulla folded around the cornu ammonis, which in its turn lies concentric to the great central ganglia (Plate 49, fig. 10). This mass of central medulla reveals two important regions, (1) the region of section of the great mass of projection fasciculi which here is lateral in position (F); (2) the region of the callosal and arcuate olfactory fasciculi which is median in position (H). The latter with which we are now concerned is wedge-shaped, in such section the base of the wedge directed towards the median cortex, the two converging sides directed outwards. The central core, so to speak, of this wedge is constituted by the callosal fasciculi subdivided into an upper segment destined for the marginal angle and tip of occipital lobe, and a lower segment which is continuous outwards towards the lateral projection system radiating to the cortex throughout this course and the divided fibres of the fornix. Superficial to it and running in the opposite direction is a small band of projection fasciculi, which has also for its ultimate destination the cortex of the outer and upper surface of the hemisphere as far as the marginal angle. Upon the upper basal angle of the wedge rides the fasciculus of the olfactory arcuate and arcuate system of the upper limbic arc, which here have accumulated into a vast collection of fibres opposed to the whole base and upper side of the wedge. These fibres, however, are no longer found in transverse section, but are seen spreading extensively on all sides to the cortex (1) of the median aspect (upper limbic arc), (2) the marginal angle of the hemisphere, and (3) the cortex of the vault as far outwards as the superior parietal sulcus. Outside the latter sulcus the coronal distribution to the cortex consists of scanty projection and callosal fasciculi; within the sulcus, on the other hand, the cortex is supplied by dense callosal, a few projection fasciculi, and the notably large supply of olfactory and limbic arcuate fibres. This rich medullated region reveals its wealth of medullated fibres compared with the rest of the hemisphere when simply examined by the naked eye, and in osmic acid preparations shows the same in the far deeper staining which it takes. The olfactory arcuate fasciculi therefore are wholly distributed to that portion of the median and superficial cortex of the occipital lobe which is characterised by the granule cell formation which here forms a deep second layer (Plate 49, fig. 1, K, D). This formation has been already fully described under the name of the modified upper limbic type. This region is moreover characterised by a further peculiarity, viz.: the two intra-cortical arcuate bands. The more superficial of these is a broad streak of medullated fibres which arising from the apical processes of the large cells in the cornu ammonis, as its first layer (nuclear—MEYNERT) is continued along the lower half of the first cortical layer of the upper limbic arc, and bending over on to the upper surface of the hemisphere is almost immediately lost here. Beneath it lies a second shallower medullated band a little over half the depth of the former, and separated from it by the second layer of granule cells. This stripe also arises from the cornu ammonis, and likewise running parallel to the cortex terminates even earlier than the more superficial stripe, as it ends abruptly at the upper marginal angle of the hemisphere. In such as we are now dealing with (fresh vertical sections), both these stripes may readily be seen by a hand lens running as delicate white streaks parallel to the surface of the cortex of the upper limbic arc, being lost beyond the upper marginal angle, whence no similar white streak can be detected until we arrive opposite the great mass of the projection system which here takes a lateral position. Here again we observe a delicate medullated stripe in the first cortical layer running towards the limbic sulcus, but lost prior to its arrival at that spot. This stripe is even shallower than the deeper arcuate stripe in the upper limbic arc.

Region of expansion of central olfactory fasciculus.—It will be remembered that the termination posteriorly of the inferior limbic arc is characterised by the large size of the nerve-cells of its second cortical layer—the *modified olfactory type.* If *horizontal* sections of the occipital end of the frozen brain be carried through this region, the following facts may be readily confirmed. Most internally is the curved border of the cornu ammonis; next to it comes the granule region, extended from the upper limbic arc; still further outwards is the peculiar region alluded to; and beyond its boundary (the limbic sulcus) is the curved surface of the extra-limbic portion of the hemisphere.

To the *naked eye* a great dissimilarity is at once observed betwixt the appearance of the extra-limbic and the limbic portion. The former exhibits a medulla which has a slight translucent aspect and is of a pale tint, but markedly contrasted with the *perfectly opaque and brilliant white colour* of the limbic medulla. The line of demarcation, again, betwixt these two is sharp and abrupt, and extends from the depths of the corpus callosum directly outwards to the limbic sulcus. Microscopic examination clearly shows this difference in colour to be due to the *vast preponderance* of medullated nerve-fibres in the limbic region at this site, which radiate outwards to the cortex from the terminal portion of the callosal commissure. In the extra-limbic portion of the hemisphere we likewise observe medullated fibres which radiate outwards to the cortex along its whole extent, and proceeding solely from the corpus callosum do not form such dense aggregations, but light radiating fibres some distance apart. In the limbic portion, however, the same fibres are present, plus a strong reinforcement from the posterior extensions of the anterior commissure, *i.e.*, the central olfactory fasciculus. Here then appears demonstrated an important fact, that the cortex of the modified olfactory type (Plate 49, fig. 1, T to W), receives medullated radiations so extremely rich in fibres that it is mapped out to the naked eye from the neighbouring extra limbic medulla by its notable white brilliant aspect, and that these fibres include the terminal expansion of the central olfactory fasciculus.

Here again we meet with *two intra-cortical arciform stripes* of medulla disposed as in the modified upper limbic arc, viz.:—

(*a*) A superficial streak at the lowest confines of the first layer.

(*b*) A deeper streak beneath the large swollen cells of second layer.*

The superficial stripe is, however, here the shallower of the two, and the deeper stripe attains a width equal to the first band in the upper limbic arc. In *horizontal sections* the medullated fibres of these two streaks will be divided obliquely. In sections such as I have just been considering a rich series of arcuate fibres will also be found extending backwards through the *extra-limbic lobe* from the anterior realms of the brain, which thin out gradually towards the abrupt line of denser medulla, commencing at the limbic sulcus, and wholly disappear here. This is the arcuate system of the extra-limbic medulla.

The above facts enable me to conclude that at the occipital pole of the hemisphere there are two important regions definitely mapped out by coarse external configuration of the brain and also by minute structural divergences.

These regions are histologically distinguished by the possession of a cortex, which is peculiar to each, and which we have named respectively the modified upper and lower limbic types. Each region is associated with that of the opposite hemisphere by the callosal commissure.

Each region is brought into relationship with the olfactory lobe and bulb; the upper limbic by means of the arcuate olfactory, the lower limbic through the medium of the central olfactory fasciculi after the decussation in the posterior commissure.

Each region is moreover supplied with the peculiar double stripe of medullated fibres, forming arcuate systems within the cortex itself.

Hence we have in these regions a reflection backwards of the olfactory sensory surface brought strongly into association with the cornu ammonis by means of the arcuate intra-cortical stripes.

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* The site of these intra-cortical bands of medulla is approximately given in Plate 50, fig. 6, *a* and *b*.

Depth of Cortical Layers on a Plane with Genu of Corpus Callosum.

	Gyrus fornicatus. mm.	Sagittal angle. mm.	Extra-limbic. mm.	Near limbic sulcus. mm.
First layer . . .	·372	·372	·348	·279
Small pyramids . .	·604	·744	1·023	·558
Granule belt . . .	. .	. .	·372	. .
Ganglionic belt . .	·232	·232	·186	·408
Spindle cells . . .	·511	·883	·883	1·069

Depth of Cortical Layers on a Plane with posterior border of Corpus Callosum.

	Gyrus fornicatus. mm.	Sagittal angle. mm.		Extra-limbic. mm.
Peripheral zone . .	·441	·325		·372
Granule layer . .	·418	·372	Small pyramids Granules . .	·372 ·372
Ganglionic layer . .	·465	·604		·604
Spindle cells . . .	·372	·604		·744

Depth of Cortical Layers of the Modified Lower Limbic Type. (Rabbit.)

	mm.
Peripheral cortical zone	·279
Inflated cell formation	·883
Ganglionic layer	·372
Pale belt.	·279
Spindle cells	·418 to ·655

Depth of Cortical Layers in the Cornu Ammonis. (Rabbit.)

	Peripheral zone. mm.	Striate and lacunar layers. mm.	Ganglionic cells. mm.	Molecular layer. mm.	Medullated layer. mm.
Anterior regions	·325	·930	·093	·523	·395
Average at six different sites.	·424	·915	·112	·397	·385

Average depth of whole cortex of ammon's horn =2·233 mm.

DIMENSIONS of Nerve-cells in Brain of Rabbits.

UPPER LIMBIC ARC.

	Second layer of small pyramids.		Ganglionic layer.	
Near frontal pole	$17\mu \times 11\mu$	Nucleus 9μ	$24\mu \times 17\mu$	$N=10\mu$
In front of corpus callosum .	$17\mu \times 12\mu$	$N=9\mu$	$24\mu \times 17\mu$	$N=13\mu$
Anterior regions of ,, .	$15\mu \times 13\mu$	$N=8\mu$	$32\mu \times 18\mu$	$N=13\mu$
Posterior regions of ,, .	$16\mu \times 11\mu$	$N=8\mu$	$27\mu \times 16\mu$	$N=13\mu$
Behind corpus callosum .	$13\mu \times 9\mu$	$N=8\mu$	$26\mu \times 16\mu$	$N=13\mu$

Granule cells $=10\mu \times 8\mu$ $N=5\mu$.

GYRUS HIPPOCAMPI.

Pyramidal cells.		Fusiform ganglionic.	
$19\mu \times 12\mu$	$N=9\mu$	$31\mu \times 13\mu$	$N=13\mu$

MODIFIED LOWER LIMBIC CORTEX.

	Second layer of inflated cells.		Third or pyramidal cells.	
From different sites	$37\mu \times 32\mu$	$N=13\mu$	$23\mu \times 13\mu$	$N=13\mu$
	$37\mu \times 23\mu$	$N=13\mu$		
	$46\mu \times 27\mu$	$N=13\mu$		

OUTER OLFACTORY CORTEX.

	Second layer.		Ganglionic layer.	
Distant from bulbus olfactorius .	$19\mu \times 12\mu$	$N=9\mu$	$31\mu \times 13\mu$	$N=13\mu$
,, ,, ,, .	$18\mu \times 10\mu$	$N=9\mu$		
Beneath bulbus olfactorius . . .	$17\mu \times 11\mu$	$N=9\mu$		

EXTRA-LIMBIC OR PARIETAL CORTEX.

	Second layer.		Ganglionic layer.	
In front of corpus callosum .	$22\mu \times 12\mu$	$N=9\mu$	$32\mu \times 16\mu$	$N=13\mu$
Anterior regions of ,, .	$21\mu \times 13\mu$	$N=9\mu$	$32\mu \times 16\mu$	$N=12\mu$
Posterior regions of ,, .	$17\mu \times 10\mu$	$N=8\mu$	Largest $60\mu \times 18\mu$	$N=13\mu$
			$31\mu \times 16\mu$	$N=12\mu$
Behind corpus callosum . .	$13\mu \times 11\mu$	$N=8\mu$	$24\mu \times 18\mu$	$N=12\mu$
Close to occipital pole . . .	$20\mu \times 11\mu$	$N=9\mu$	$31\mu \times 19\mu$	$N=13\mu$

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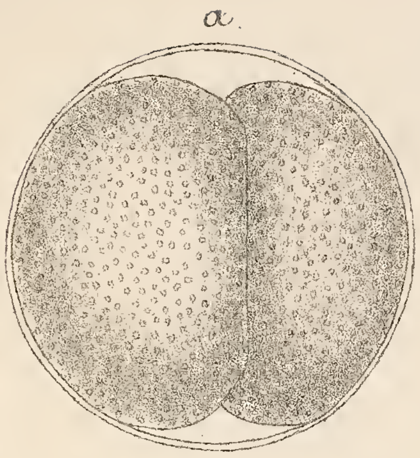

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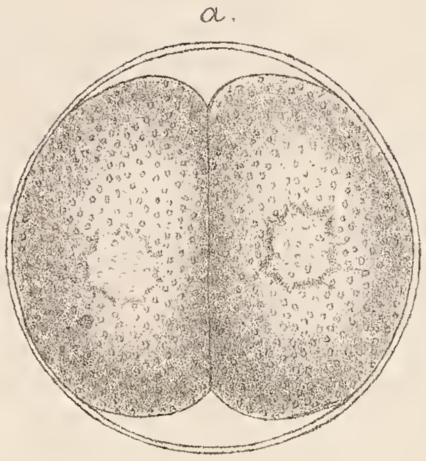

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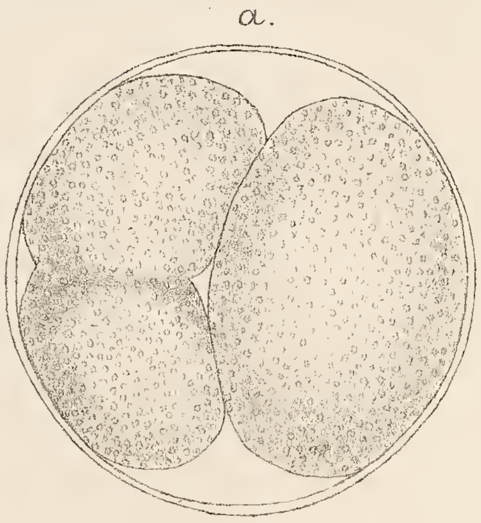

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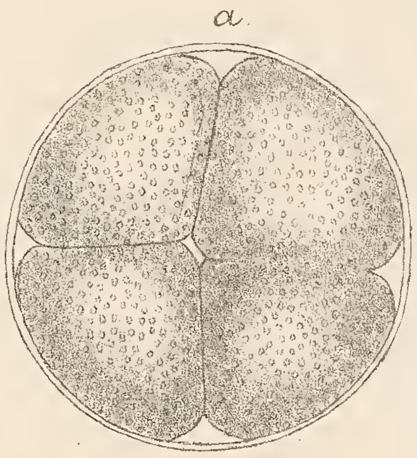

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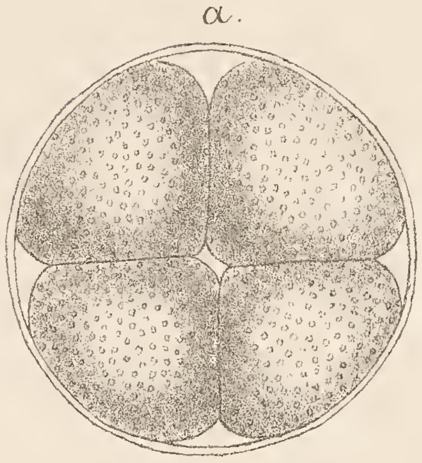

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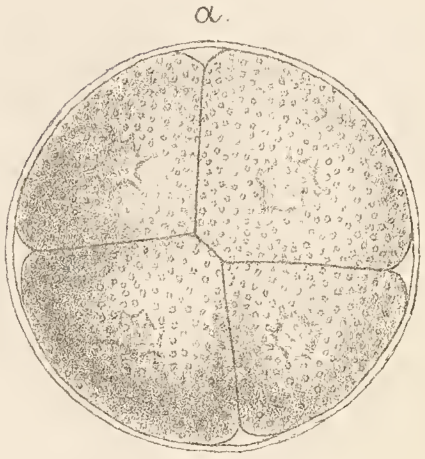

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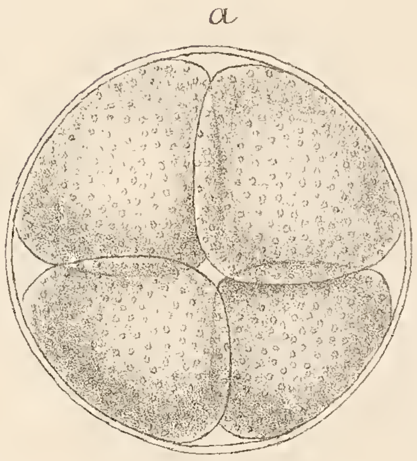

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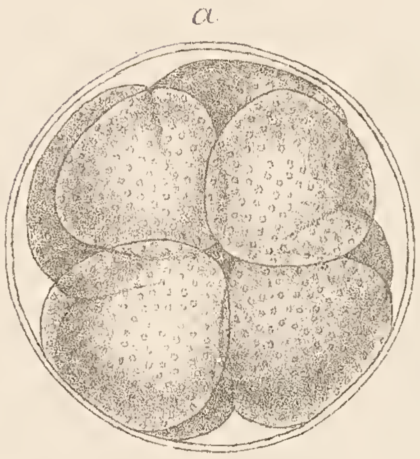

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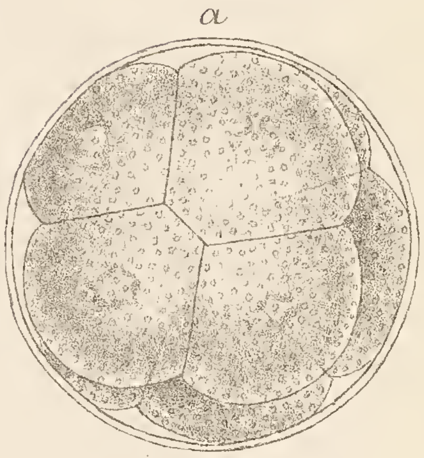

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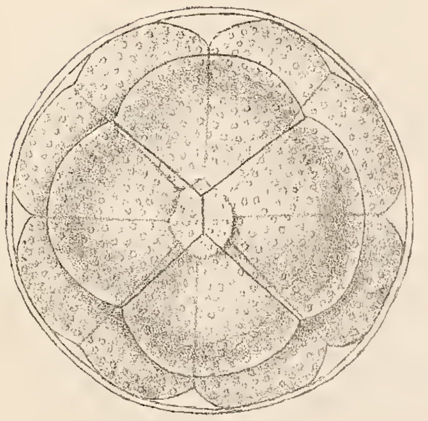

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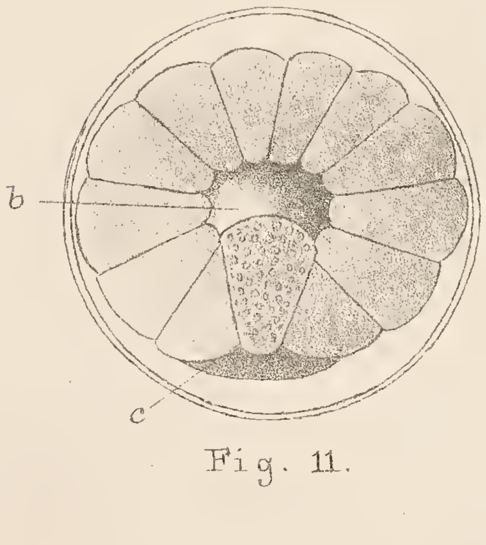

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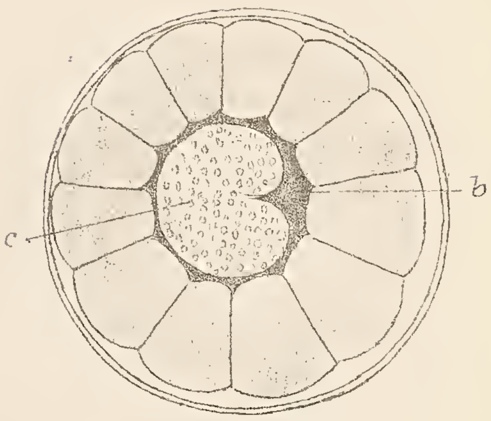
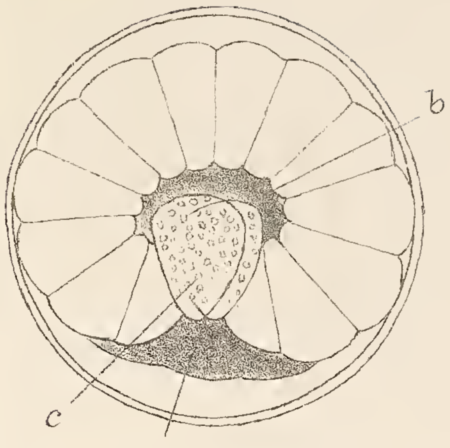


Fig. 12.

W.K.B. ad nat. del.

West Newman & Co. lith



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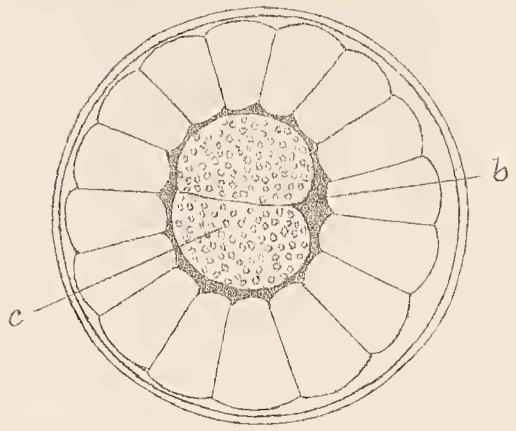

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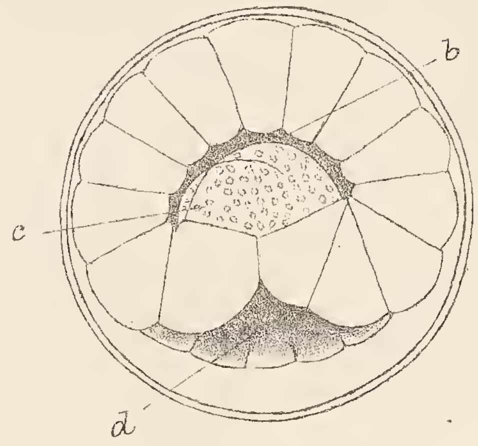

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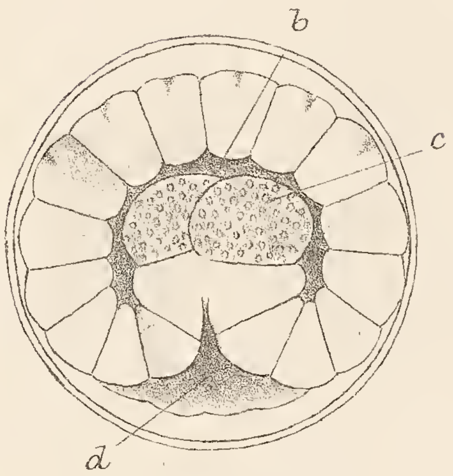

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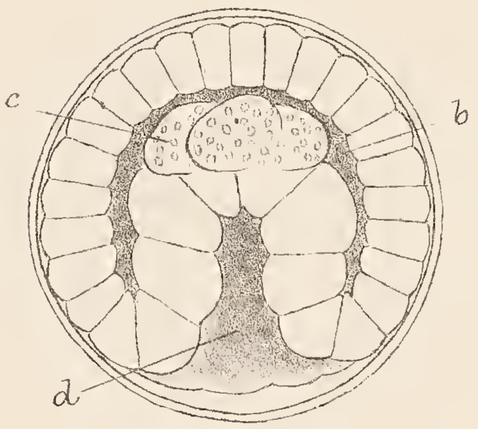

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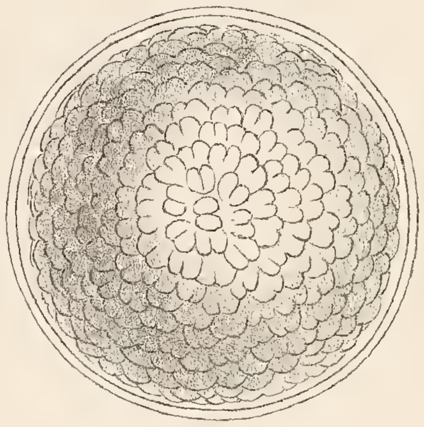

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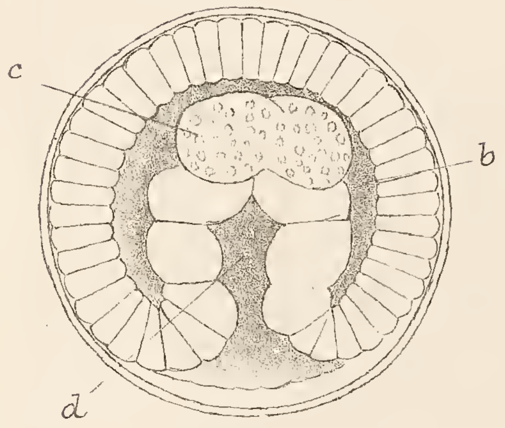

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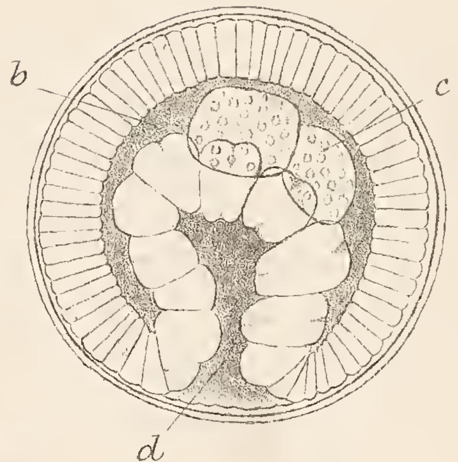

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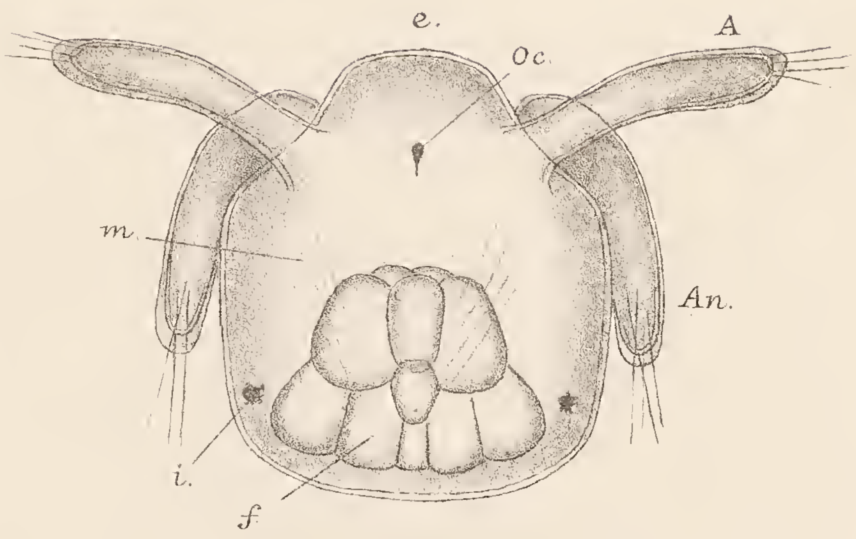

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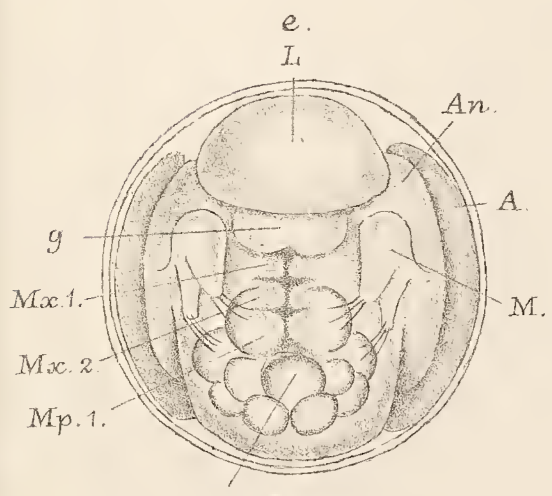

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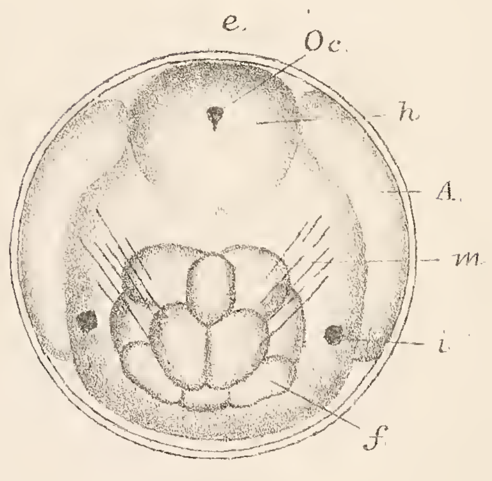
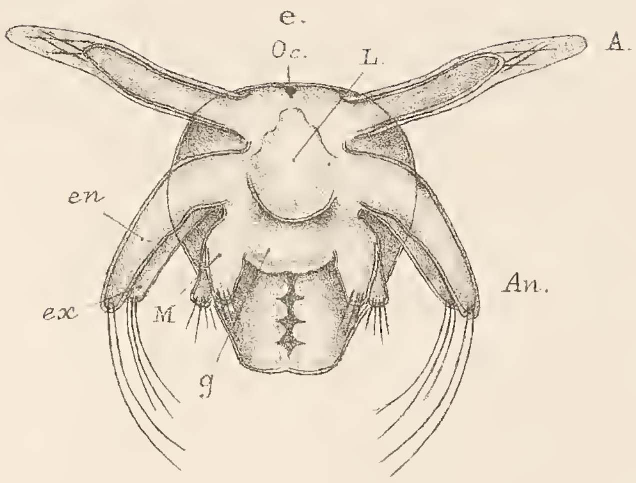


Fig. 22.

W. K. B. ad nat. del.

West Newman & C° lith.



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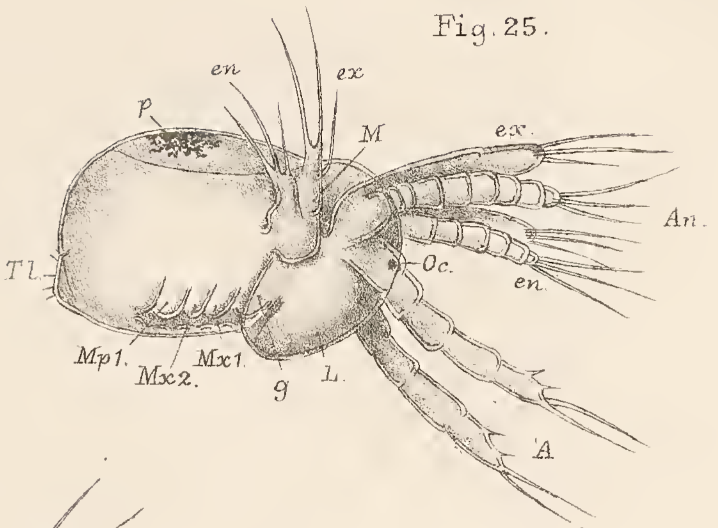

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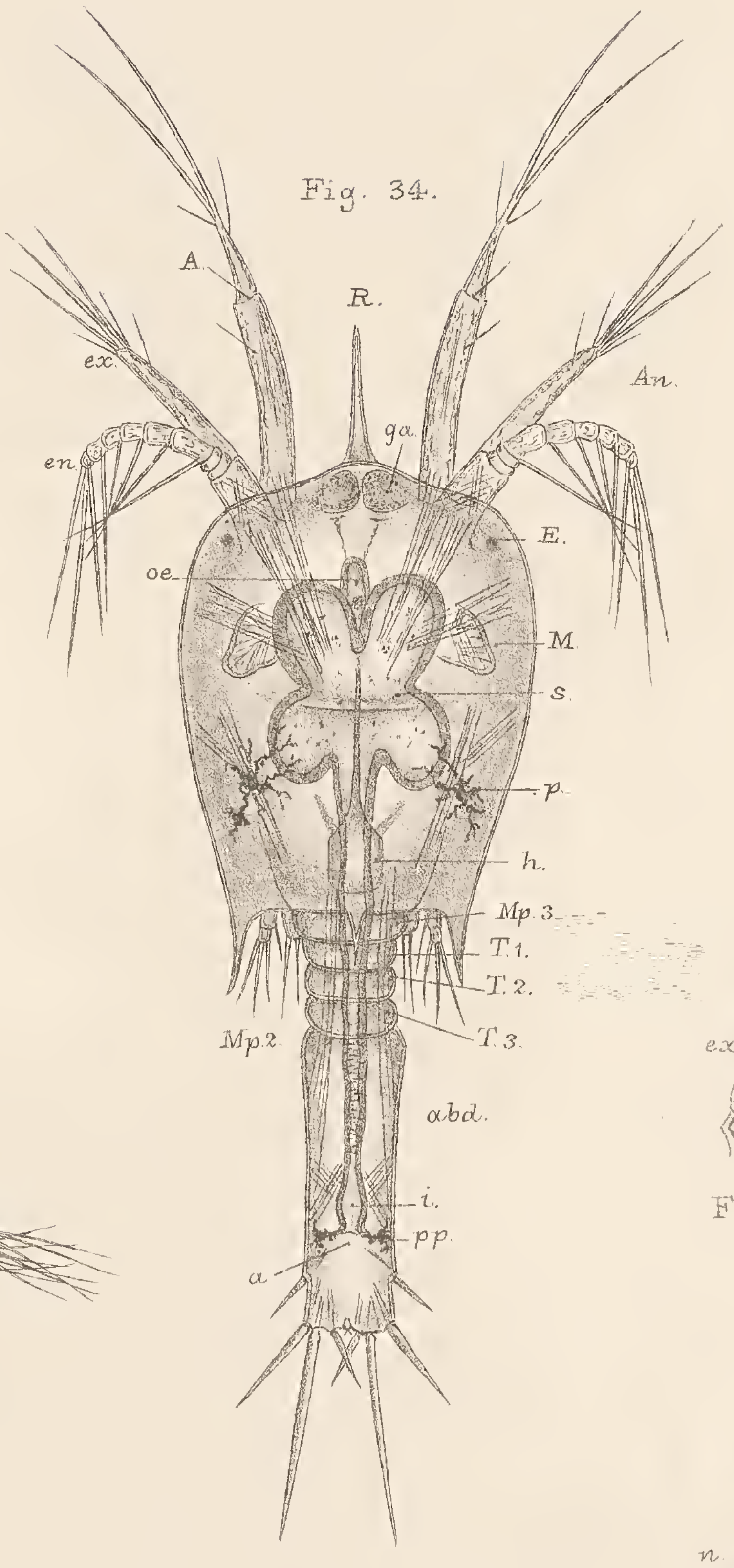

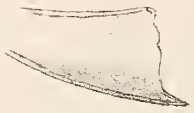

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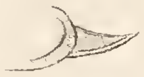

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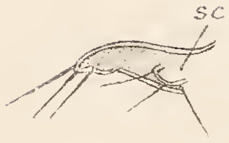
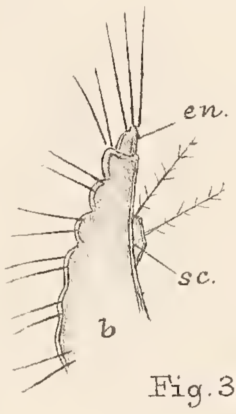


Fig. 30.



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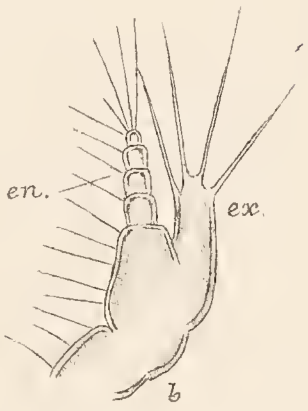

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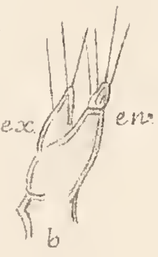


Fig. 33

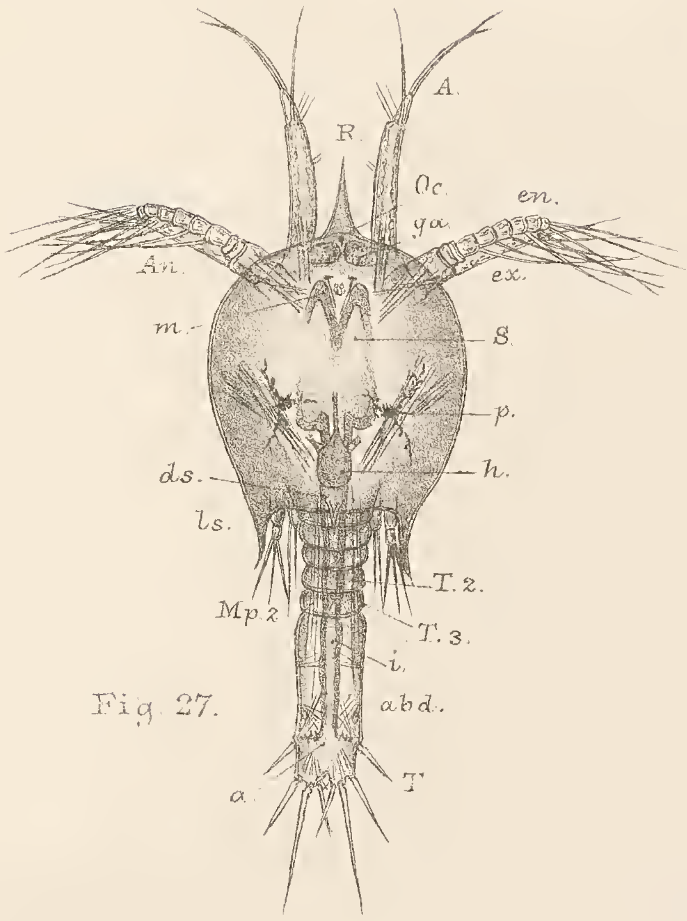
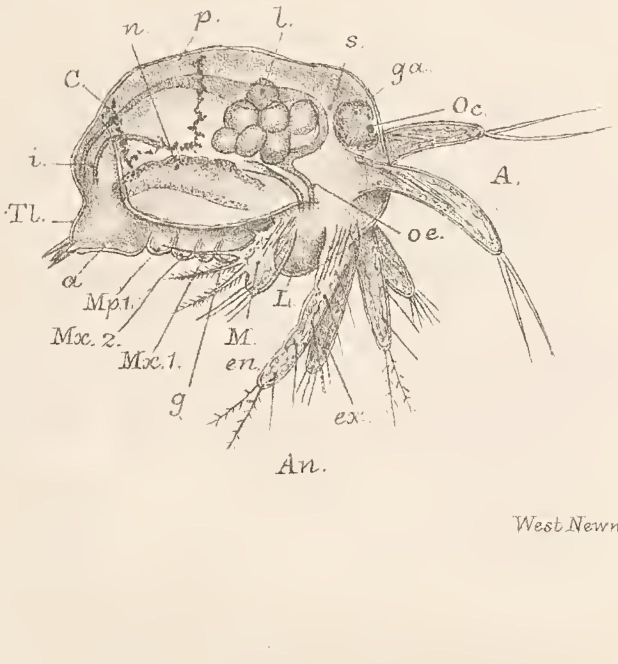


Fig. 27.

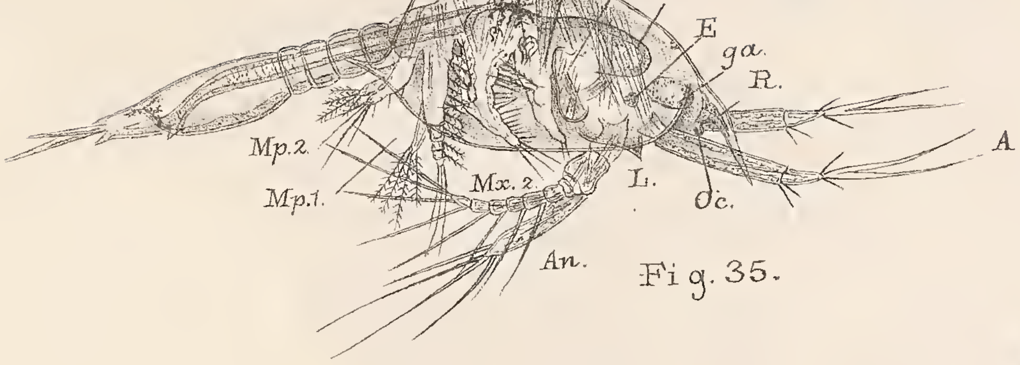
Fig. 26.



W. K. B. ad nat. del.

West Newman & Co. imp.

Fig. 35 a.



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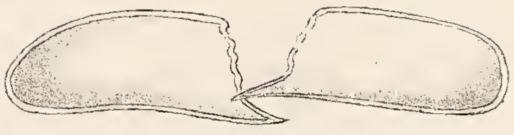

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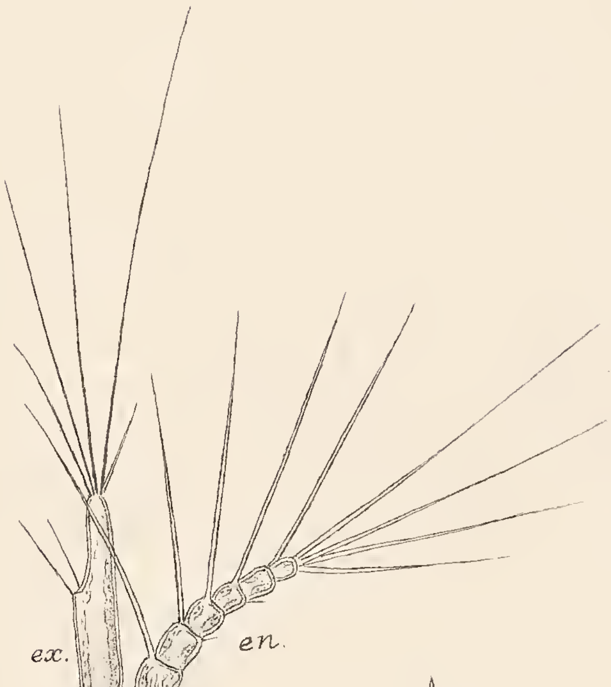

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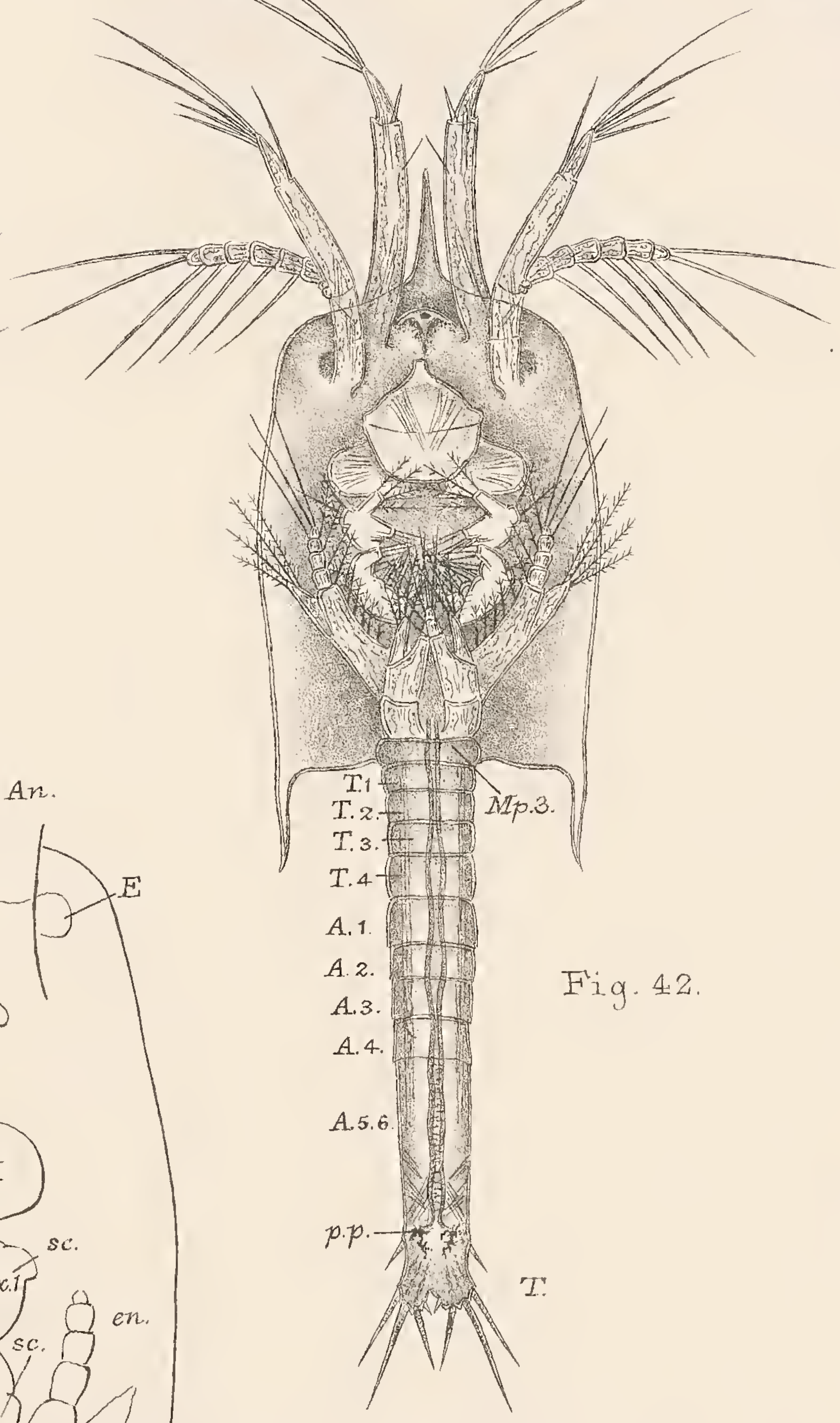

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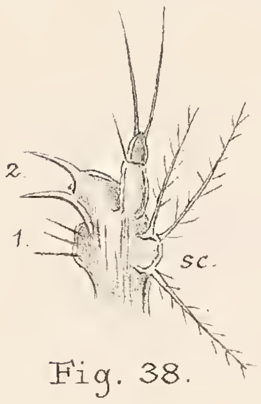

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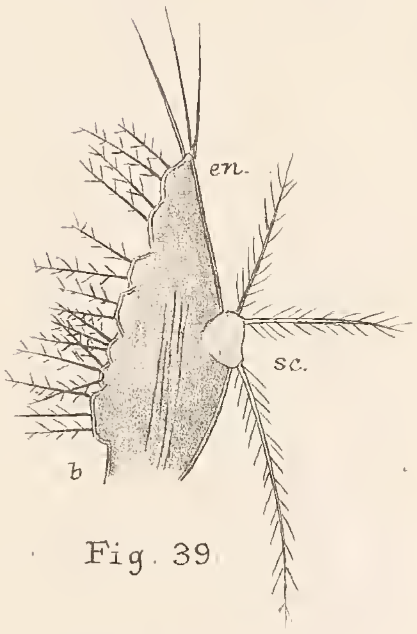


Fig. 39

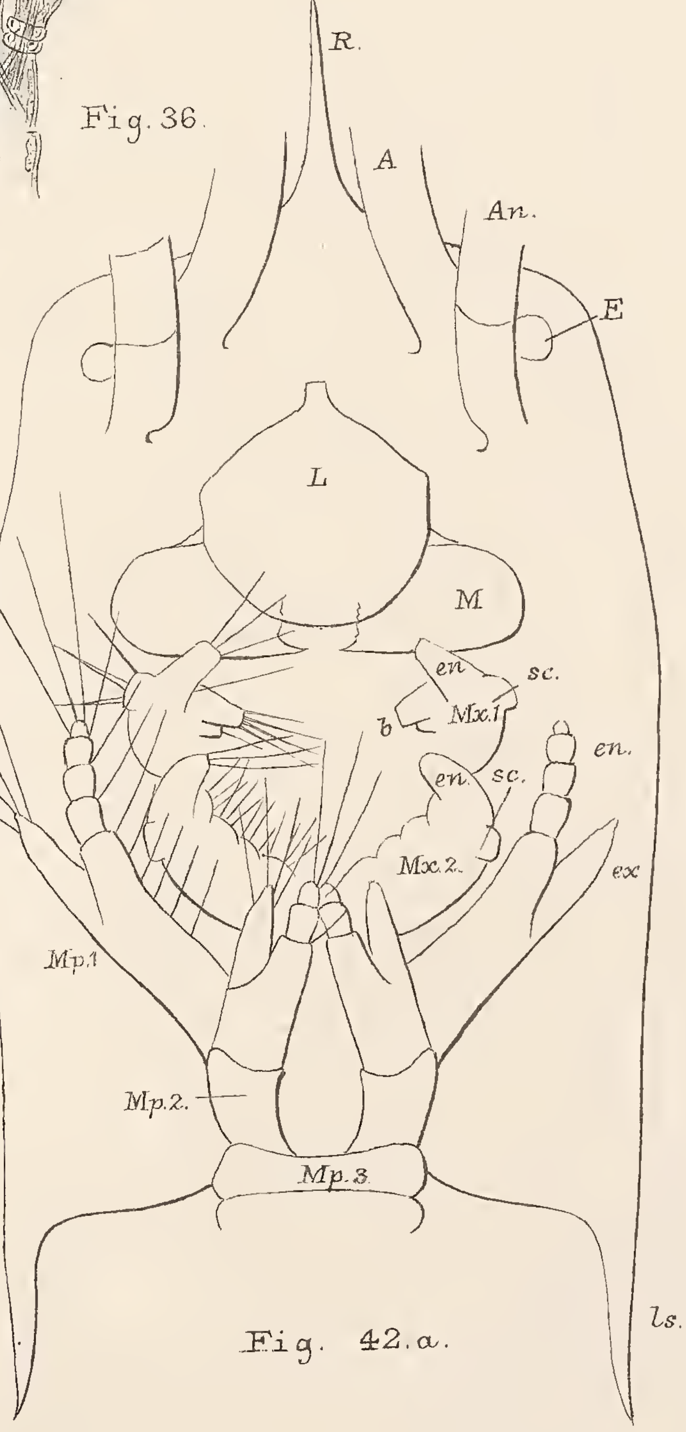
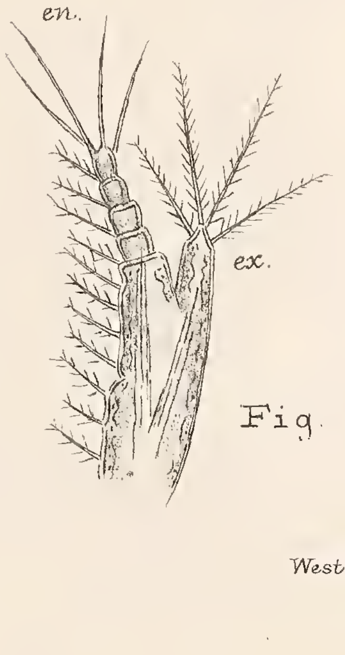


Fig. 42.a.



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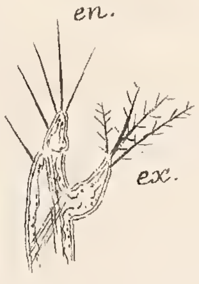
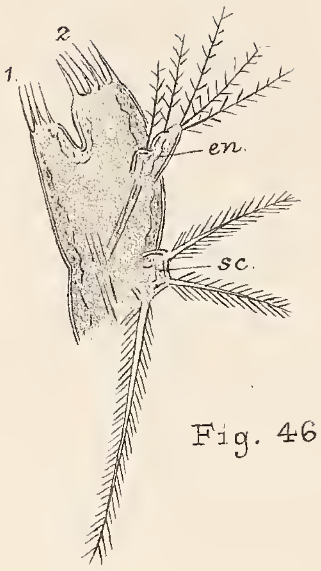


Fig. 41.

W. K. B. ad nat. del.

West Newman & Co. lith.



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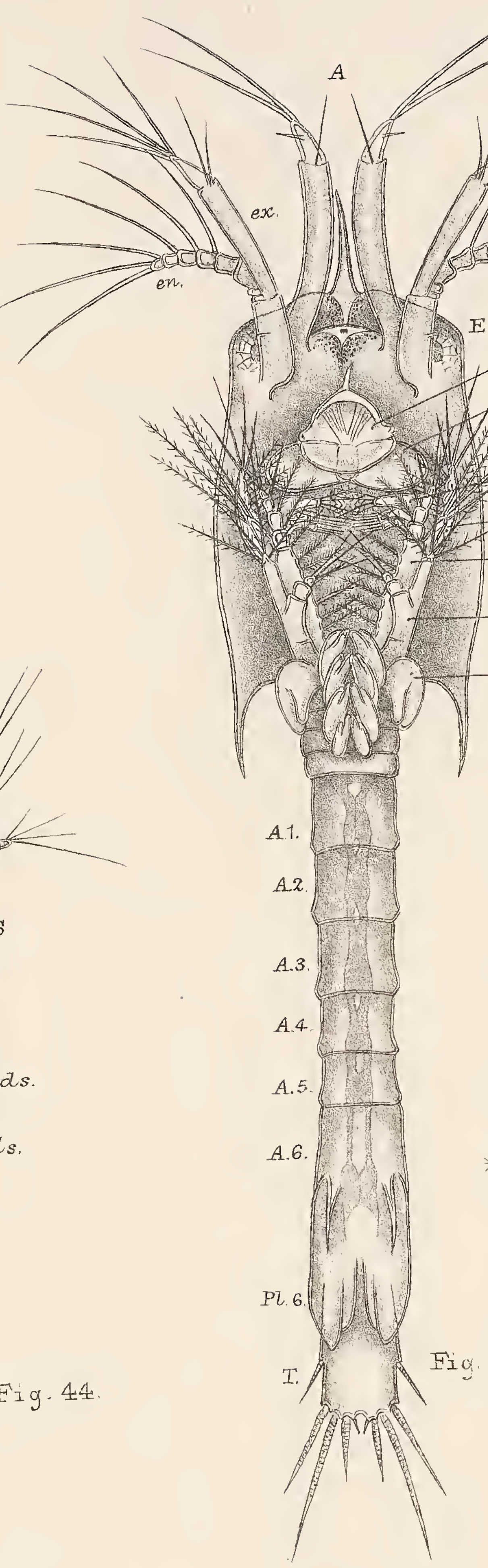

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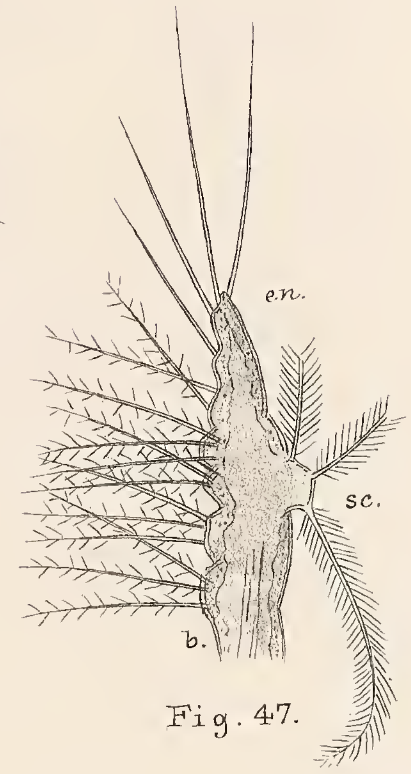

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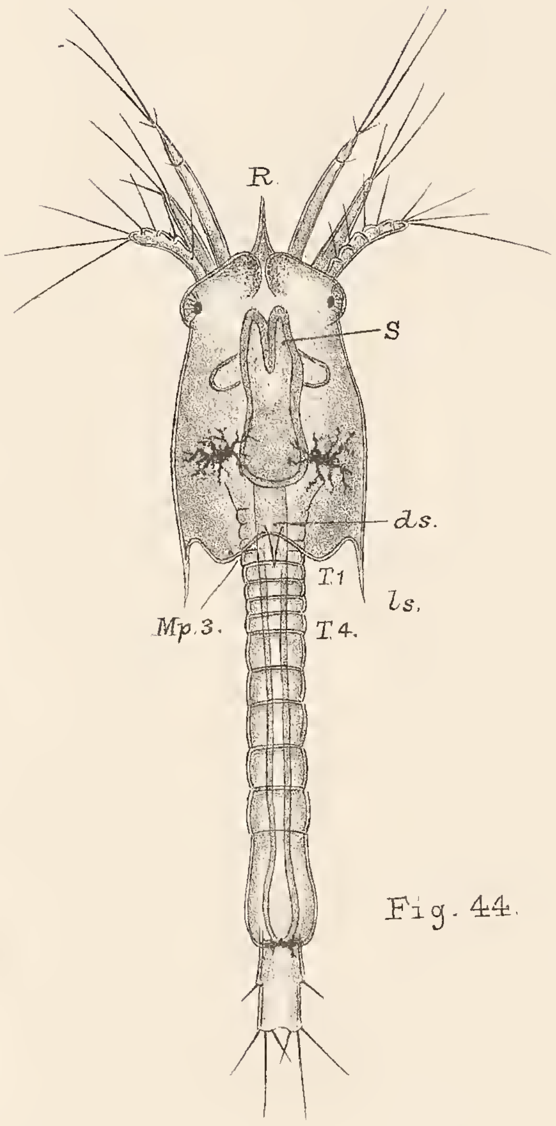

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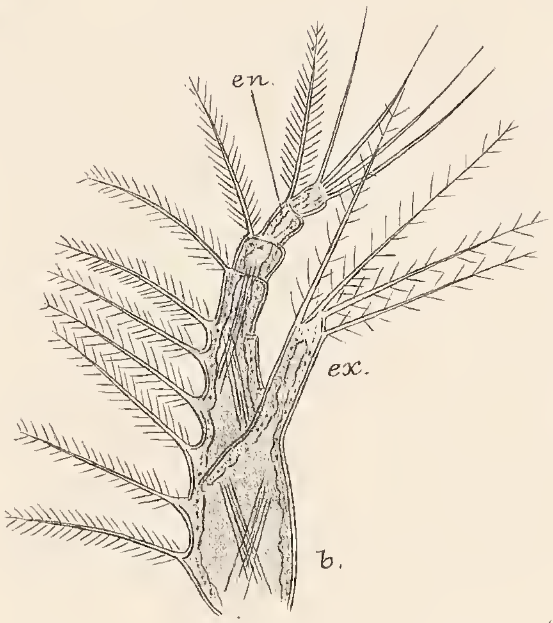

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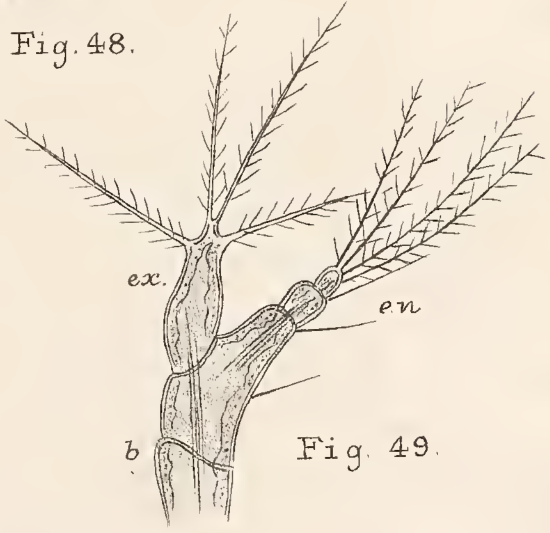

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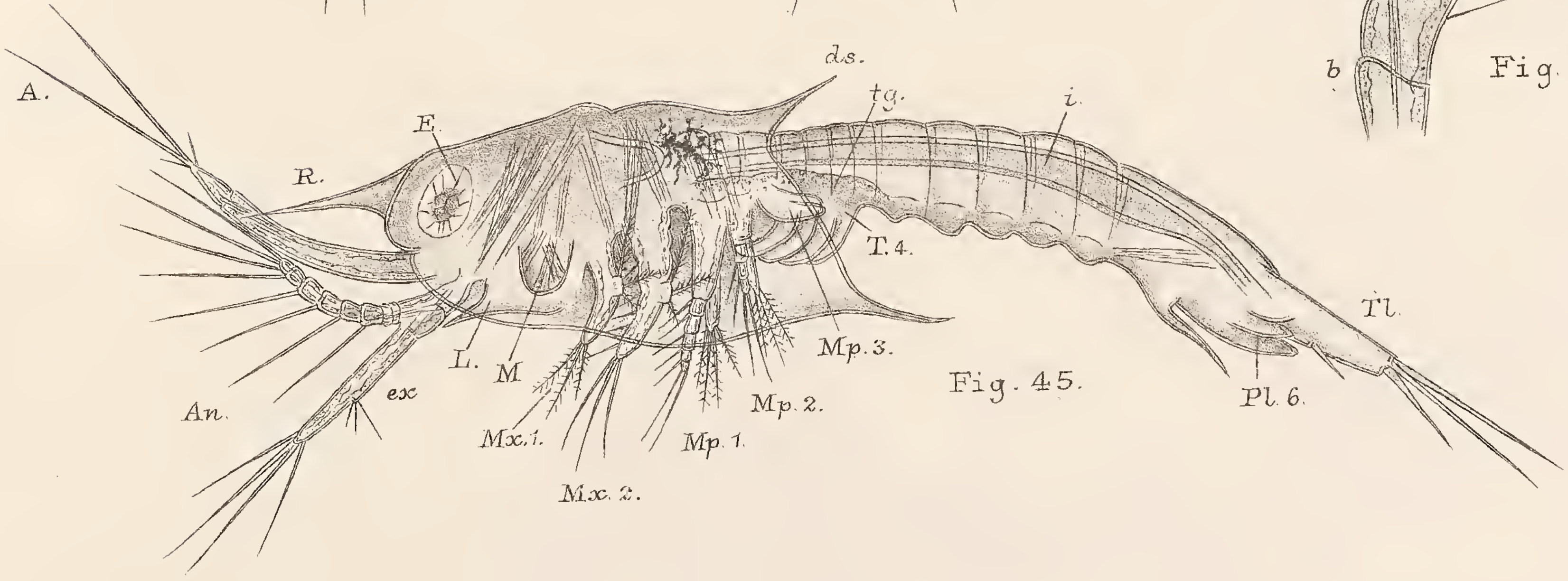
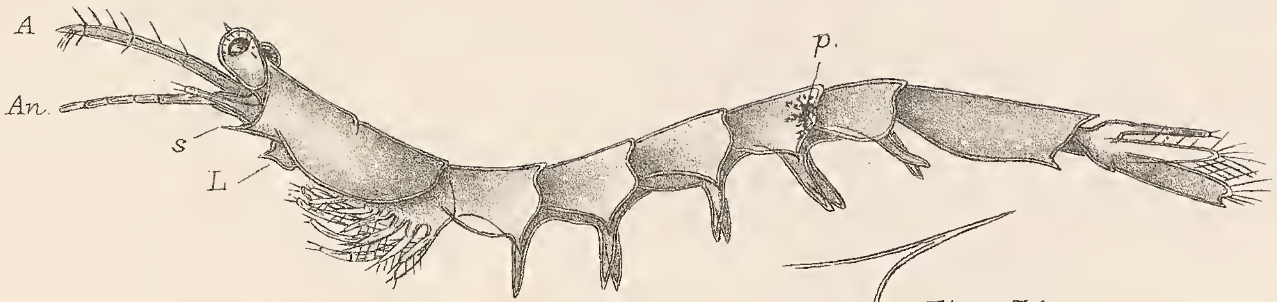


Fig. 45.

W.K.B. ad nat. del.

West Newman & Co. lith.



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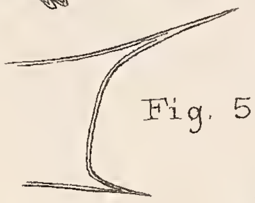
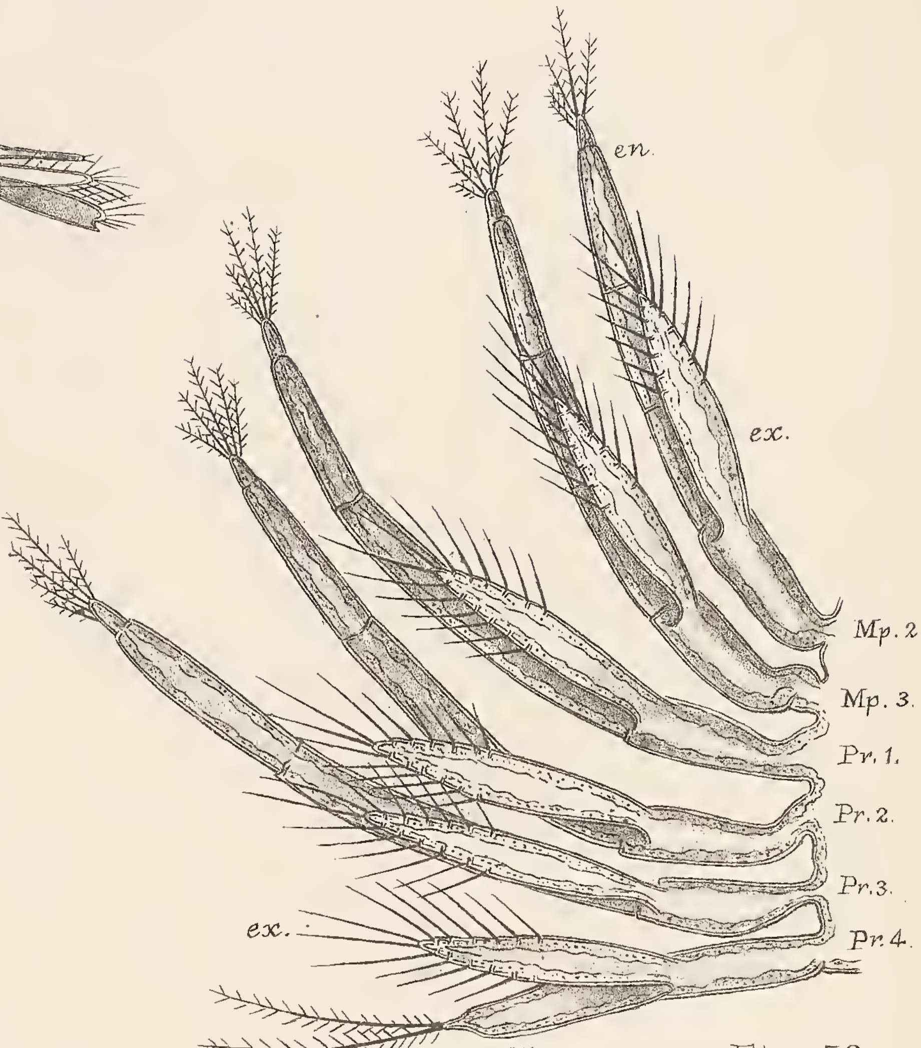


Fig. 54a.



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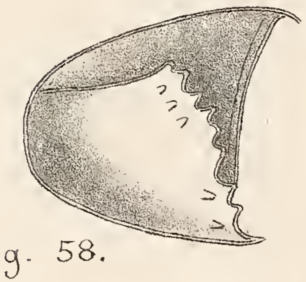

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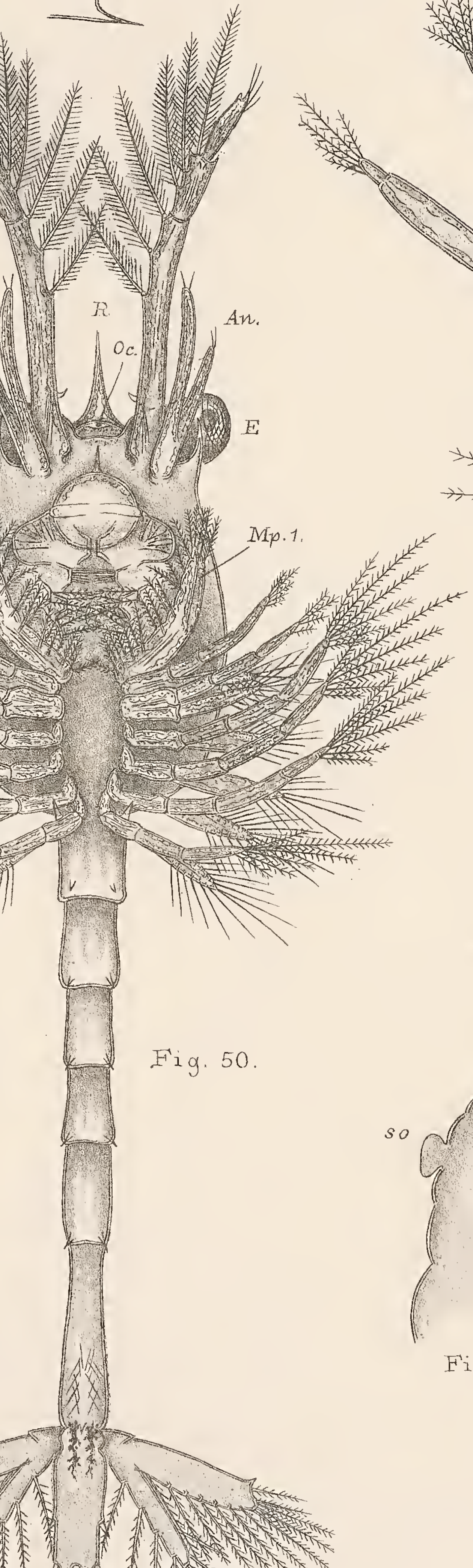

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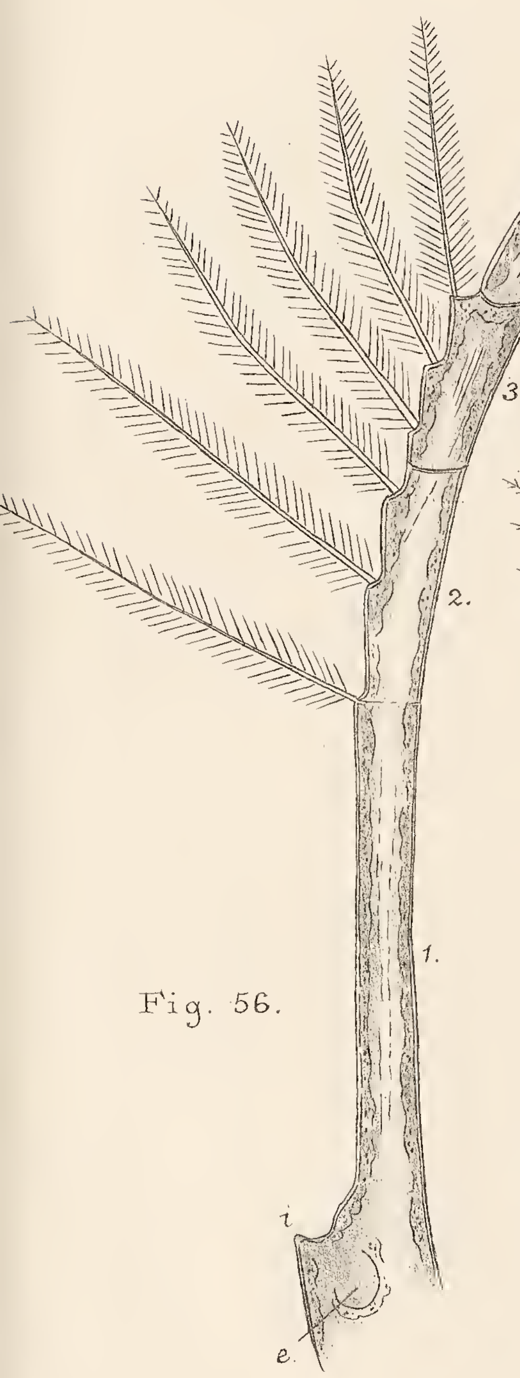

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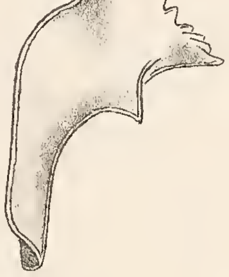

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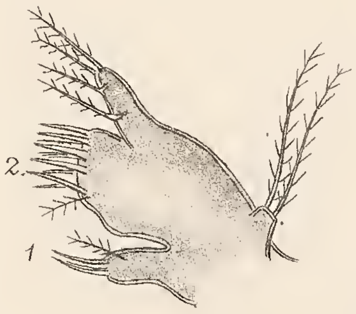

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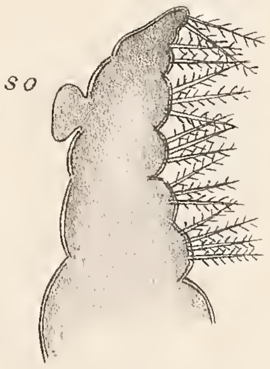

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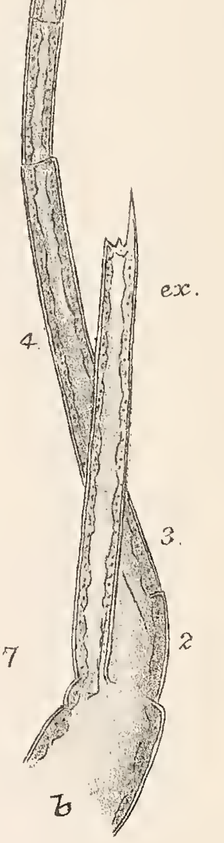


Fig. 57

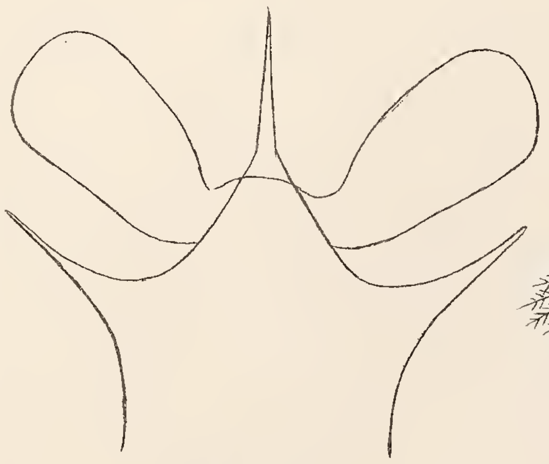


Fig. 55.

W.K.B. ad nat. del.

West Newman & Co. lith.

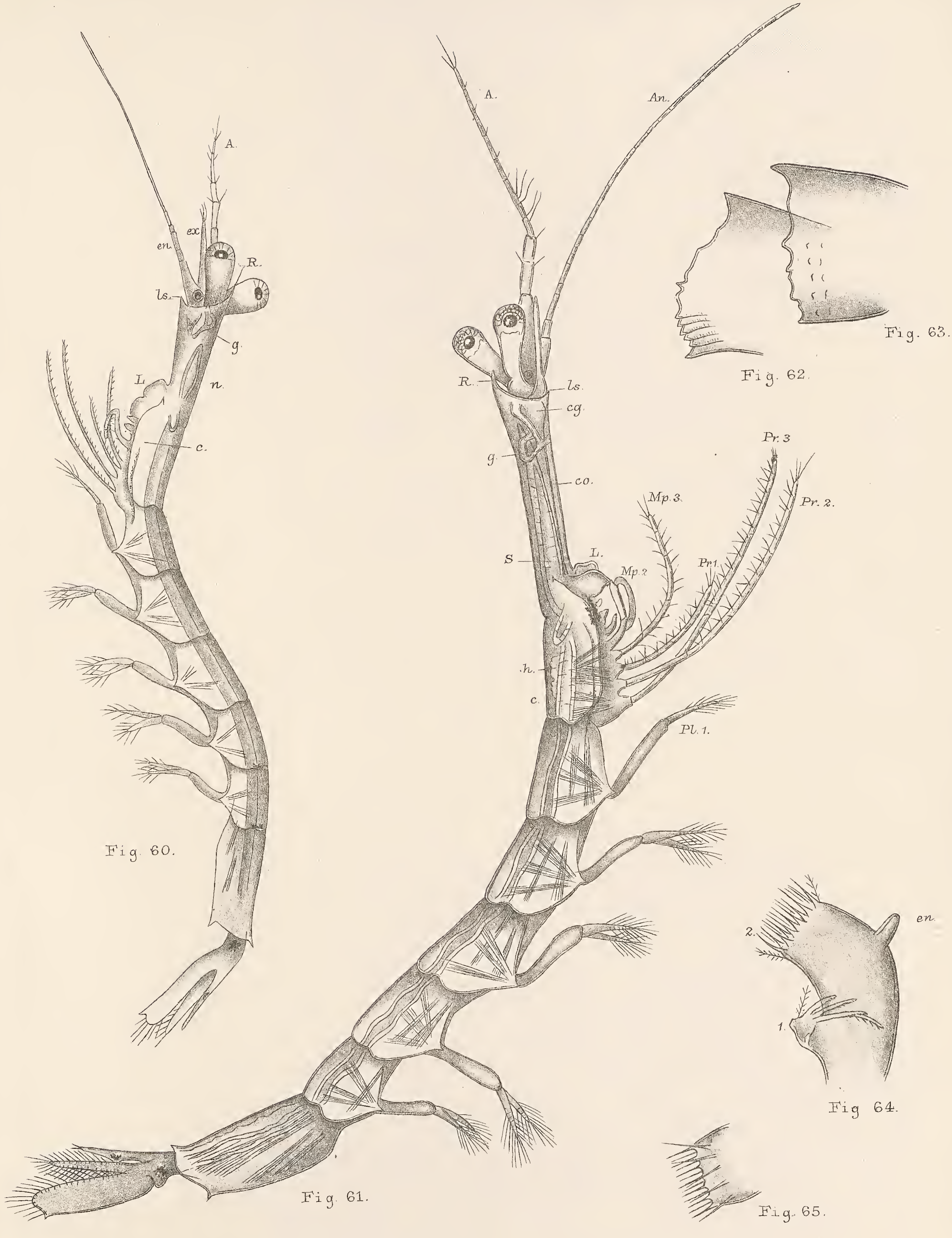


Fig. 60.

Fig. 61.

Fig. 62.

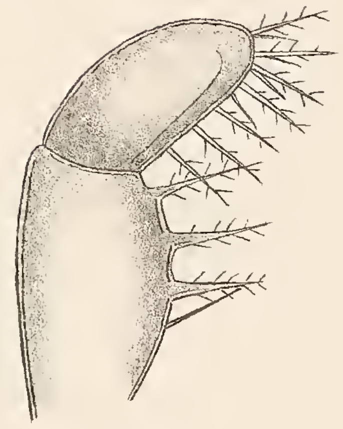
Fig. 63.

Fig 64.

Fig. 65.

W.K.B. ad nat. del.

West Newman & Co lith.



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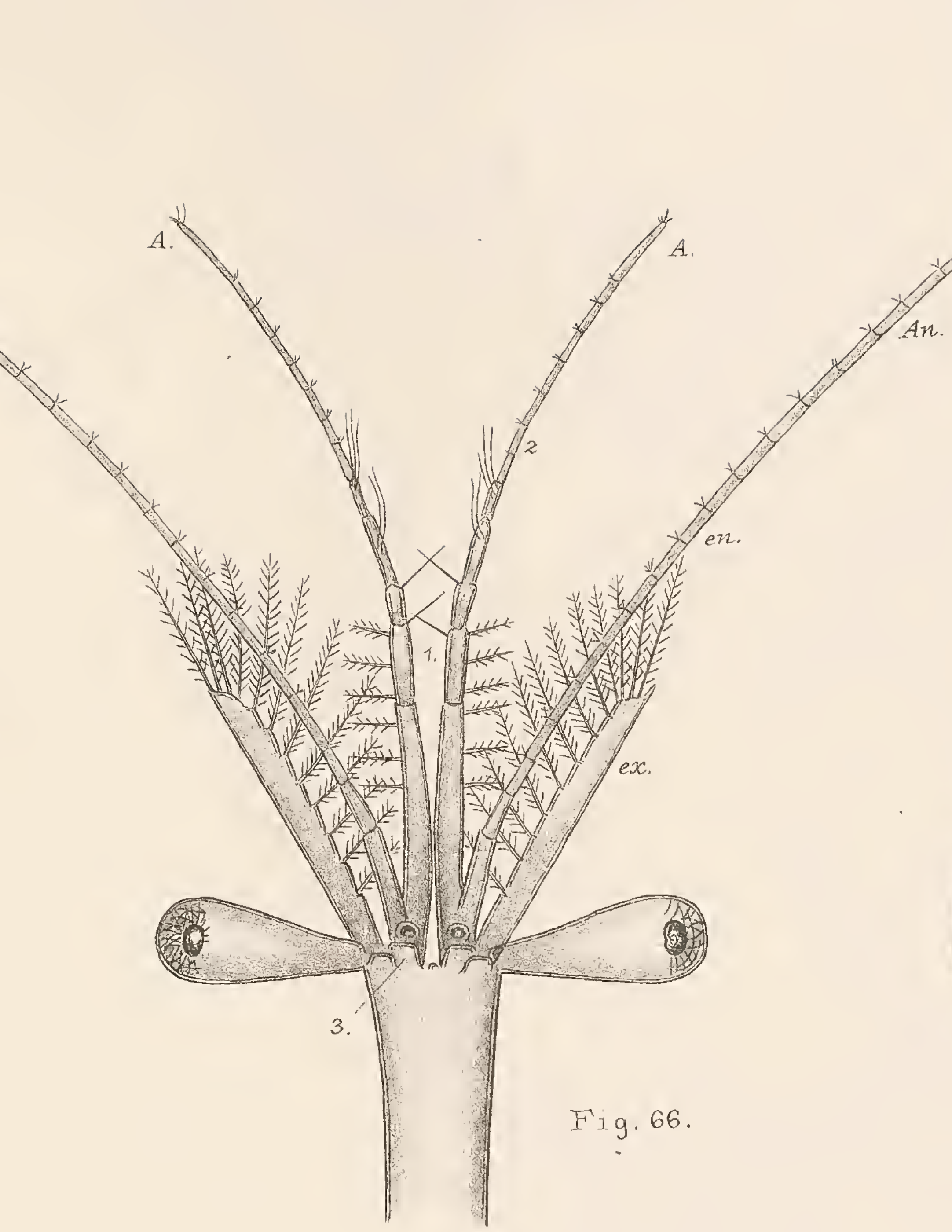

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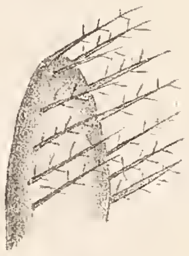

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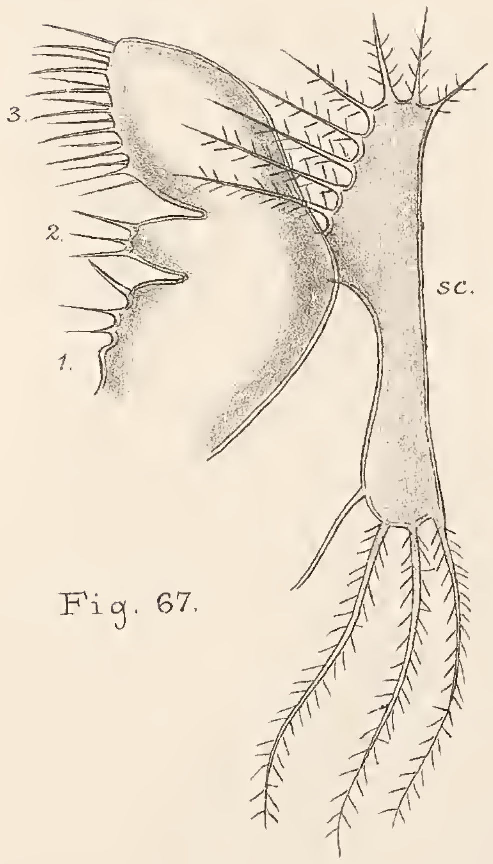

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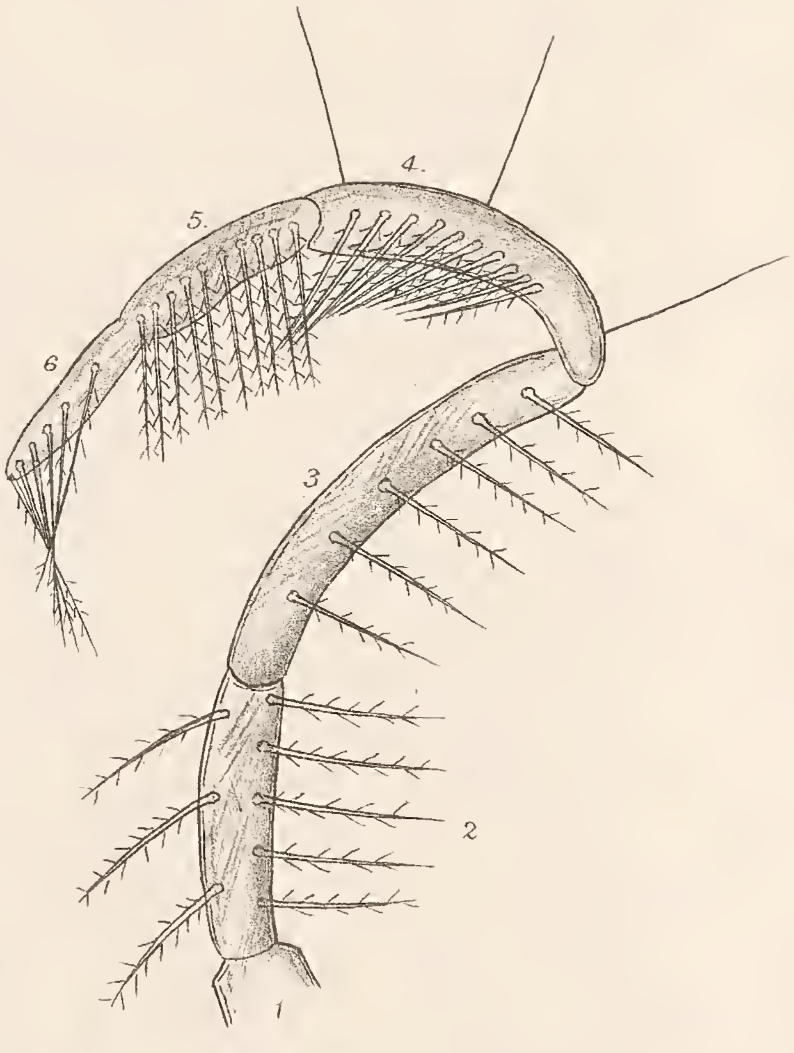
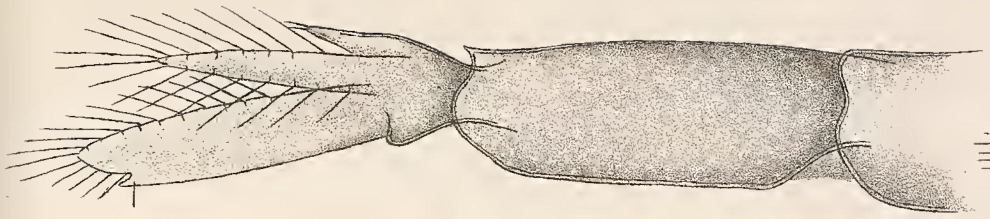


Fig. 70.

W. K. B. ad nat. del. West Newman & Co lith.



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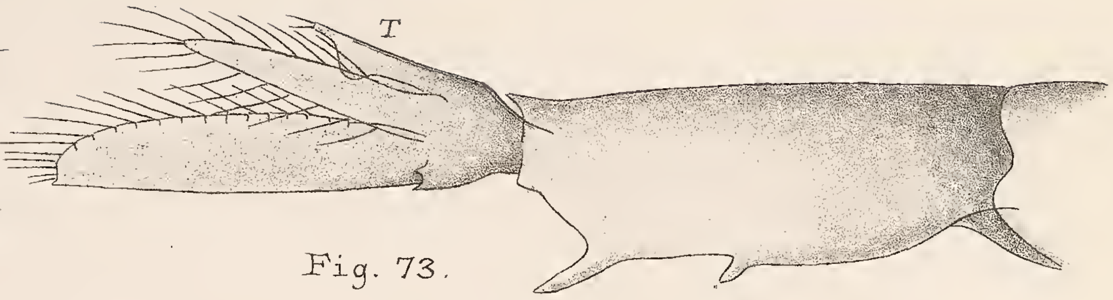

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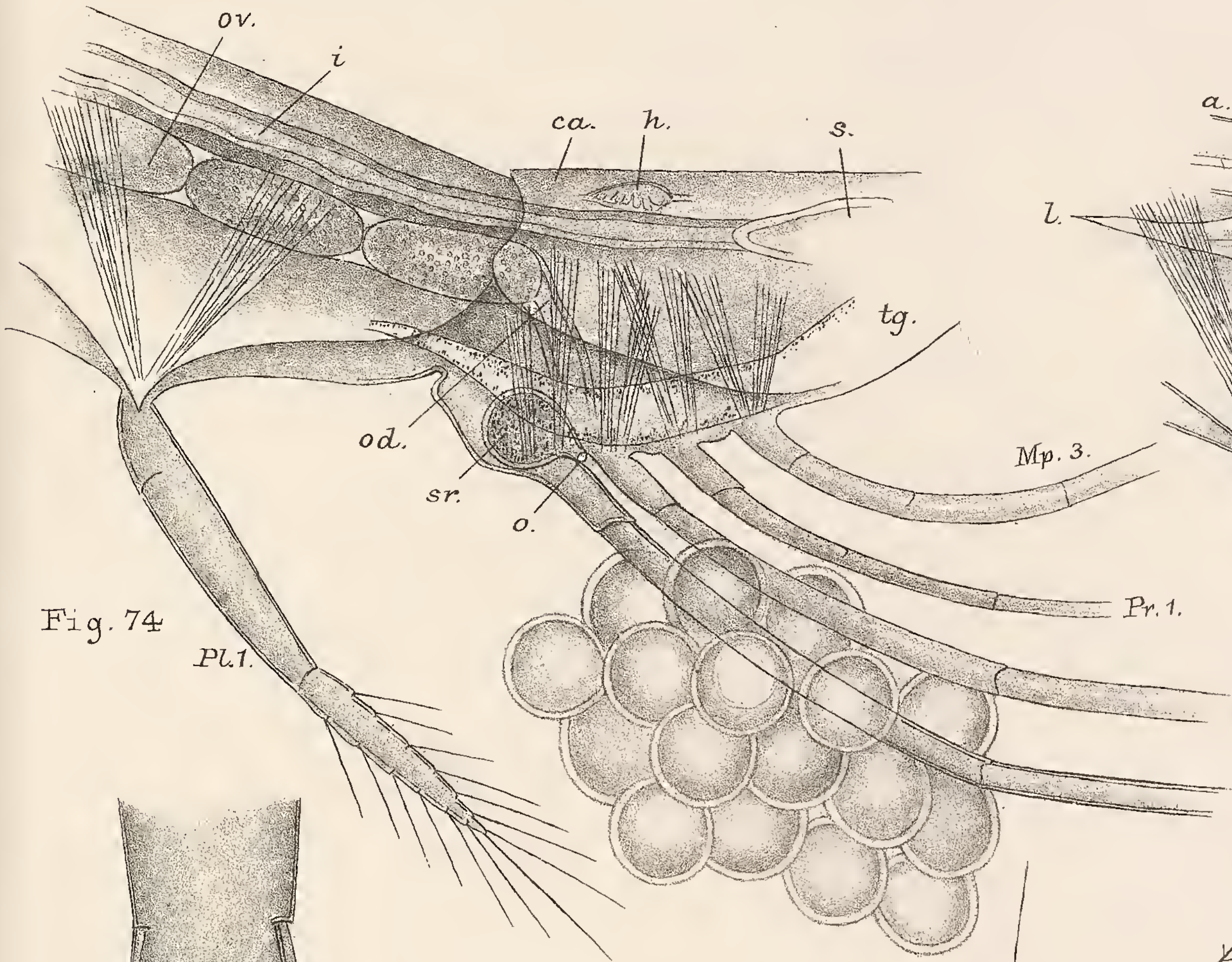


Fig. 74

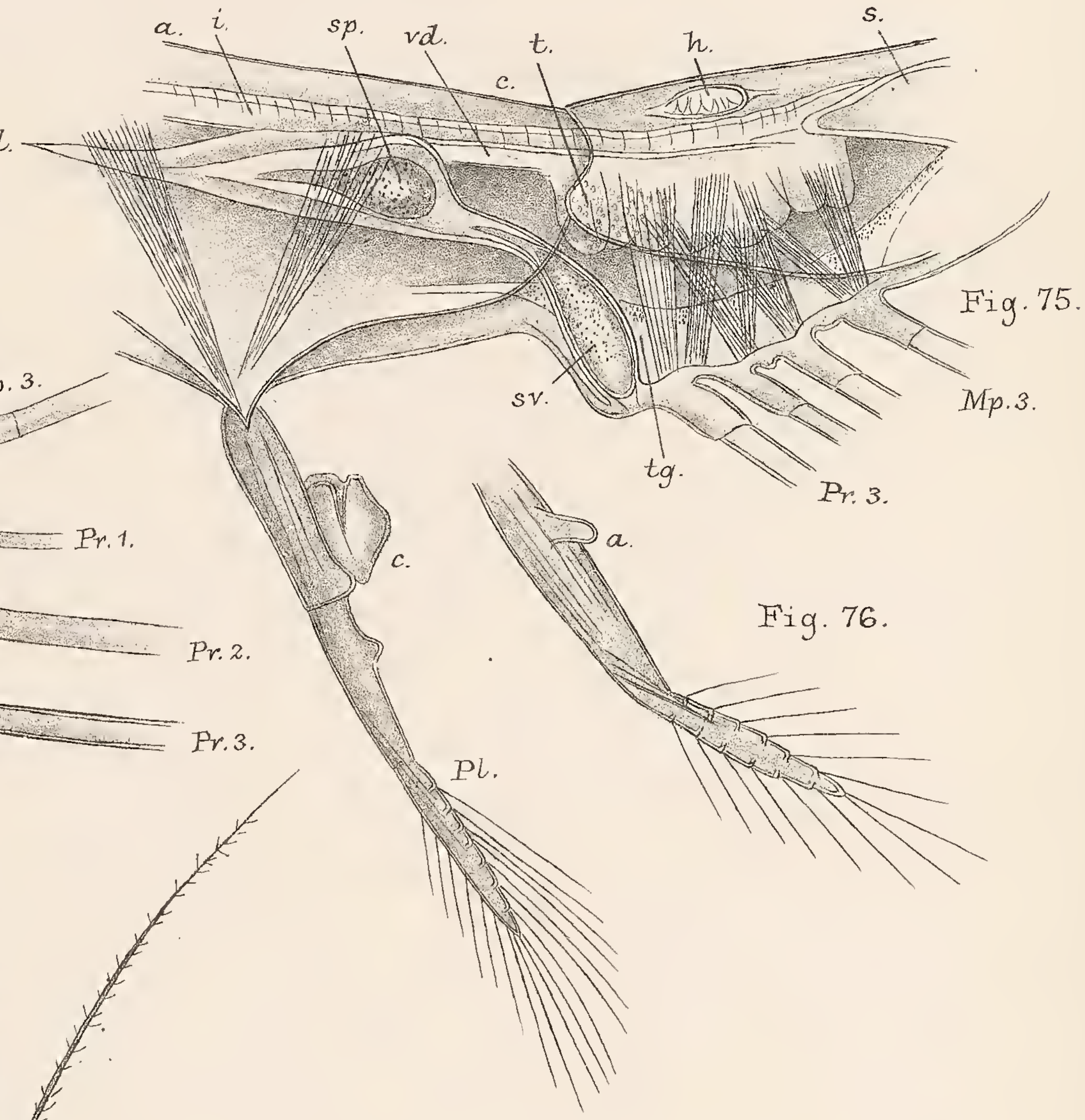
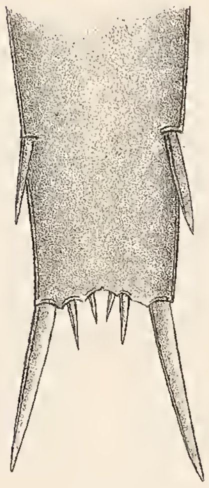


Fig. 75.

Fig. 76.



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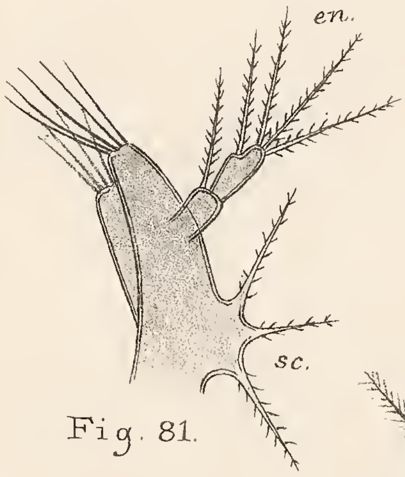

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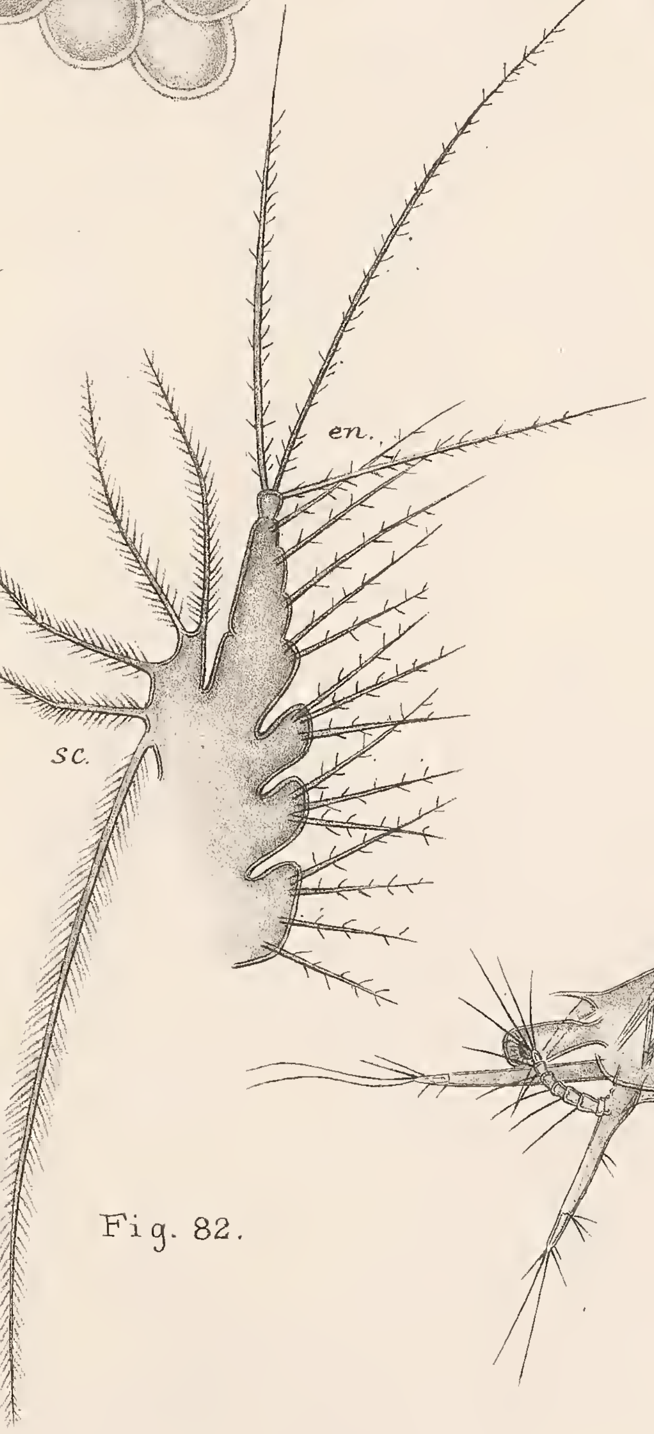
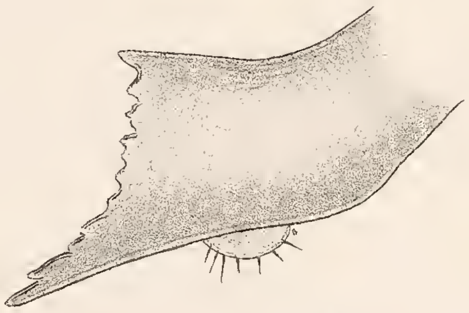


Fig. 82.



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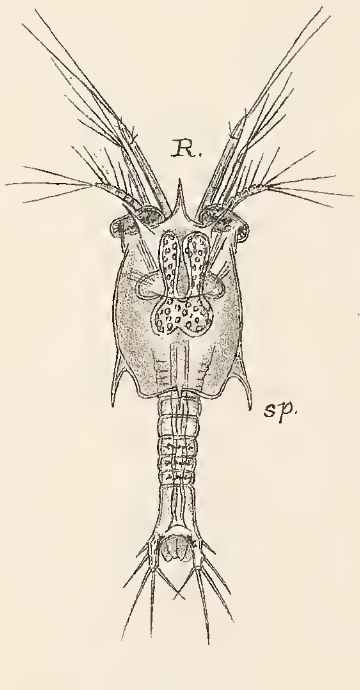

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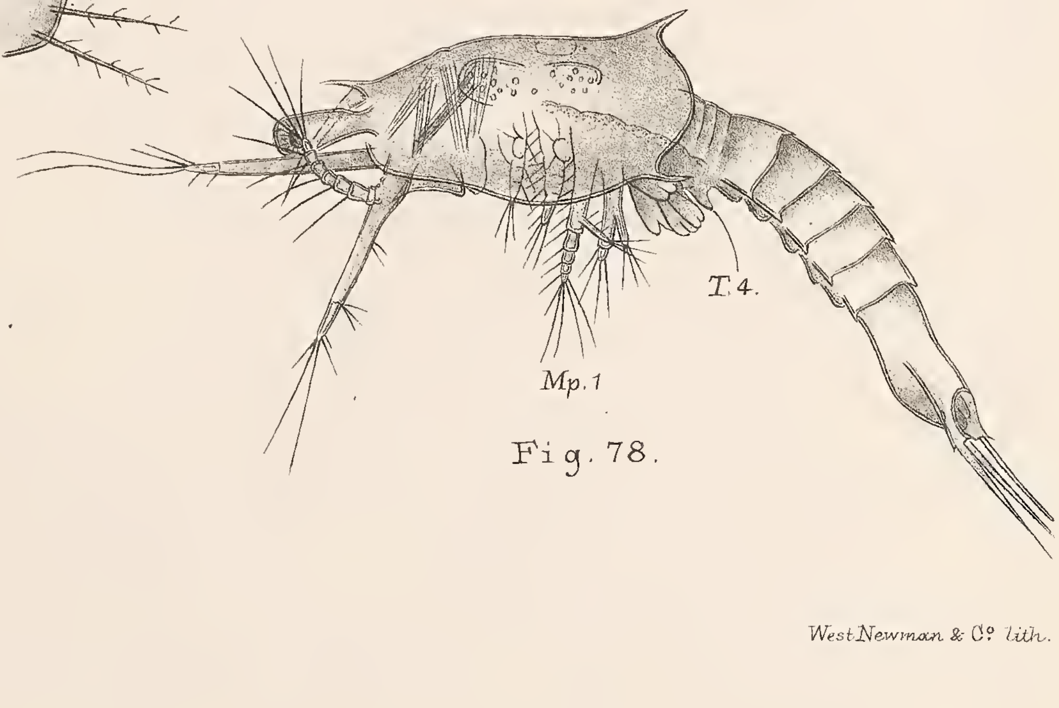


Fig. 78.

W.K.B. ad nat. del.

West Newman & Co. lith.

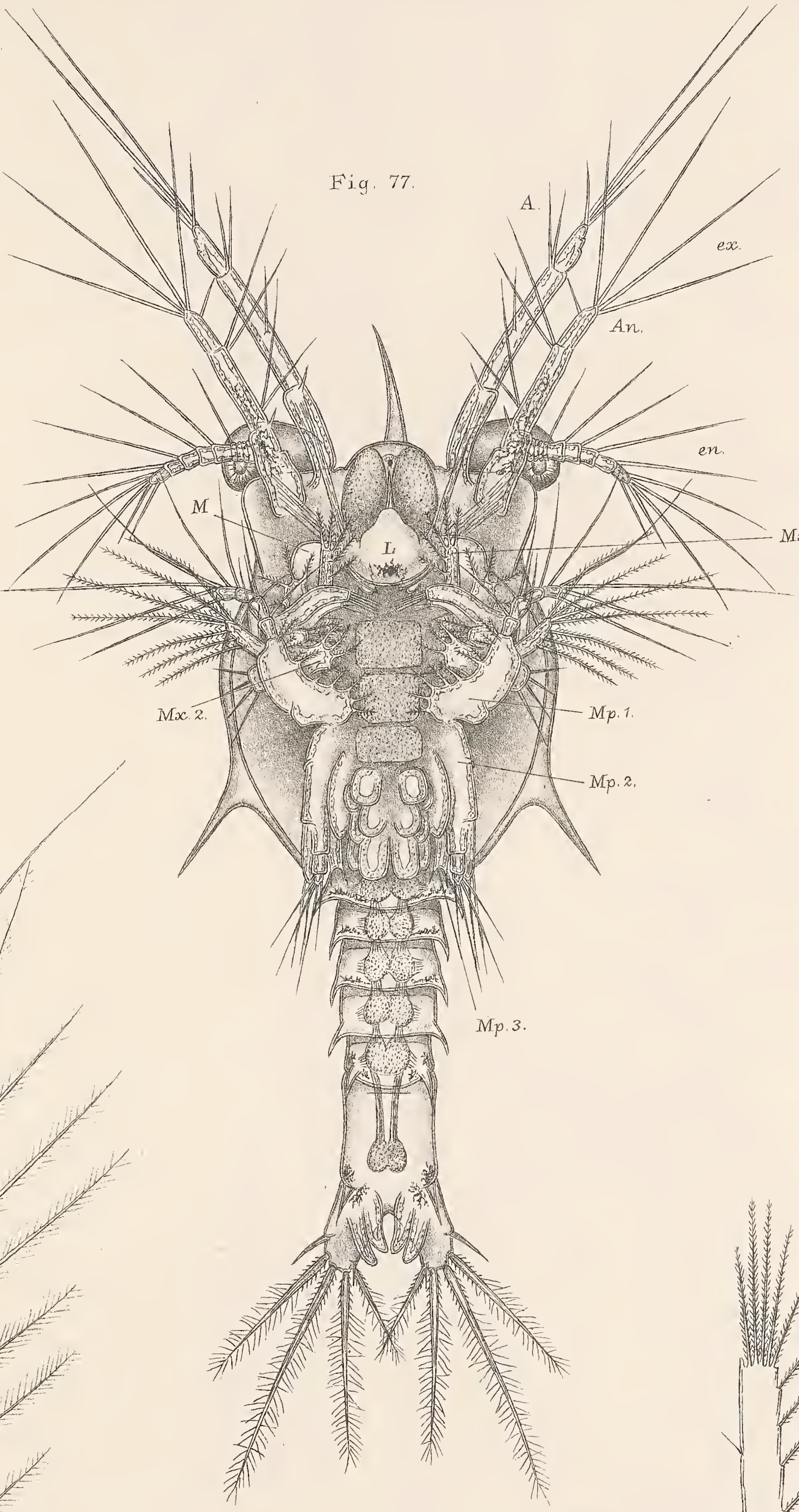
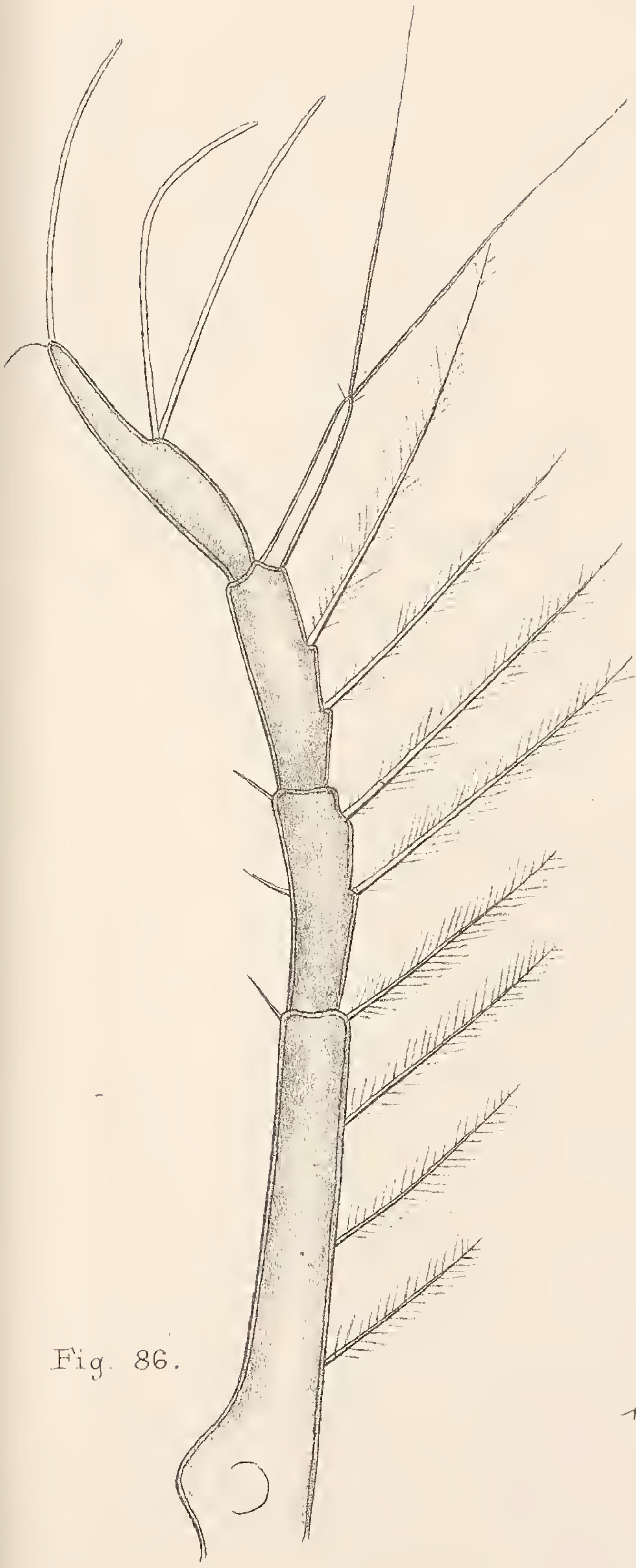


Fig. 77.



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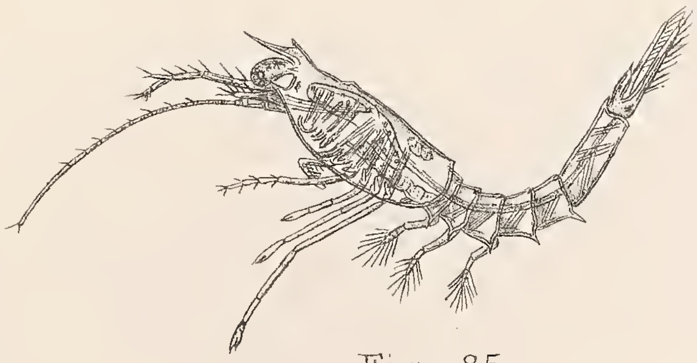

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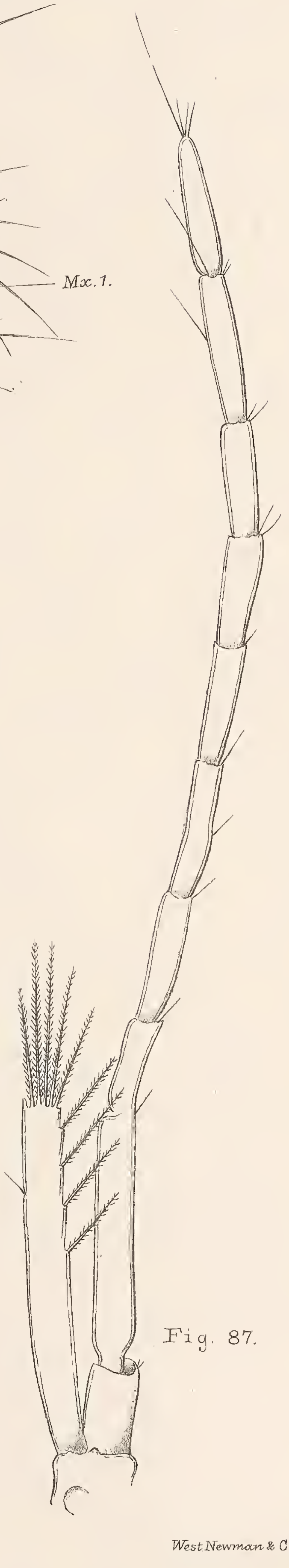
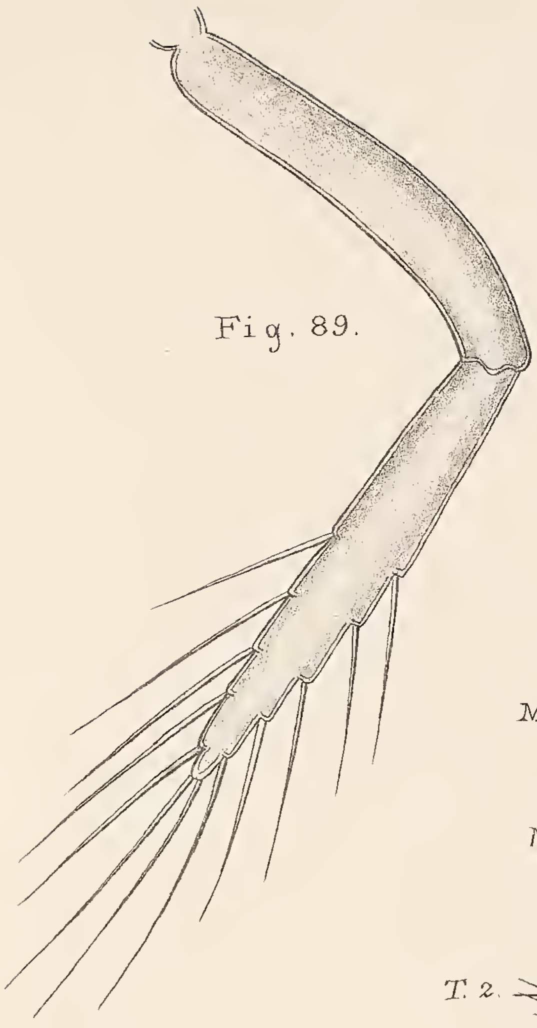


Fig. 87.

W.K.B. ad nat. del.

West Newman & Co lith.



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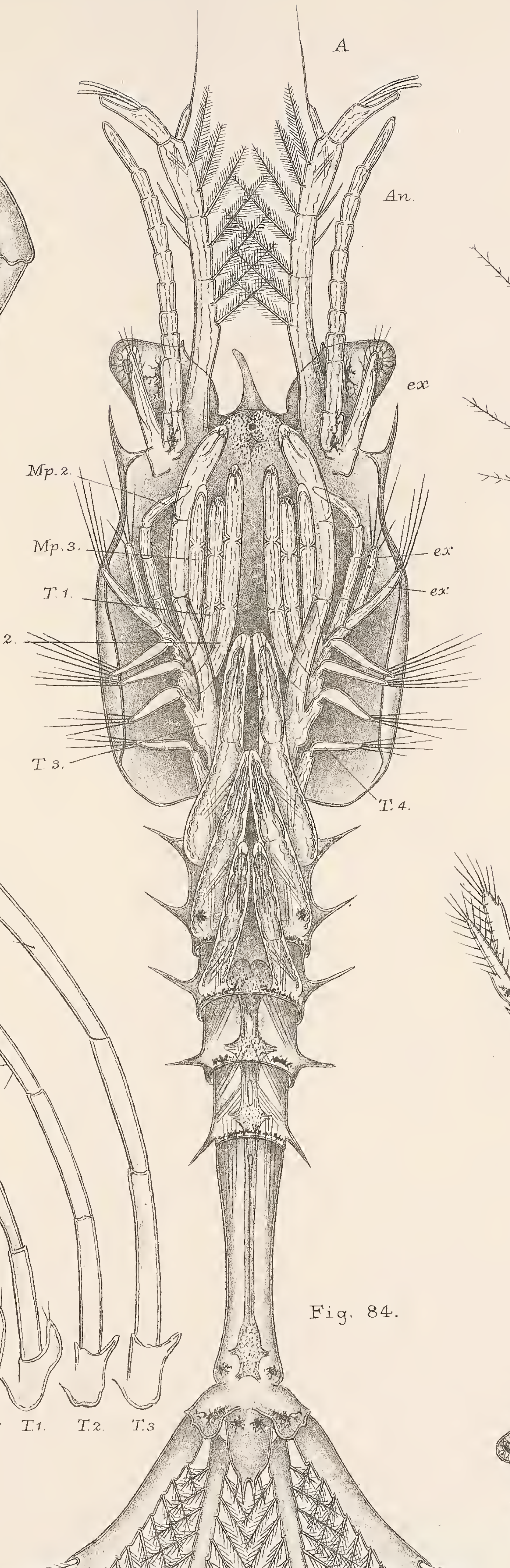

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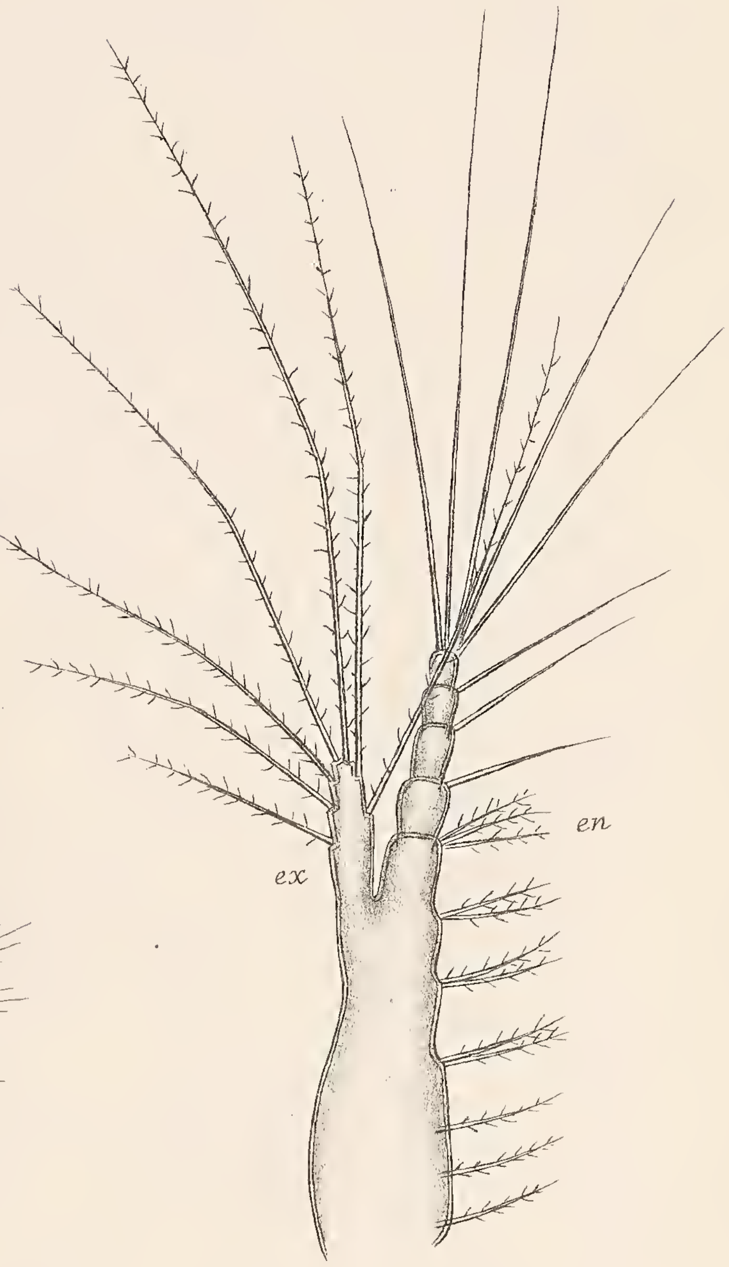

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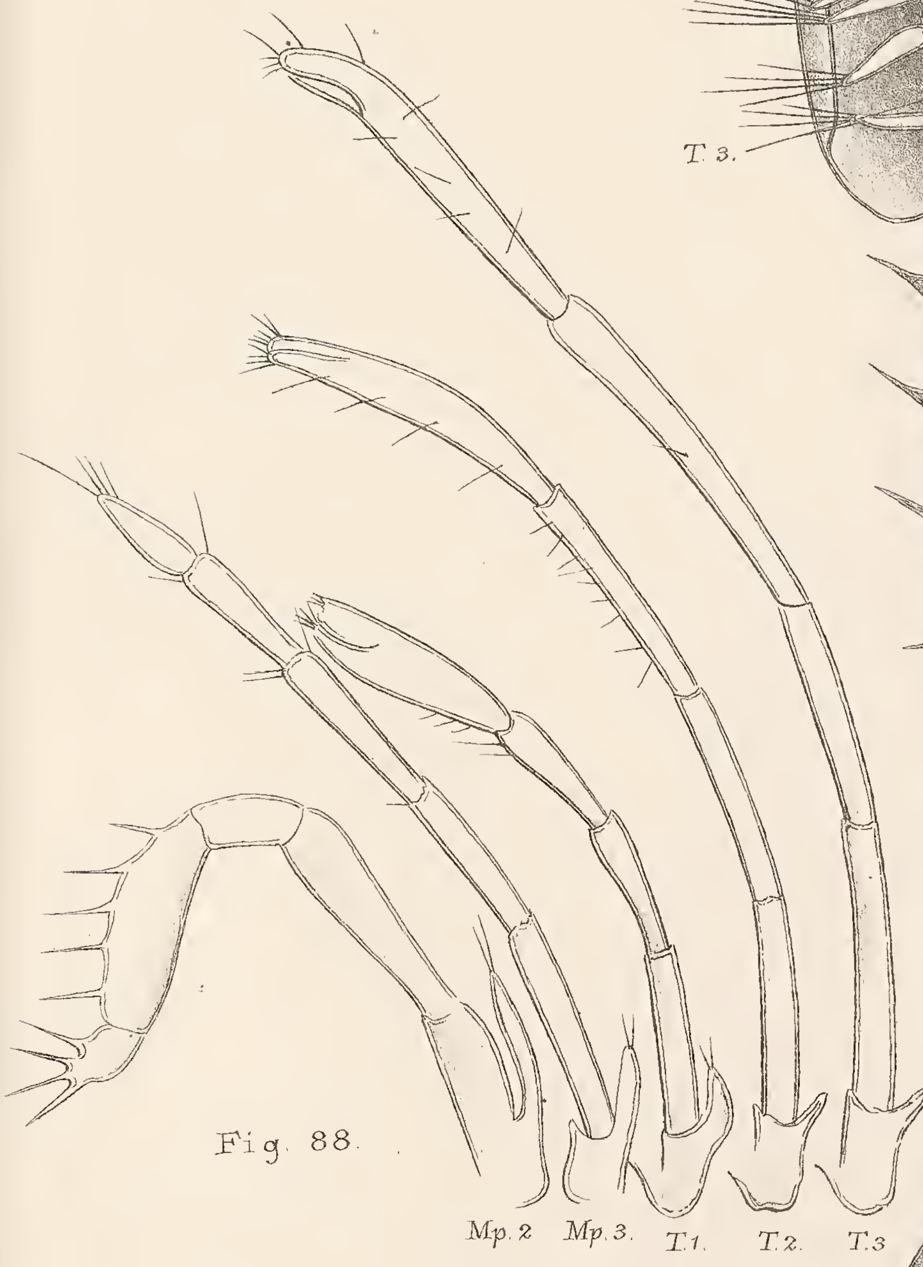


Fig. 88.

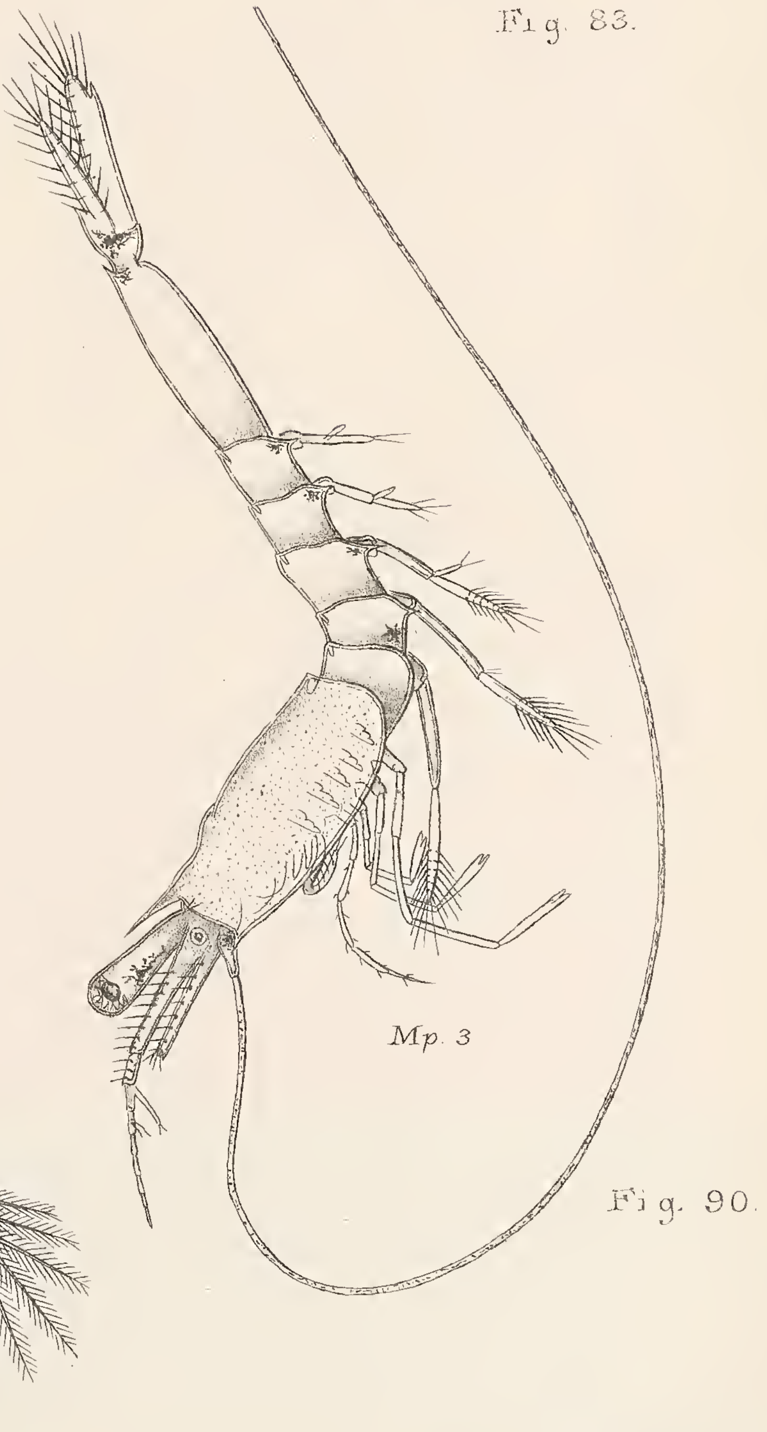
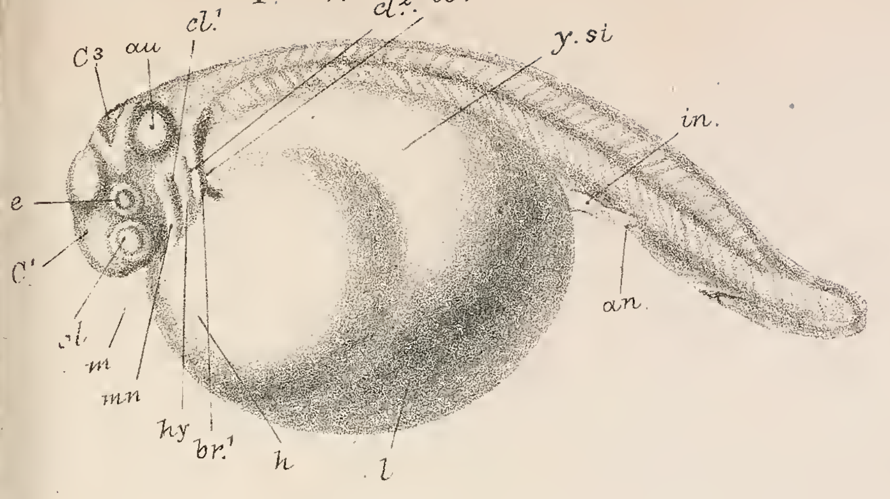
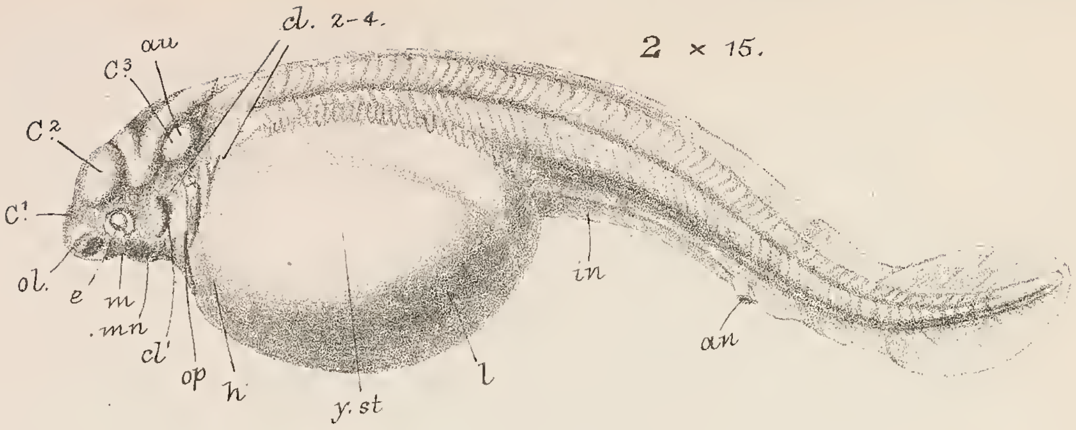


Fig. 90.

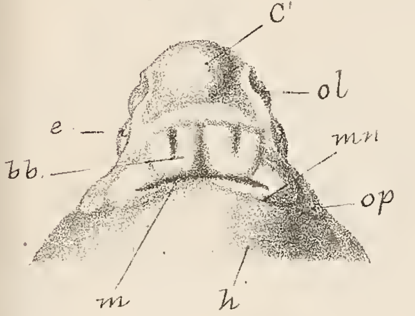
1. ×17



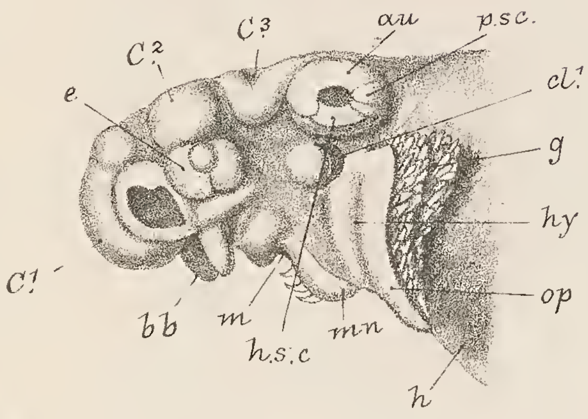
2 × 15.



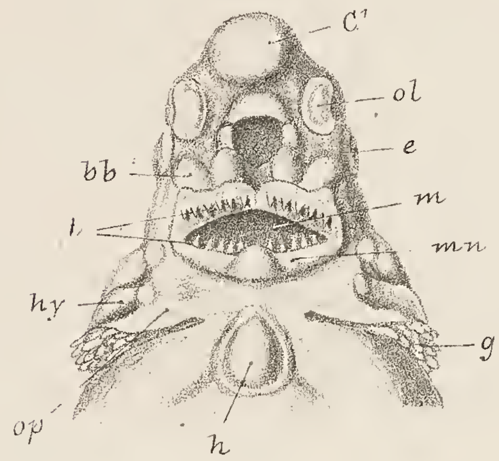
3 ×15



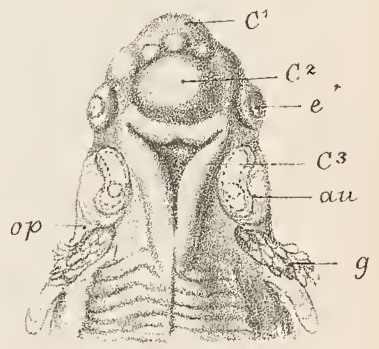
4 ×15



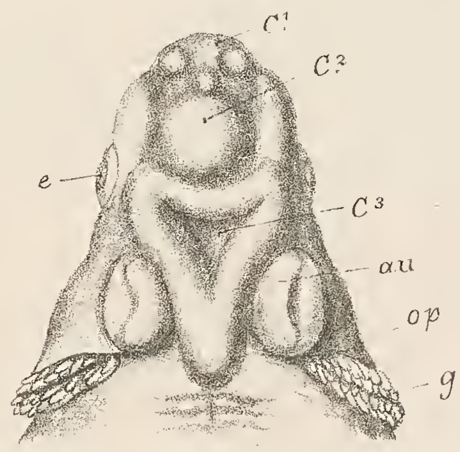
5 ×15



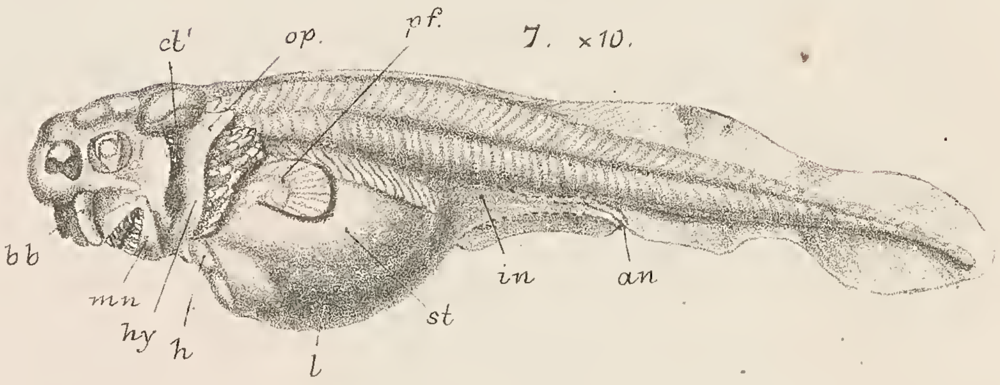
8 ×10



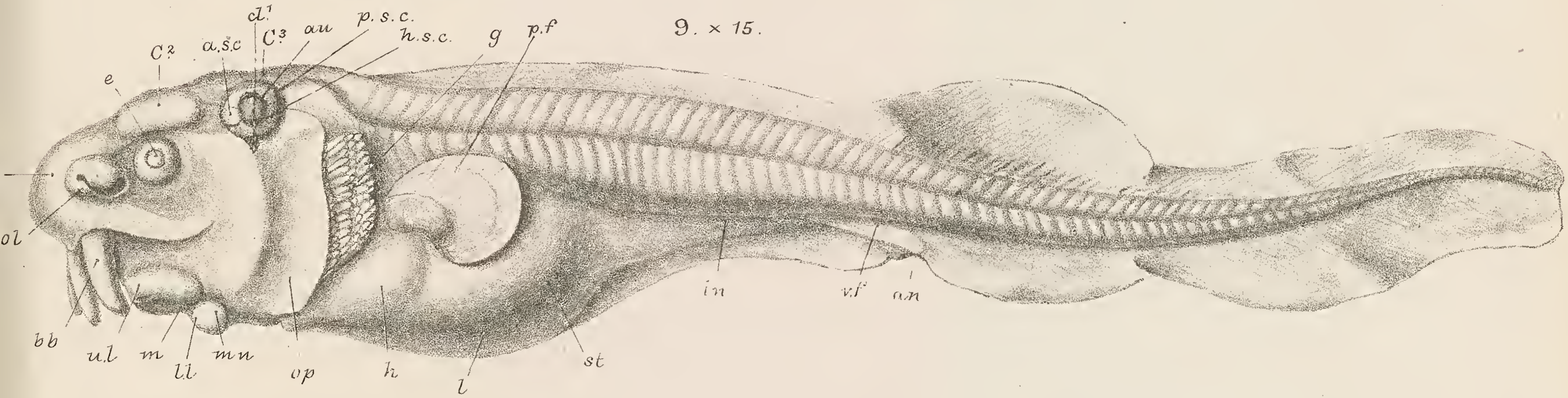
6 ×15



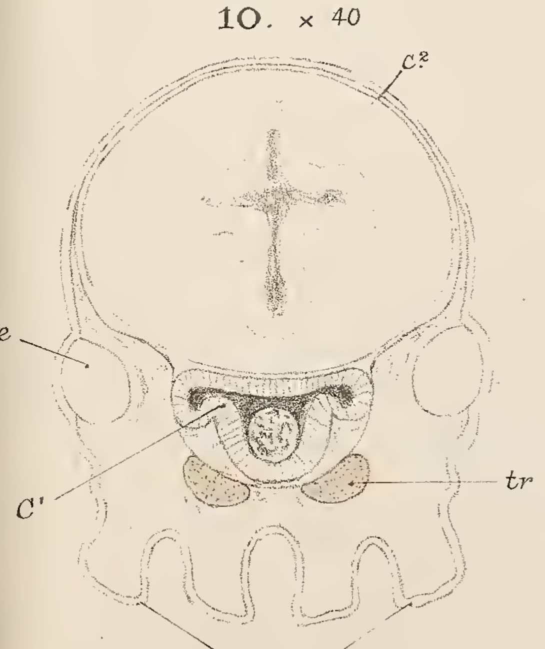
7. ×10.



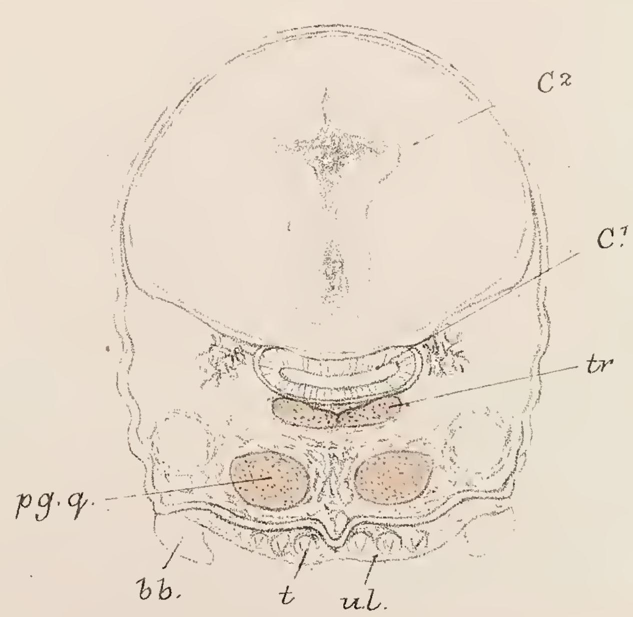
9. × 15.



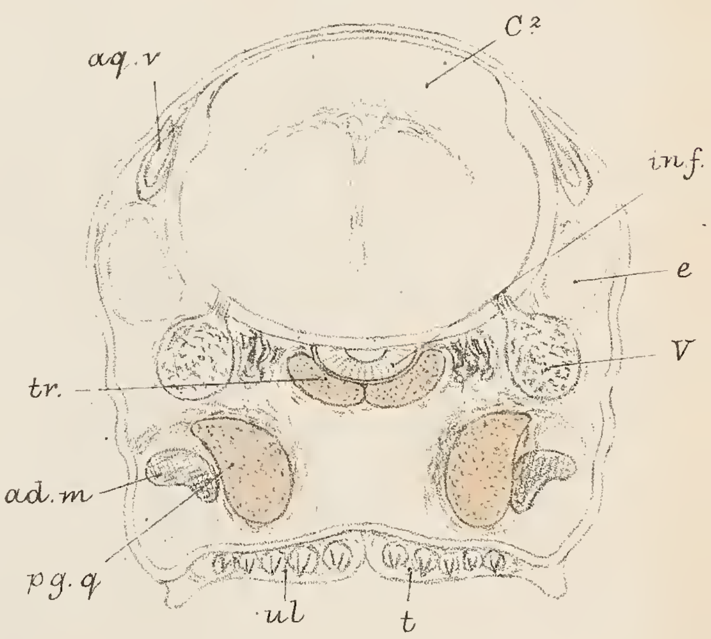
10. × 40



11 × 40



12 × 40

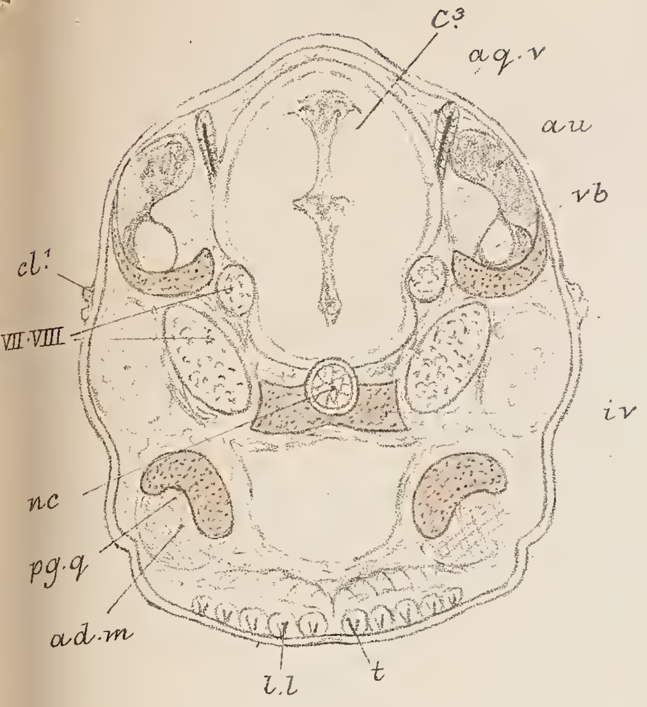


W.K.P. del. ad nat.
M.P. Parker lith.

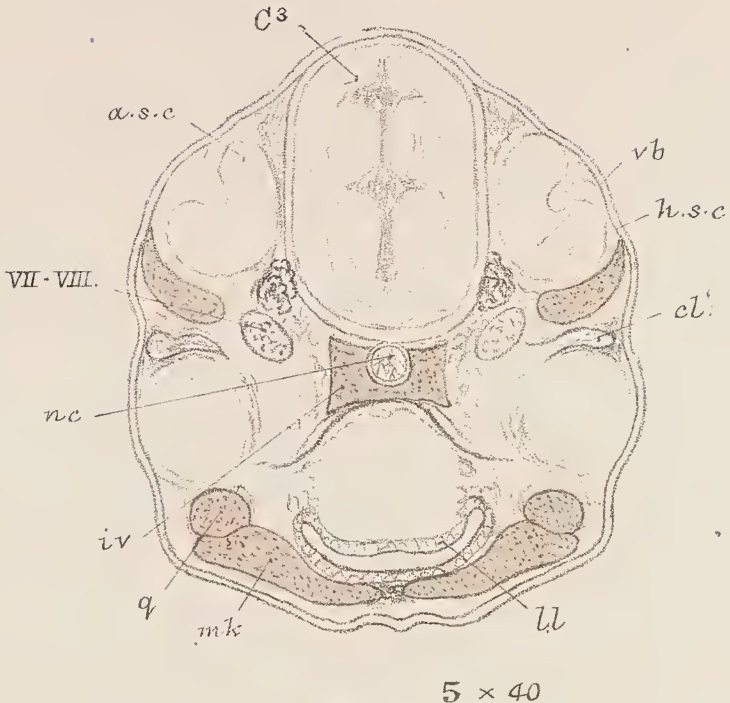
ACIPENSER.

West Newman & Co. lith.

1 × 40



2 × 40



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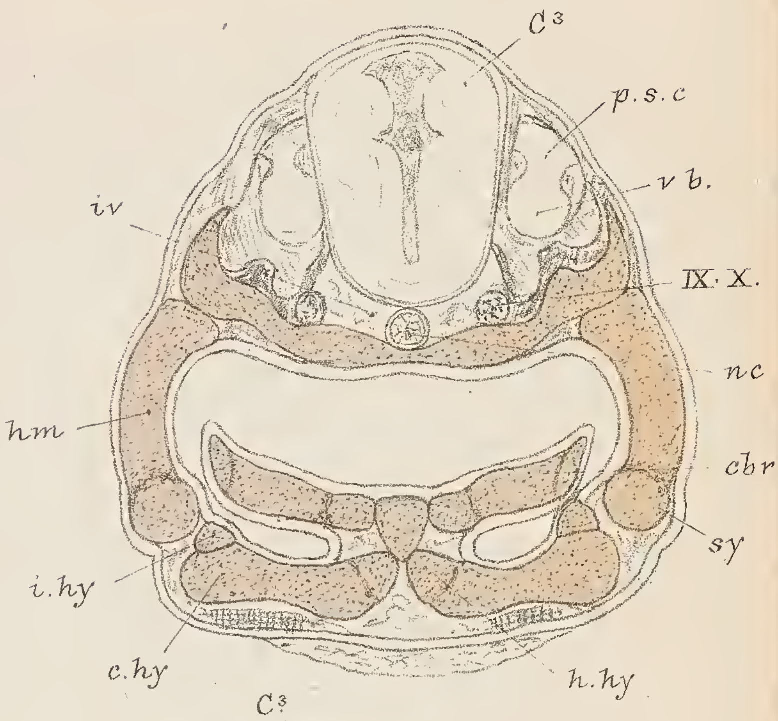

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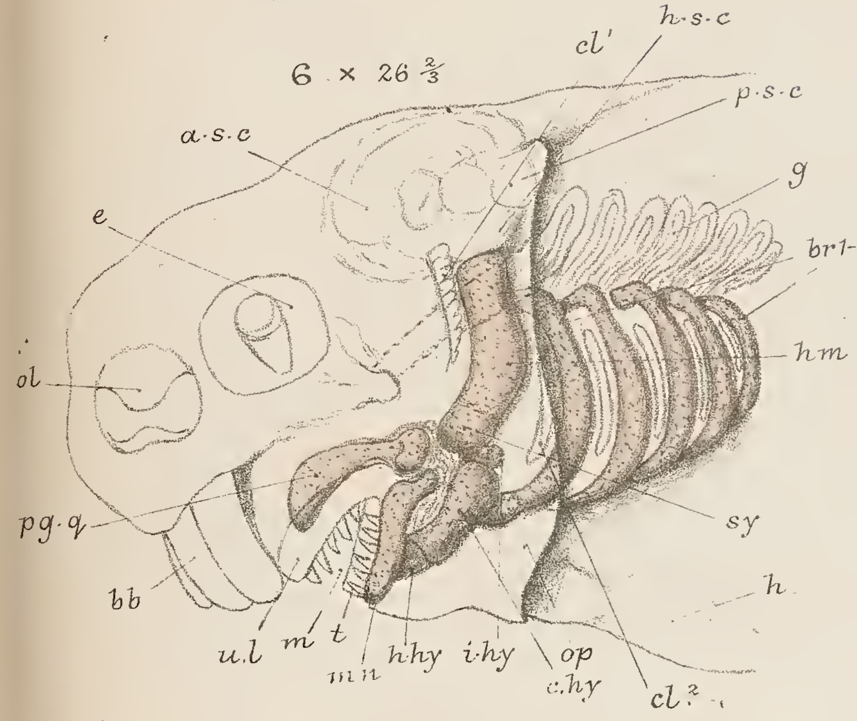

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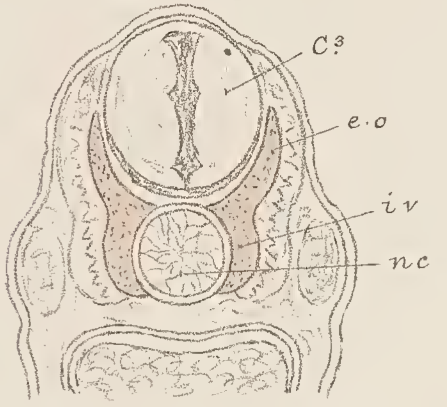

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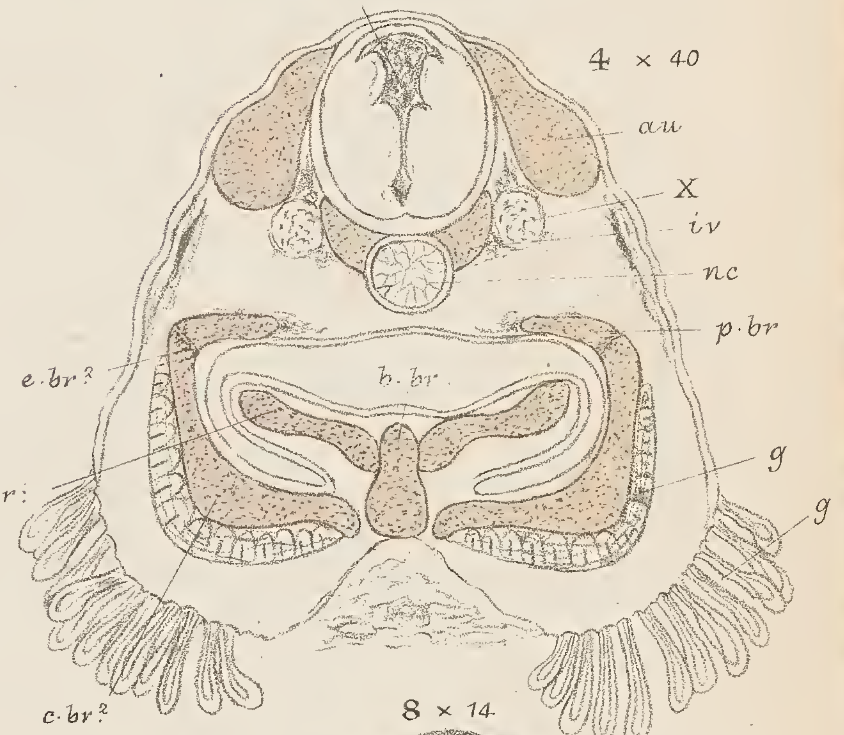

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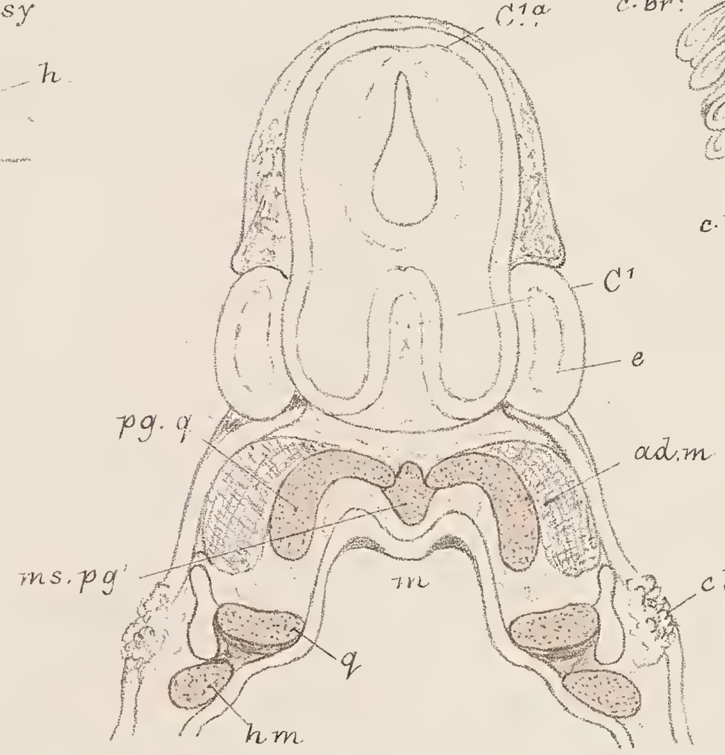

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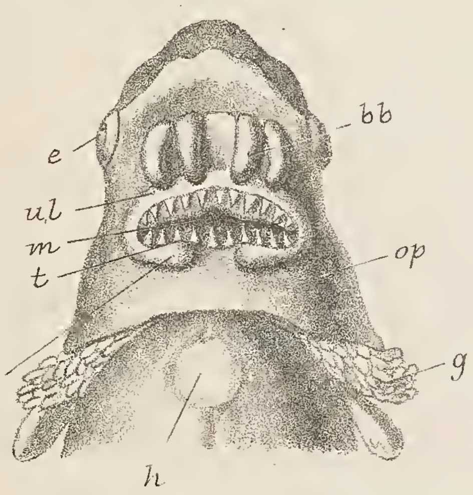

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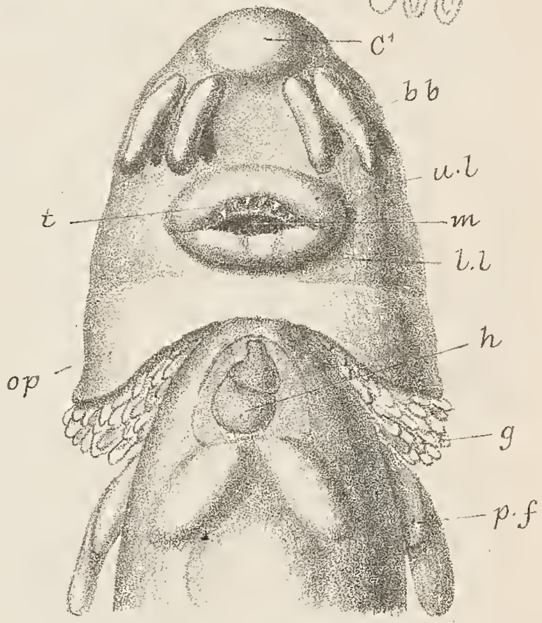

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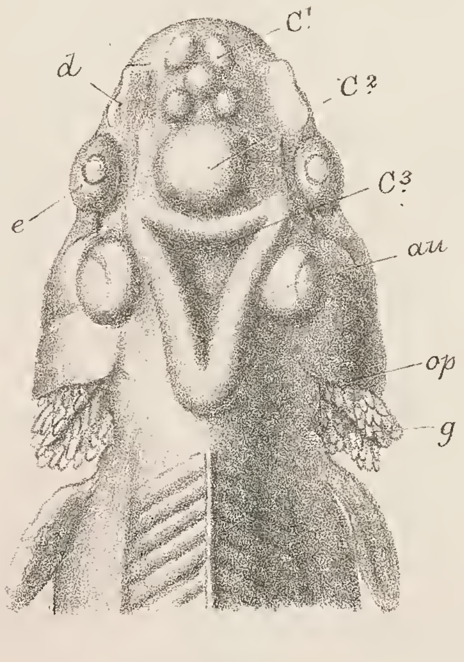

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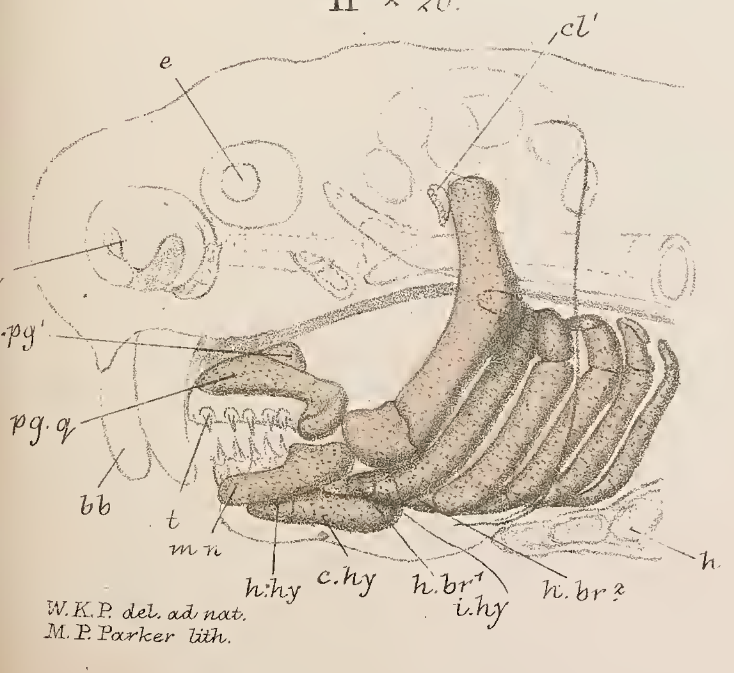

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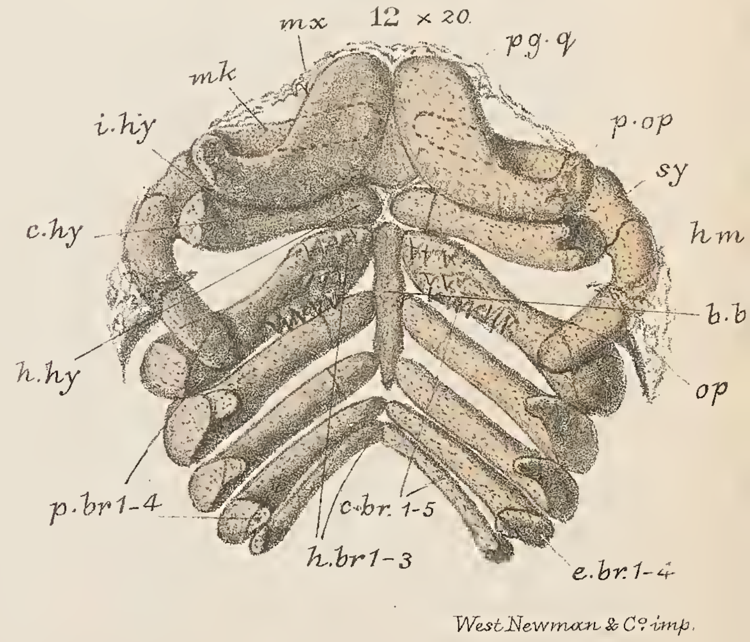
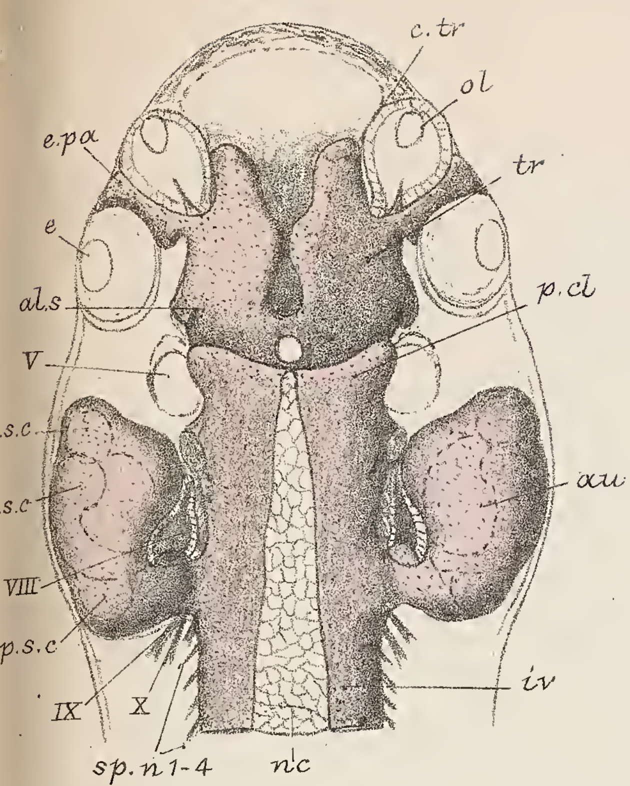


W.K.P. del. ad nat.
M.P. Parker lith.

ACIPENSER.

West Newman & Co. imp.

4 × 20



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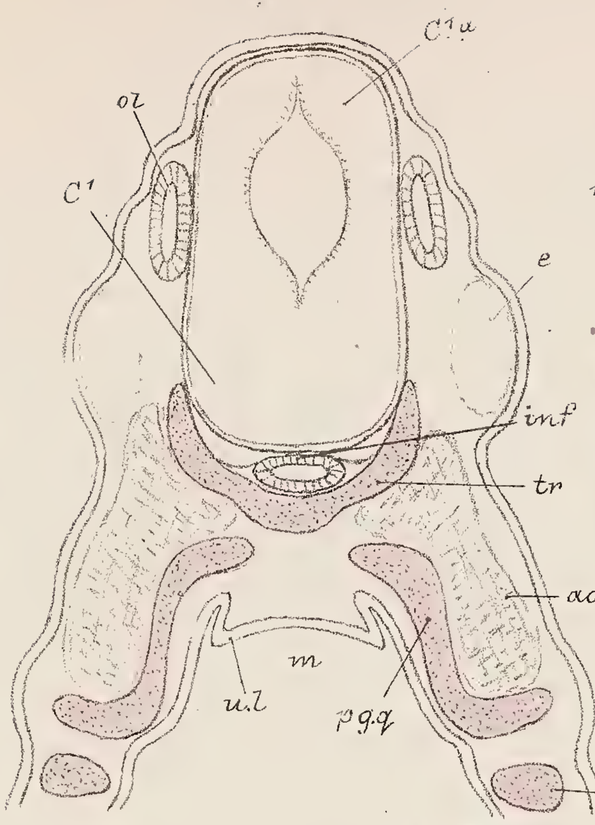

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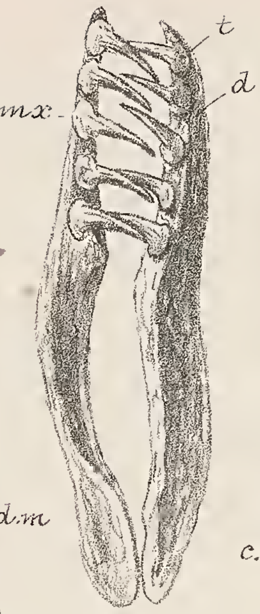

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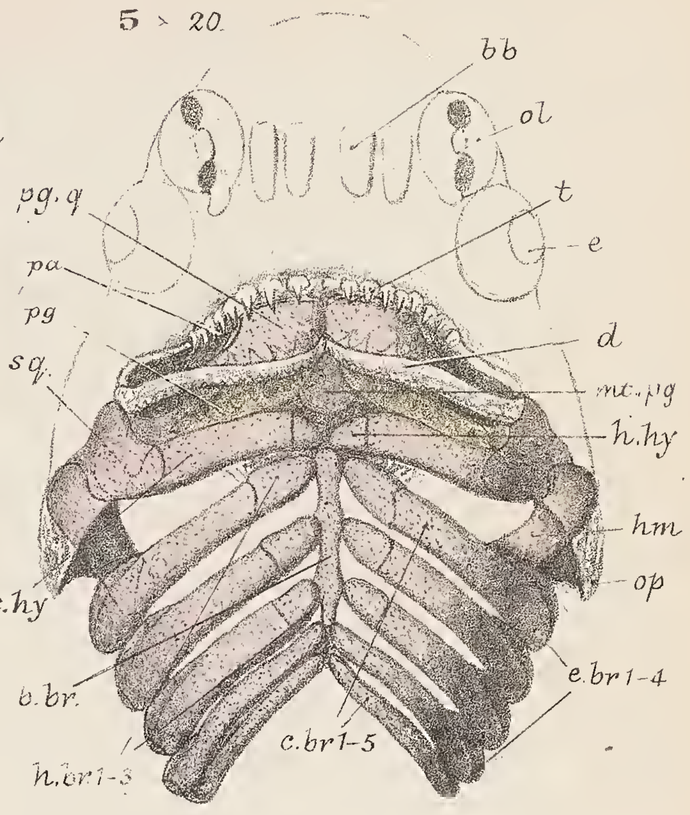

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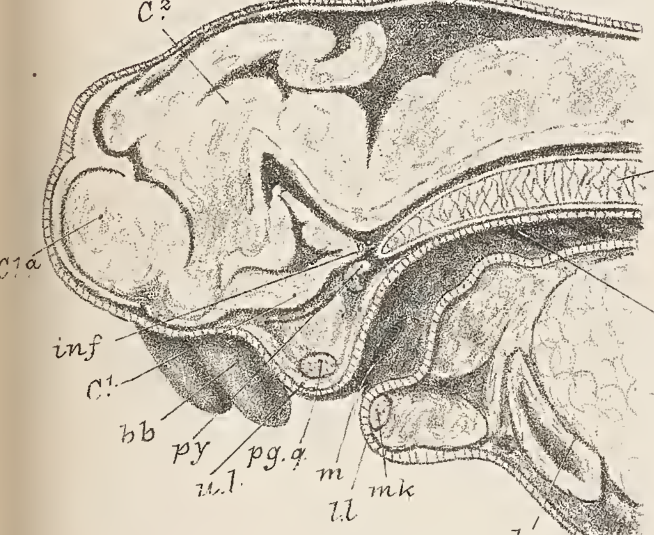

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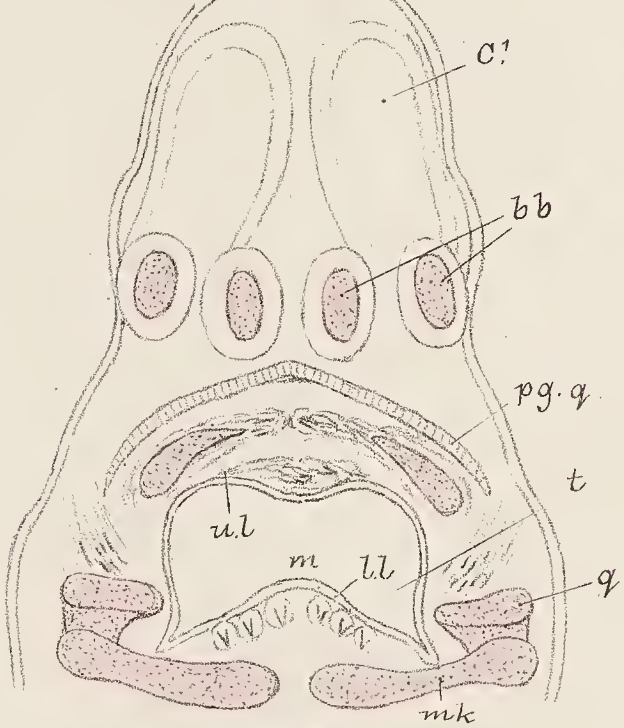

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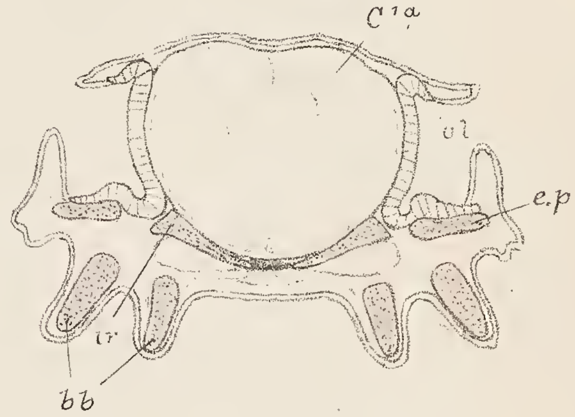

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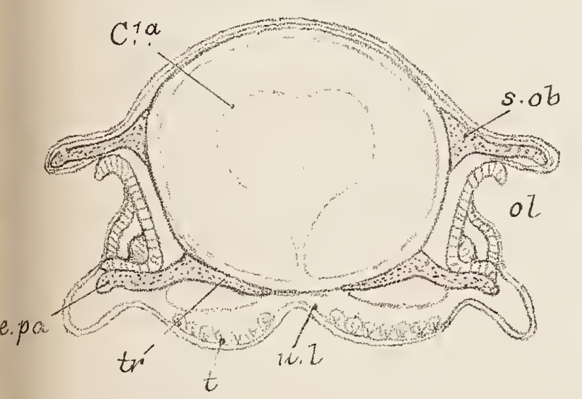

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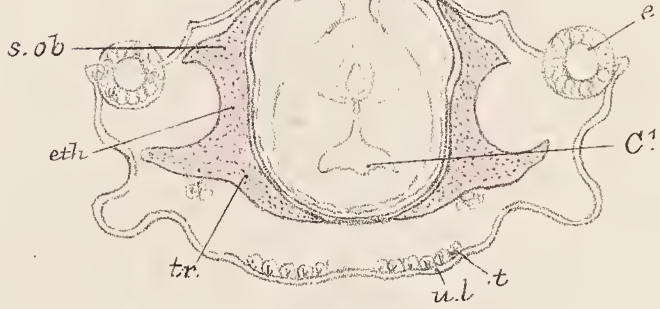

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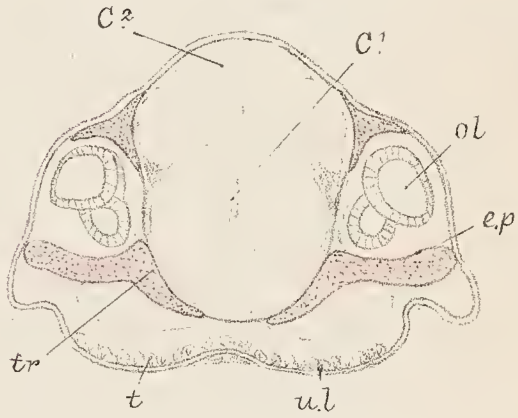

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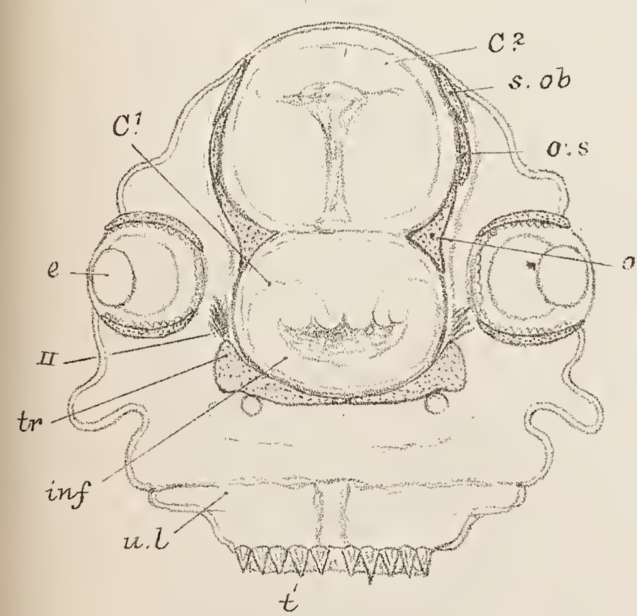

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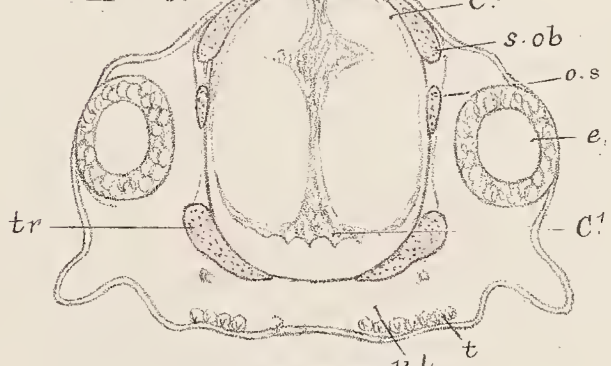

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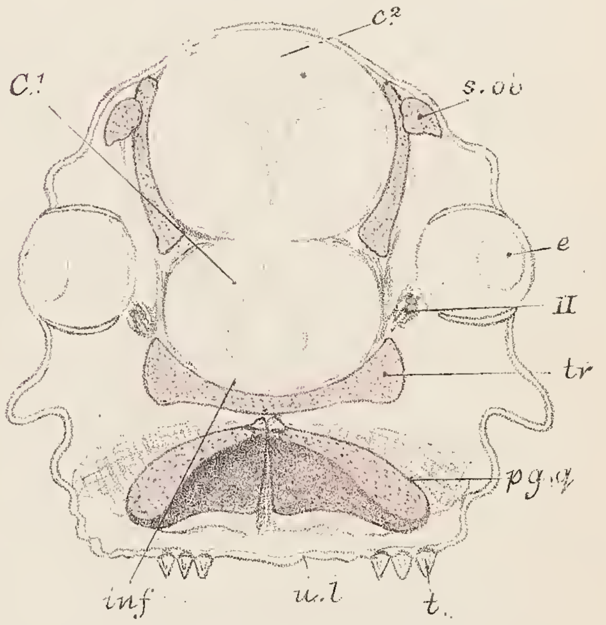

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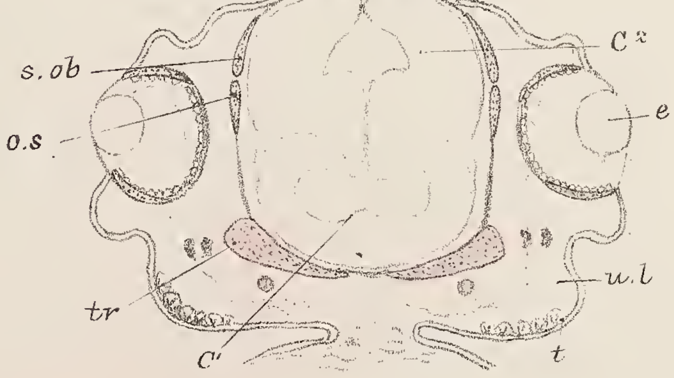
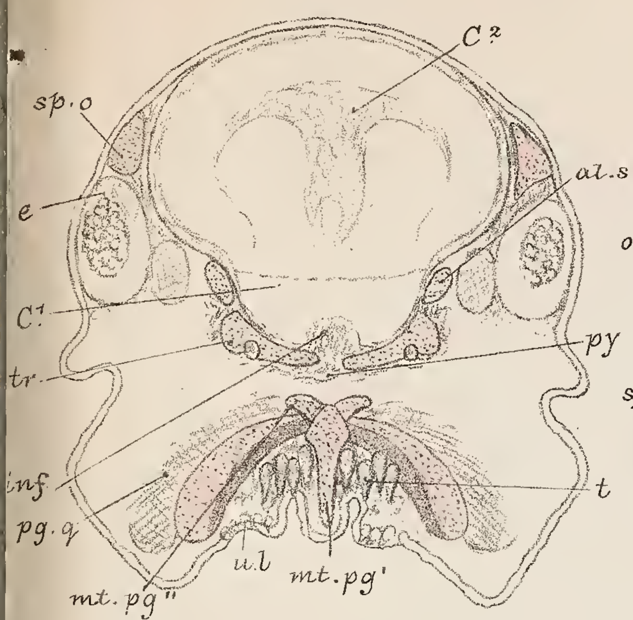


W.K.P. del. ad nat.
M.P. Parker lith.

ACIPENSER.

West Newman & Co imp.

1 × 30



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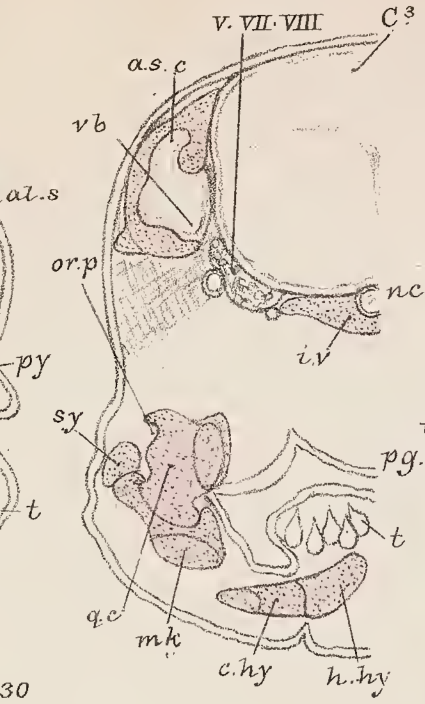

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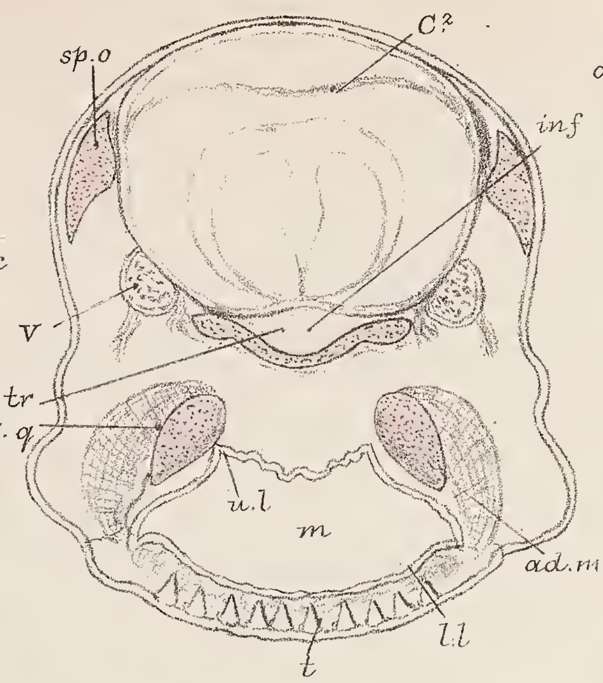

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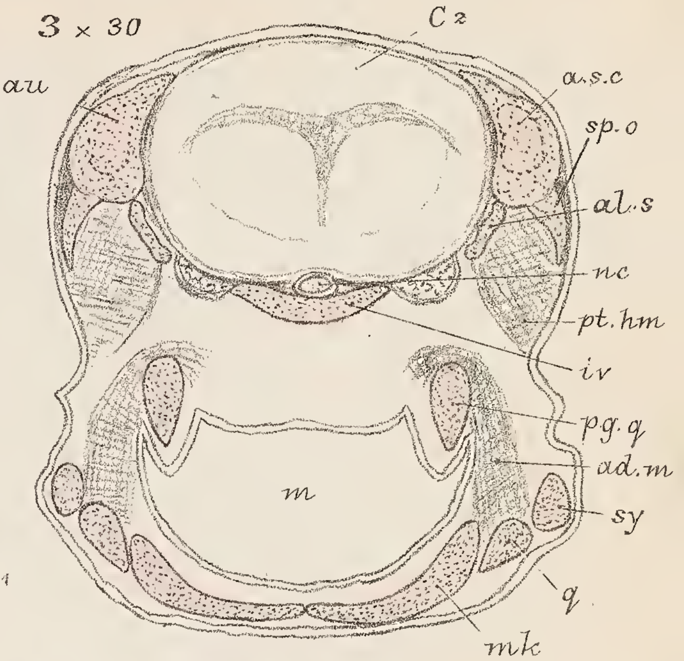

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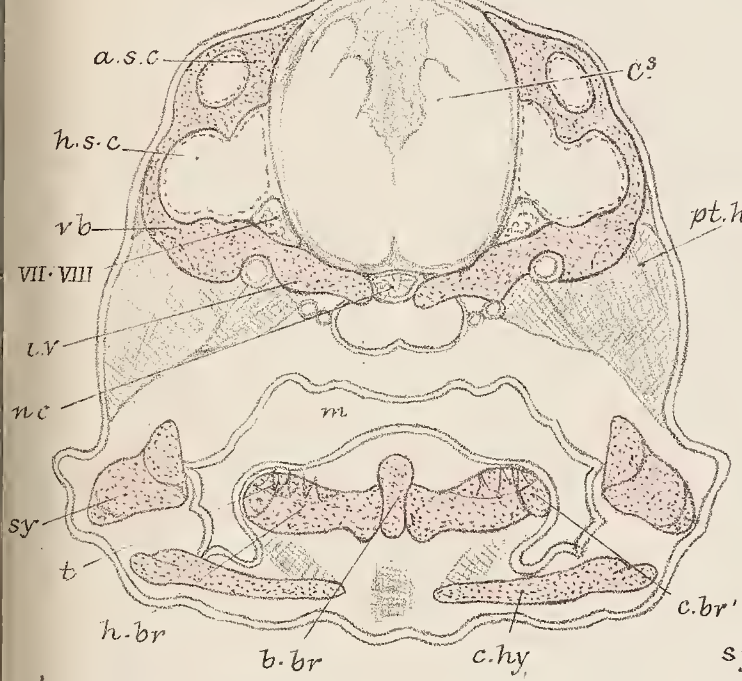

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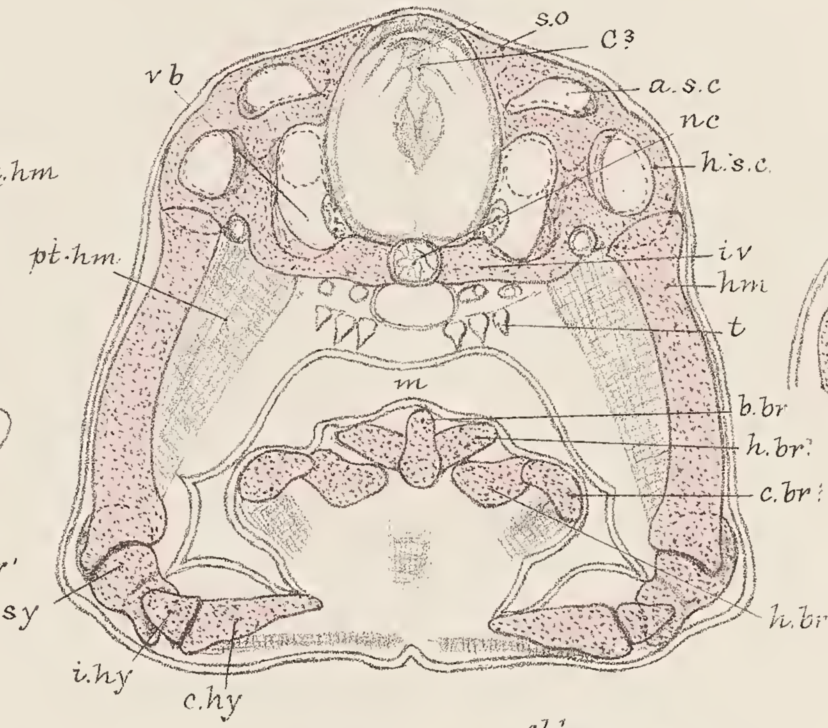

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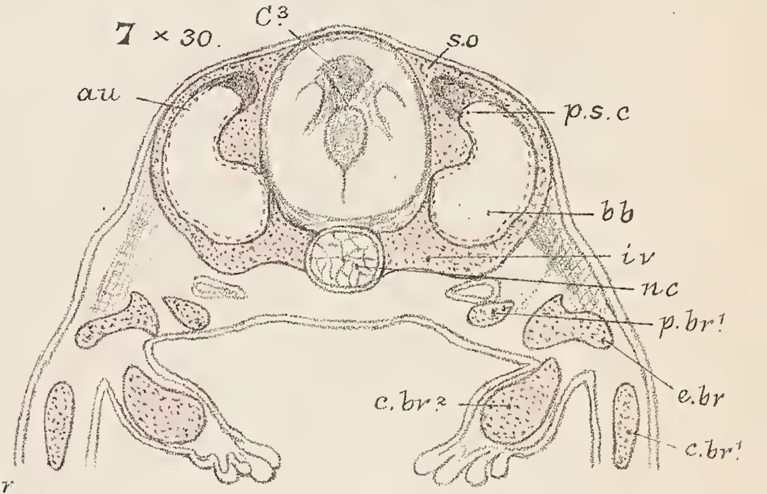

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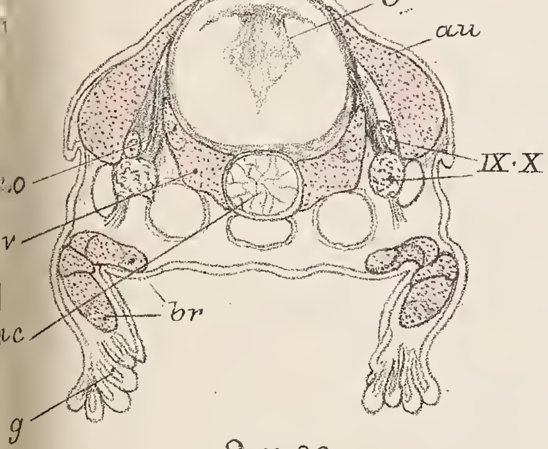
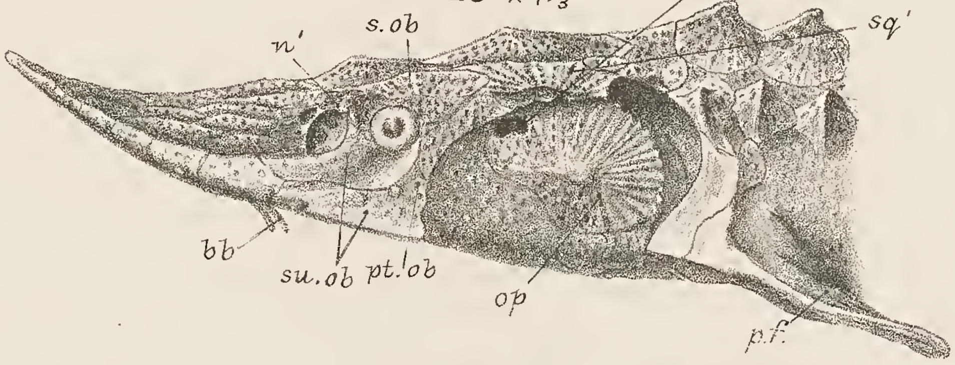


10 × 1⅓



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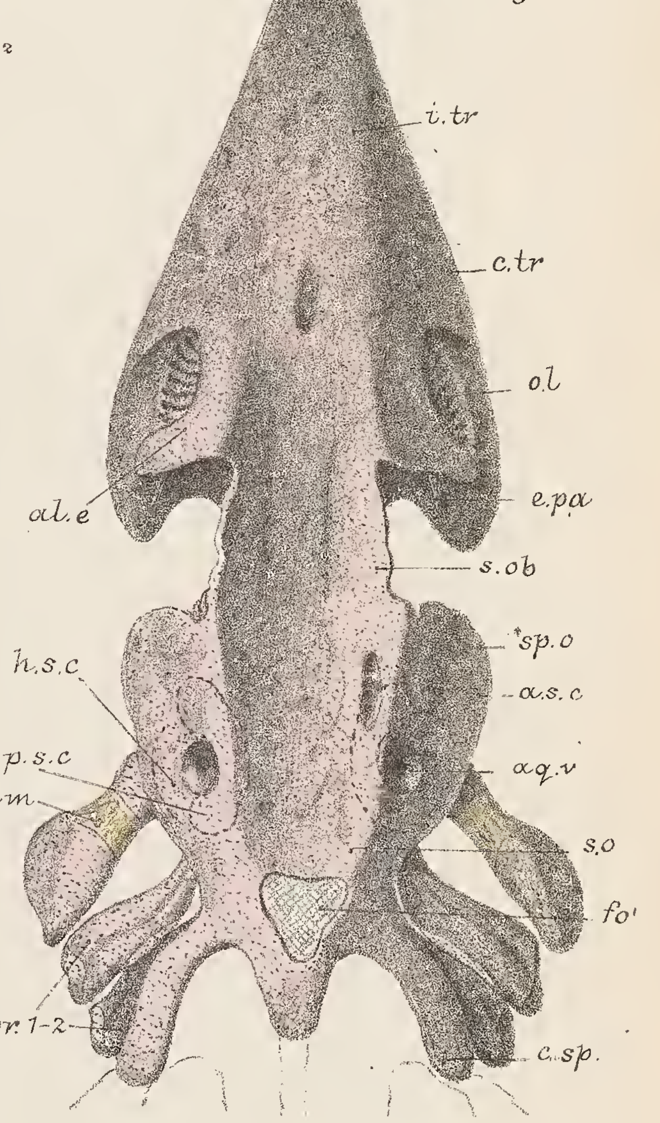

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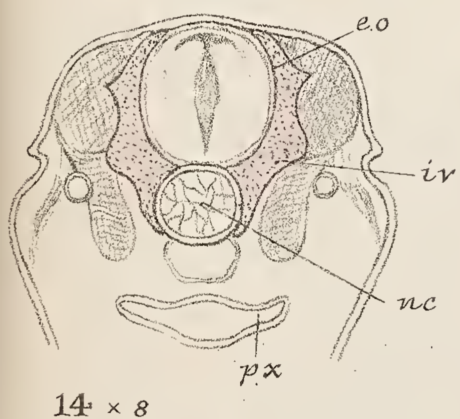

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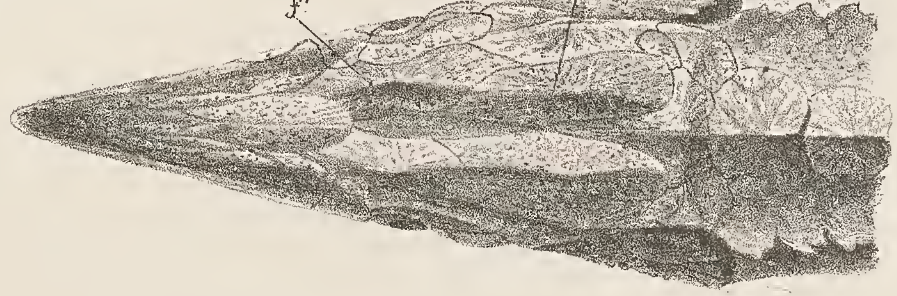

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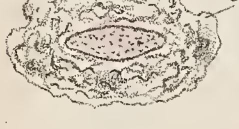

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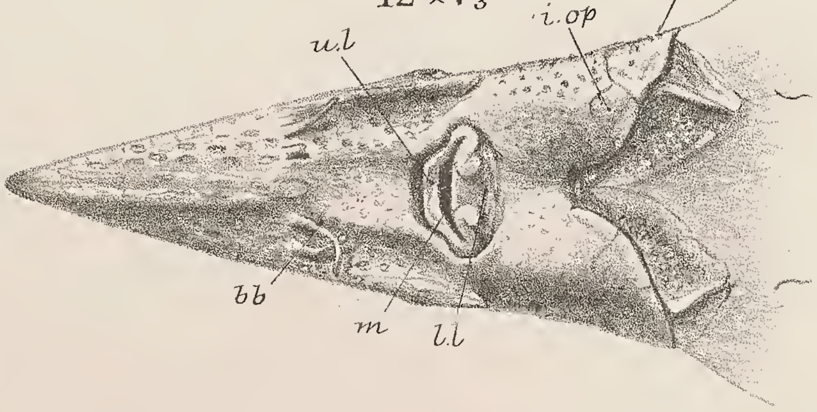

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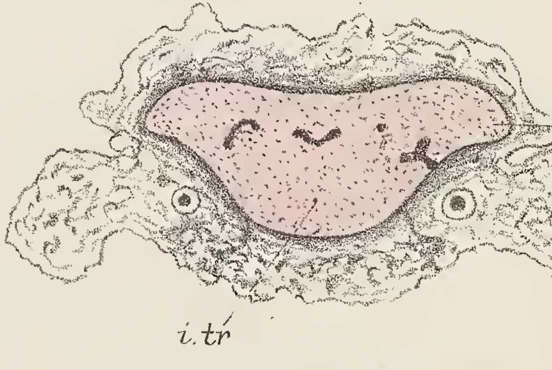

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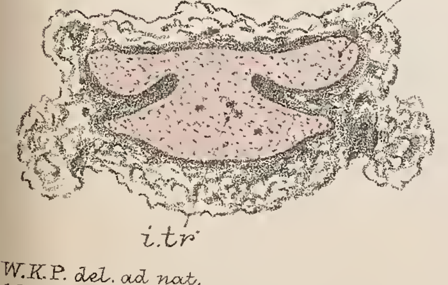
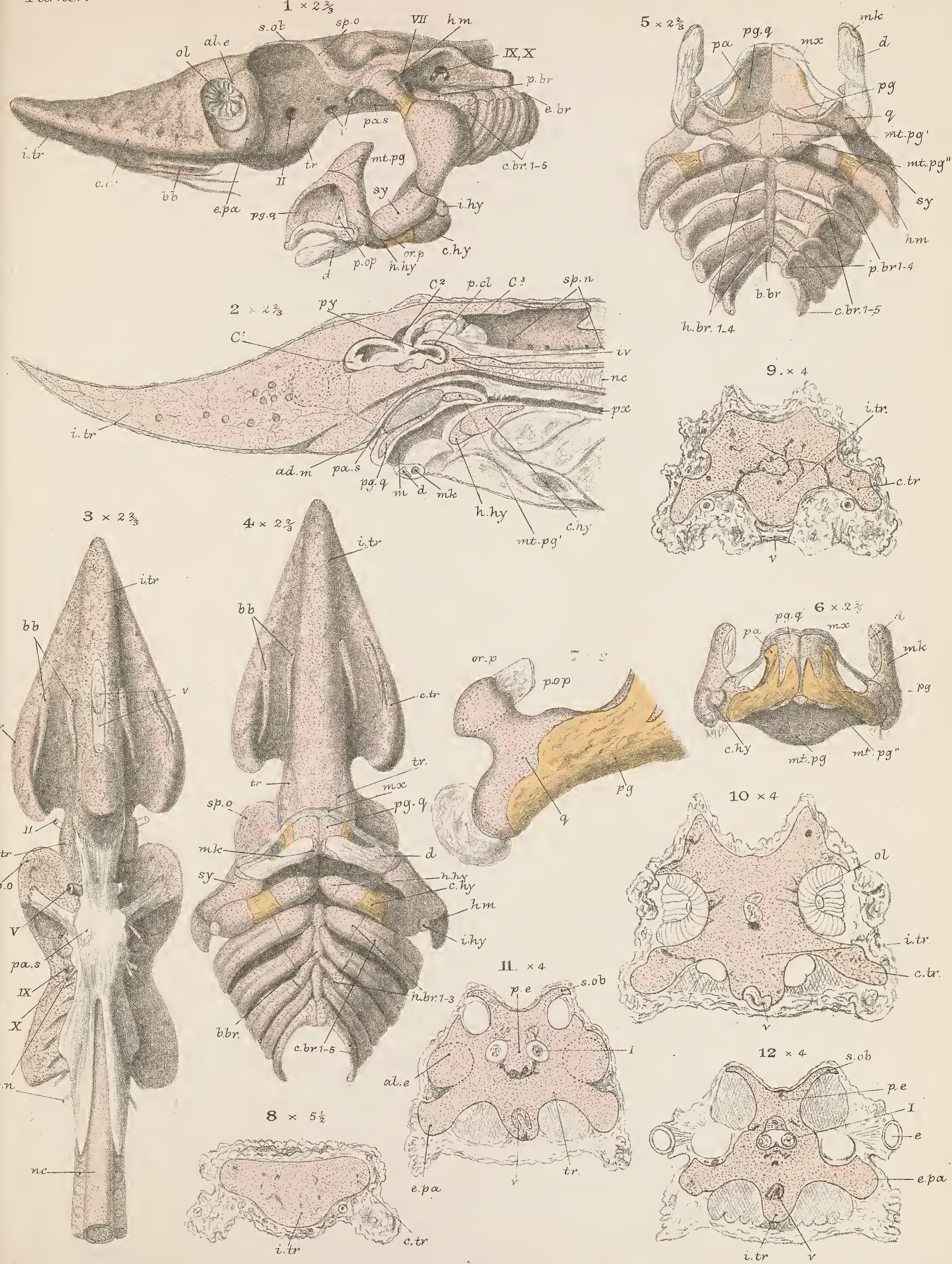


W.K.P. del. ad nat.
M.P. Parker lith.

ACIPENSER.

West Newman & Co imp.



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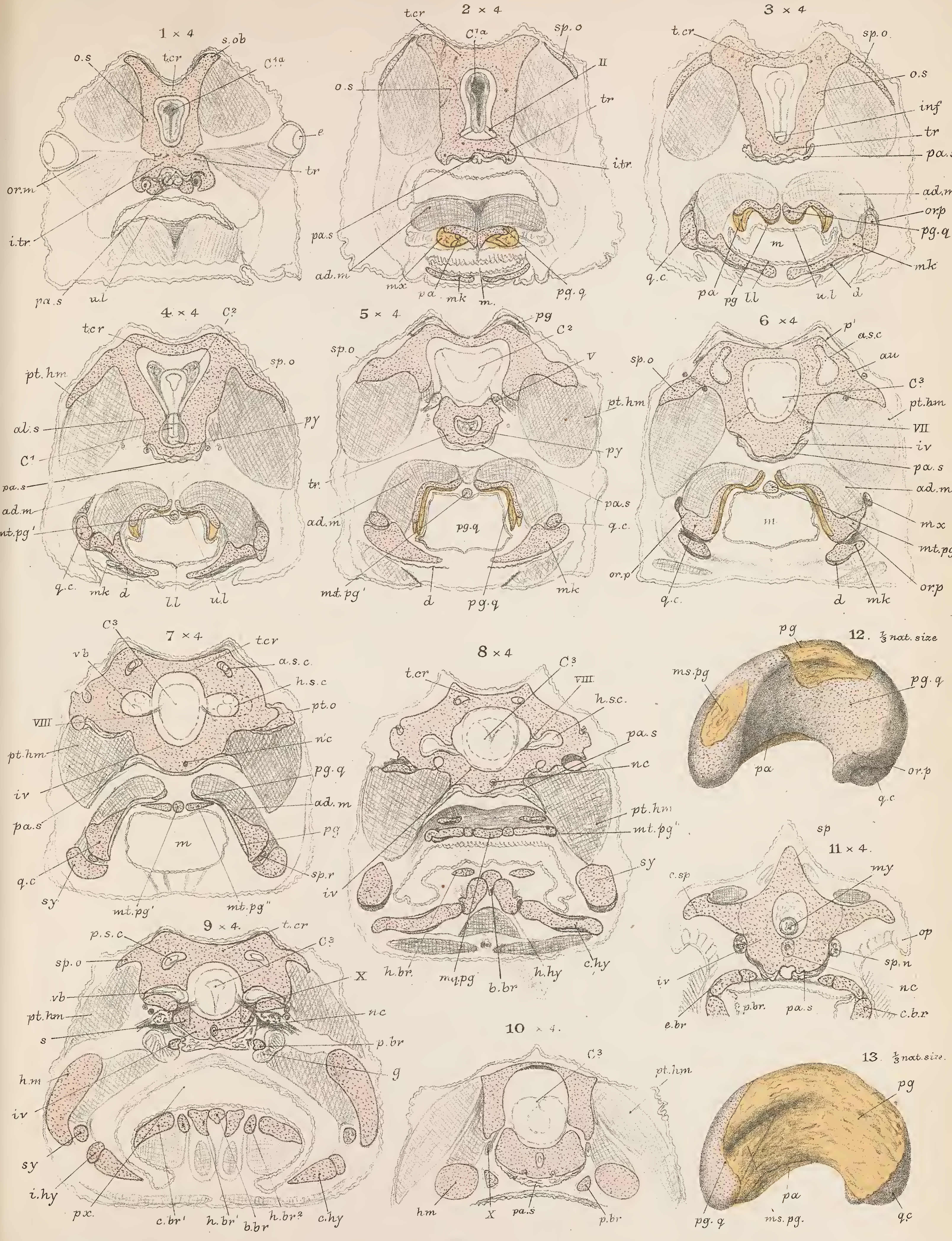
M.P. Parker lith.

ACIPENSER.

West Newman & Cº imp.

Parker.

Phil. Trans. 1882. Plate 17.

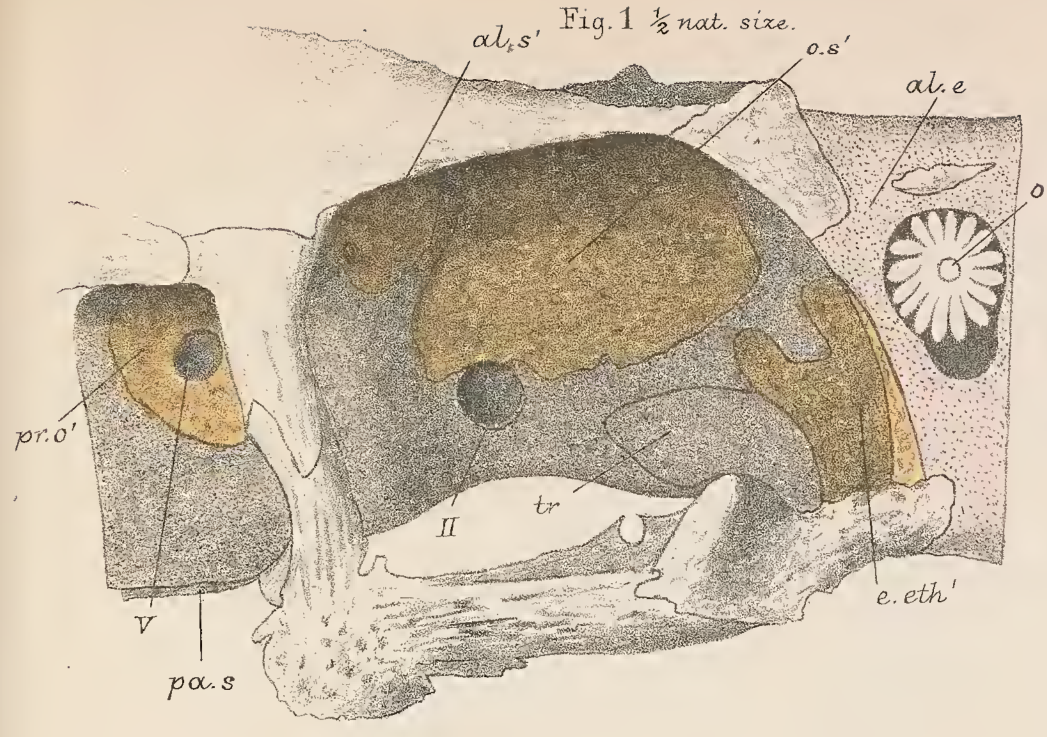


W.K.P. del. ad nat.
M.P. Parker lith.

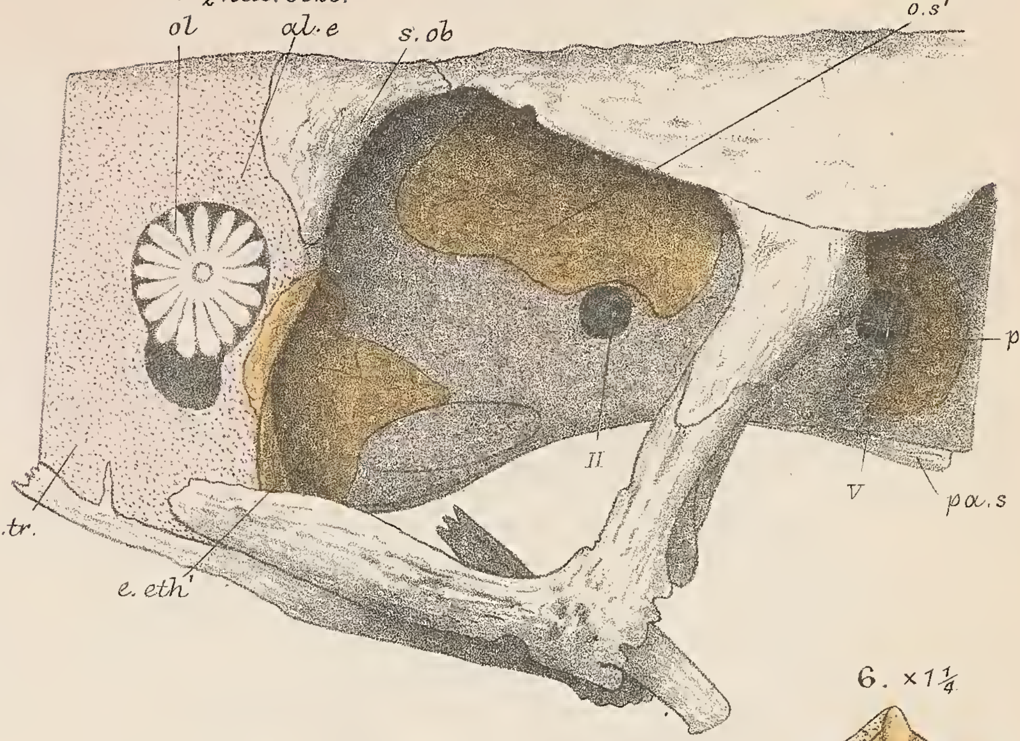
ACIPENSER.

West Newman & Co. imp.

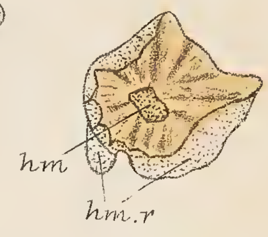
Fig. 1 ½ nat. size.



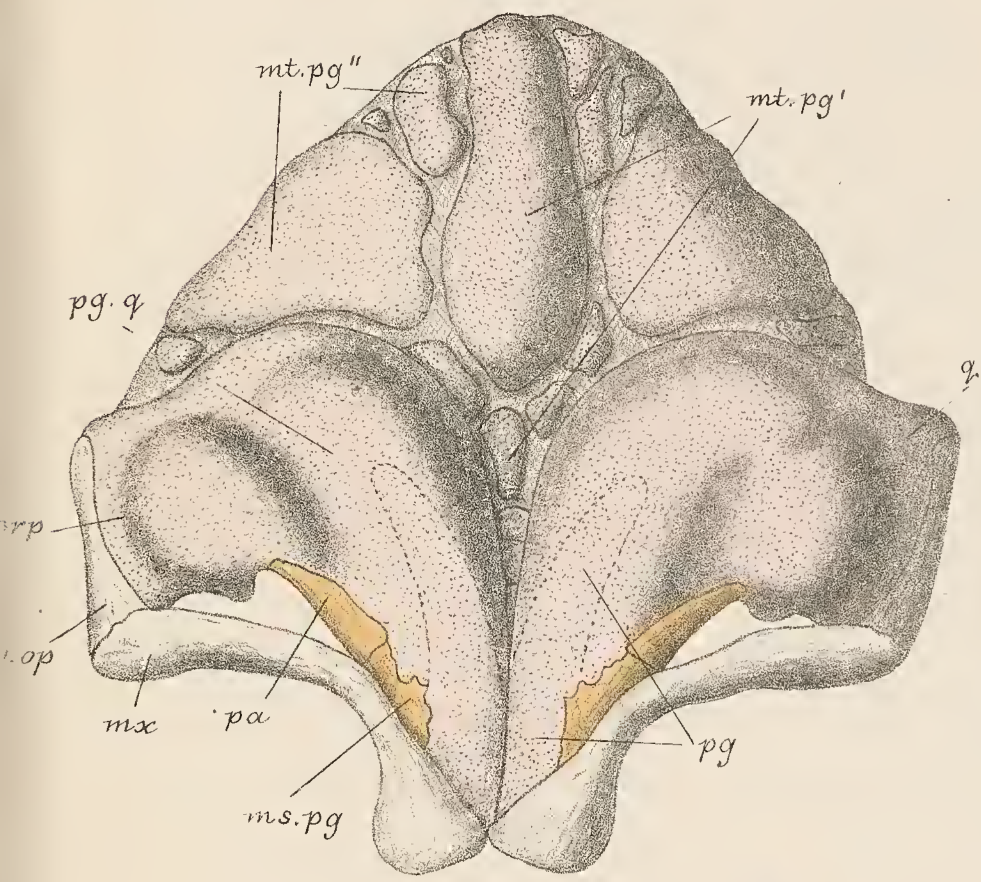
2. ½ nat. size.



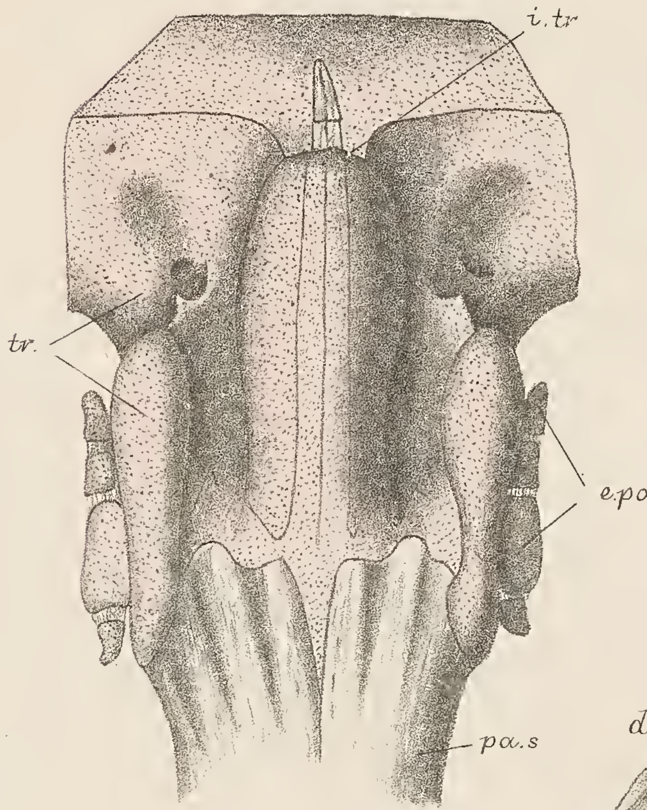
6. × 1¼



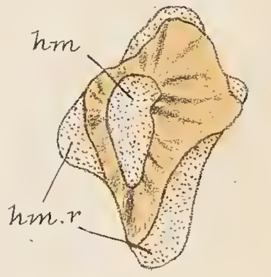
4 × 1¼



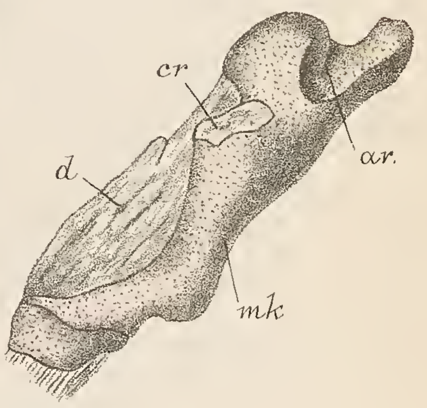
3 × 1¼



7. × 1¼



8. × ¼



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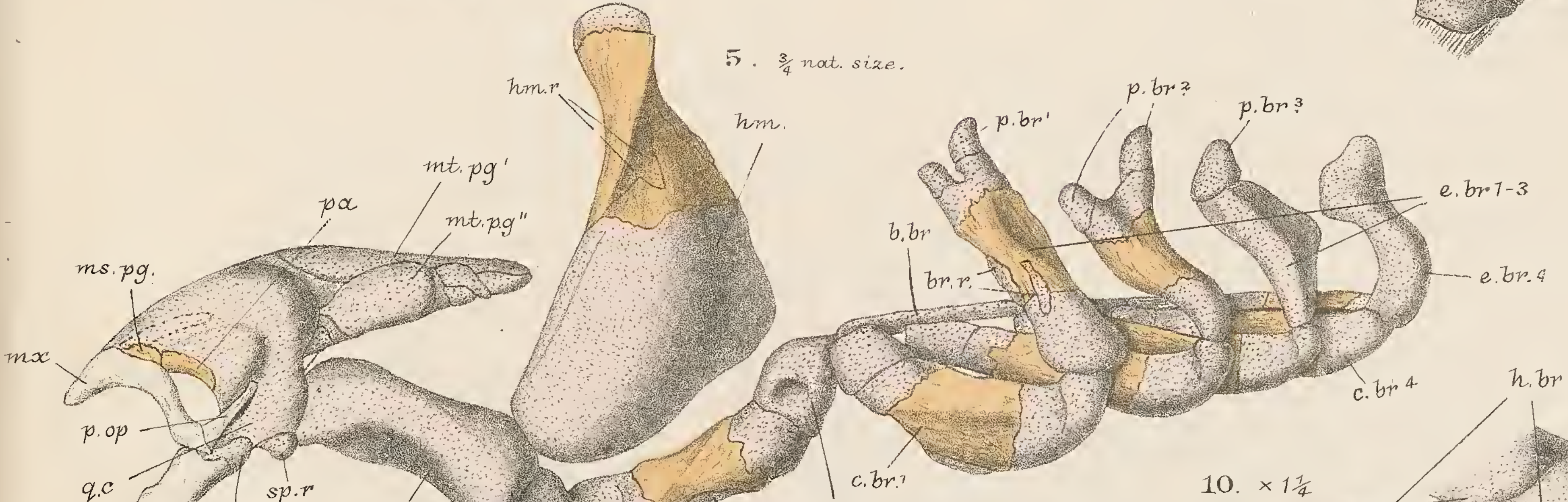
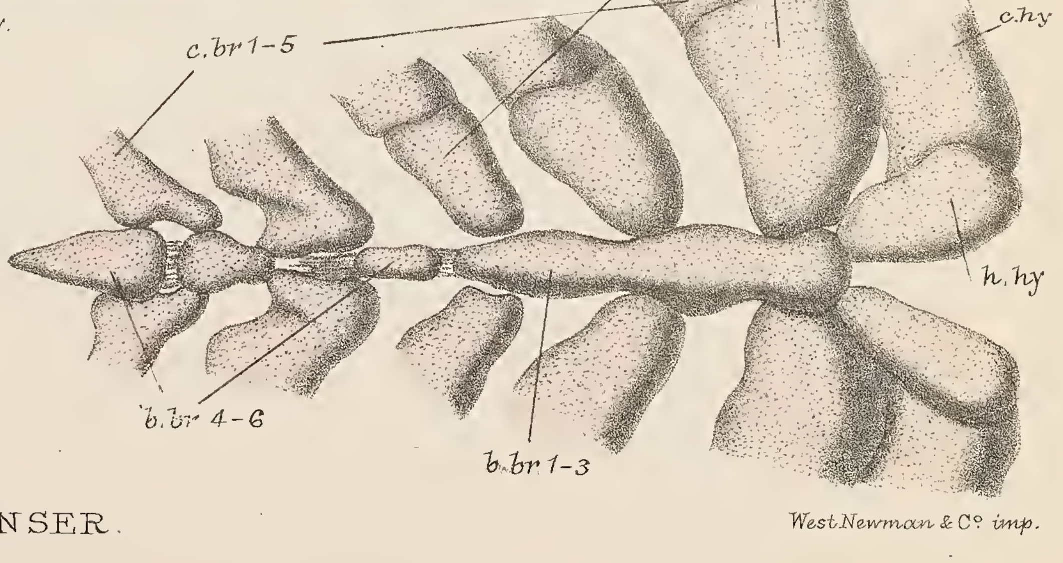
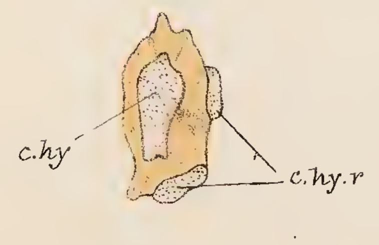


10. × 1¼



9 × 1¼



G.B. Howes del. ad nat.
M.P. Parker lith.

ACIPENSER.

West Newman & Co. imp.

Fig. 1

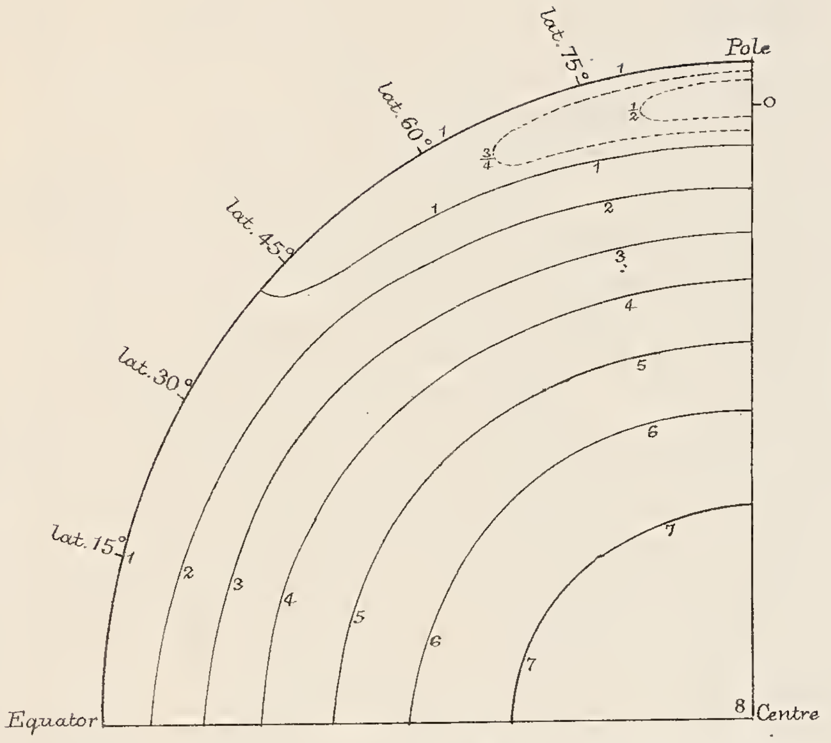


Diagram showing curves of equal stress-difference due to the weight of 2nd harmonic inequalities or to tide-generating force.

Fig. 3.

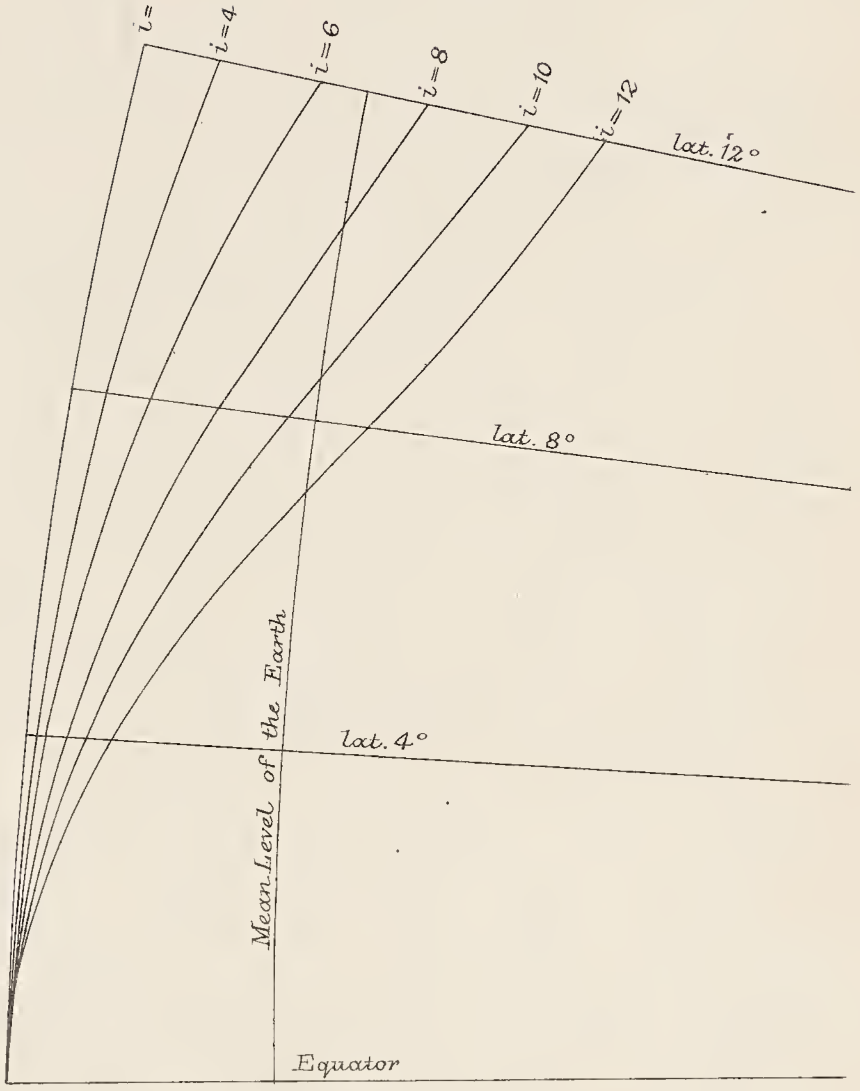


Diagram showing the profile of the even harmonics near the Equator. Radius of sphere 18 inches.

Fig. 2.

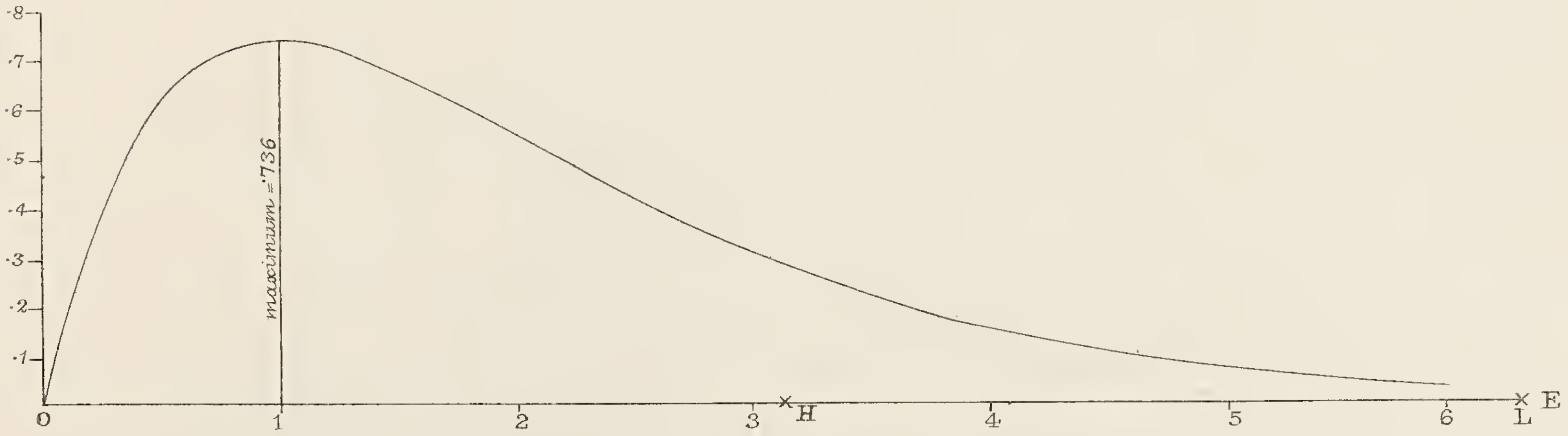


Diagram showing the difference between the principal stresses due to harmonic mountains and vallies on a horizontal plane.

West Newman & Co. lith.

Fig. 4.

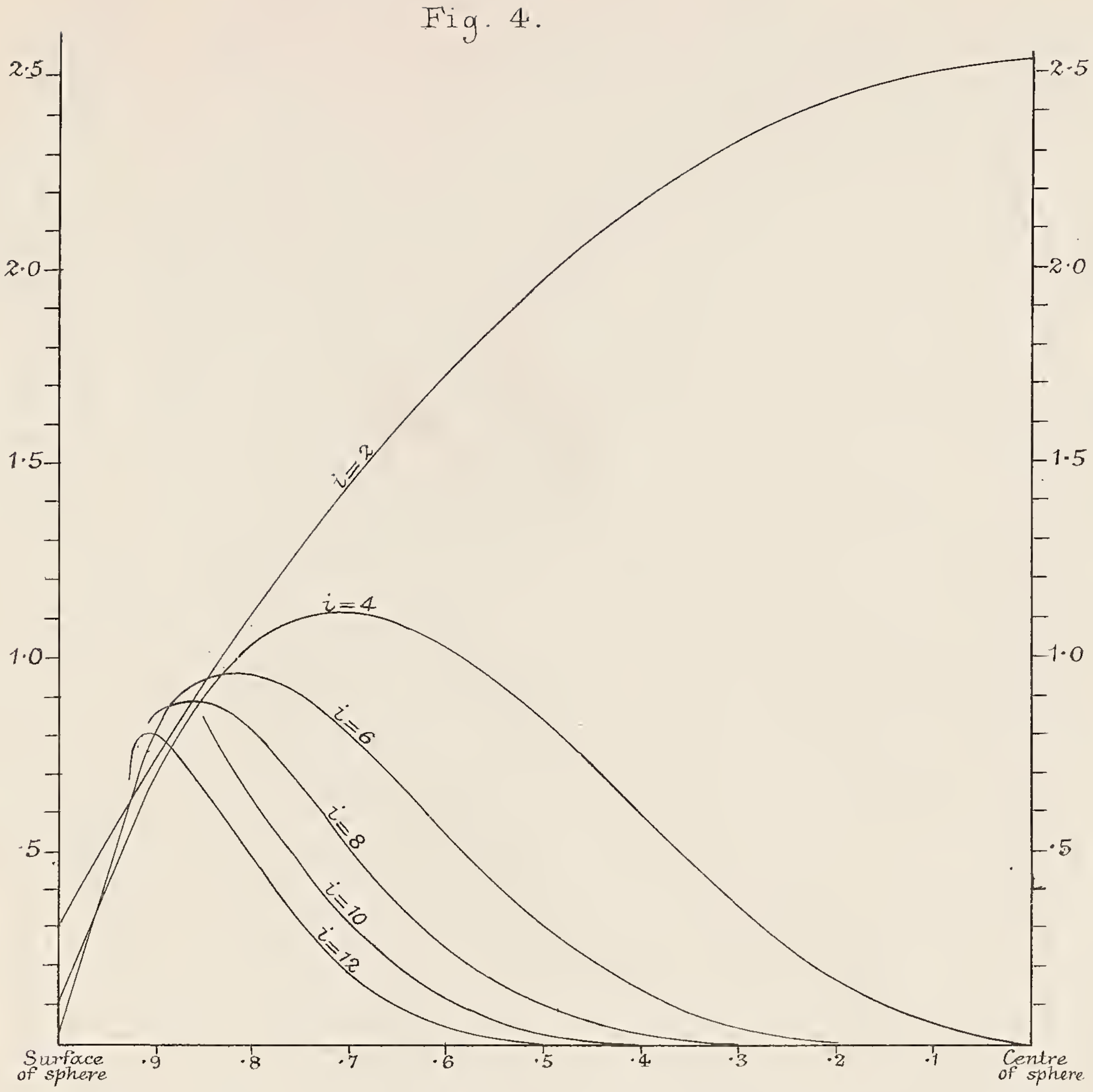


Diagram showing the difference of the principal stresses at the equator, due to inequalities represented by the even zonal harmonics.

Fig. 5.

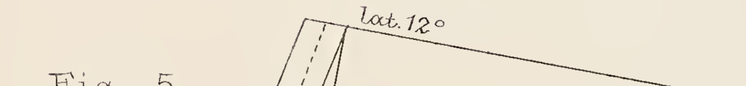

lat. 12°
lat. 8°
lat. 4°
Equator
Inequality to be represented
Representation
Repn. without 2nd harmonic constituent
Mean Level of Earth.

Diagram showing the profile of isolated equatorial continents.
Radius of sphere 18 inches.

Fig. 6.

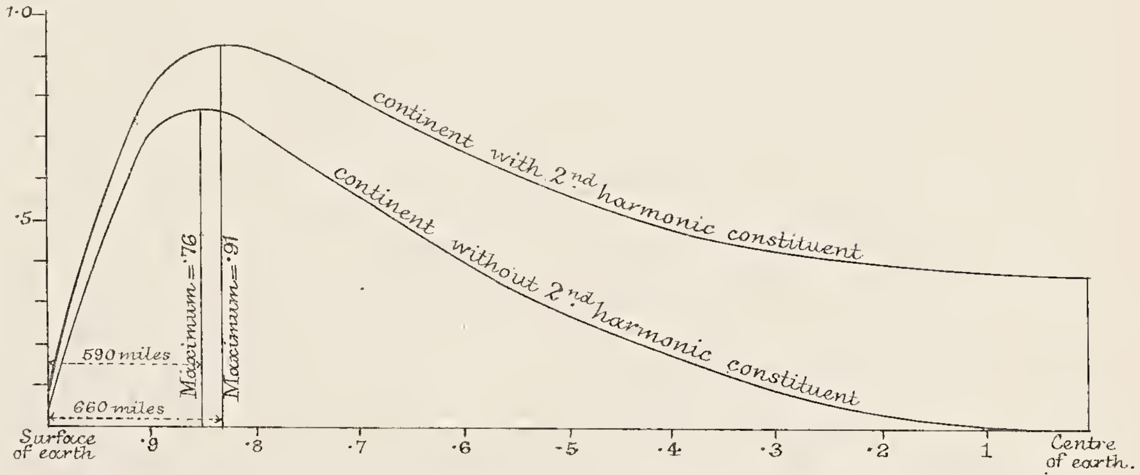


Diagram showing the difference of principal stresses at the equator due to isolated equatorial continents.

West, Newman & Co. lith.

Balfour & Parker.

Phil. Trans. 1882. Plate 21.

Fig. 1.

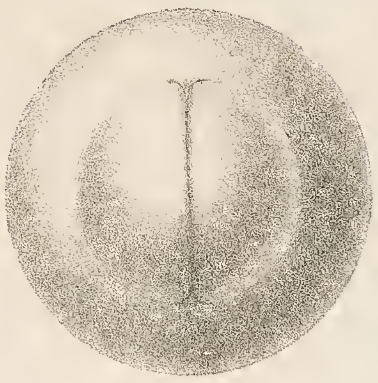


Fig. 2.

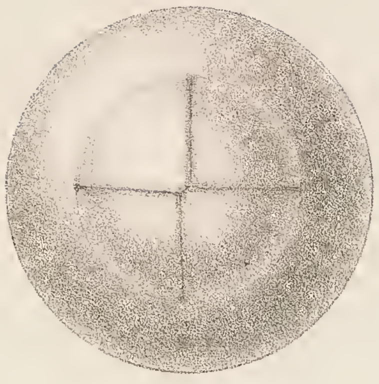


Fig. 3.

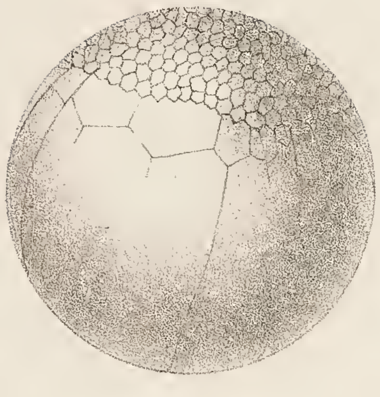


Fig. 4.

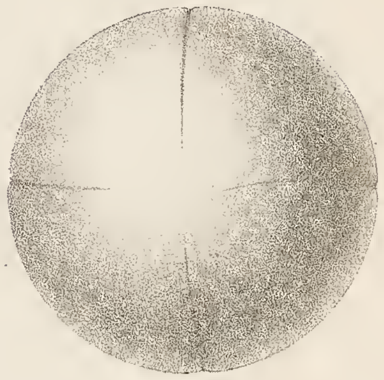


Fig. 5.

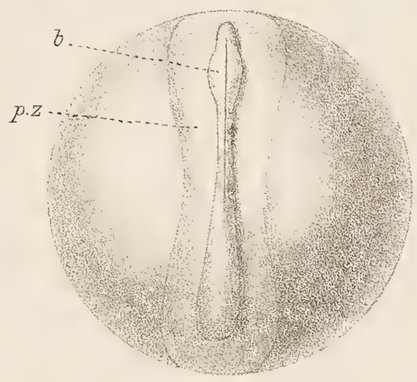


Fig. 6.

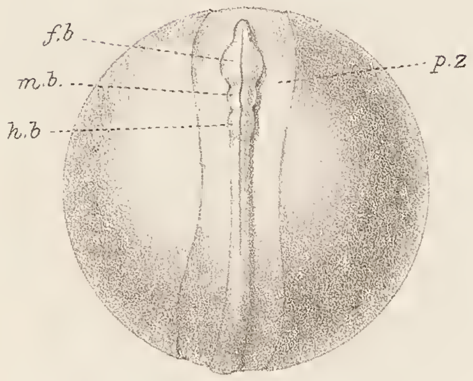


Fig. 7.

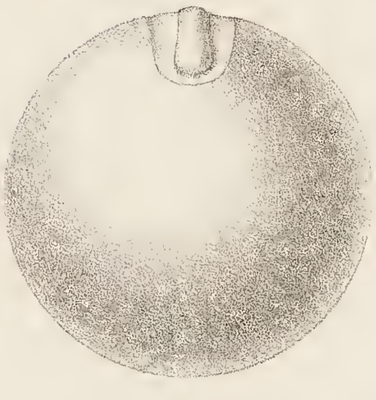


Fig. 8.

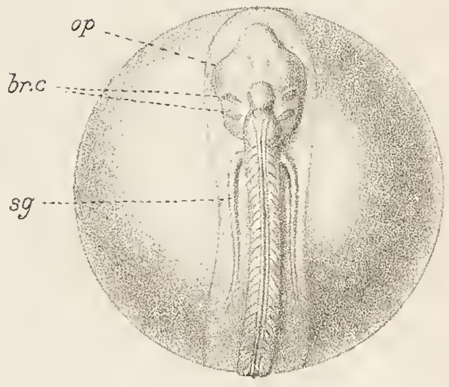


Fig. 12.

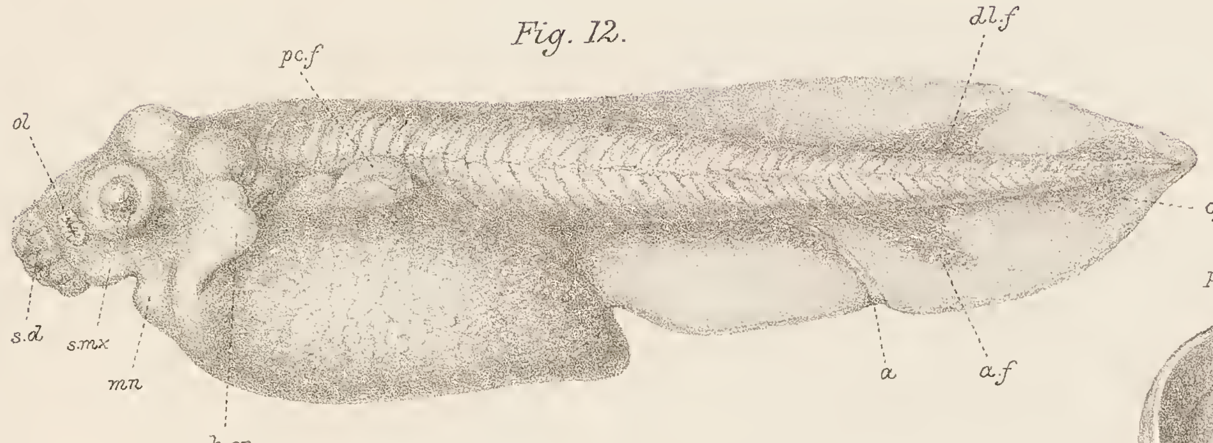


Fig. 9.

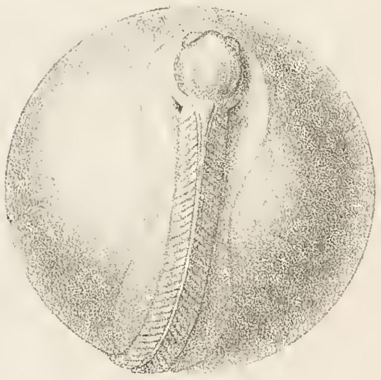
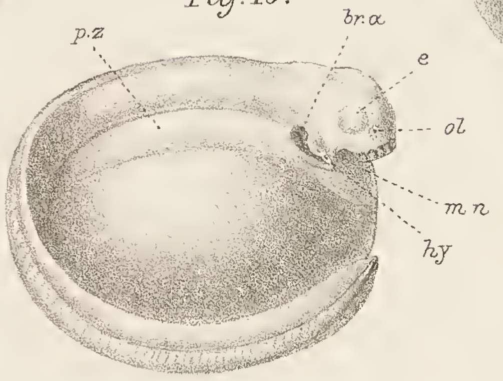


Fig. 10.



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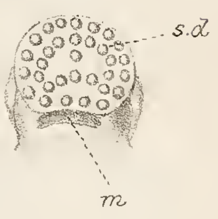

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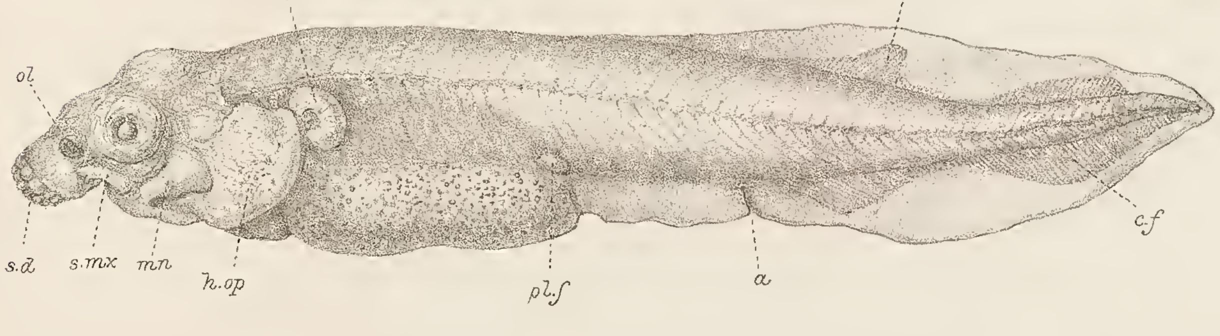

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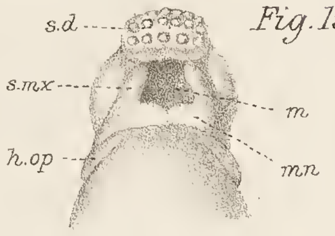

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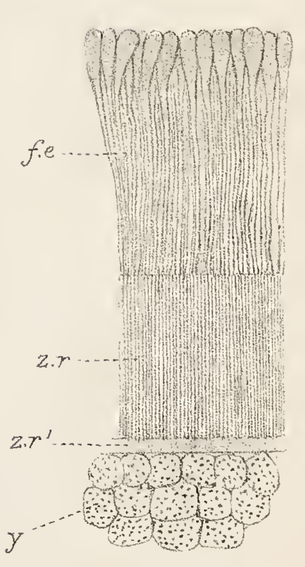

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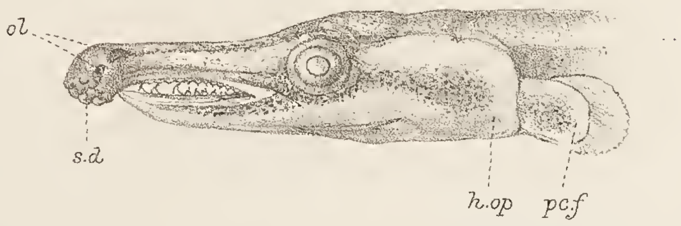

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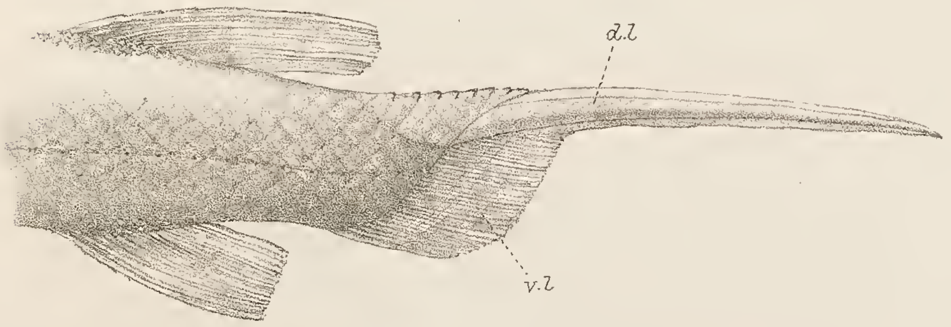
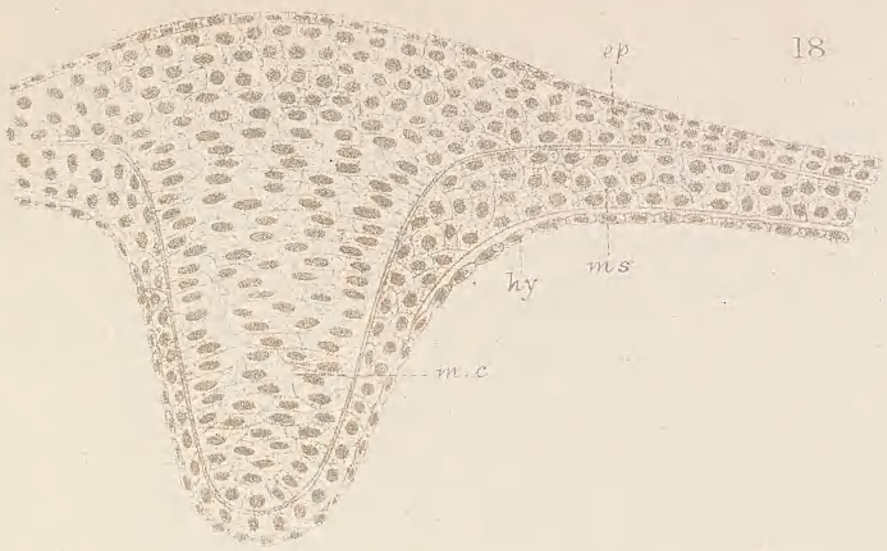


W.K.P, & W.N.P del. ad nat.

F. Huth, Lith^r Edin^r

18



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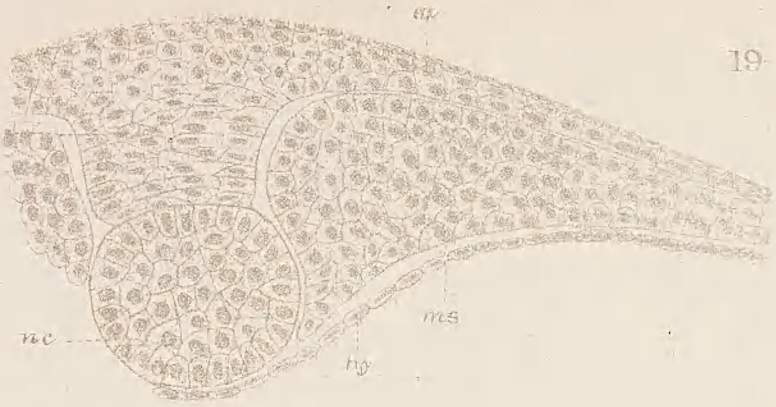

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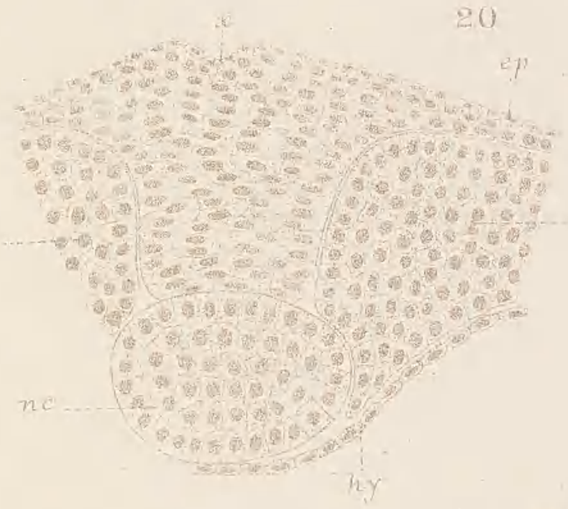

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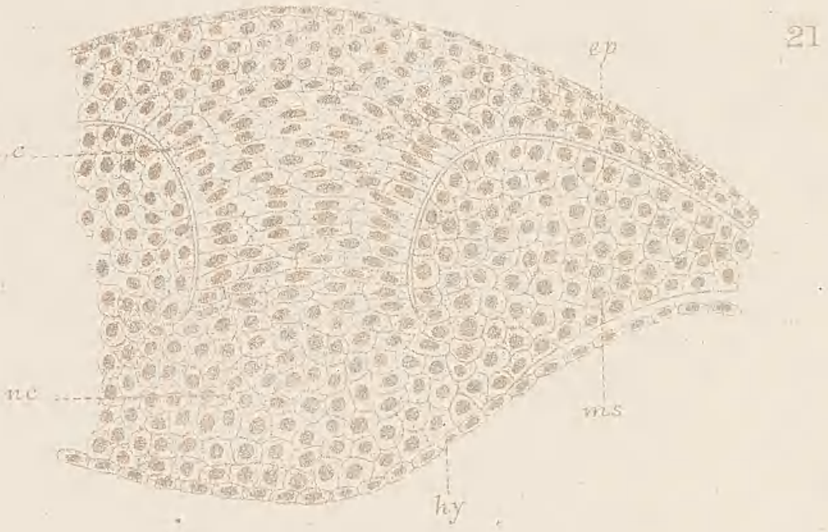

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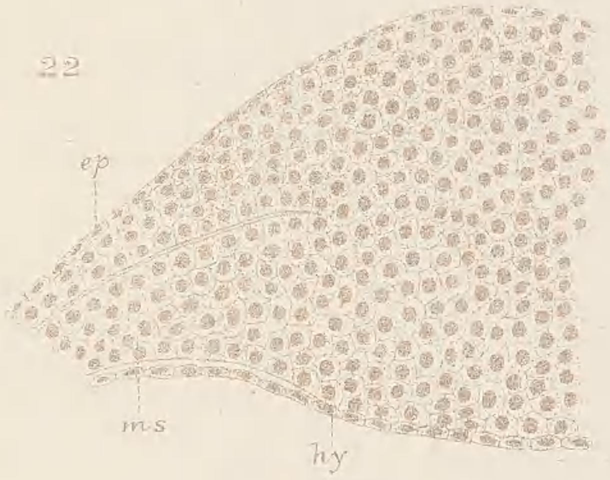

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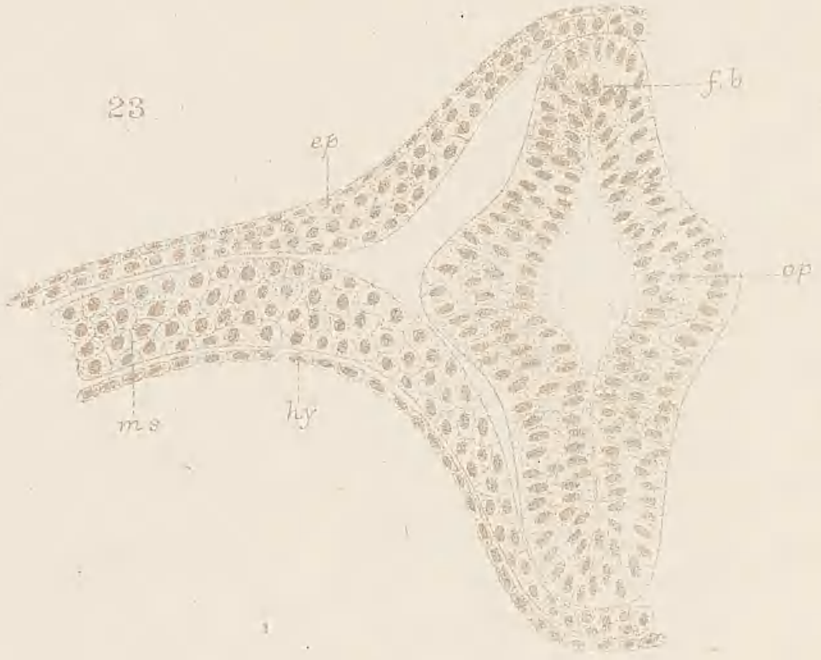

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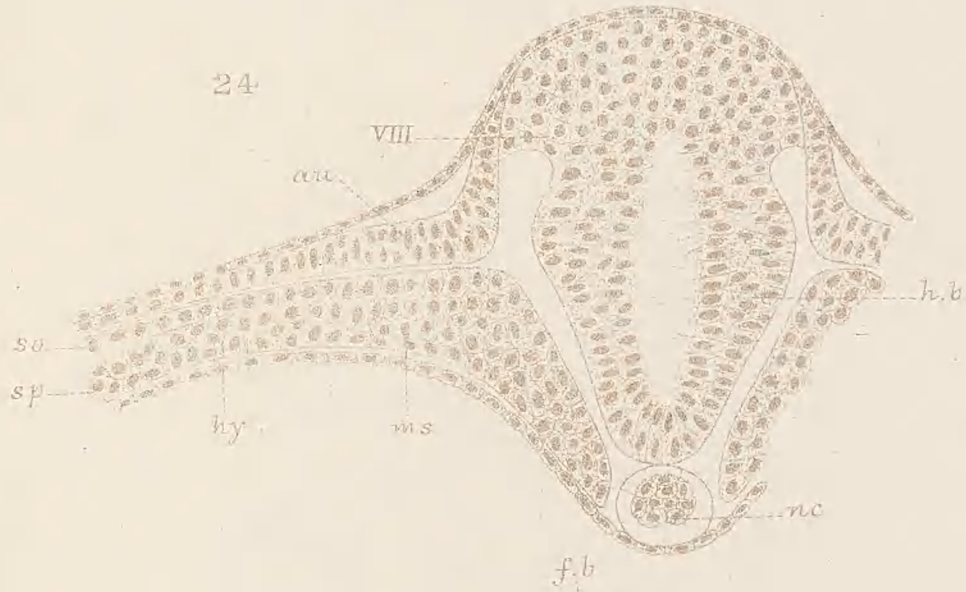

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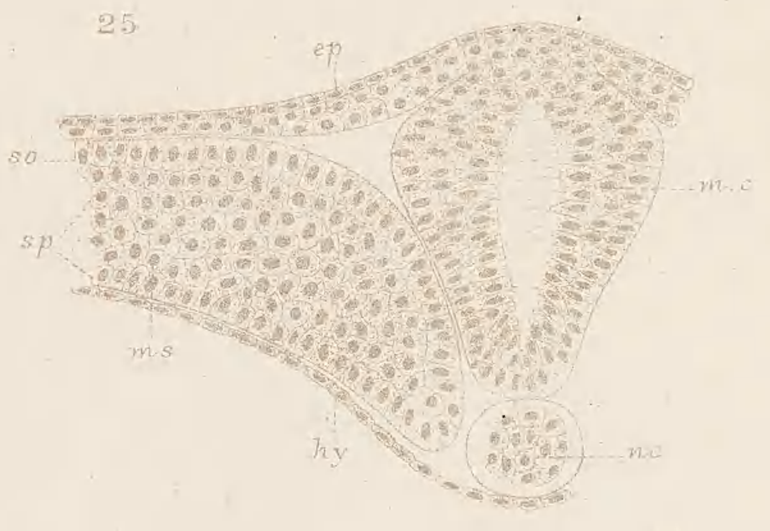

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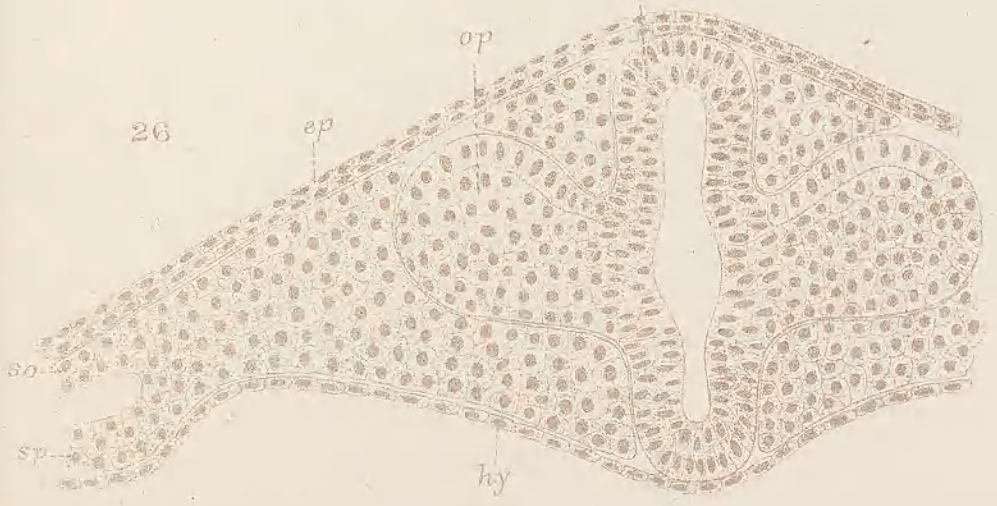

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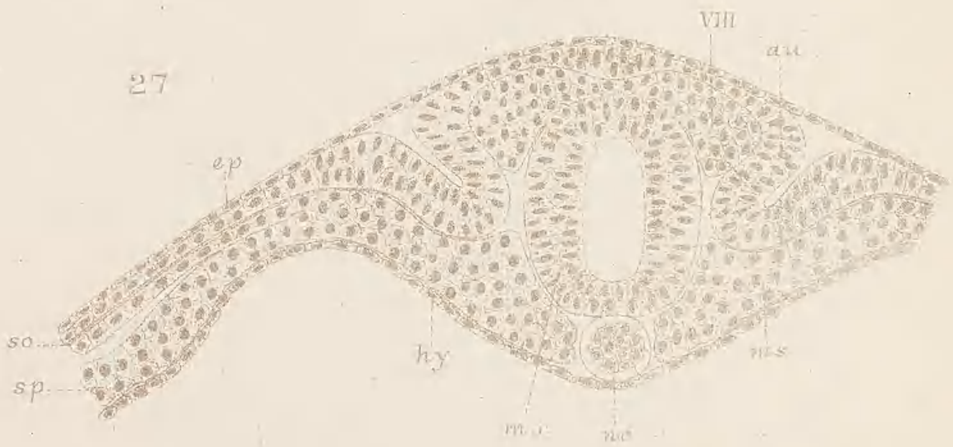
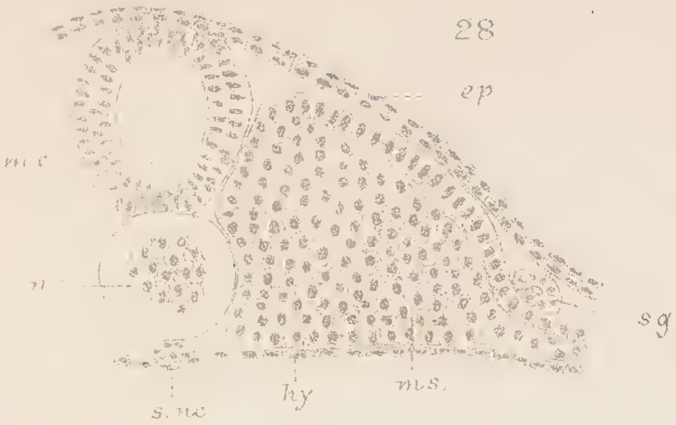


W.N.P. del ad nat.
M.P. Parker lith.

West Newman & Co imp.

28



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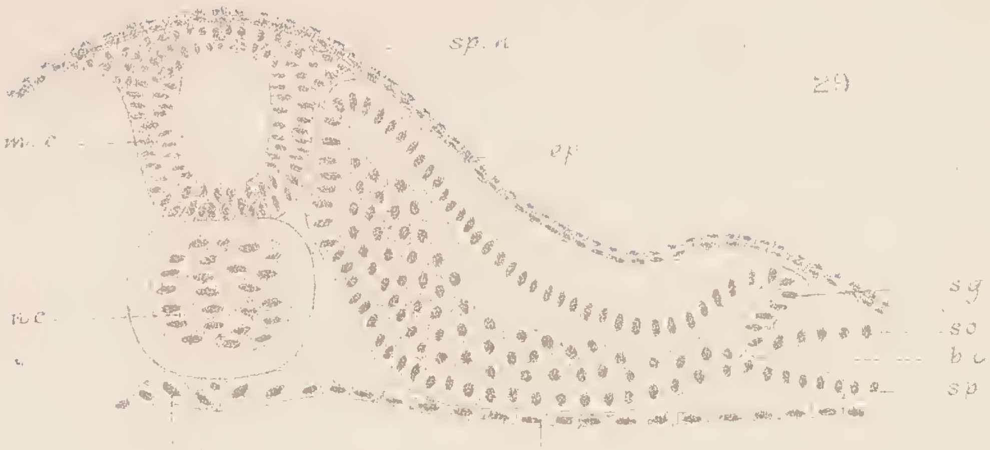

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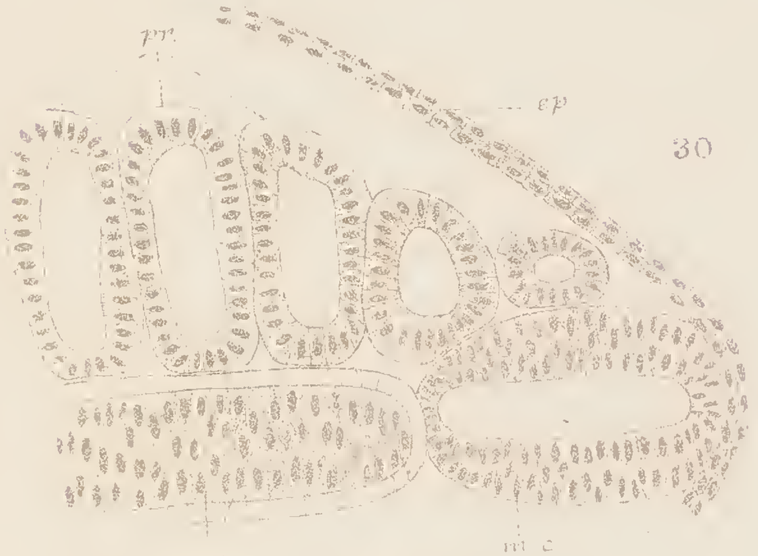

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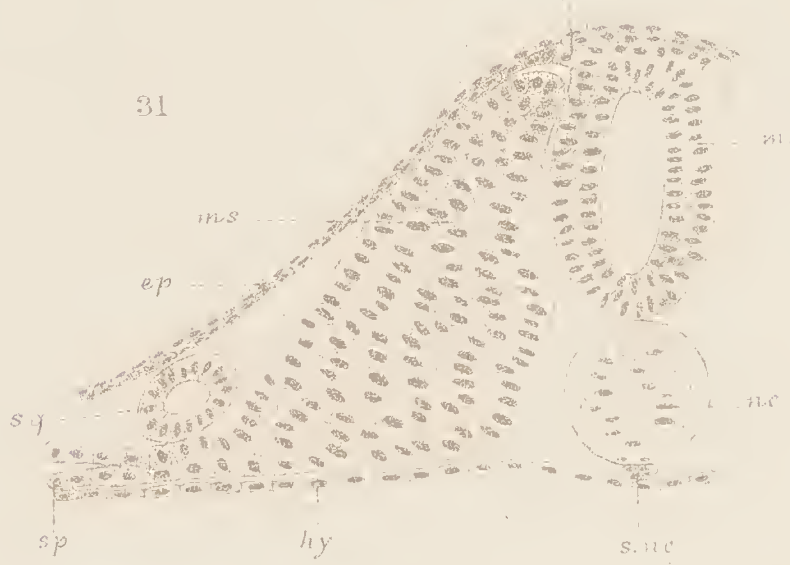

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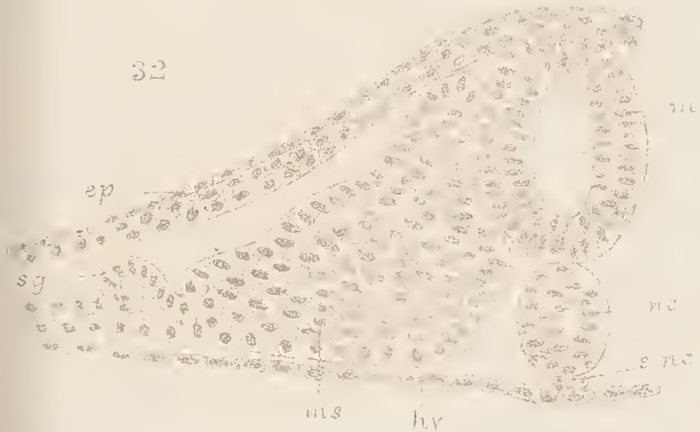

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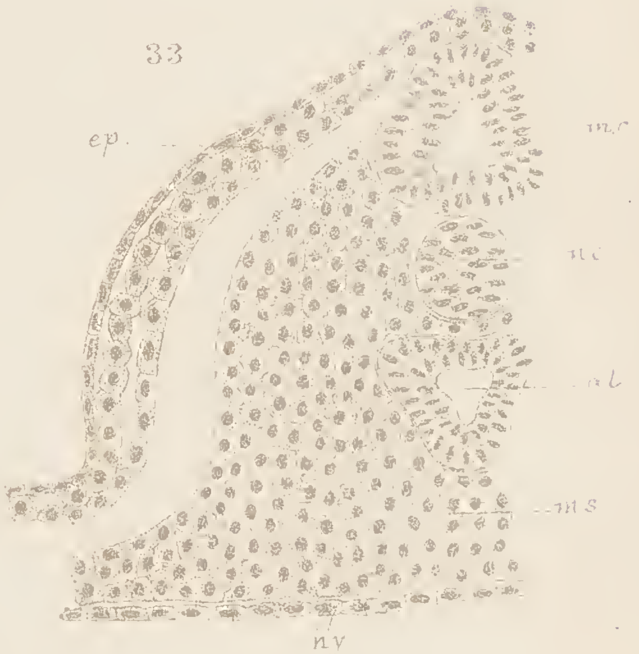

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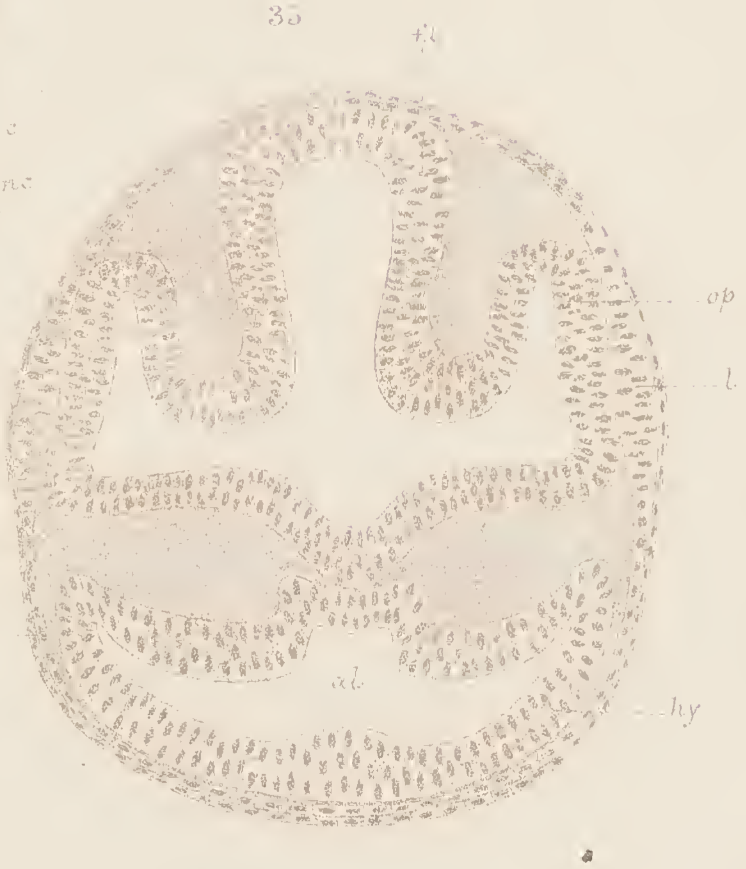

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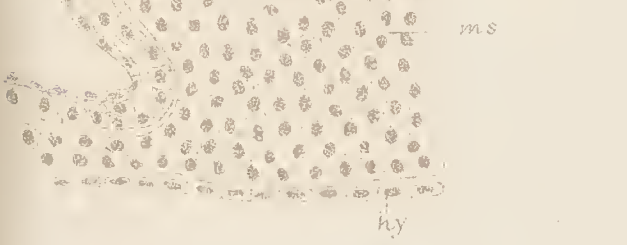

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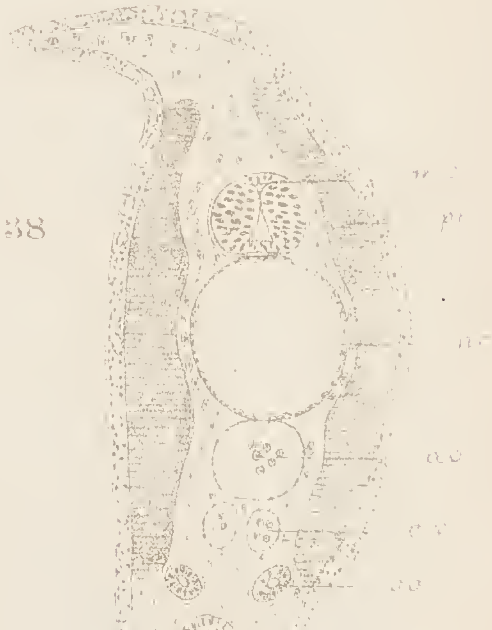

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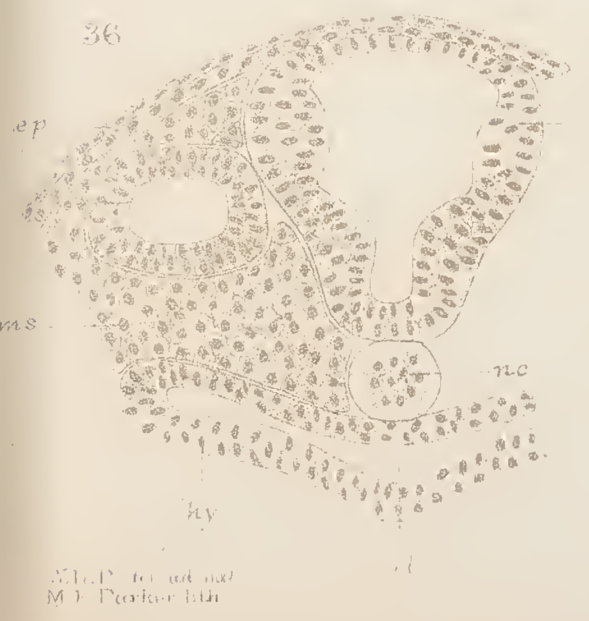

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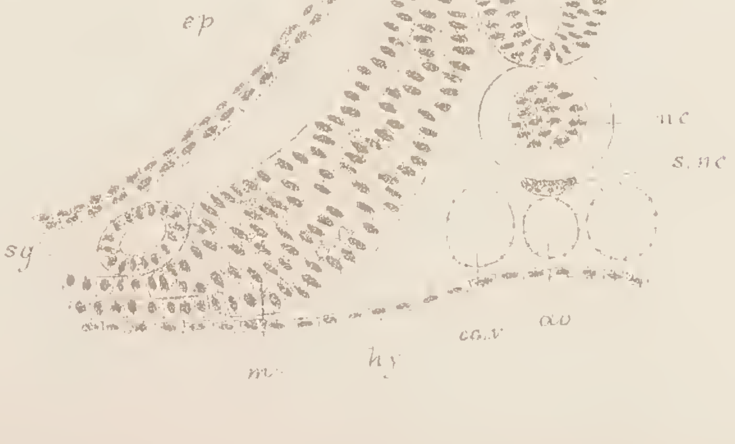

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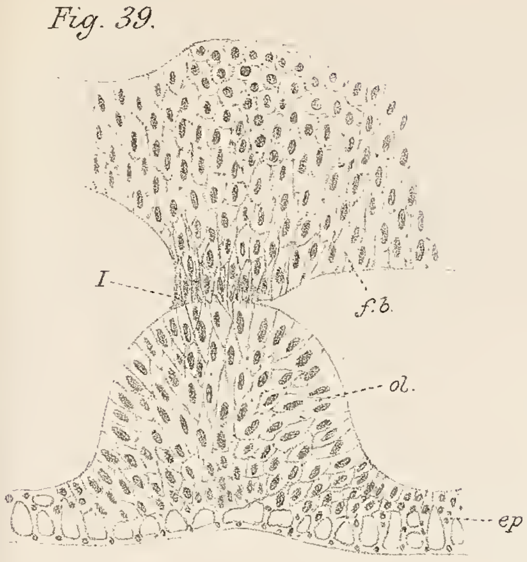

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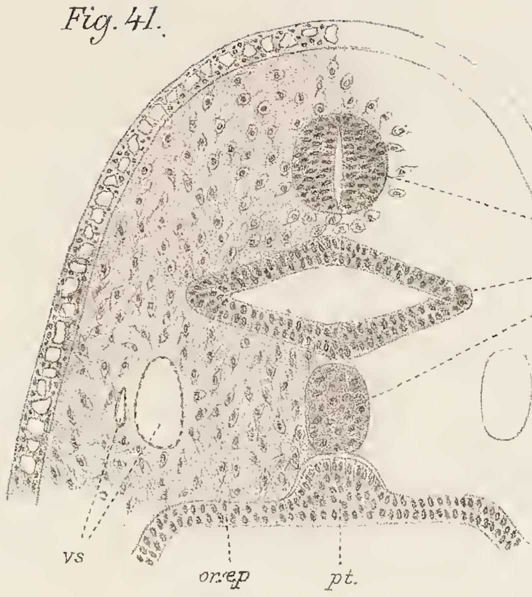

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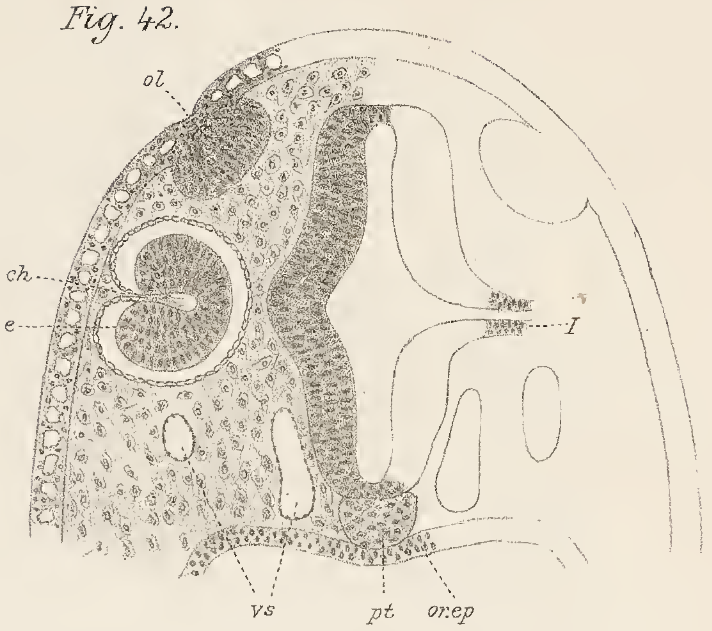

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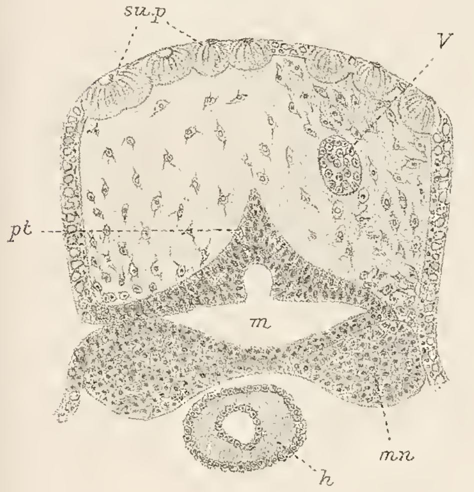

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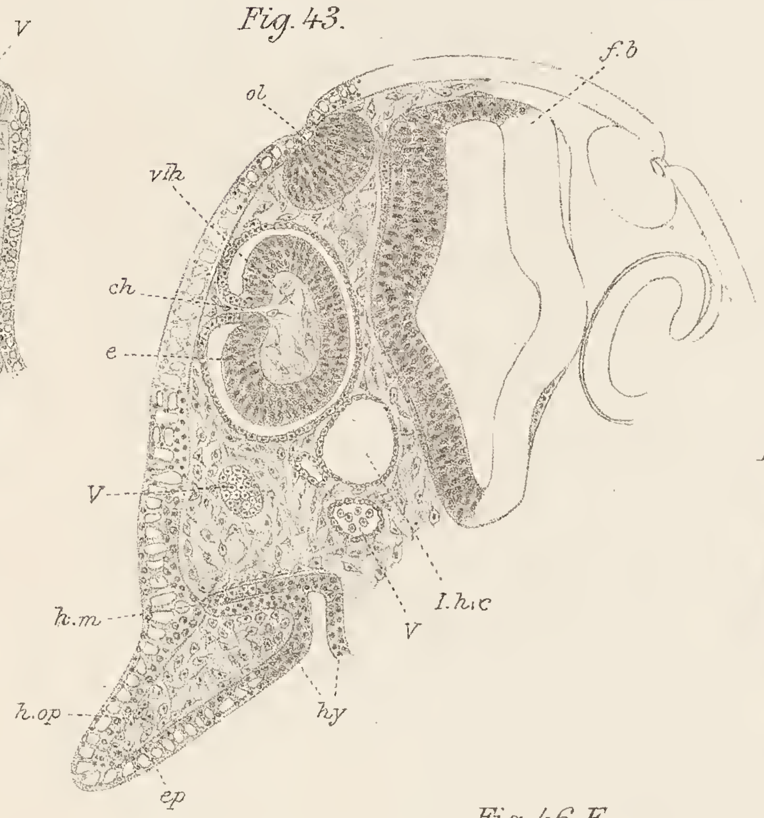

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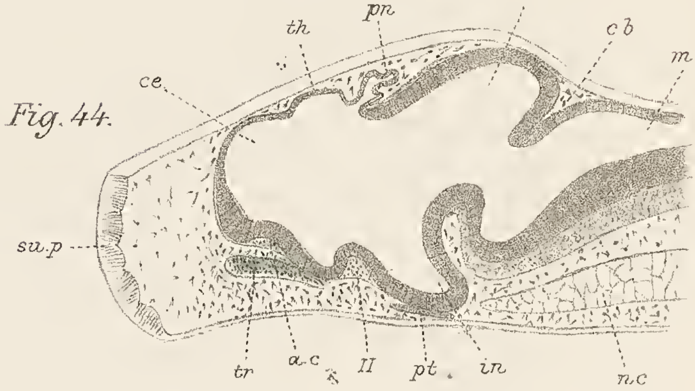


Fig. 46 A.

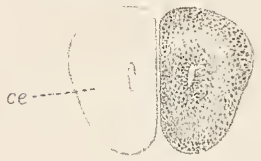


Fig. 46 B.

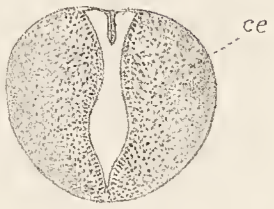


Fig. 46 C.

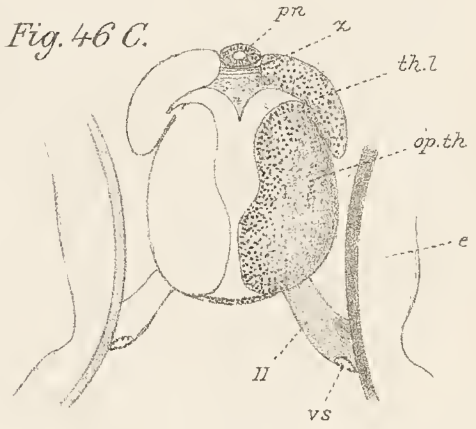


Fig. 46 D.

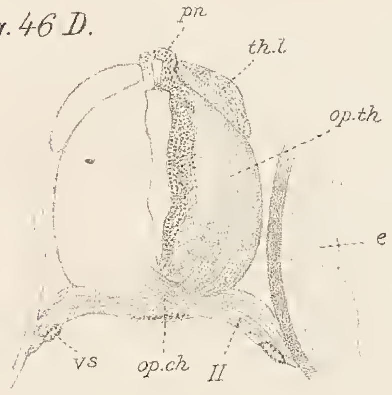


Fig. 46 E.

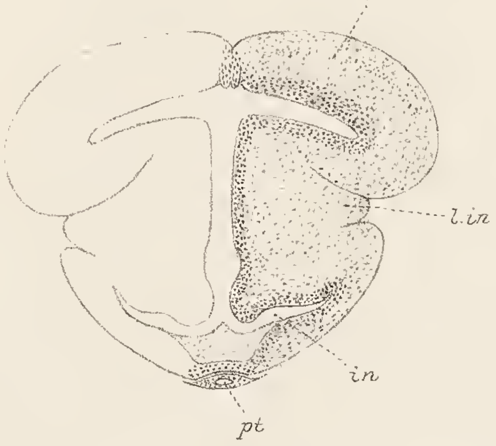


Fig. 46 F.

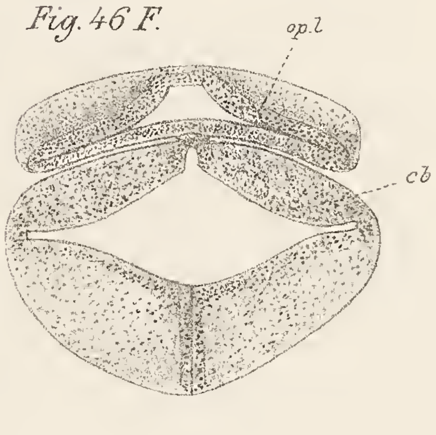
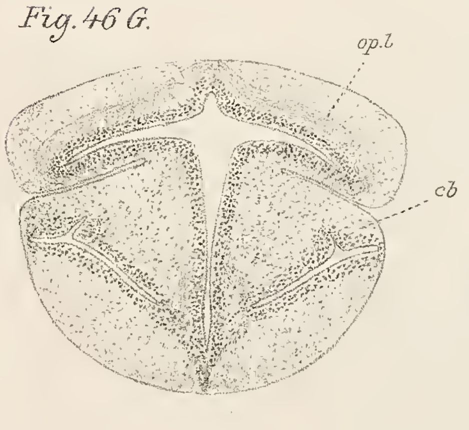


Fig. 46 G.



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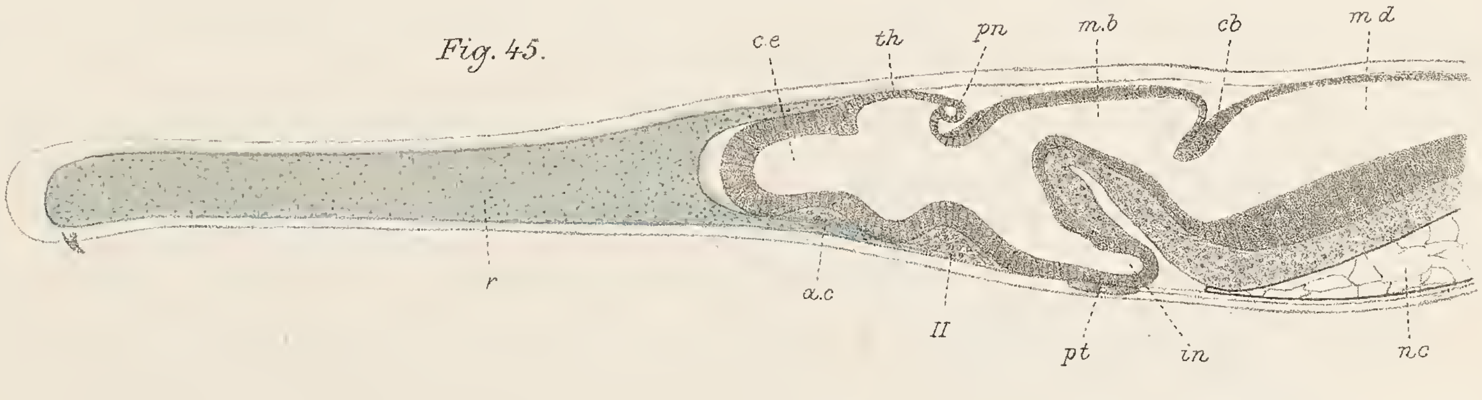


W. N. P. del. ad nat.

F Huth, Lith^r Edin^r

Fig. 47 A.

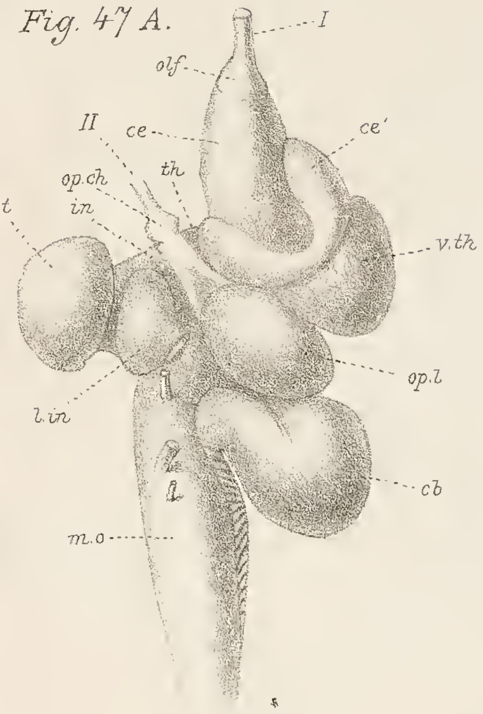


Fig. 47 B.

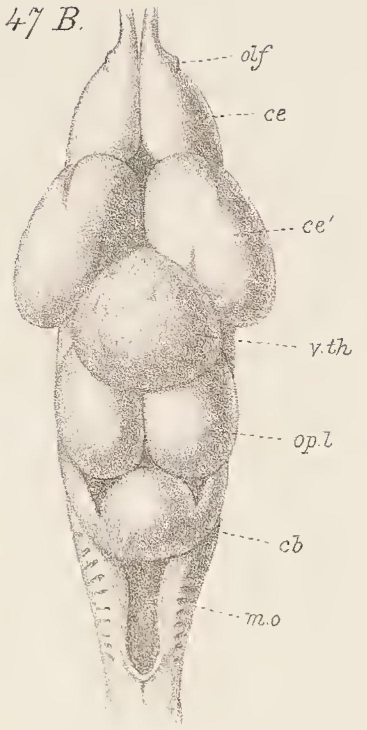


Fig. 47 C.

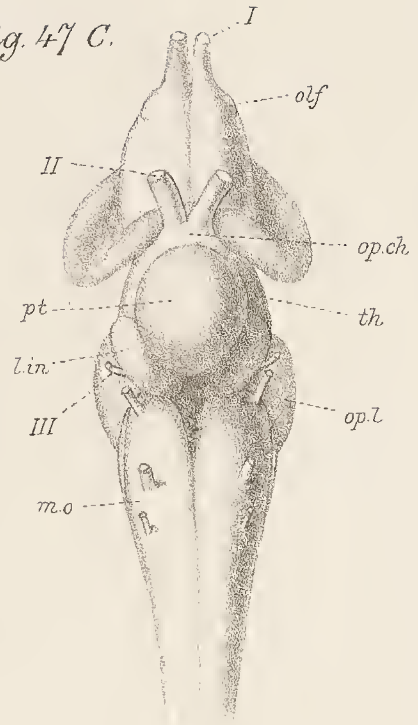
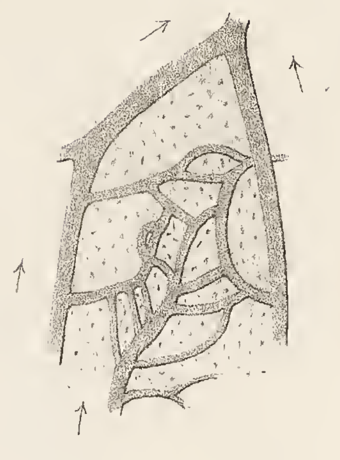


Fig. 50.



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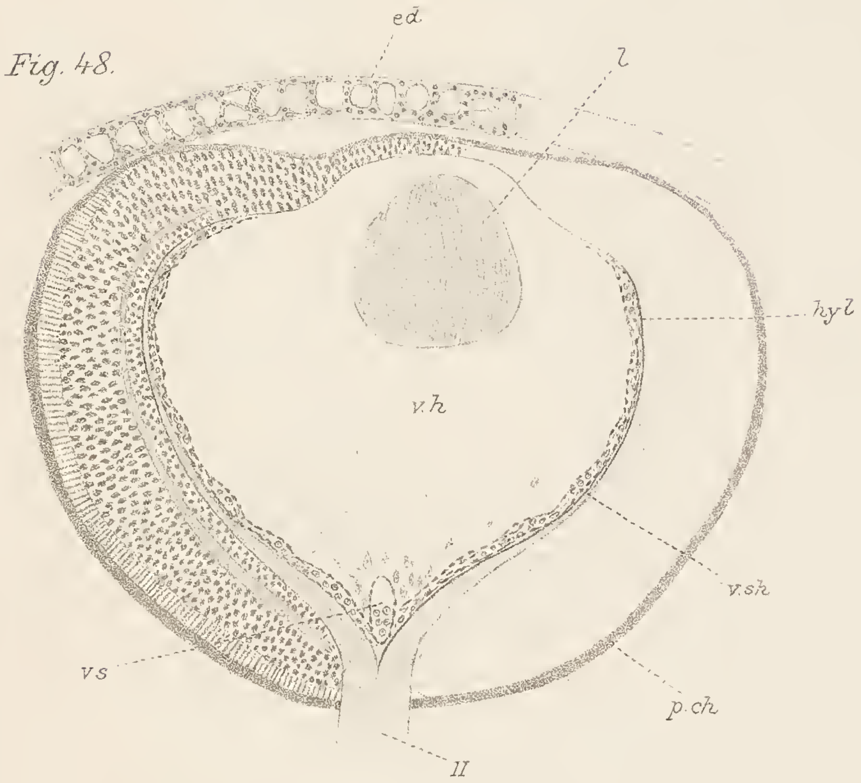

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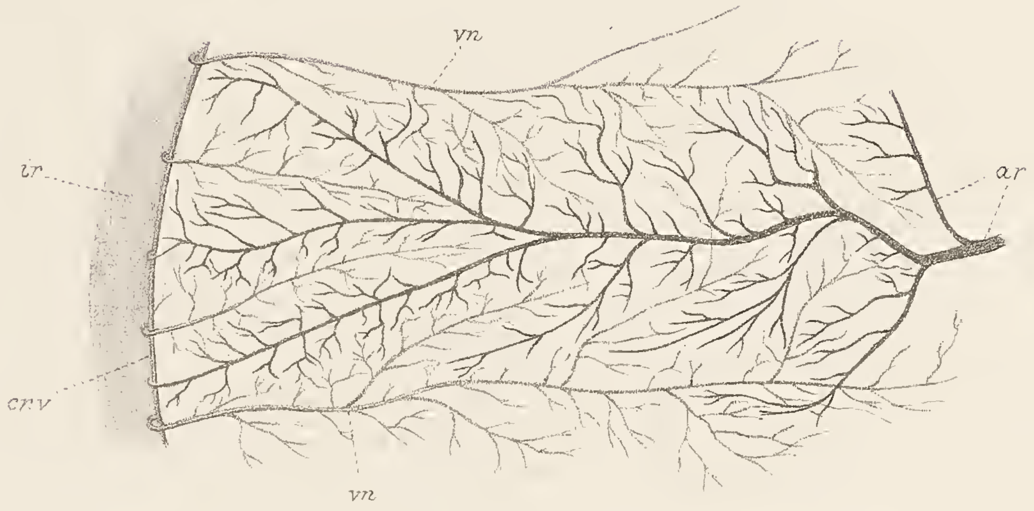

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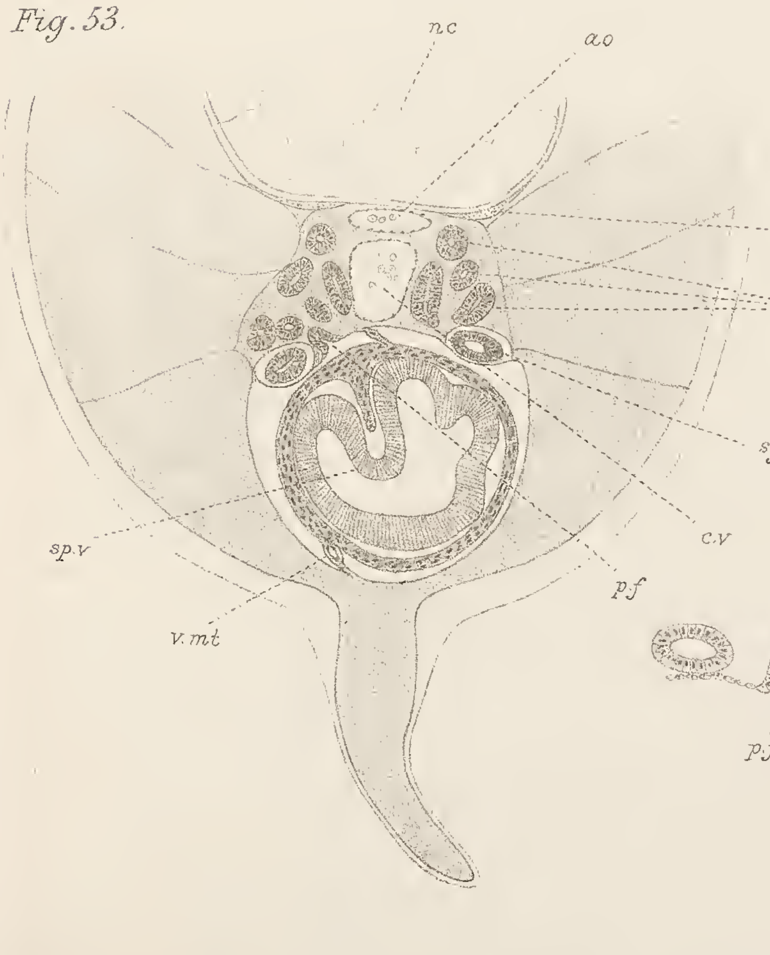

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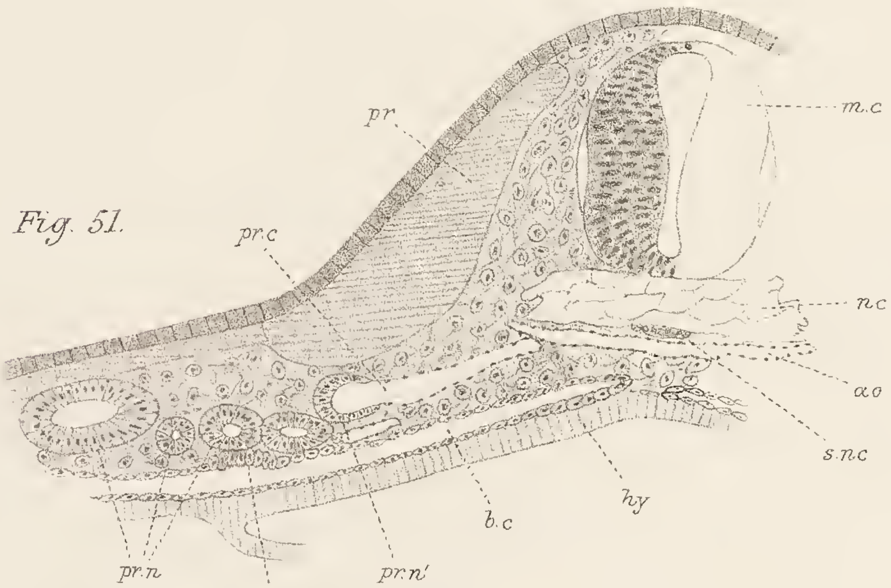

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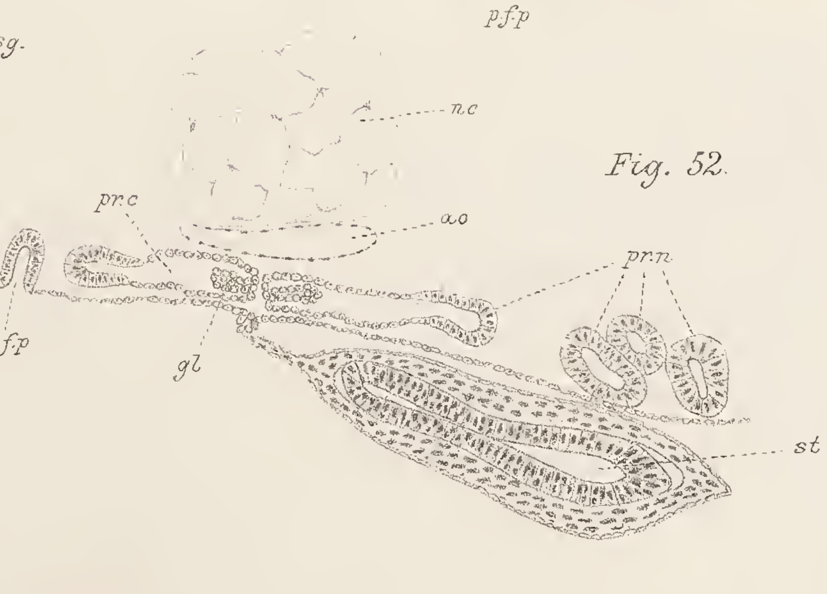
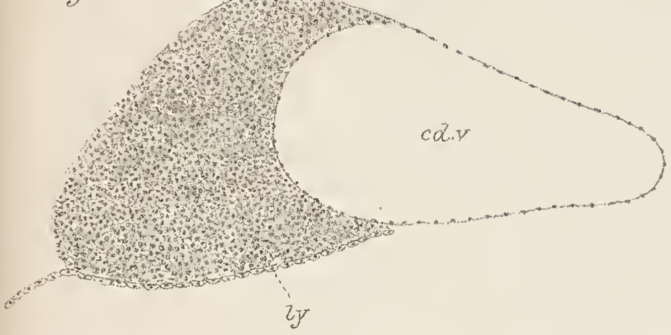


W N P., & F. M. B, del. ad nat.

F. Huth, Lith^r Edin^r

Fig. 54.



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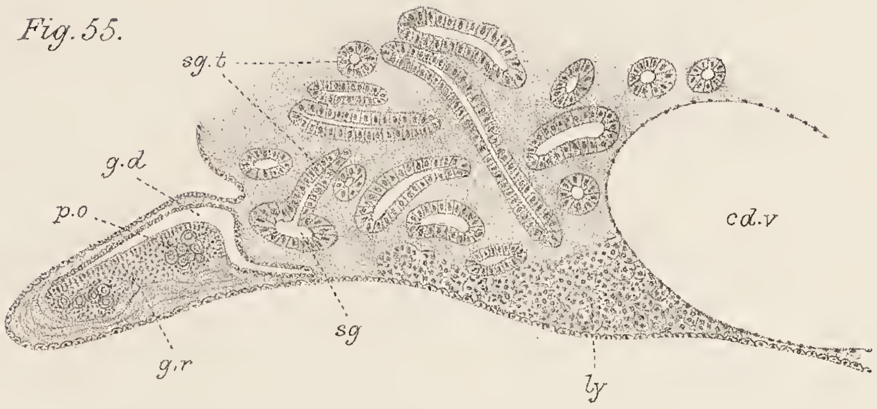

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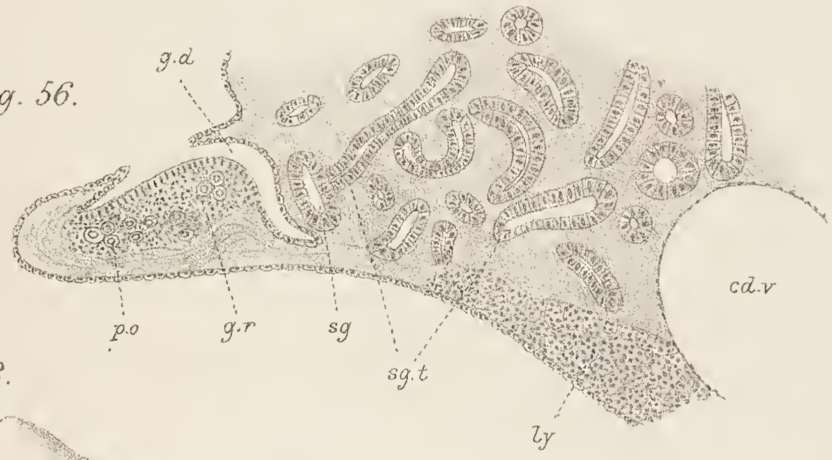

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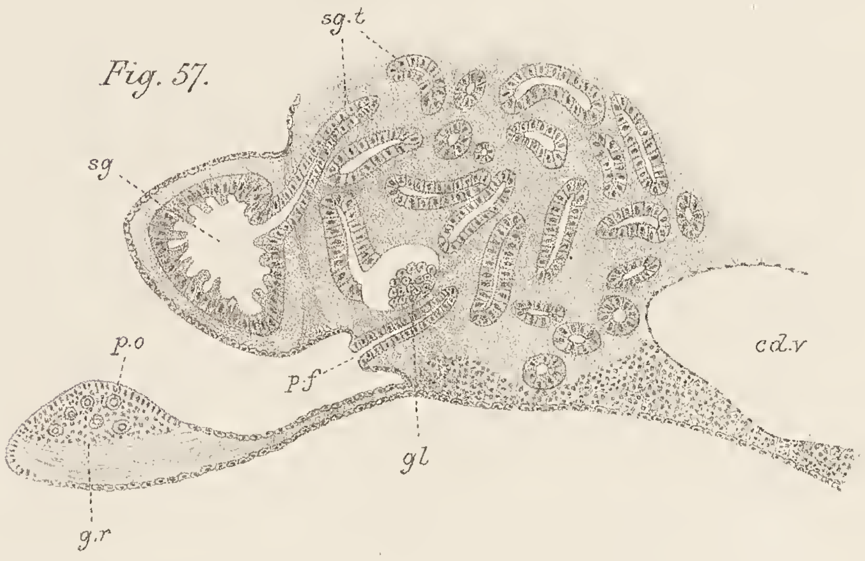
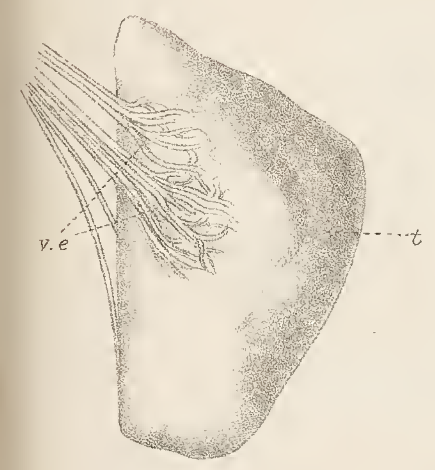


Fig. 58 B.



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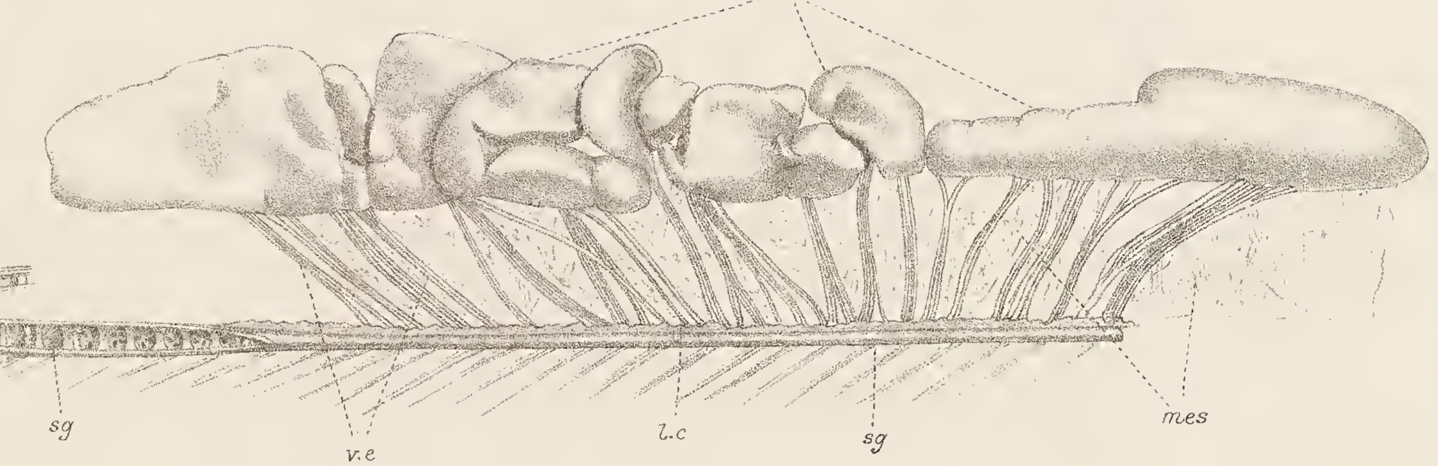
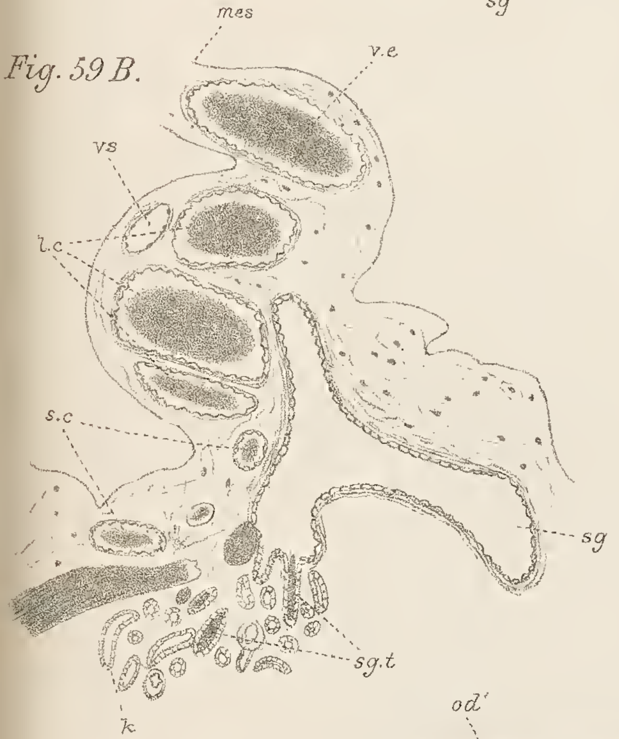


Fig. 59 B.



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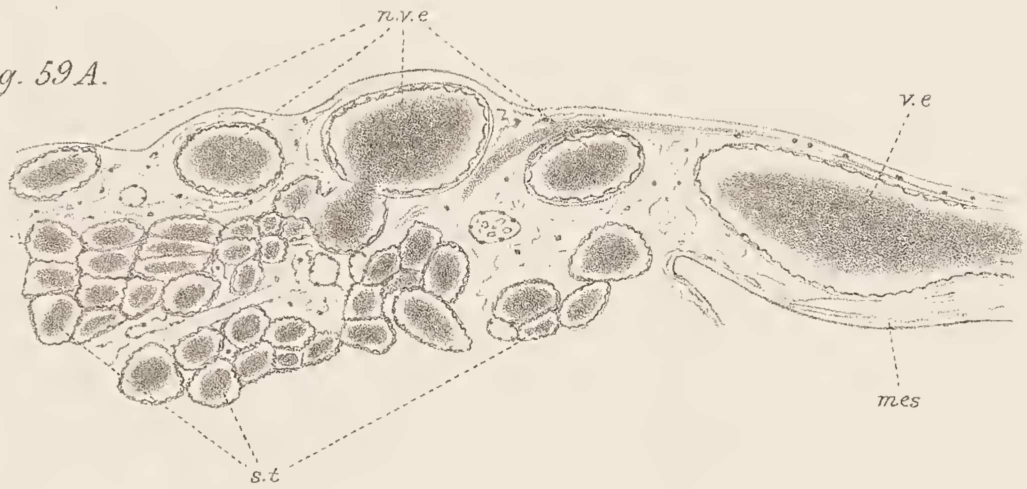
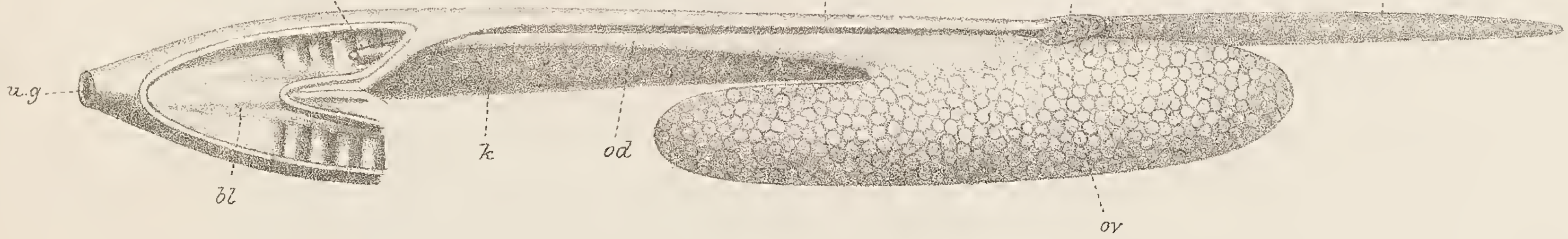


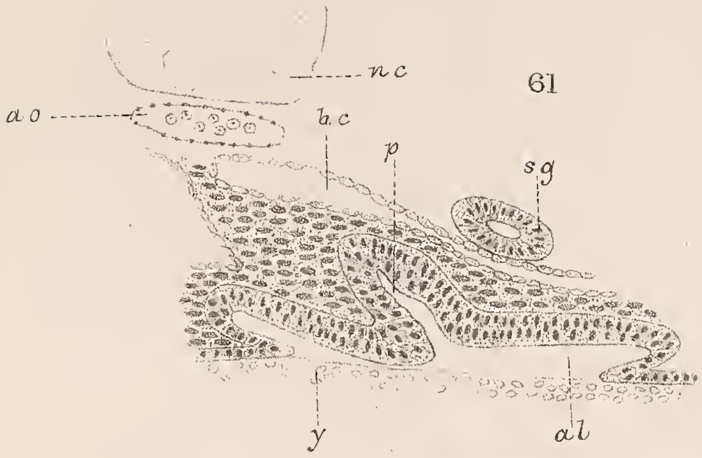
Fig. 60.



F.M.B., W.N.P. & M.P.P. del. ad nat.

F. Huth, Lith^r. Edin^r.

61



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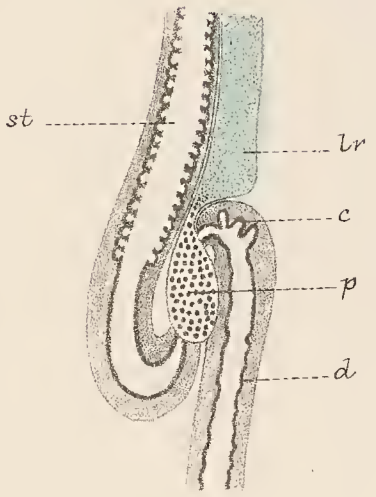

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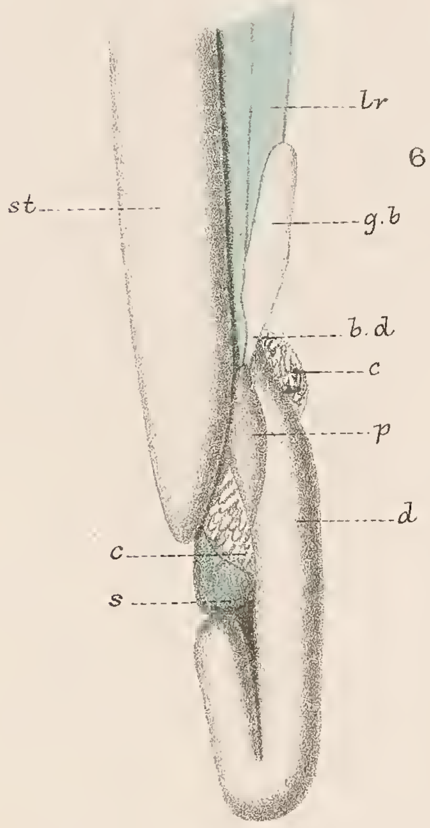

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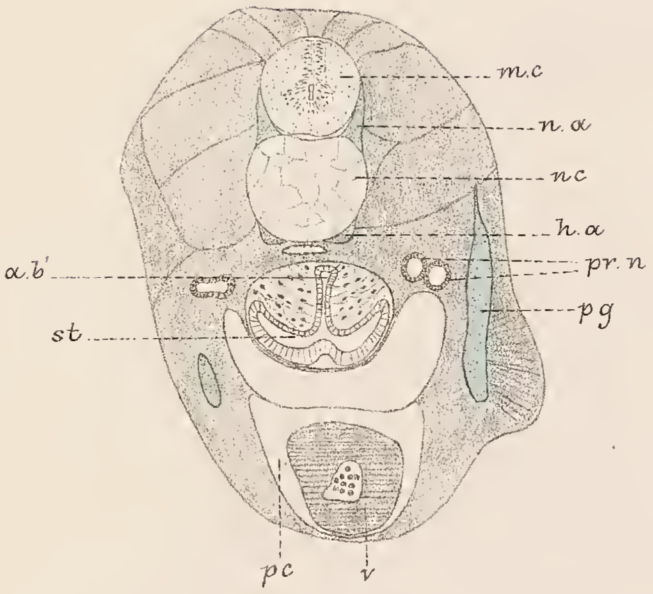

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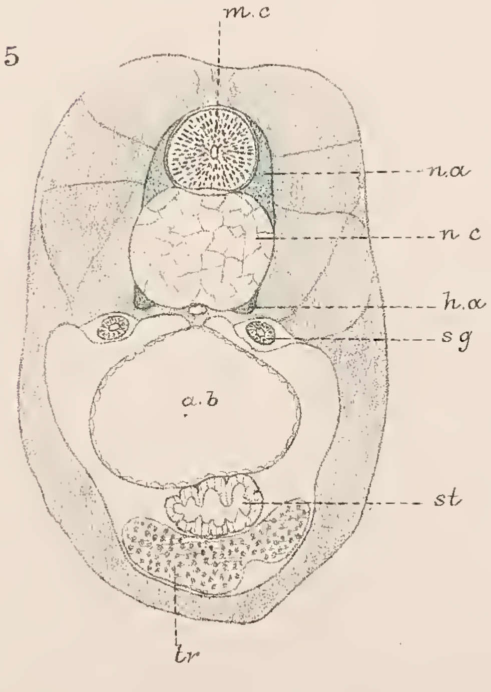

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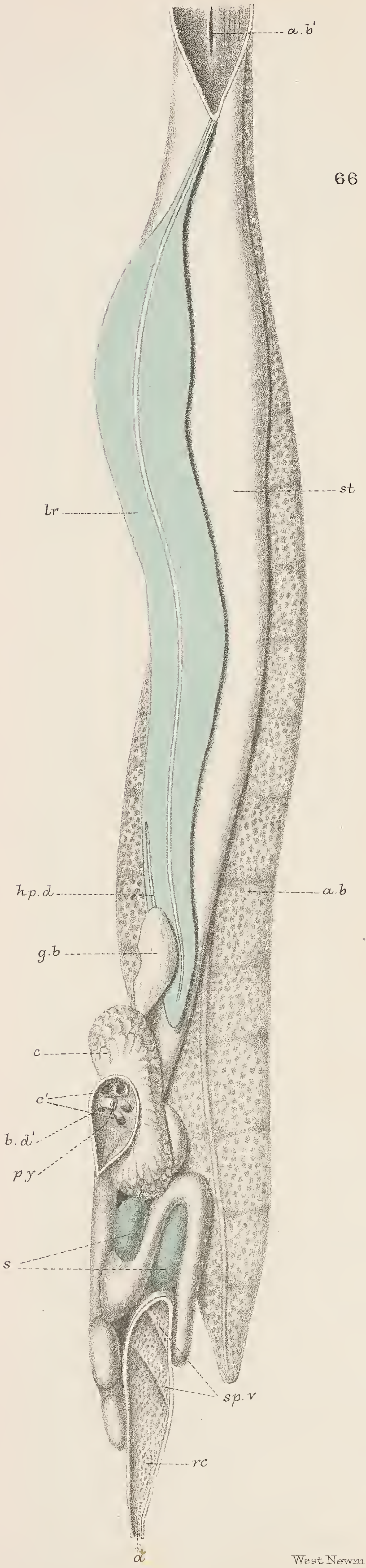
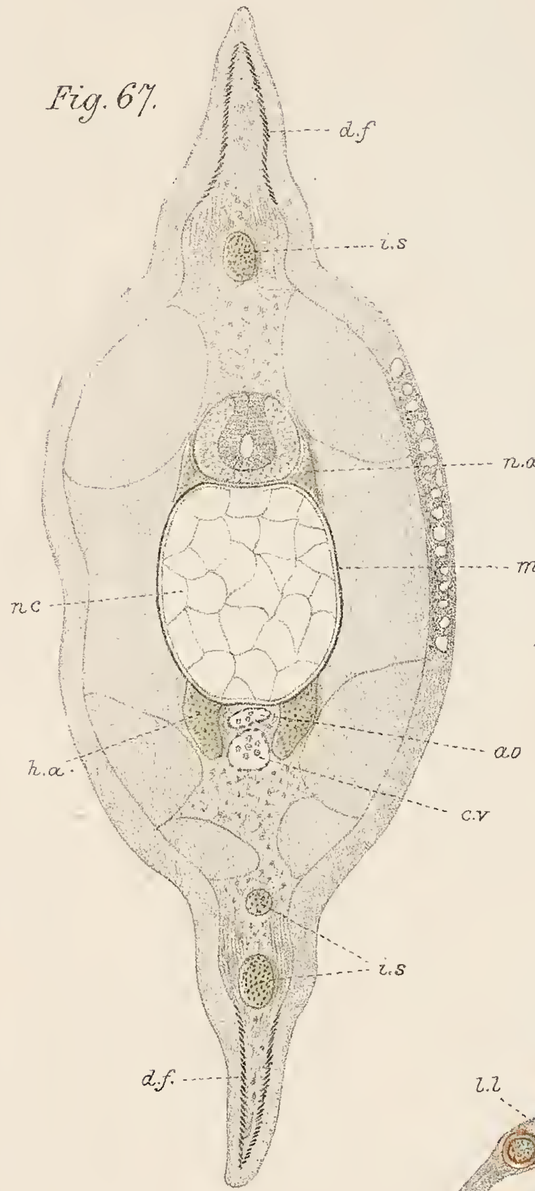


W.N.P. & A.C.H. del. ad nat.
M.P. Parker lith.

West Newman & Co. imp.

Fig. 67.



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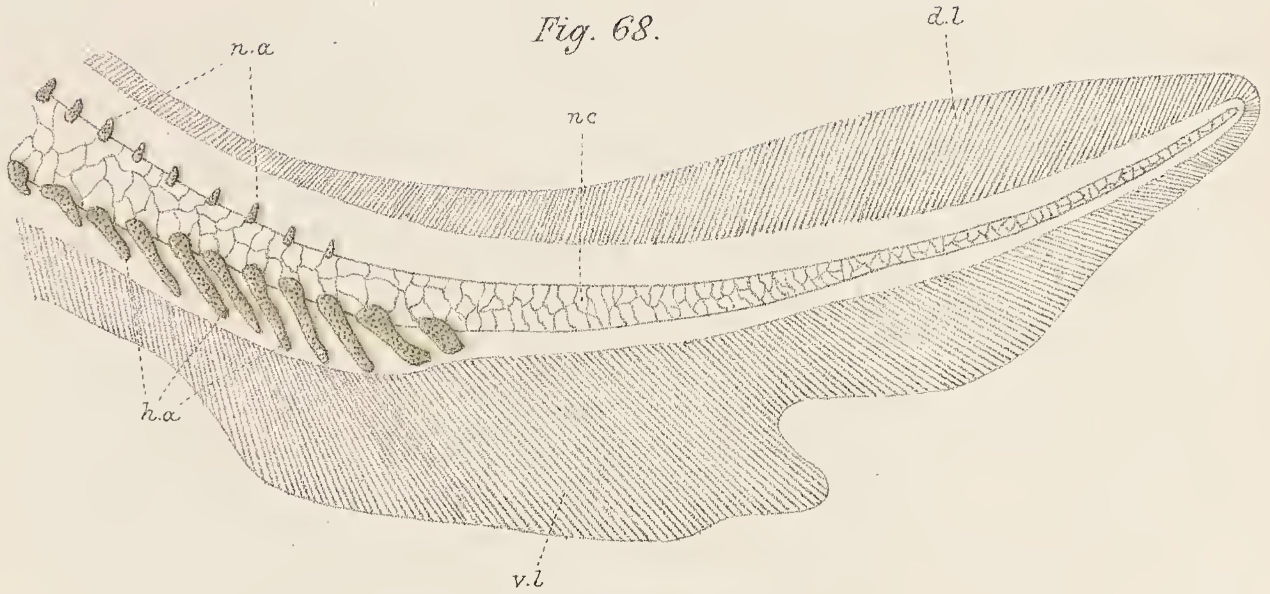


Fig. 69.

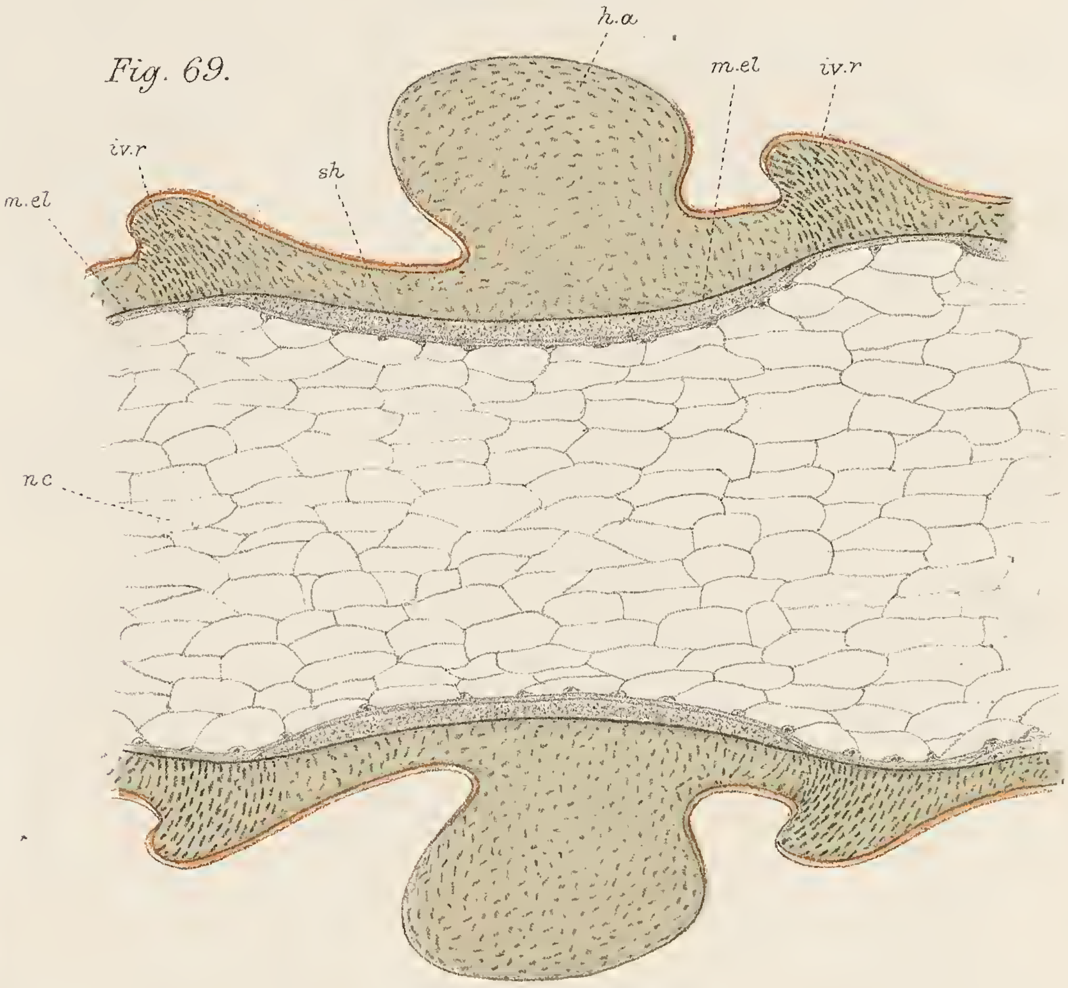
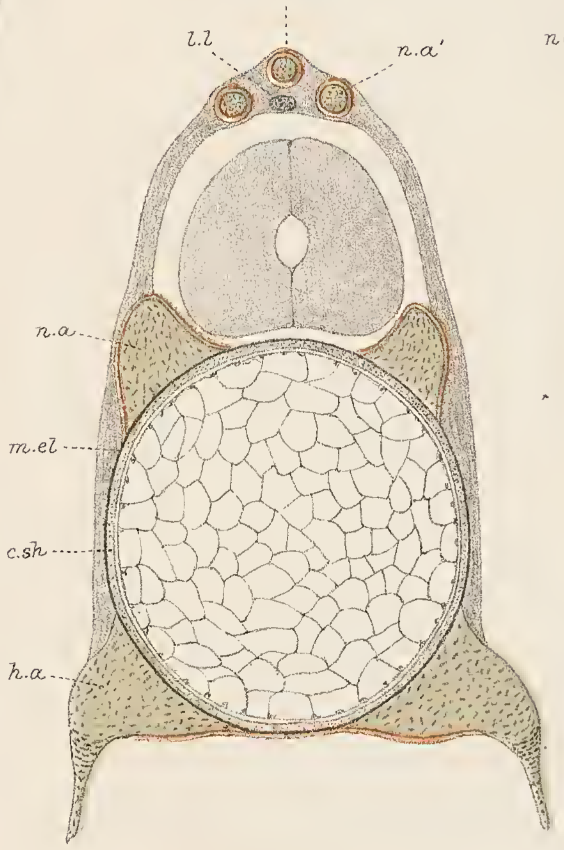


Fig. 70.



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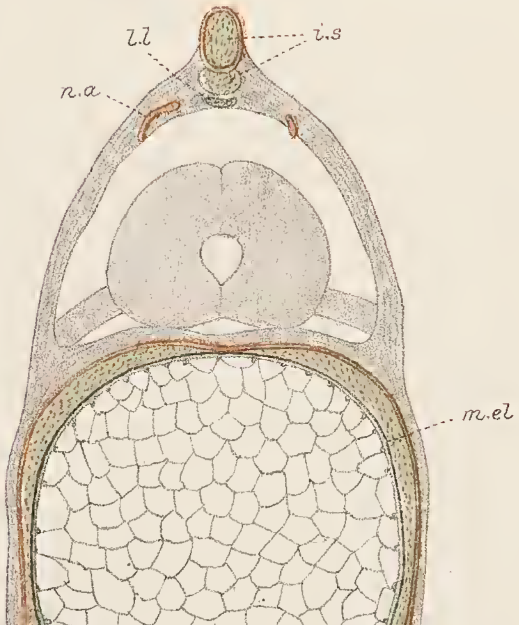
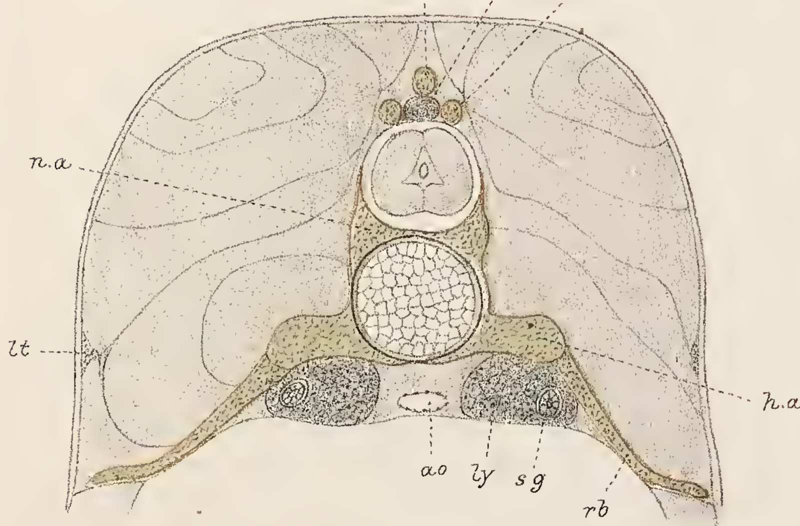


Fig. 72.



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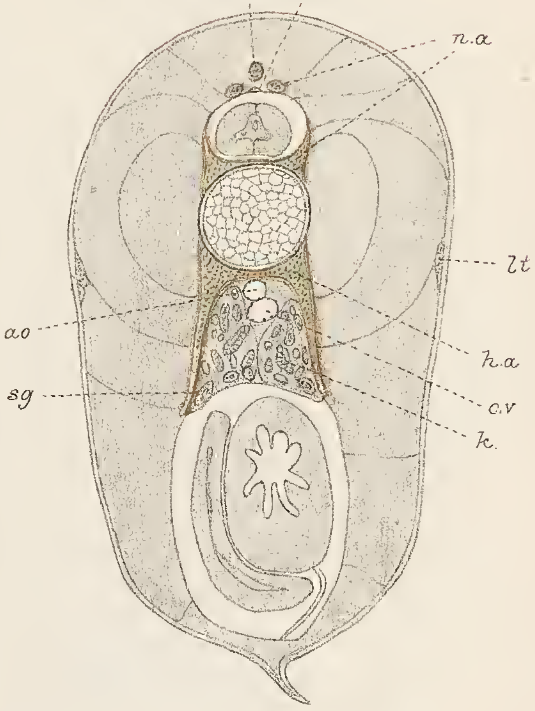

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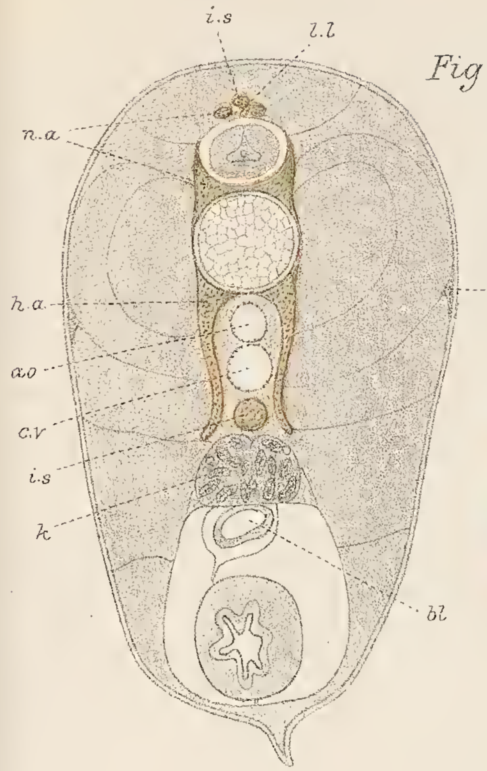

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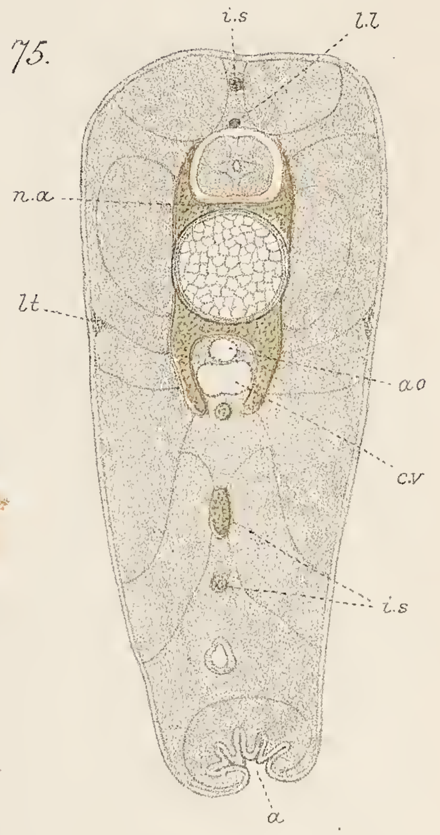

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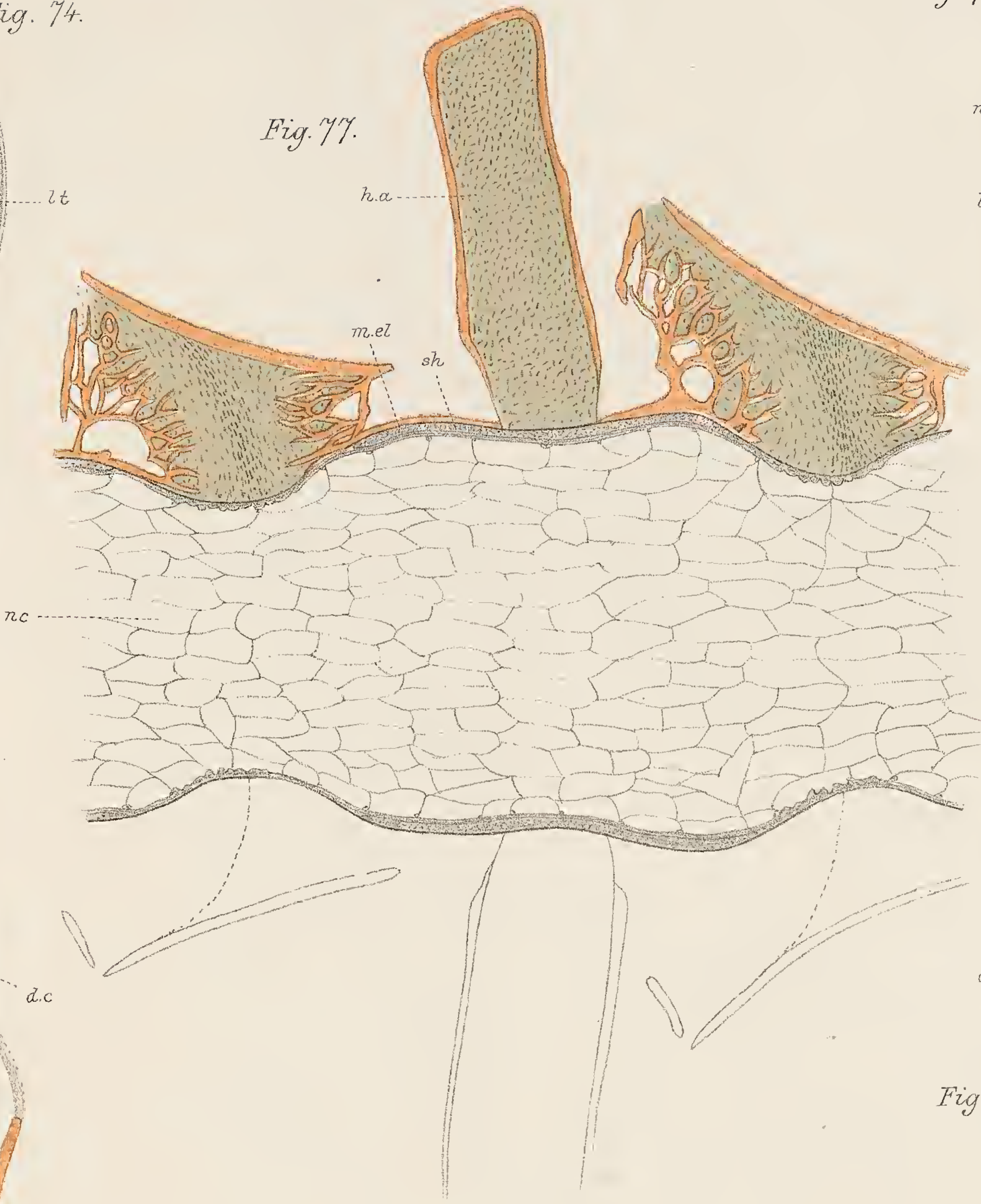

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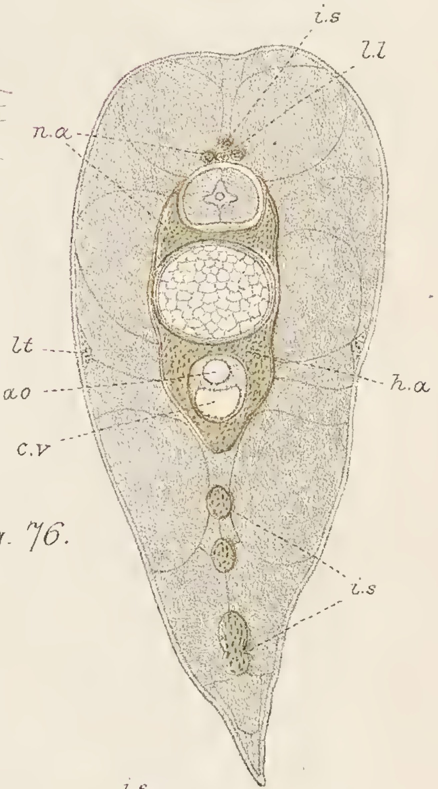

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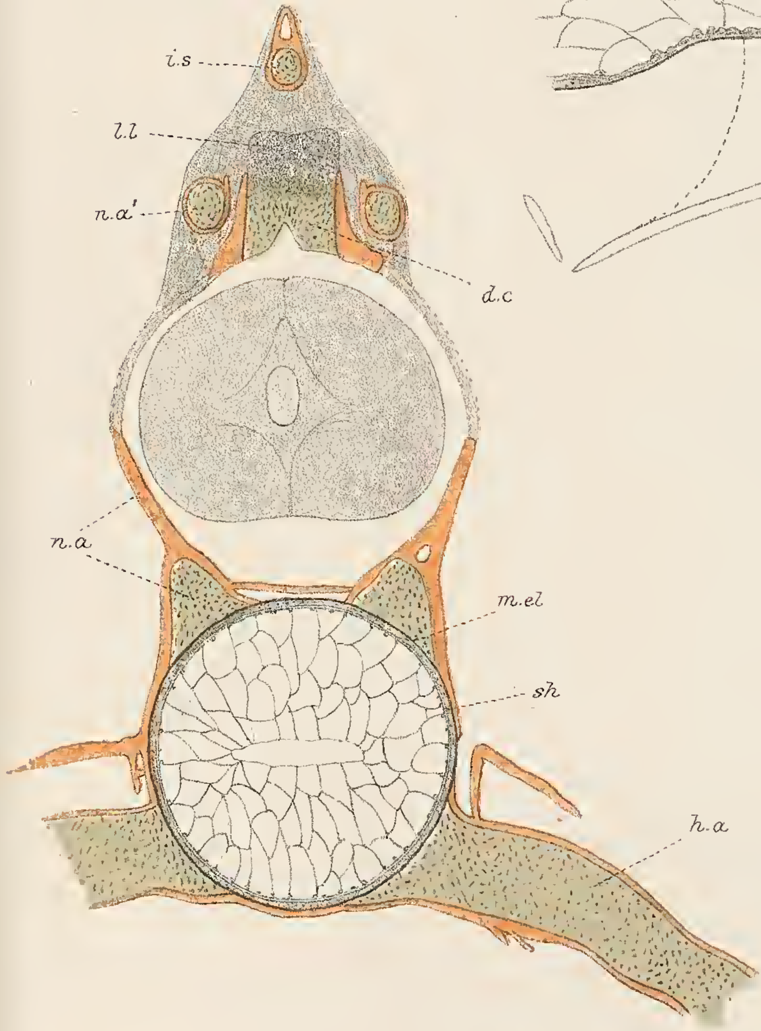

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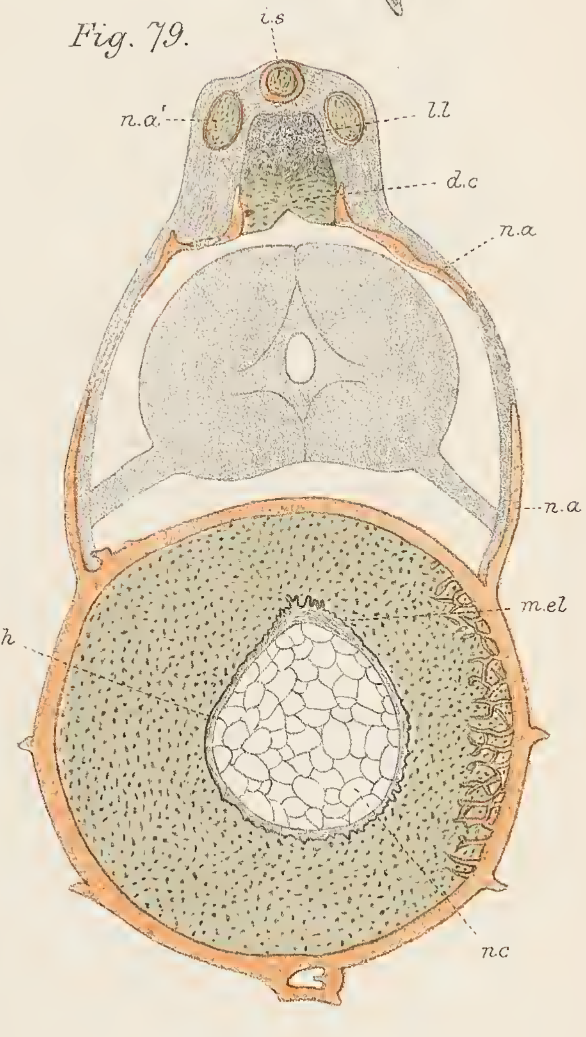

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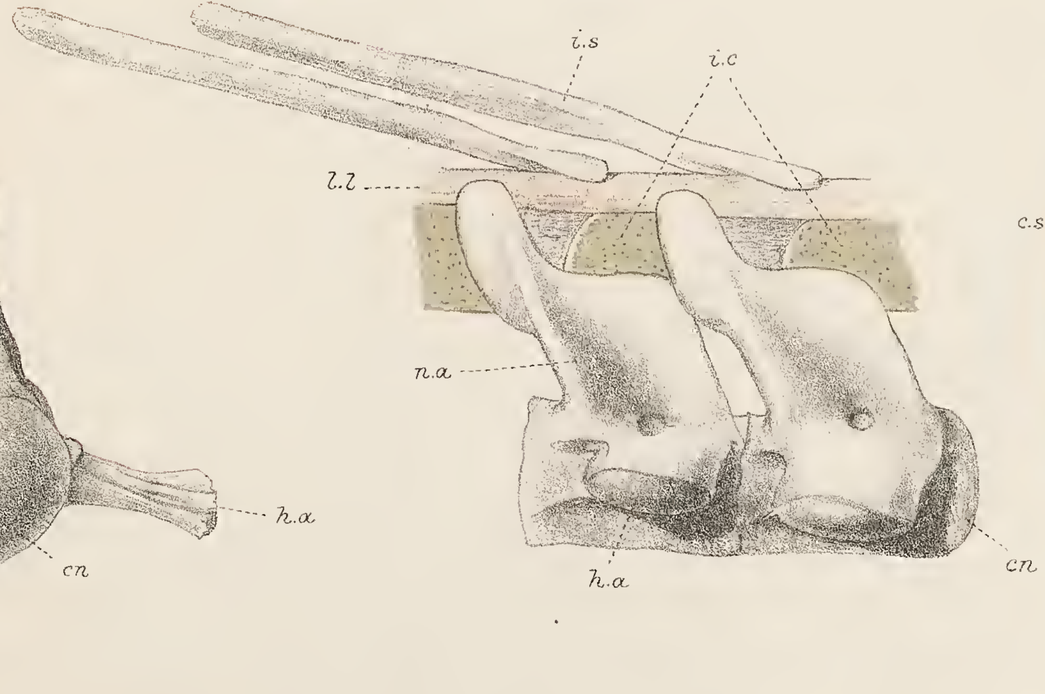

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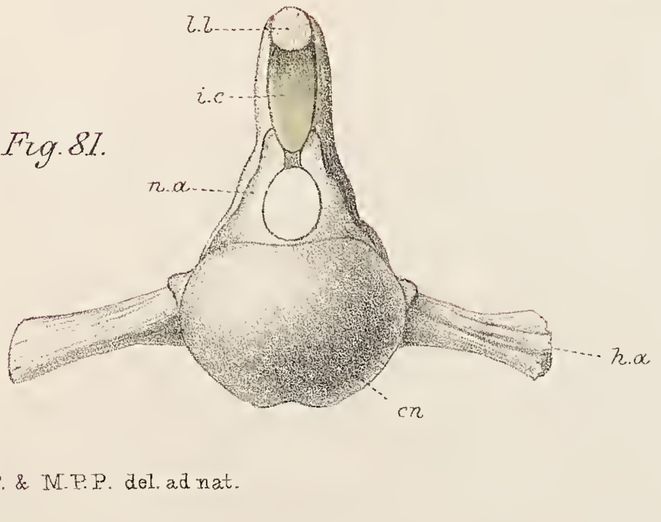
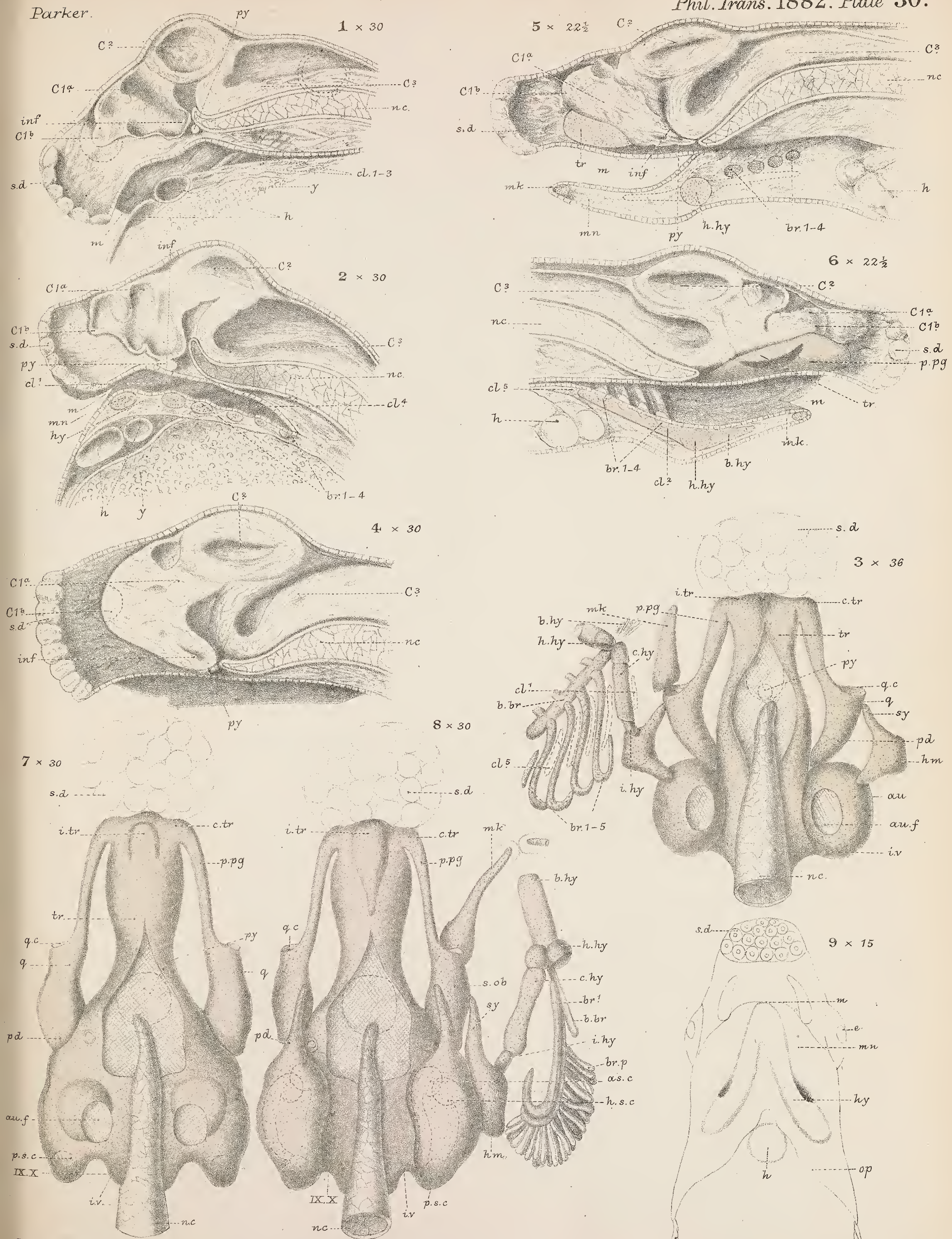


Parker.

Phil. Trans. 1882. Plate 30.



W.K.P. del. ad nat.
M.P. Parker lith.

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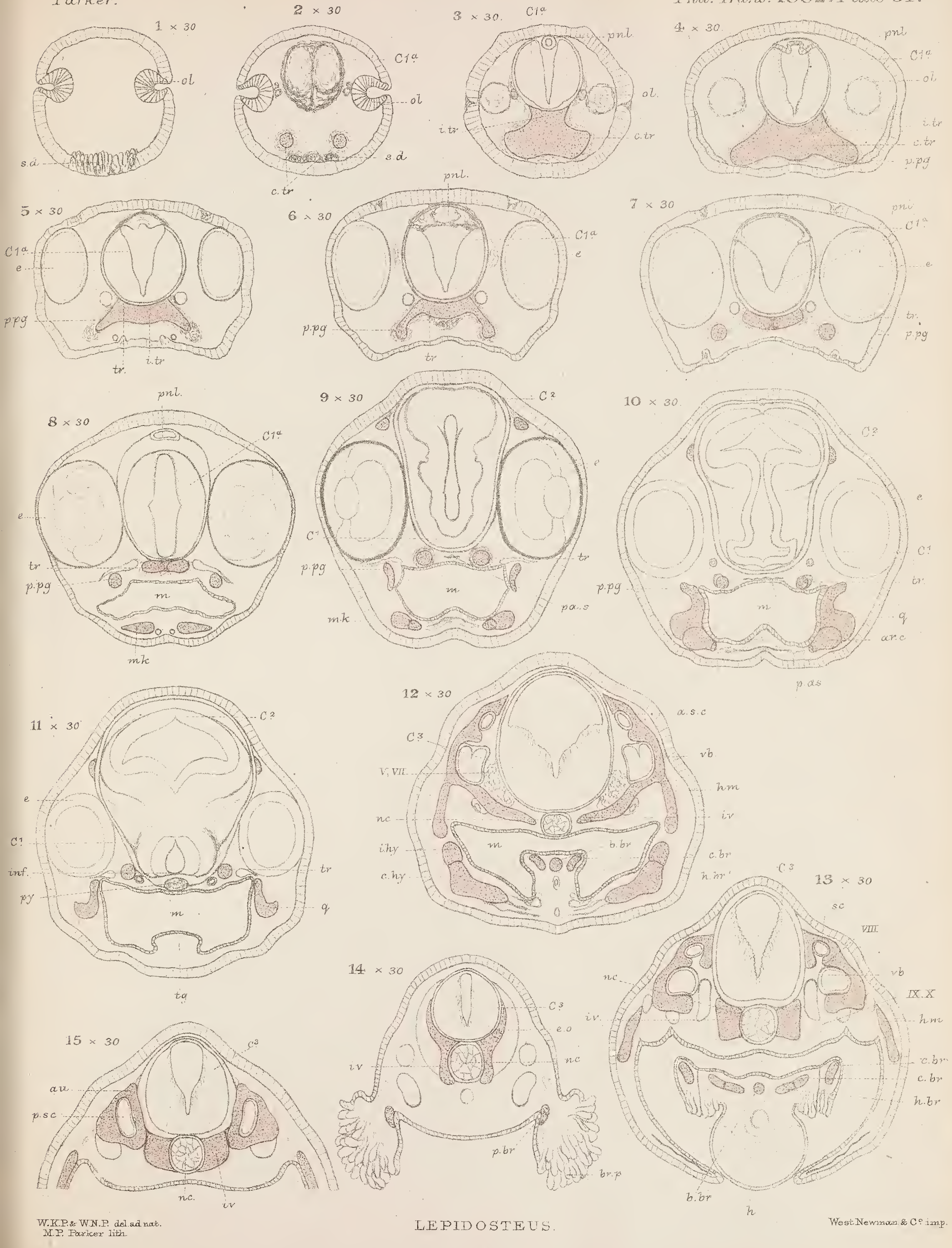
West Newman & Co imp.



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M.P. Parker lith.

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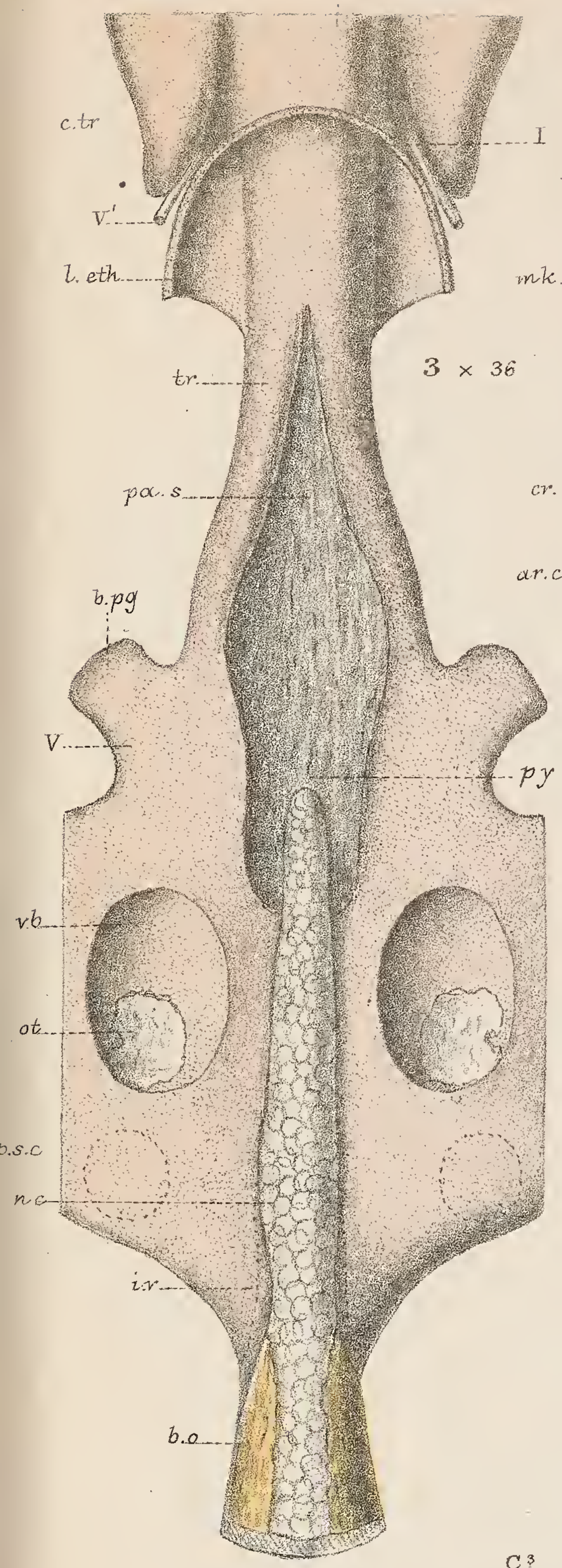

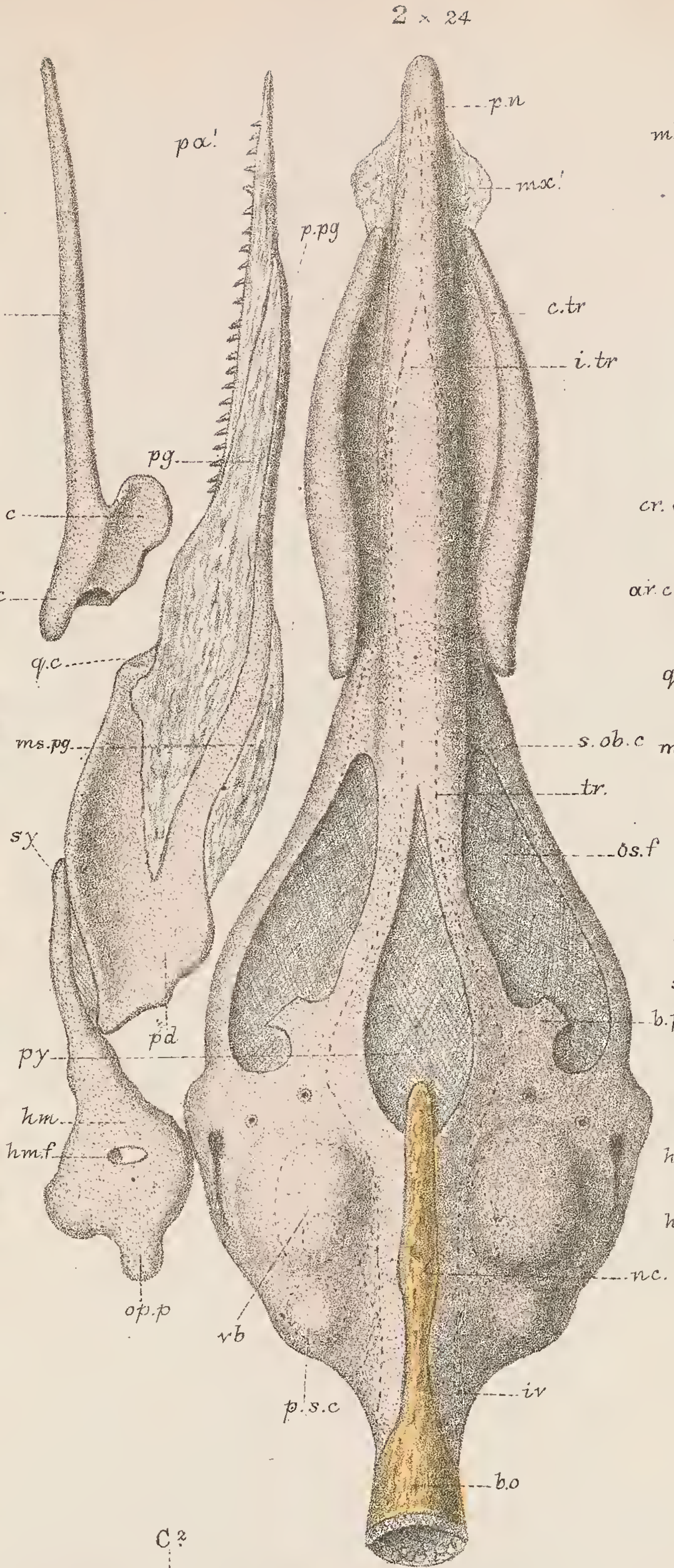

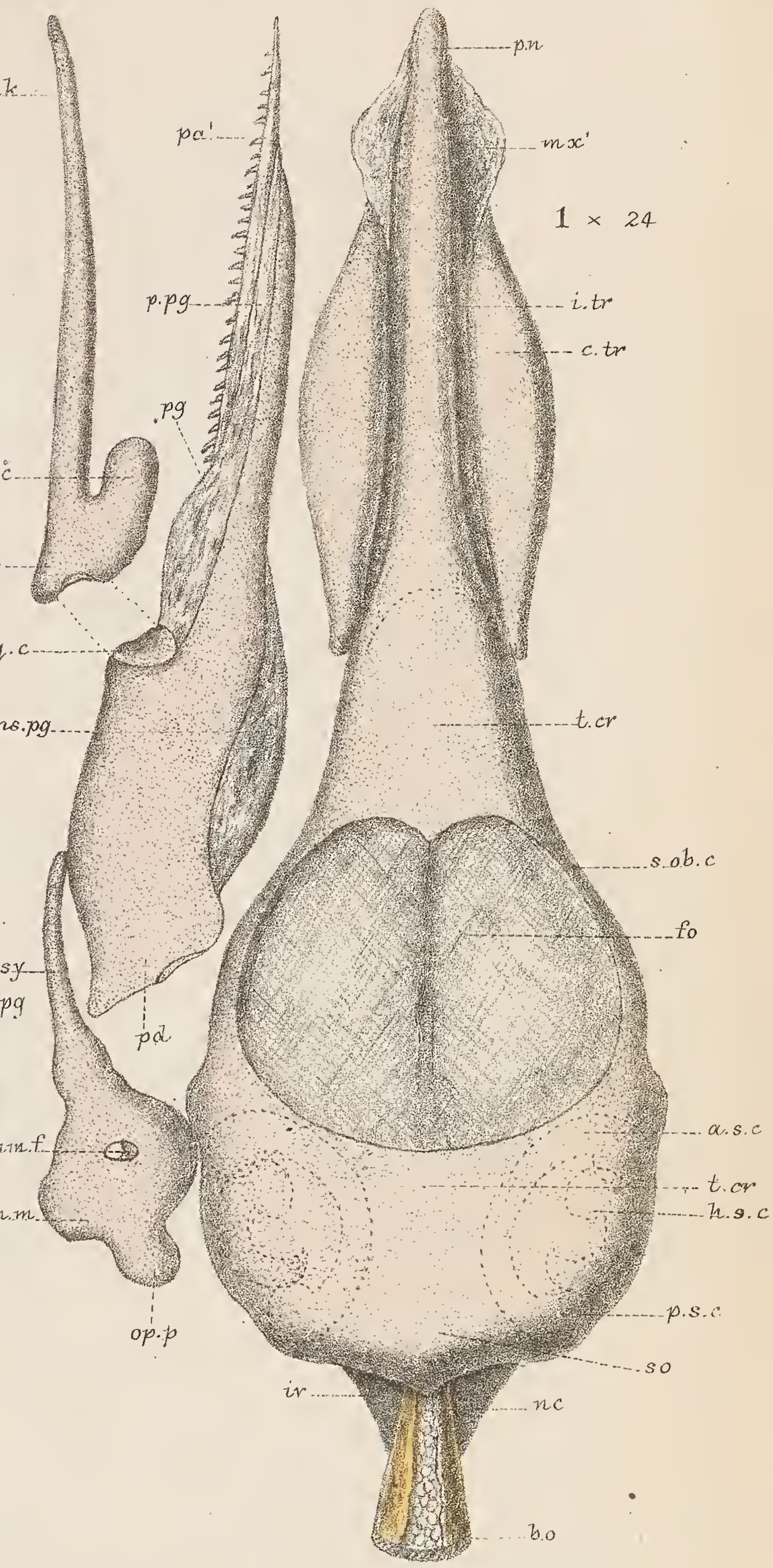

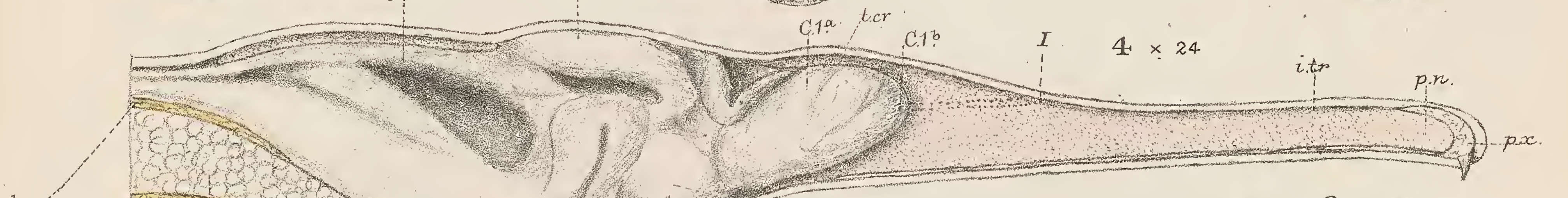

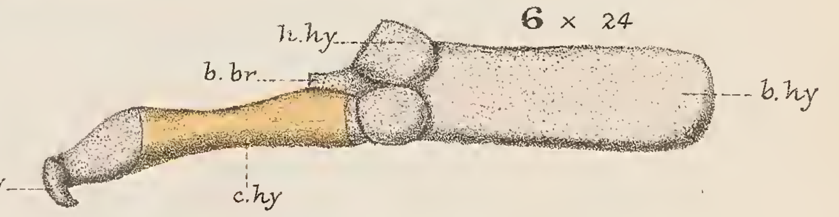

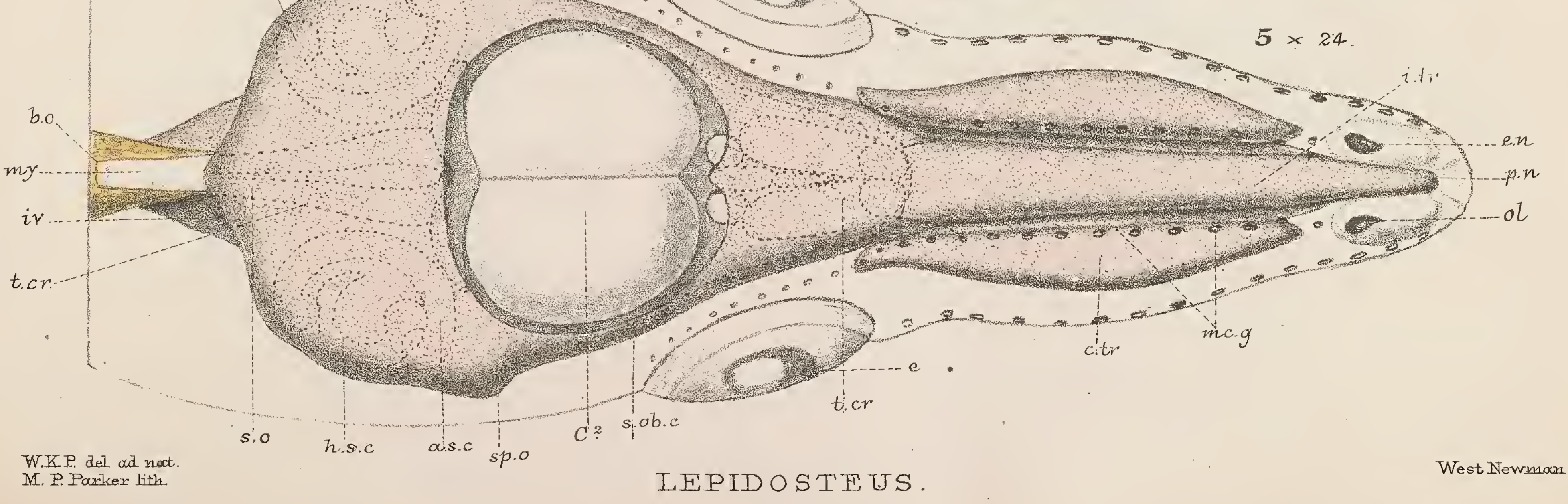

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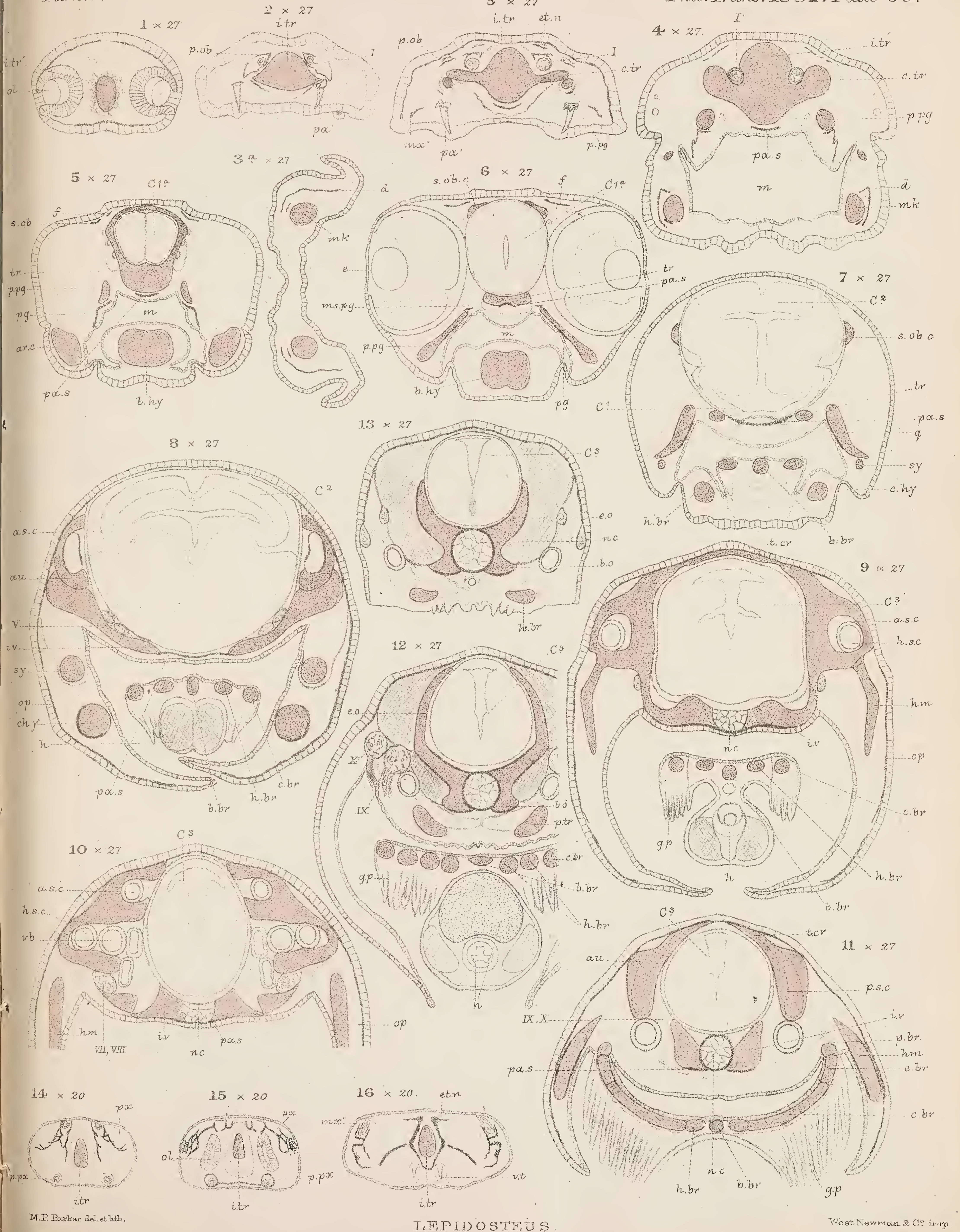
M. P. Parker lith.

LEPIDOSTEUS.

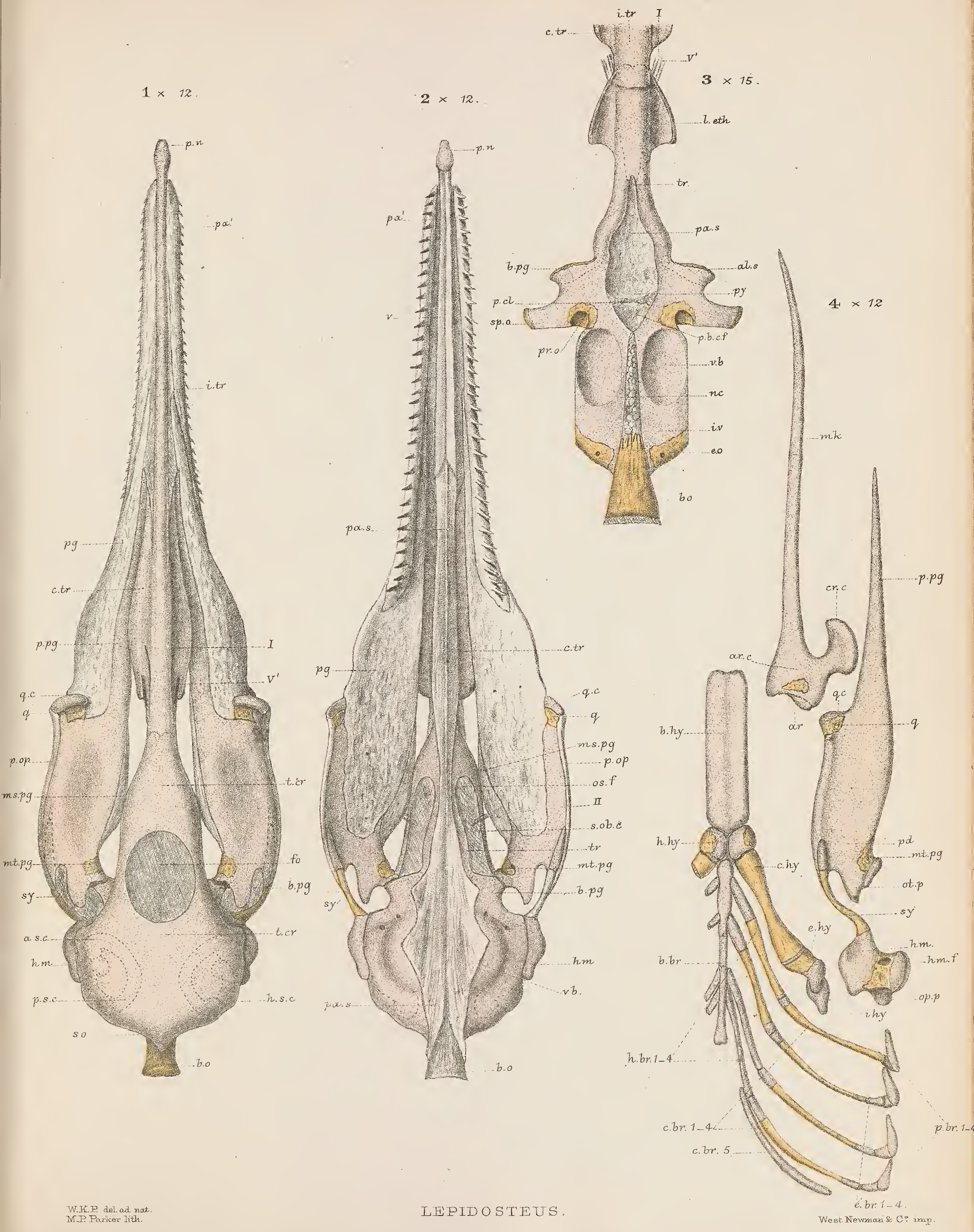
West Newman & Co. imp.



M.P. Parker del. et lith.

LEPIDOSTEUS.

West Newman & Co. imp.



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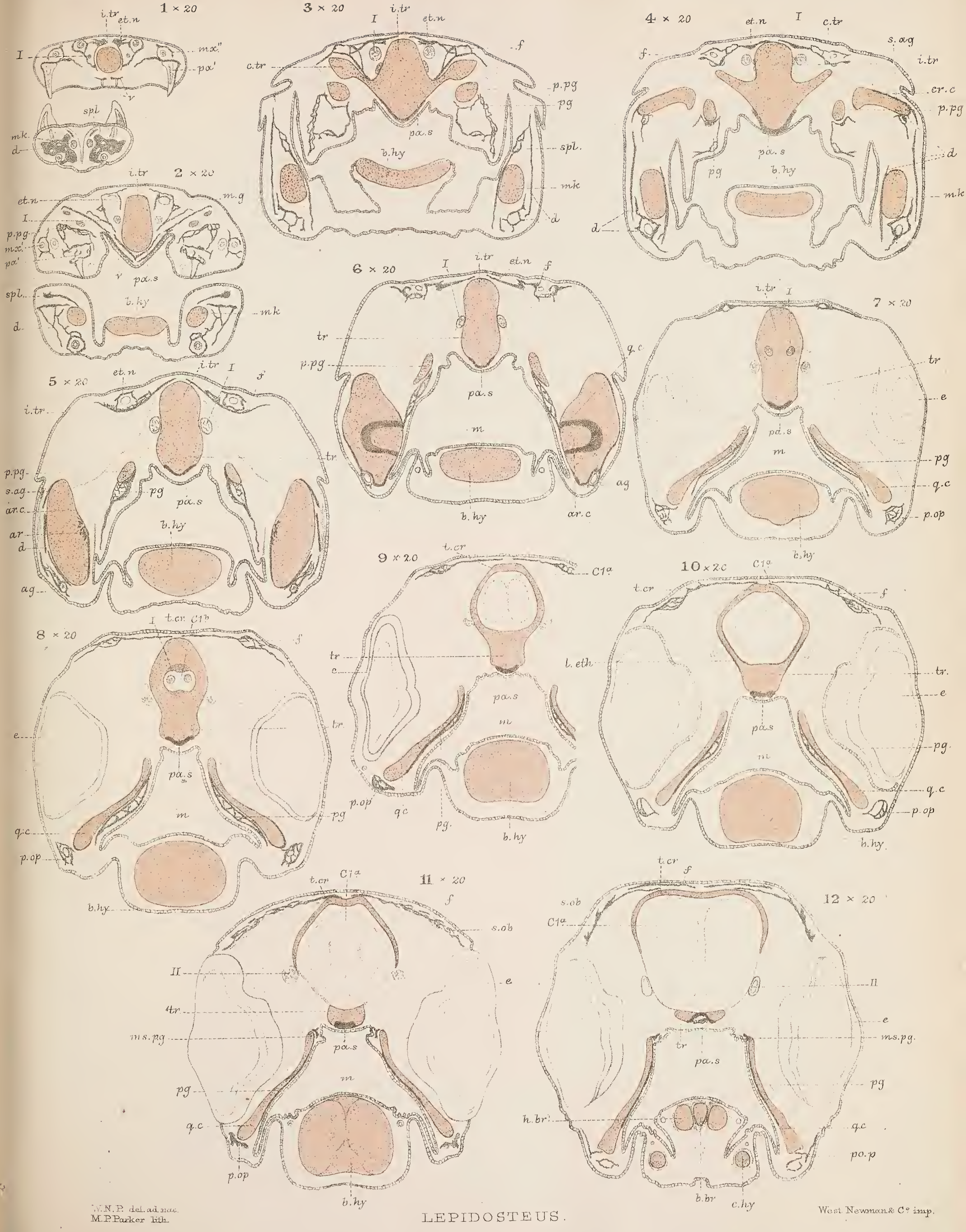

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M.P. Parker lith.

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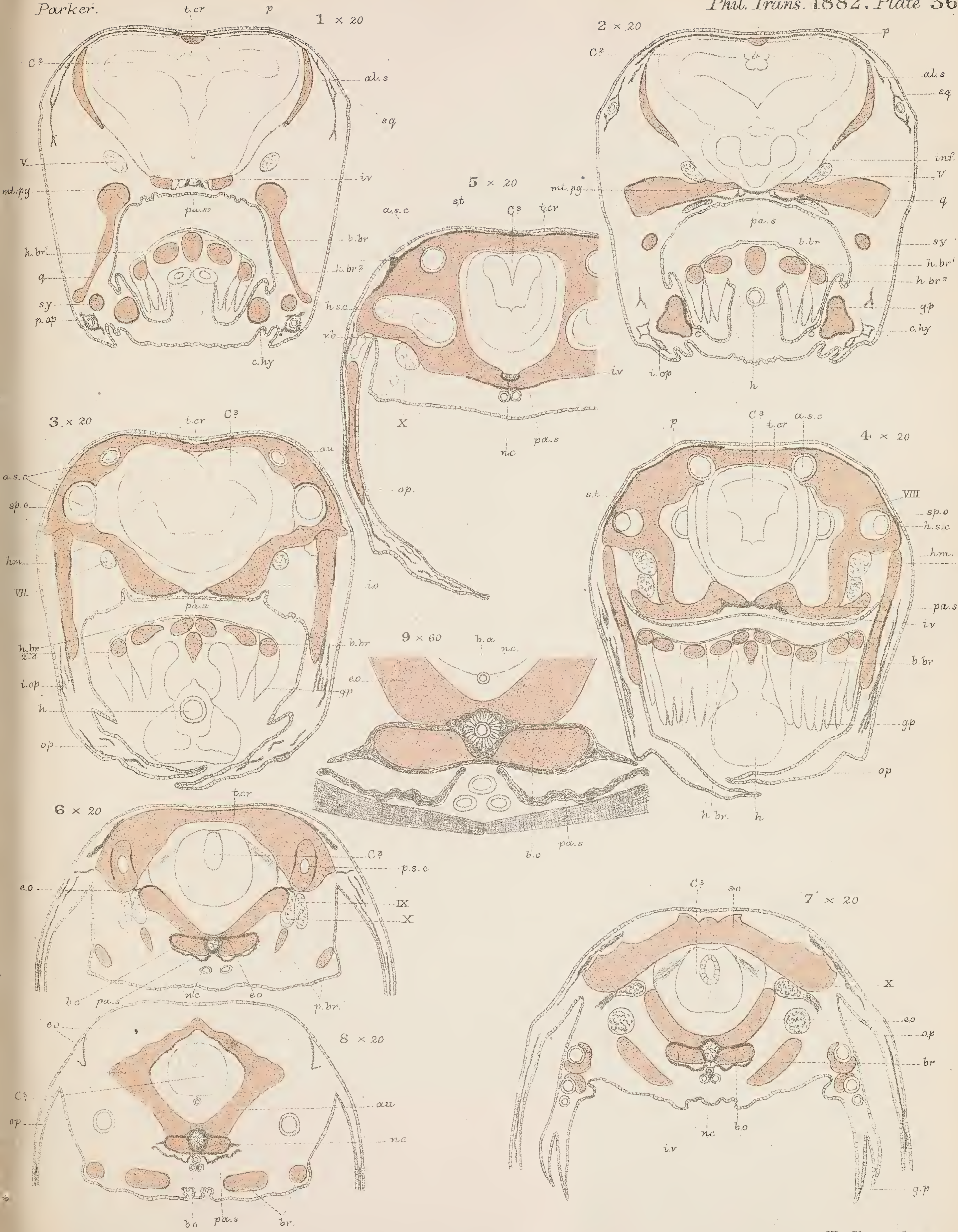
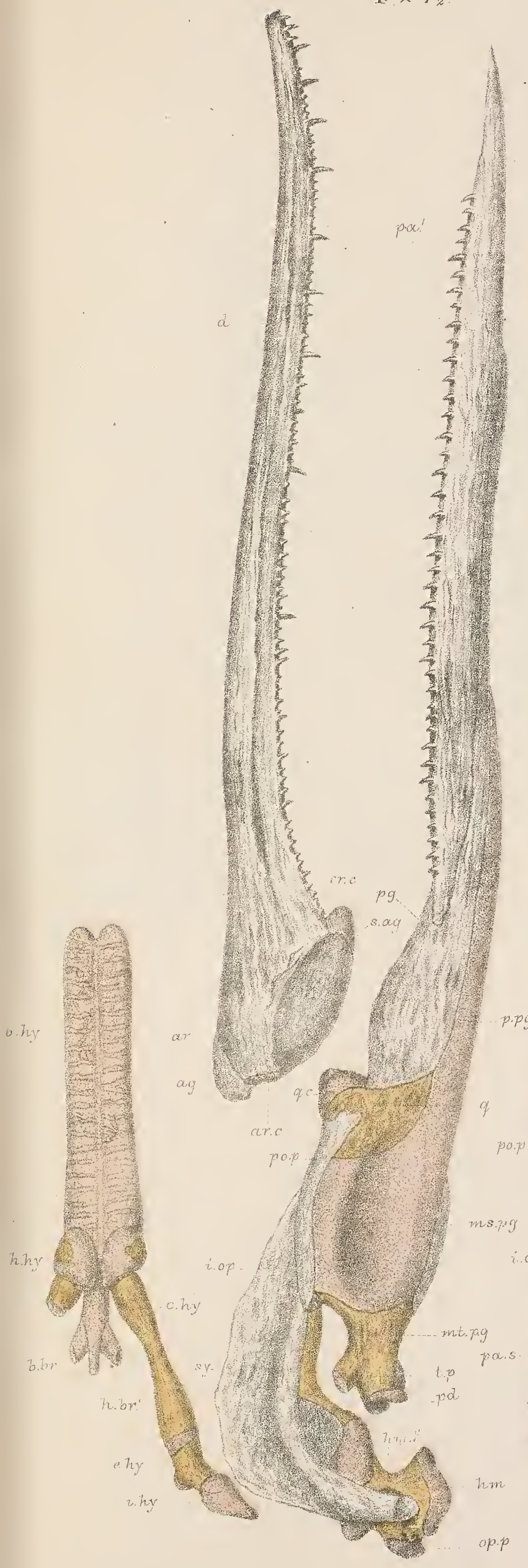


M.P. Parker del & lith.

LEPIDOSTEUS.

West Newman & Co imp

4 × 7½.



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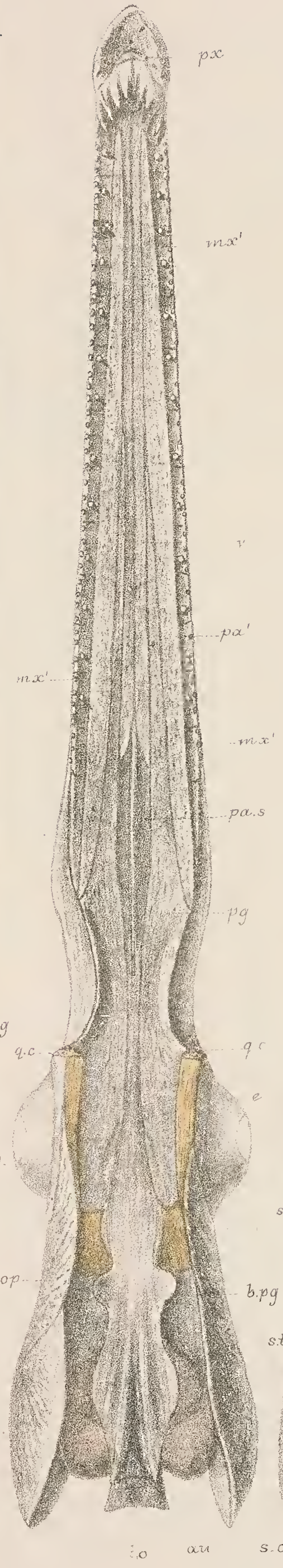

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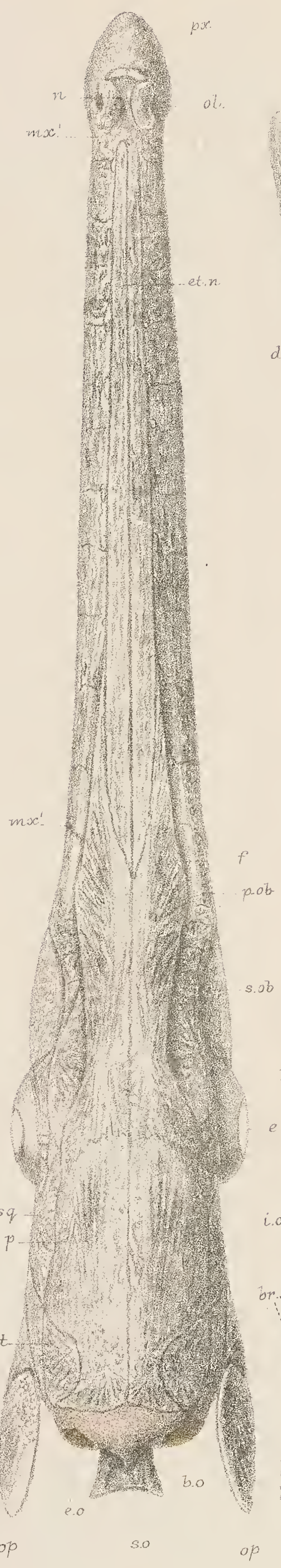

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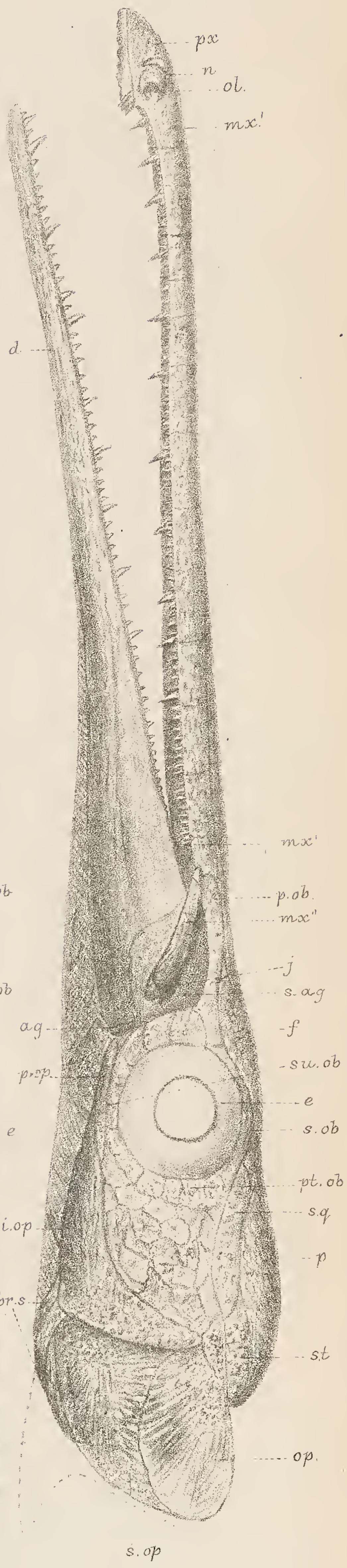
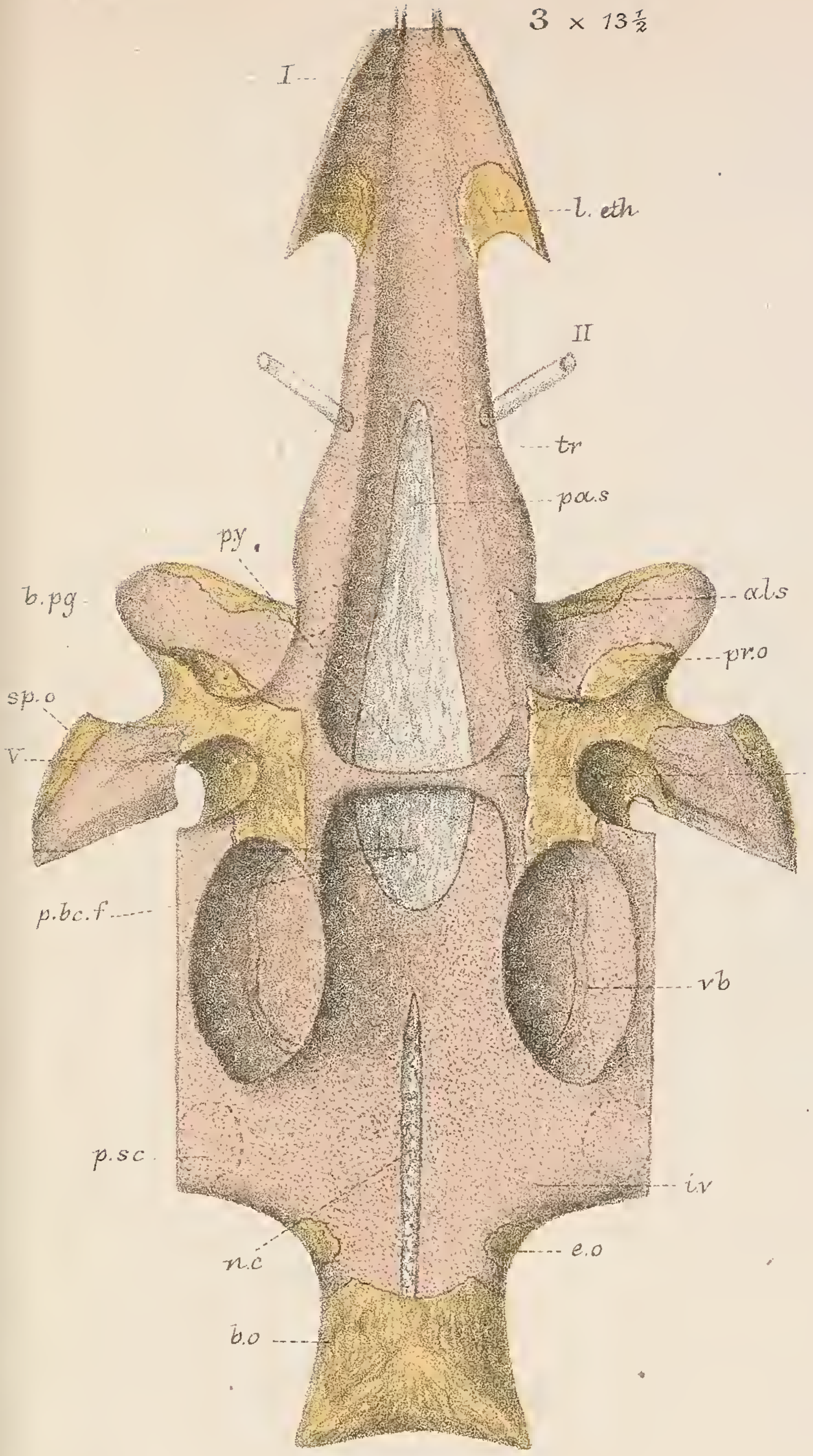


W.K.P. del. ad nat.
M.P. Parker lith.

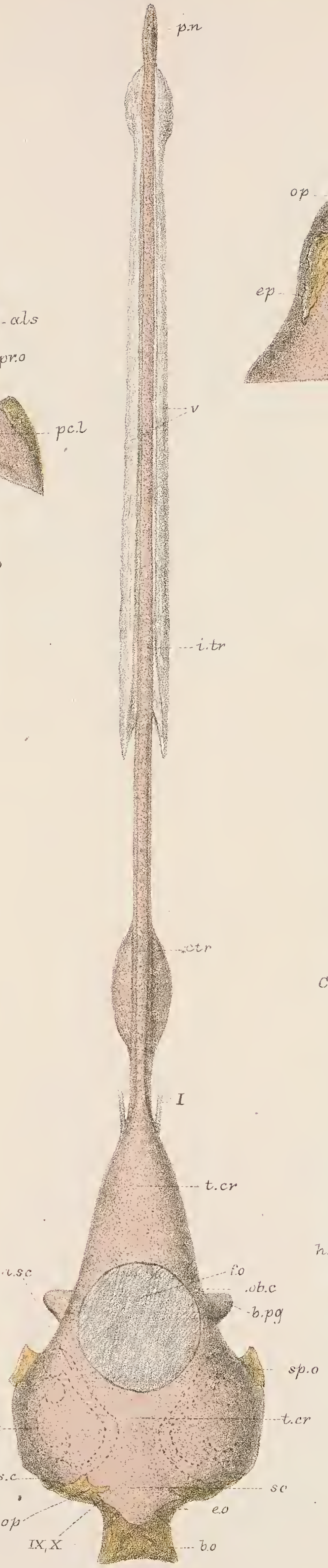
LEPIDOSTEUS.

West Newman & Co imp.

3 × 13½



1 × 6¾



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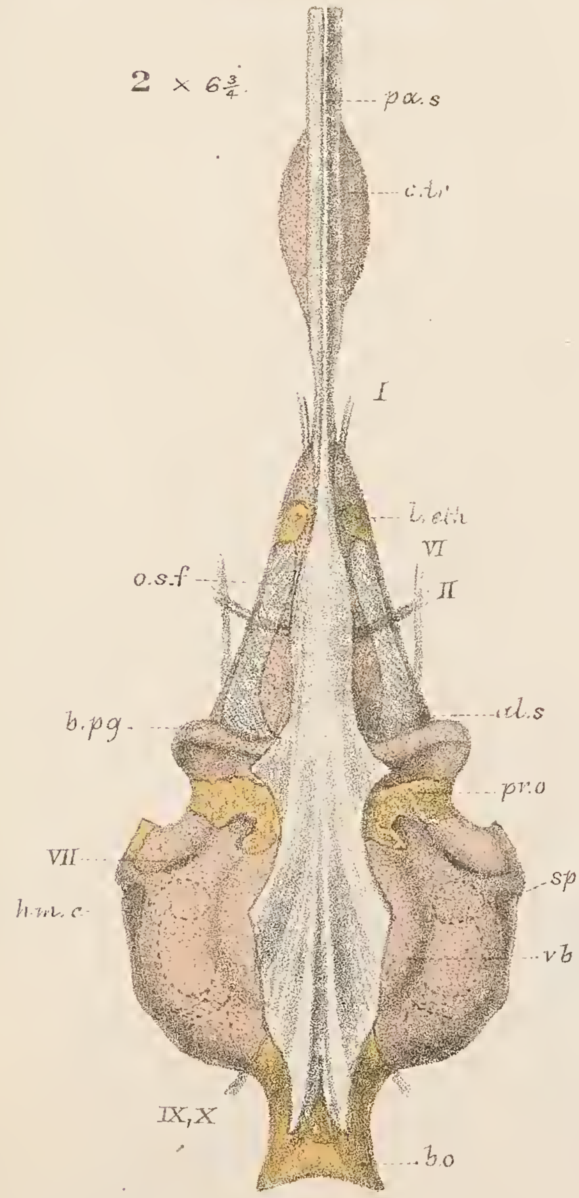
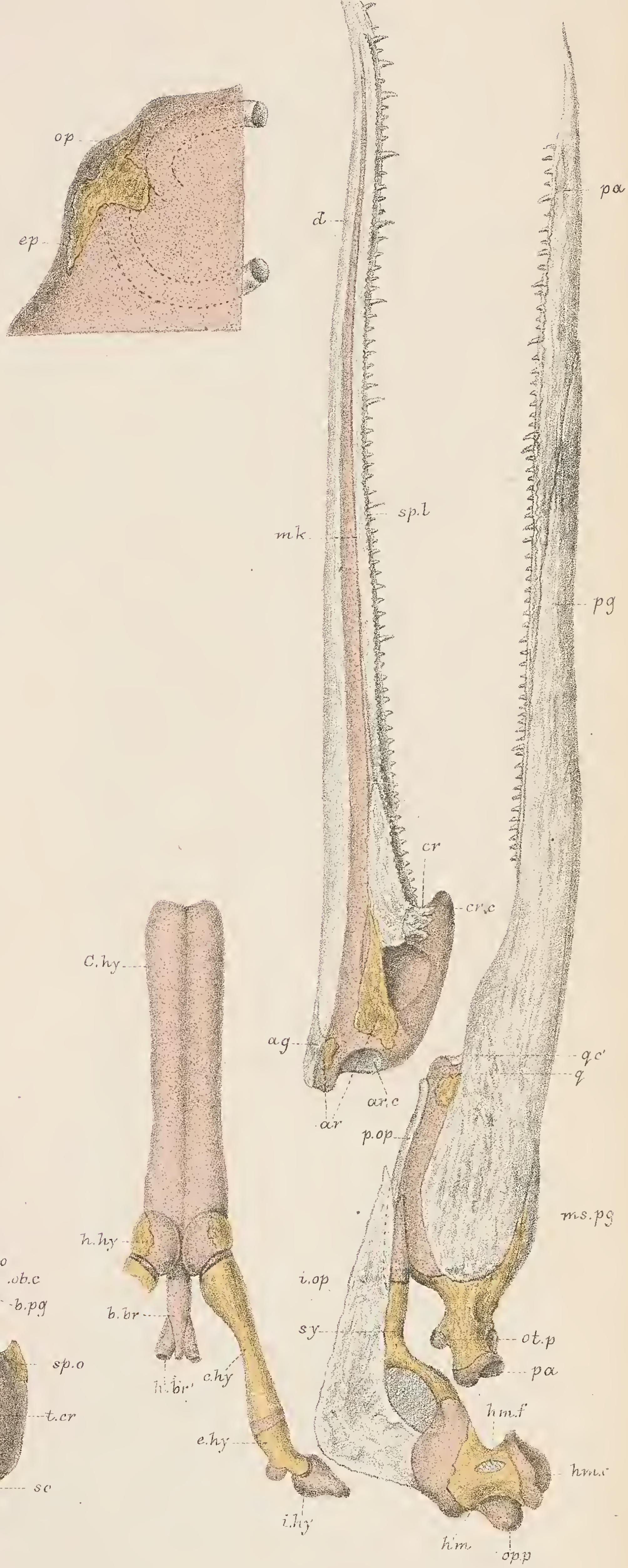


4 × 13½. 5 × 7½.



W.K.P. del. ad nat.
M.P. Parker lith.

LEPIDOSTEUS.

West Newman & Co. imp.

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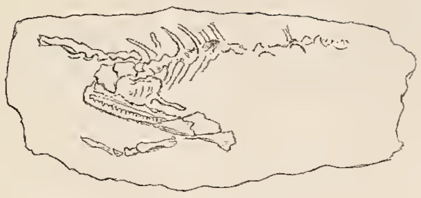

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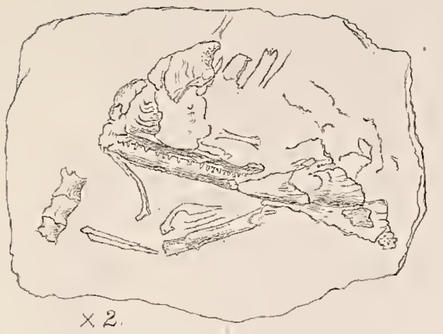

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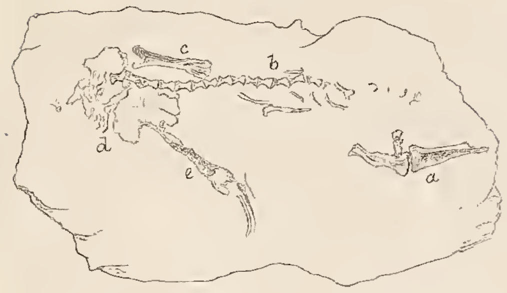

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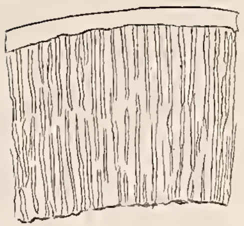

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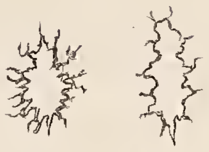

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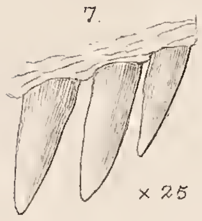

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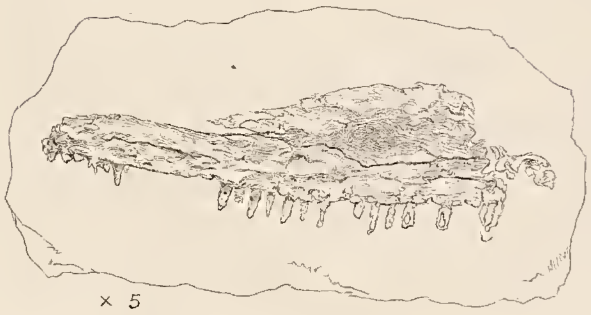

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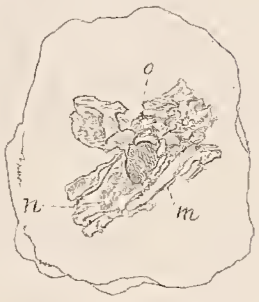

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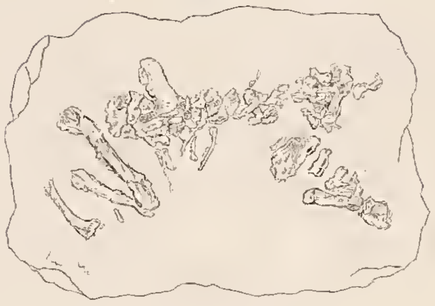

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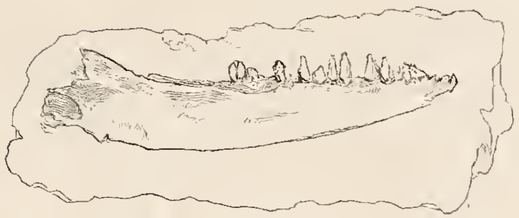

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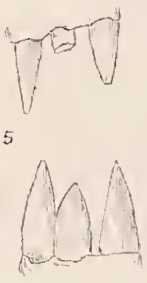

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17.

× 5

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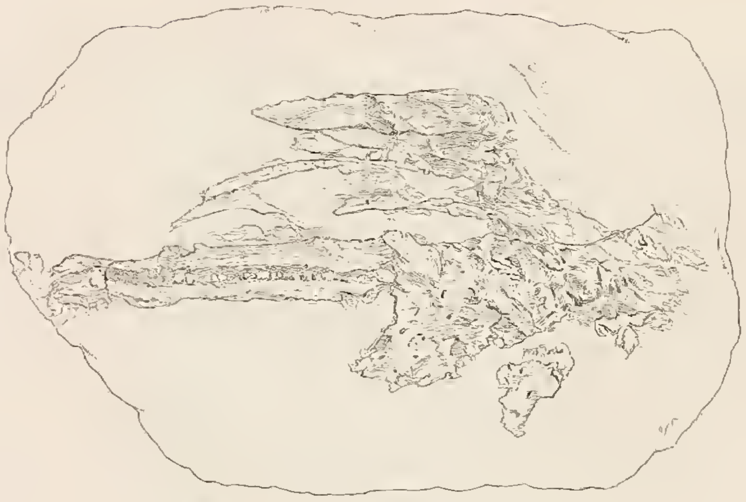

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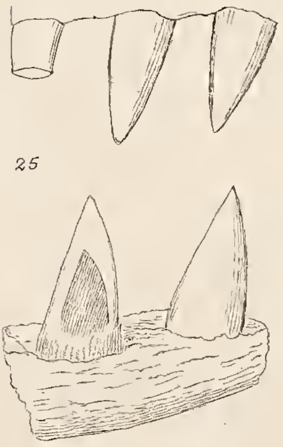

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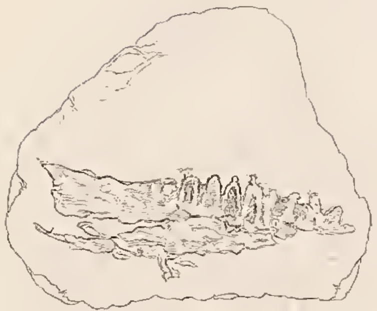

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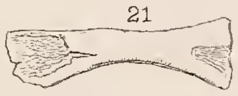
20.

× 2

21



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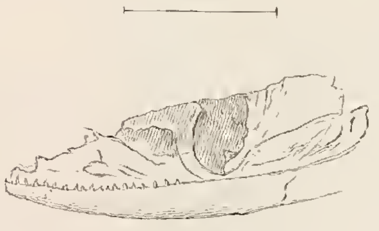

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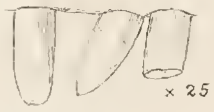

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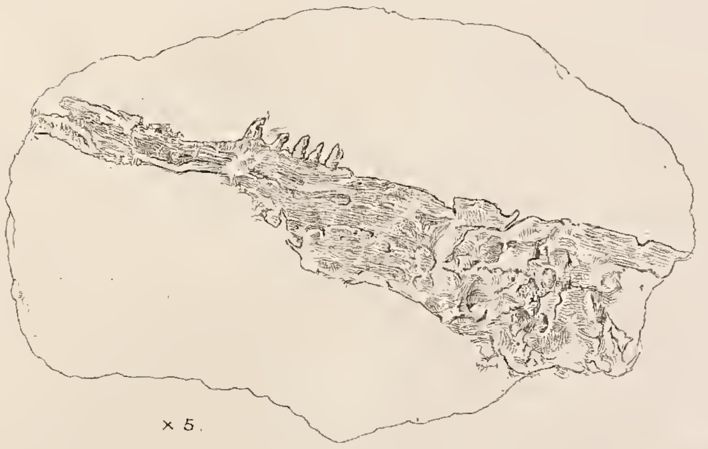

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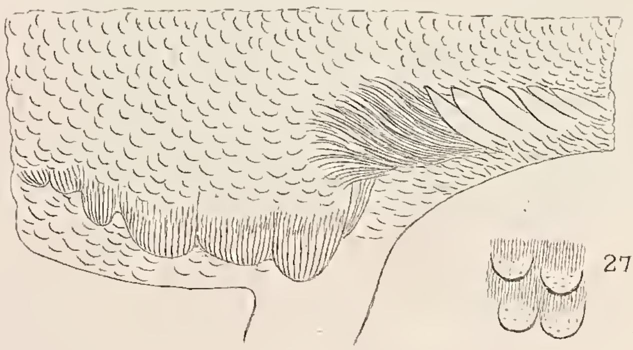
27 b

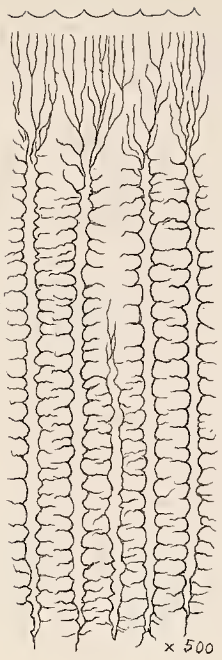
27



HYLONOMUS.

West Newman & Co. sc.

28.



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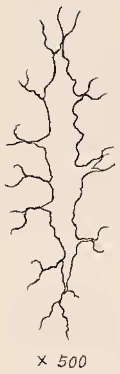

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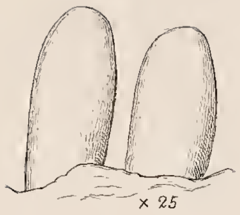

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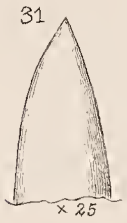

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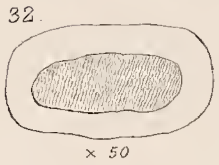

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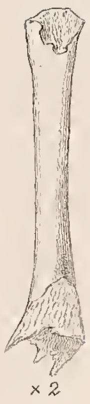

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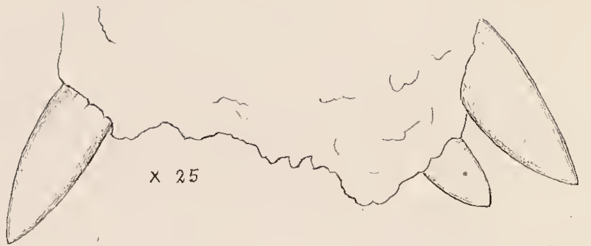

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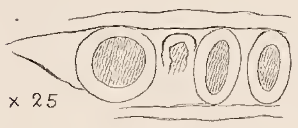

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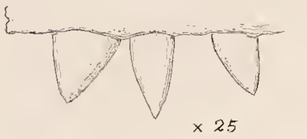

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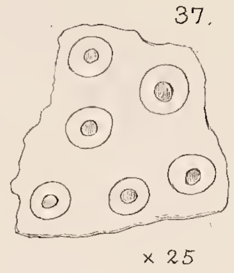

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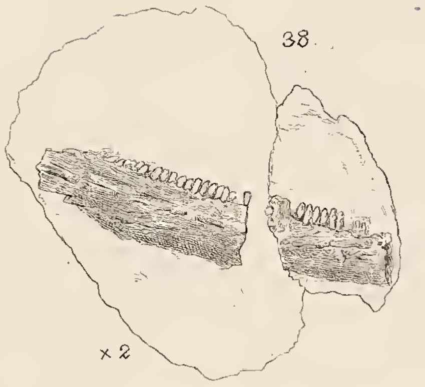

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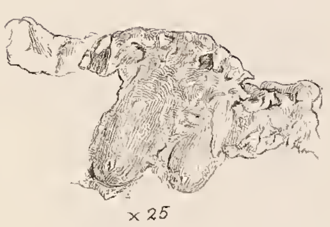

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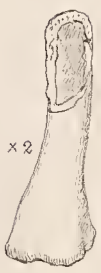

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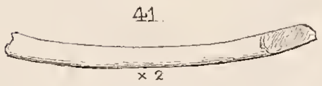

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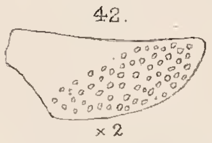

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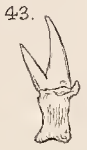

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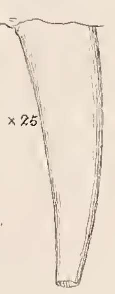

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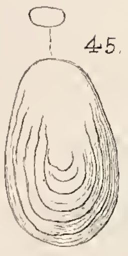

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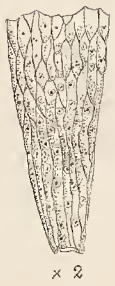

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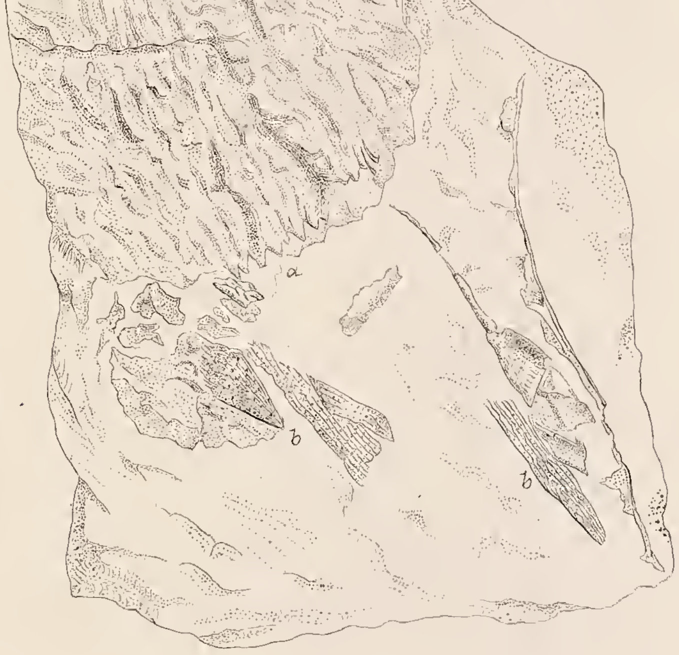

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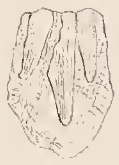

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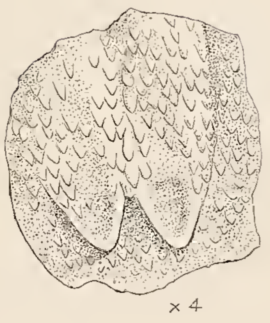

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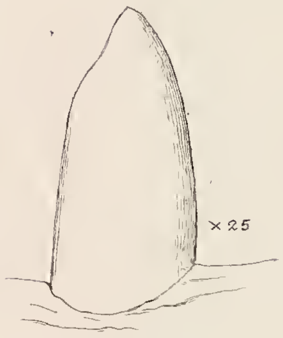

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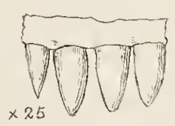

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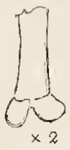

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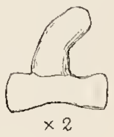

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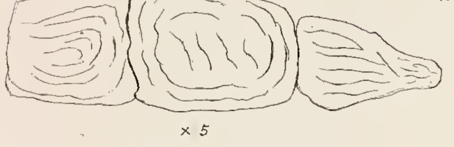

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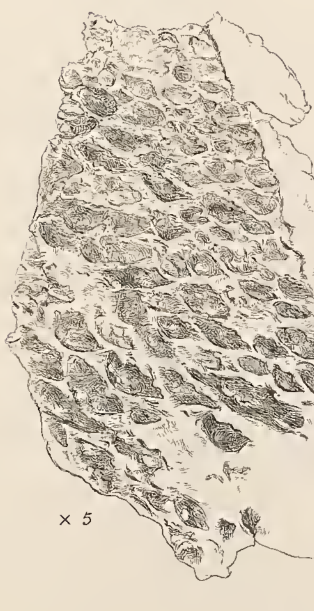

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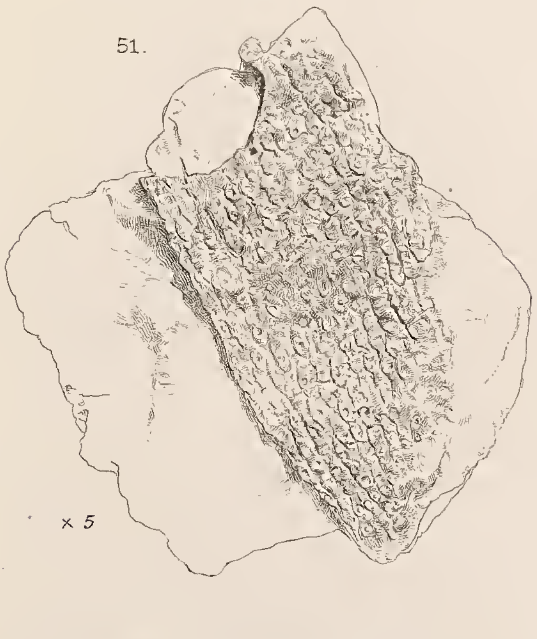

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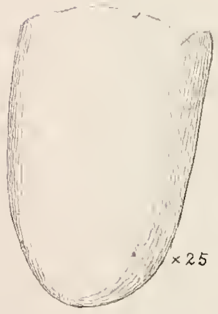

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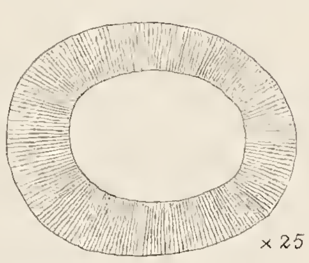

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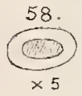

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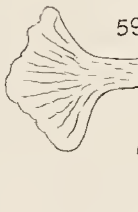

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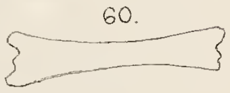

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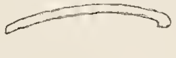
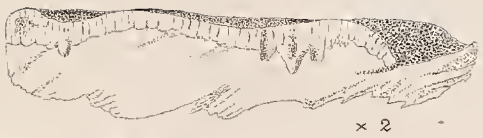


West Newman & Co. sc.

SMILERPETON (28 to 45) DENDRERPETON (46 to 51) SPARODUS (52 to 56) AMBLYODON (57 to 61)

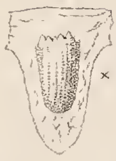
62.

62ª.



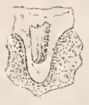
× 2

63ª.

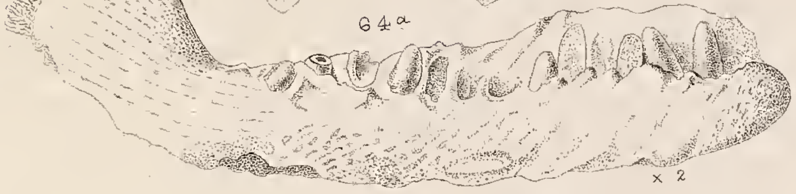


× 2

63.

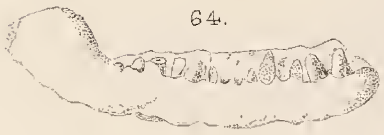


64ª.



× 2

64.



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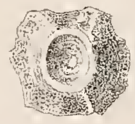

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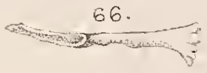

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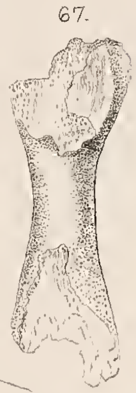

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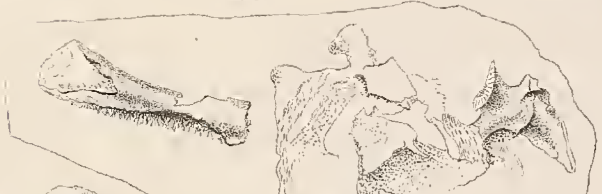

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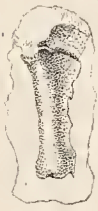

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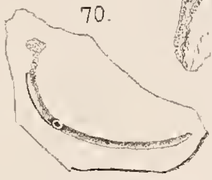

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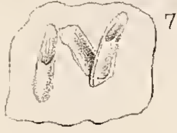

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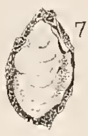

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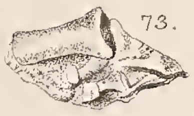

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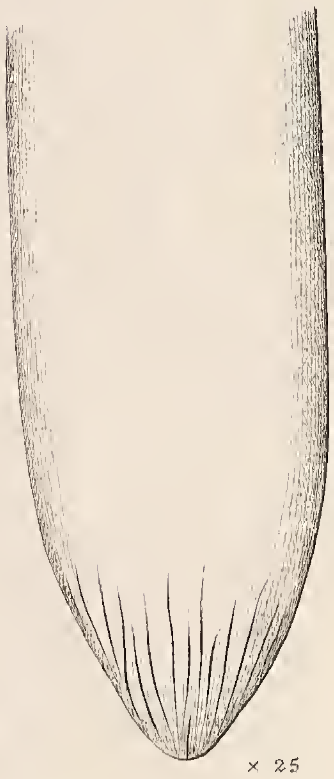
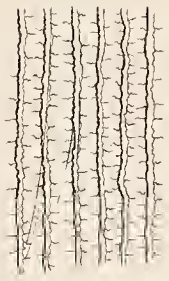


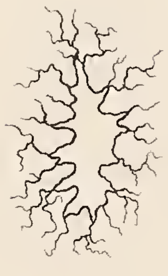
× 25

75



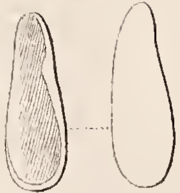
× 500

76.



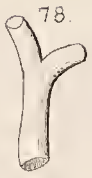
× 500

77.



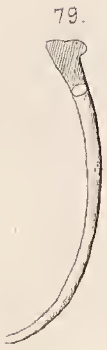
× 2

78.



× 2.

79.



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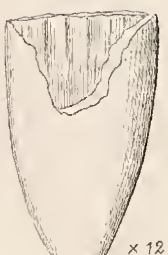
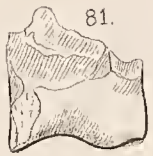


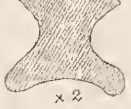
× 12

81.



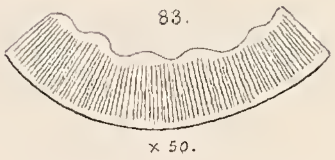
× 2

82

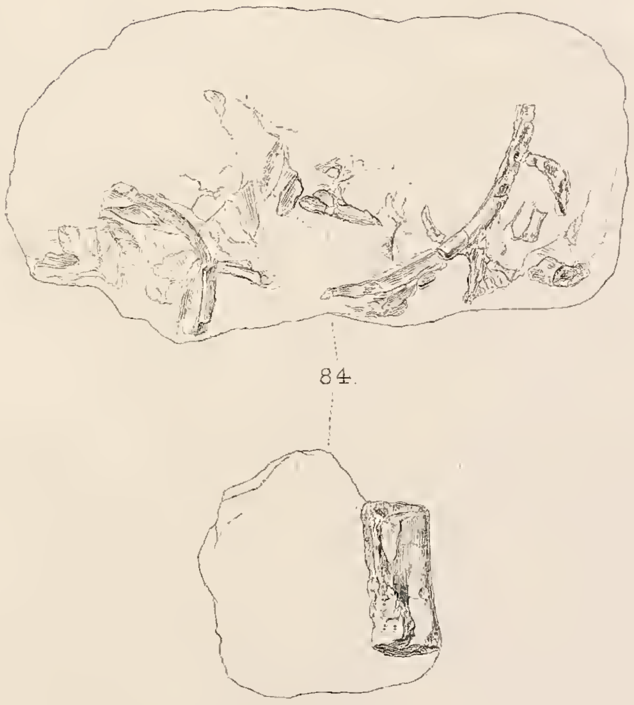


× 2

83.

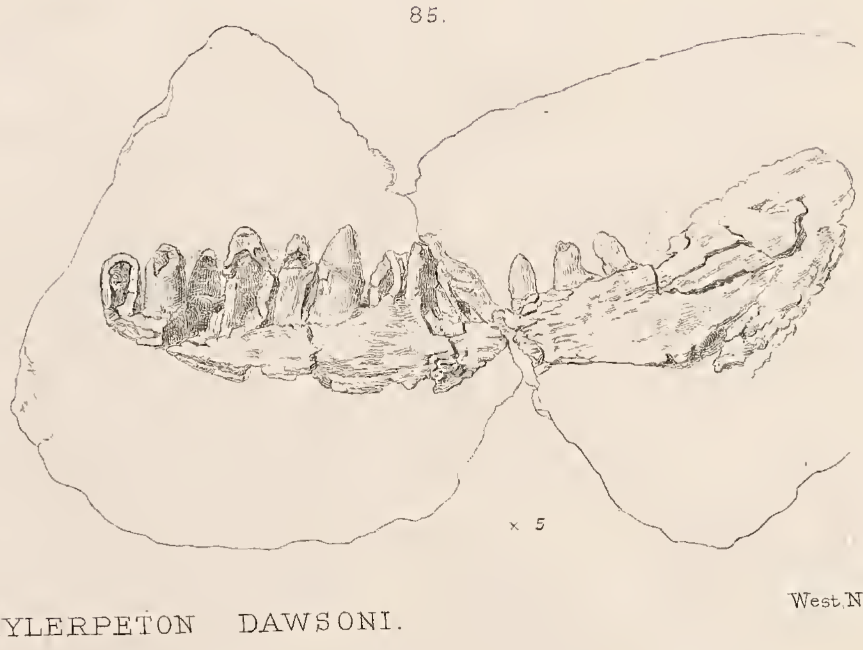


× 50.



84.

85.



× 5

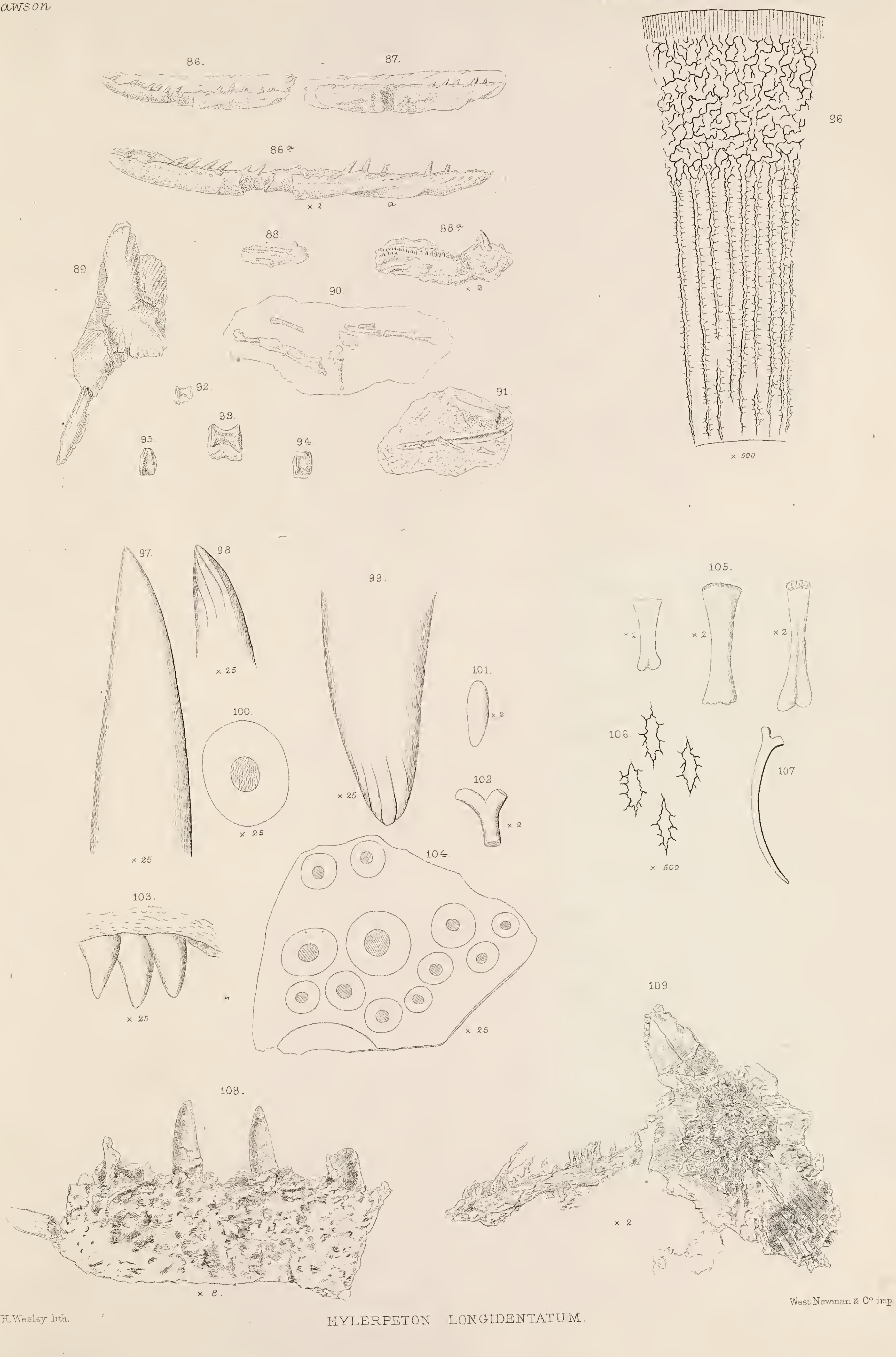
W. H. Wesley lith.

HYLERPETON DAWSONI.

West Newman & Co. imp.

Dawson.

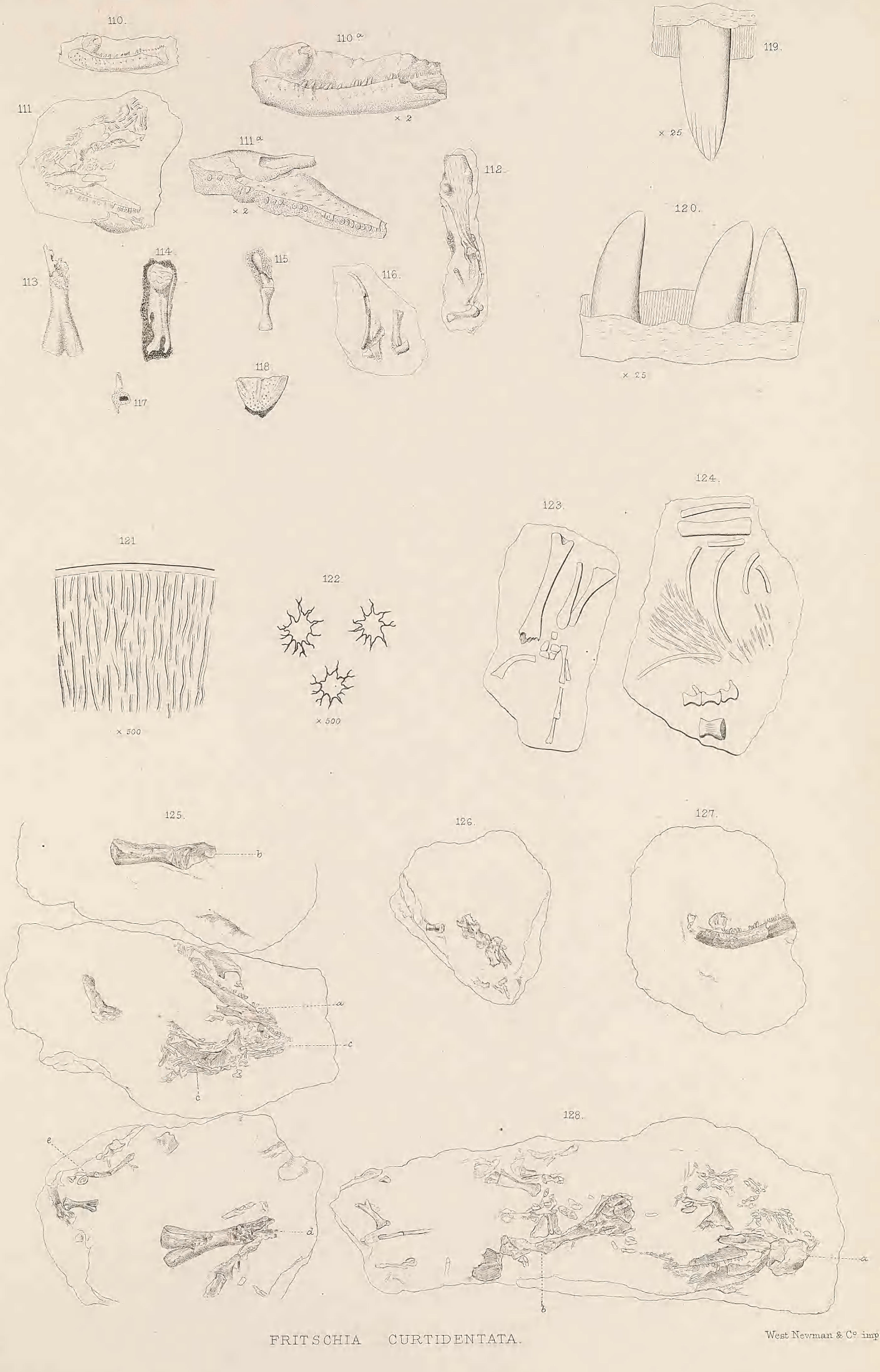
Phil. Trans. 1882. *Plate* 42.



W. H. Wesley lith.

HYLERPETON LONGIDENTATUM.

West Newman & C^o imp.

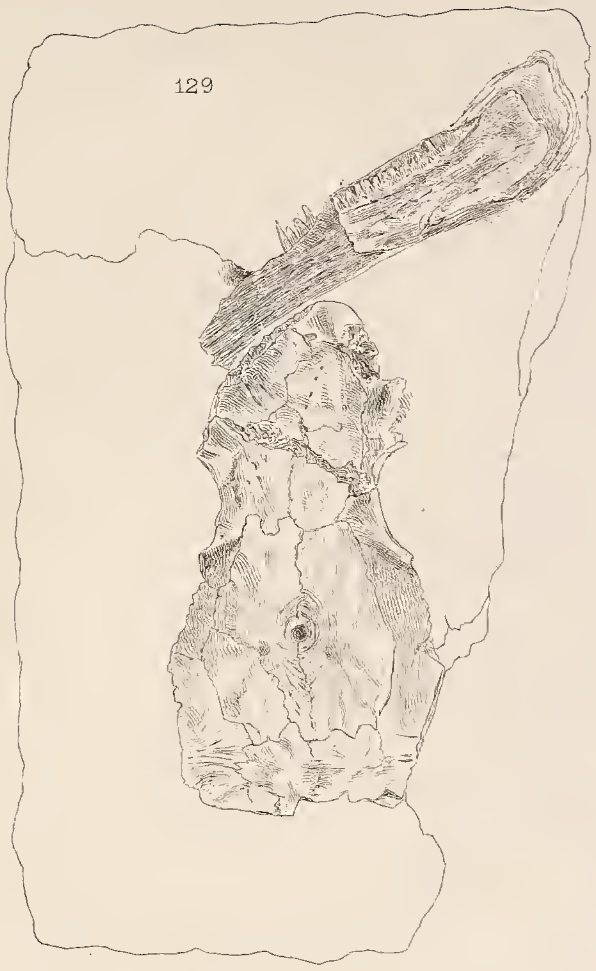


W.H. Wesley lith.

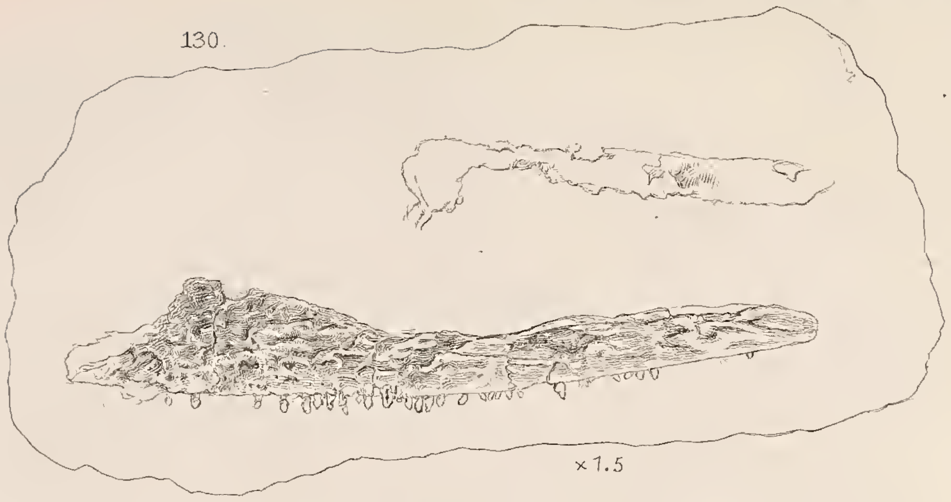
FRITSCHIA CURTIDENTATA.

West Newman & Co. imp.

129

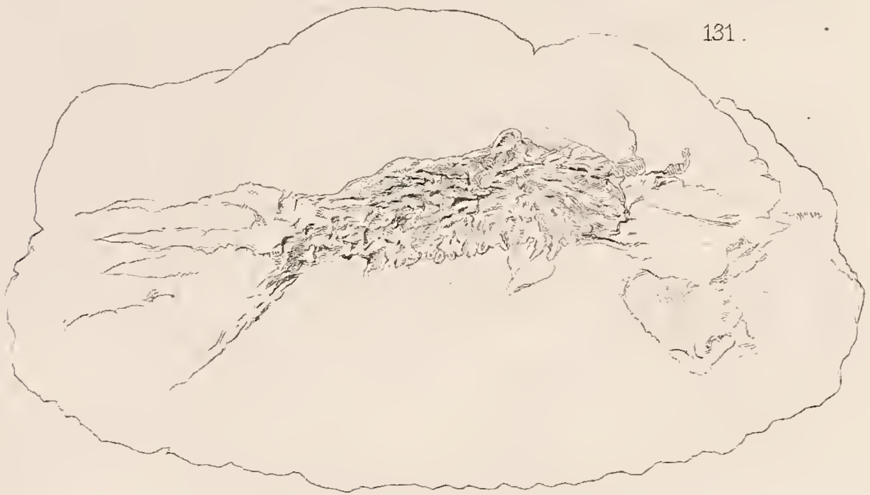


130.



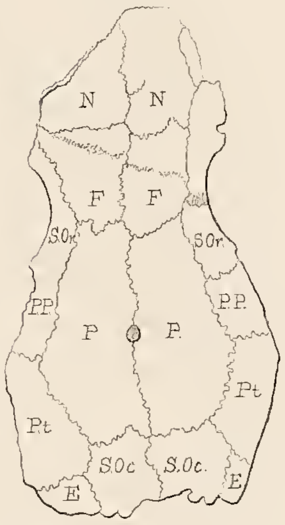
× 1.5

131.

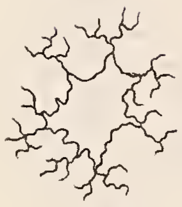


× 2.5

132.

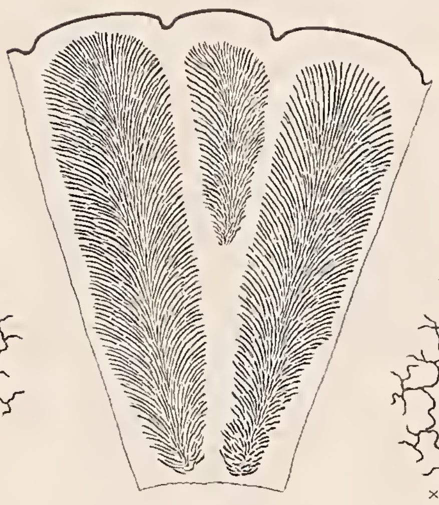


134.



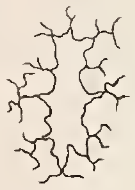
× 500

133.



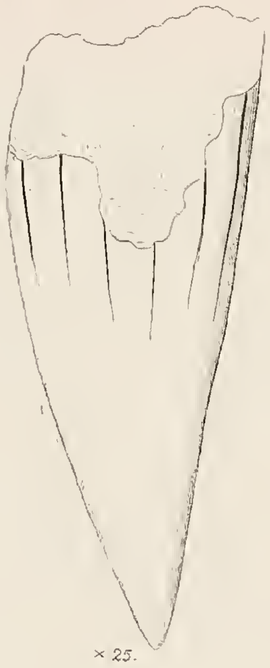
× 250

134



× 500

135

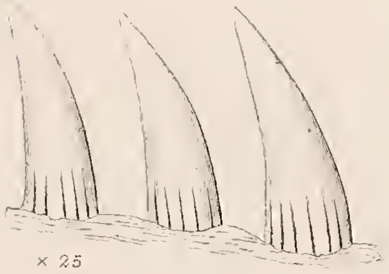


× 25.

136.

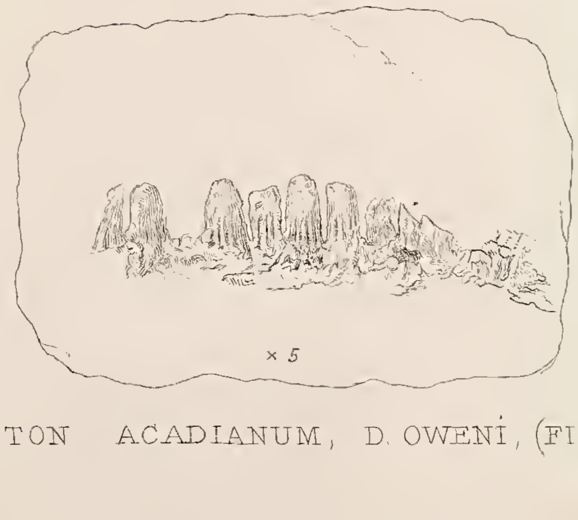
× 5

138



× 25

137.



× 5

139.

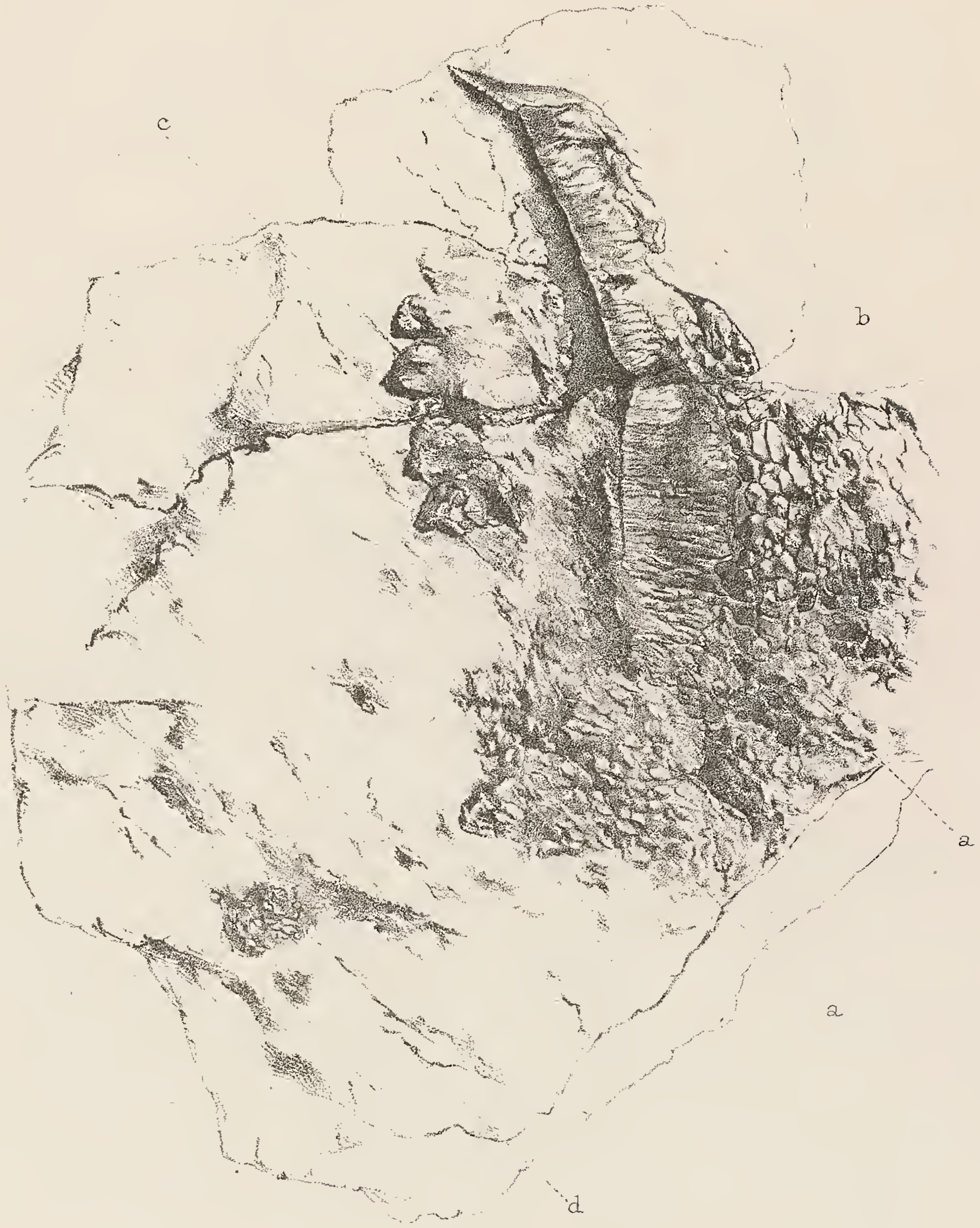
× 2.

W.H. Wesley lith.

DENDRERPETON ACADIANUM, D. OWENI, (FIGS 131, 138, 139.)

West Newman & Cº imp.

Fig. 140.

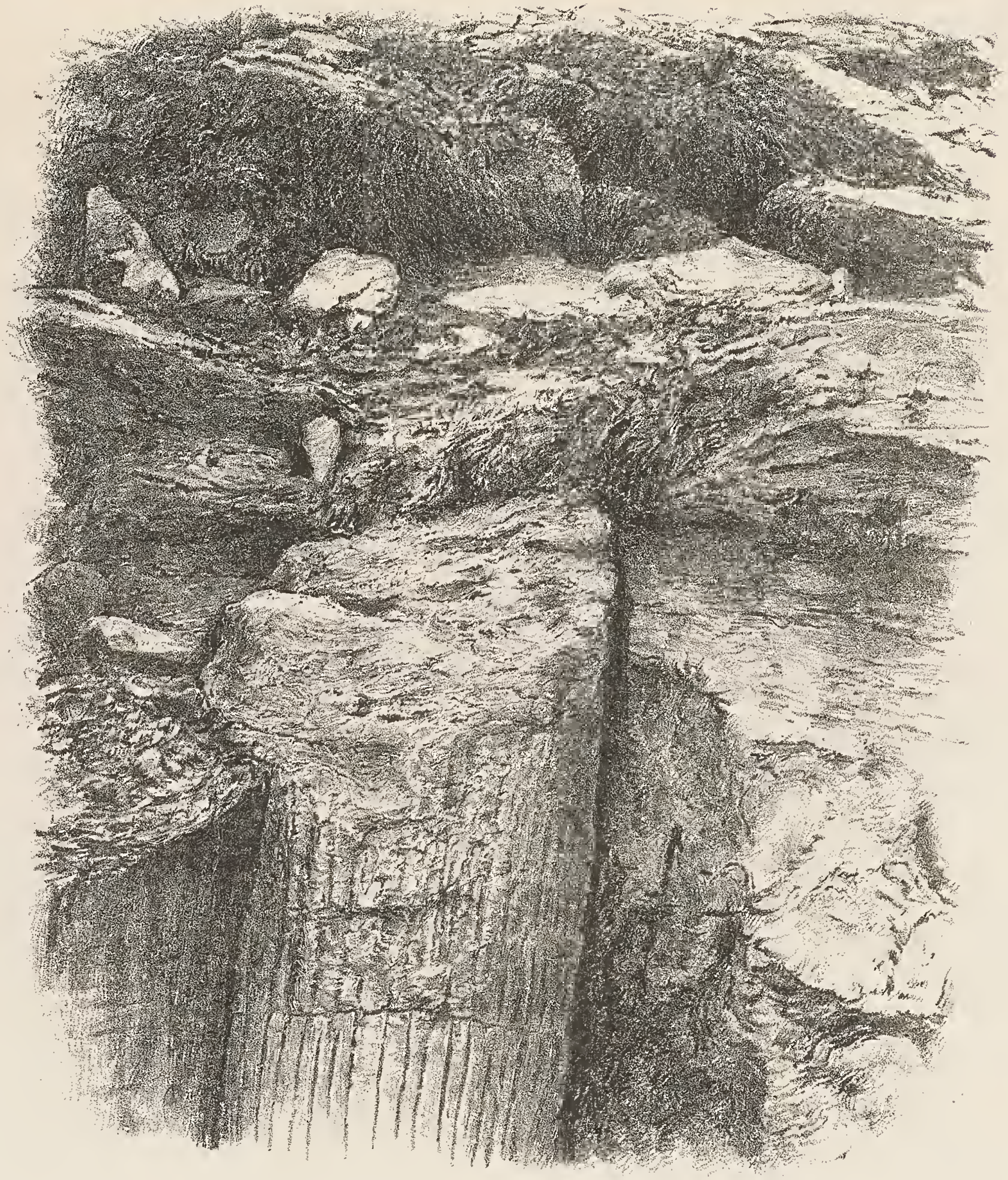


HORNY ARMOUR OF BACK OF HYLONOMUS.
× 5.

(a) *Imbricated Scales*; (b) *Horny Plates*; (c) *Horny Spines or Tubercles*; (d) *Small imbricated Scales.*

From a Photograph by Inglis, Montreal.

Fig. 141.



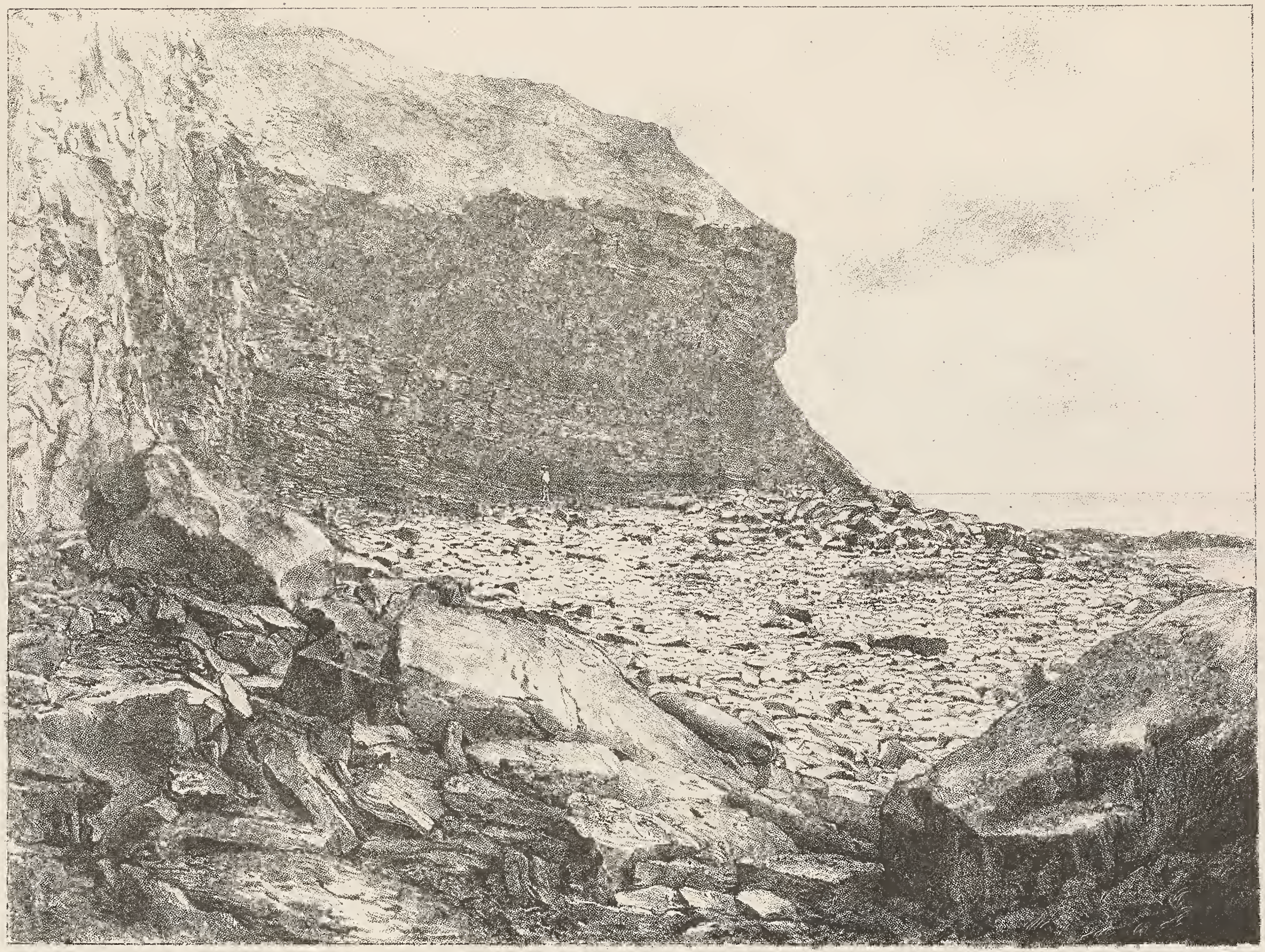
TREE No 13.

Upper part, in situ, in the reef, after it had been exposed by blasting.

From a Photograph by Weston.

W.H. Wesley lith.

West Newman & Co imp.



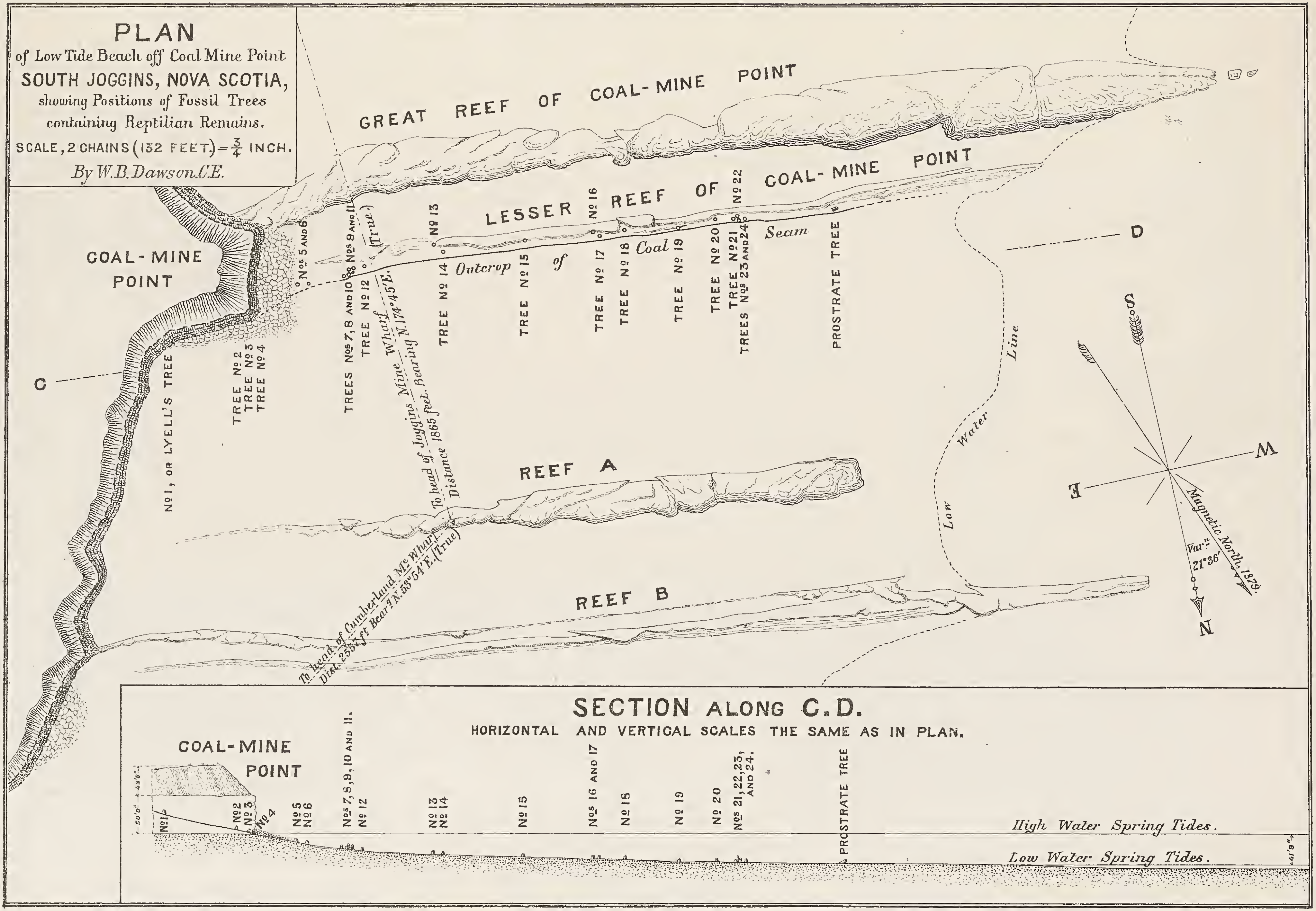
W. H. Wesley lith.

West Newman & C° imp.

COAL MINE POINT, S. JOGGINS, 1879.

The head of the human figure is immediately below the Outcrop of the Six-inch Coal, on which the fossiliferous trees stand.

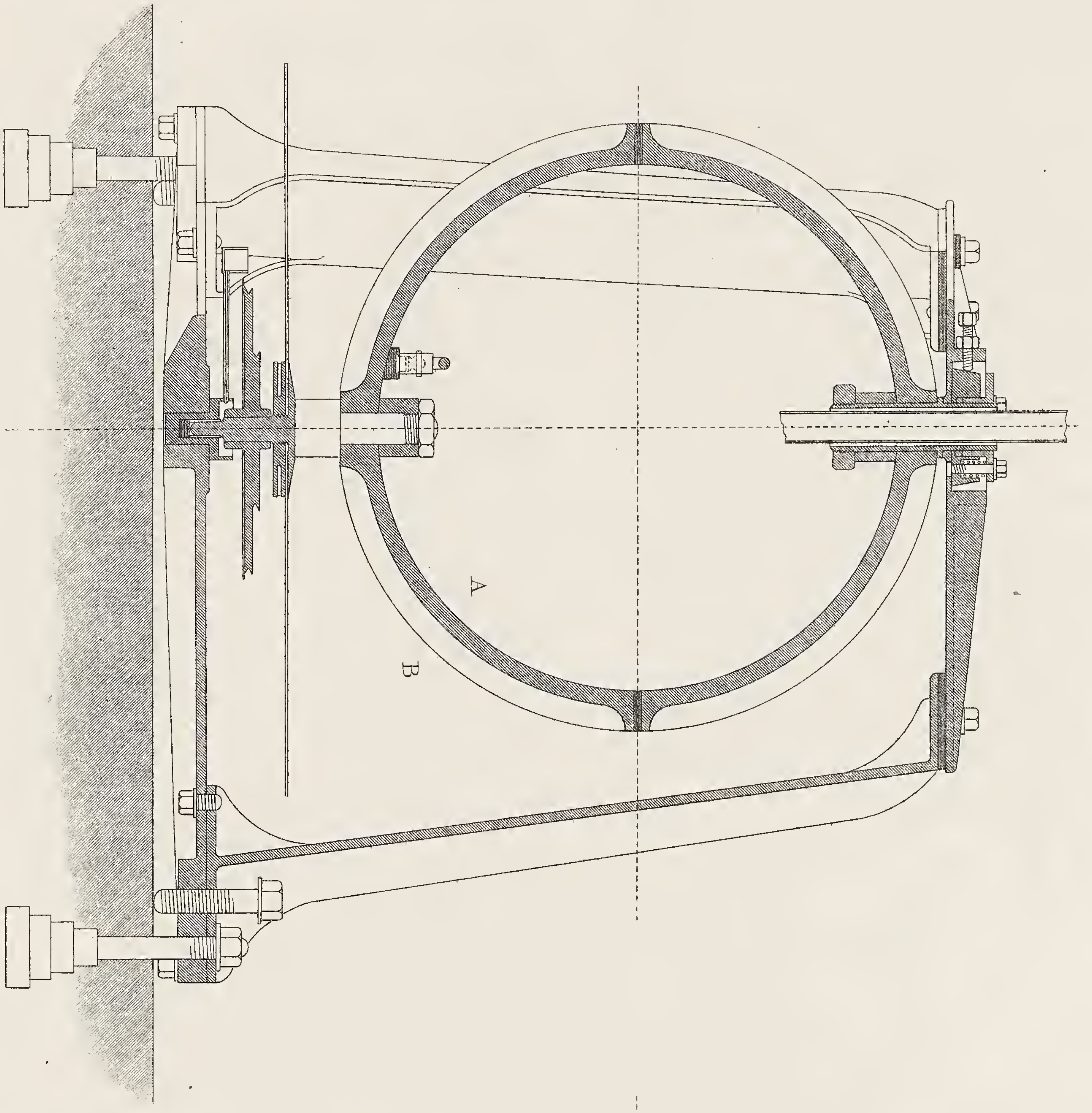
From a Photograph by Weston.

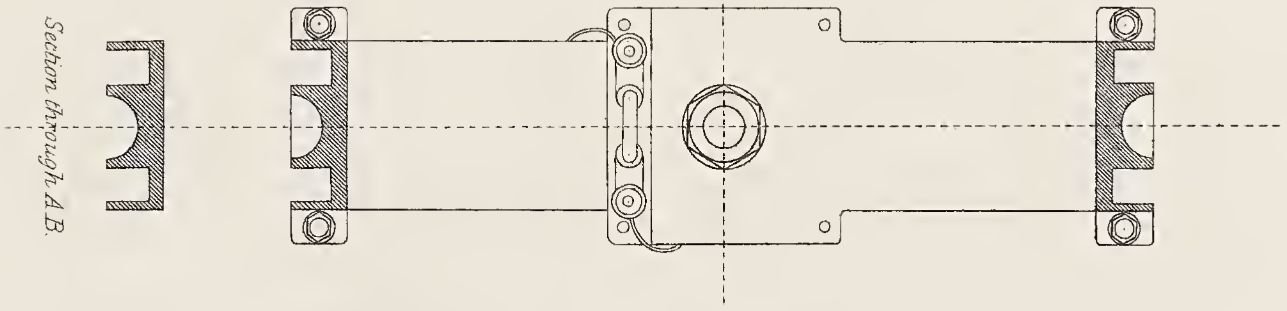


PLAN
of Low Tide Beach off Coal Mine Point
SOUTH JOGGINS, NOVA SCOTIA,
showing Positions of Fossil Trees
containing Reptilian Remains.
SCALE, 2 CHAINS (132 FEET.) = $\frac{3}{4}$ INCH.
By W. B. Dawson, C.E.

SECTION ALONG C.D.
HORIZONTAL AND VERTICAL SCALES THE SAME AS IN PLAN.

Harrison & Sons, Lith, St Martins Lane, W.C.





Section through A.B.

Scale one sixth

Harrison & Sons, Lith. St Martins Lane, W.C.

Fig. 1.

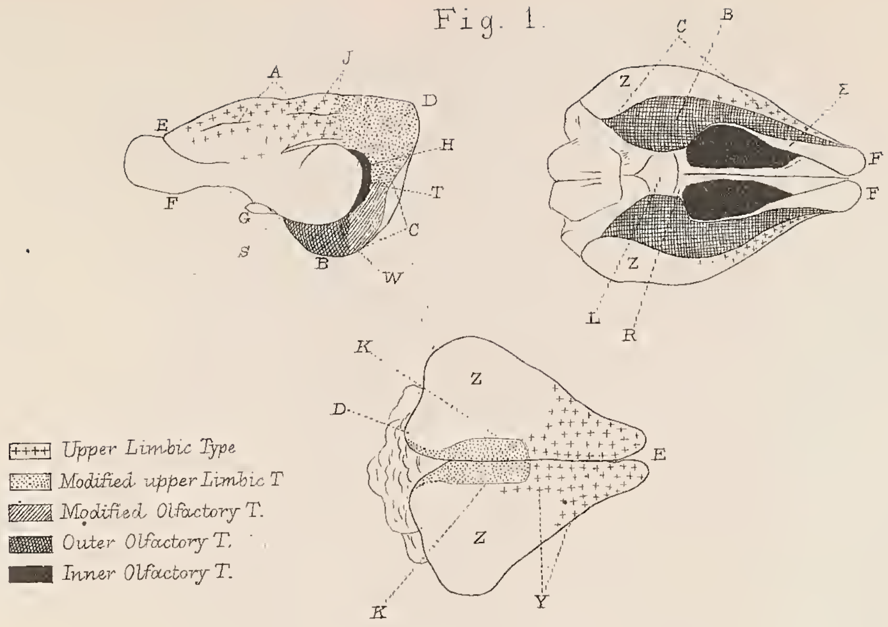


Fig. 2.

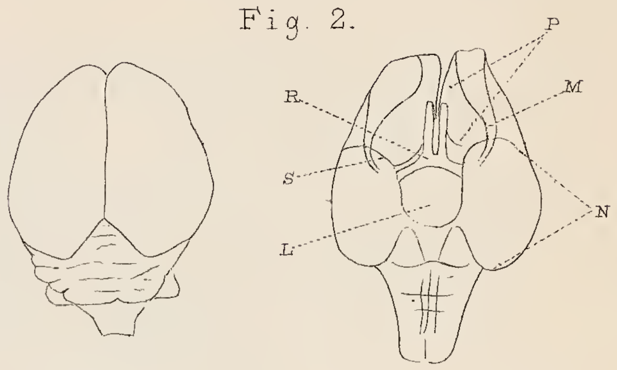


Fig. 7.

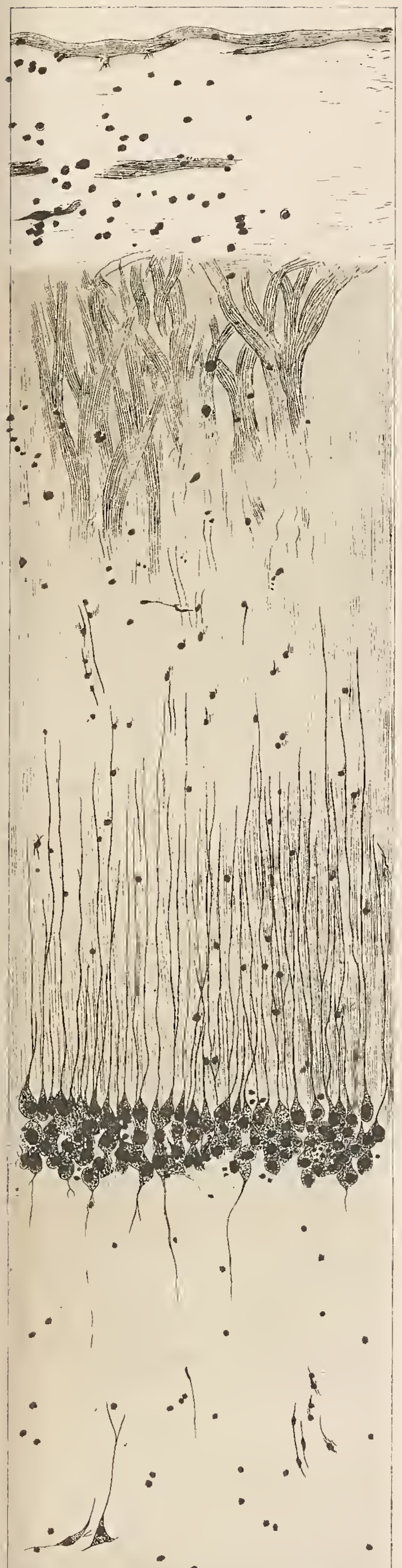


Fig. 3.

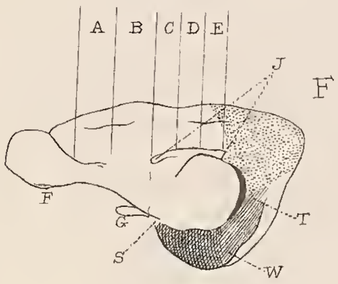


Fig. 8.

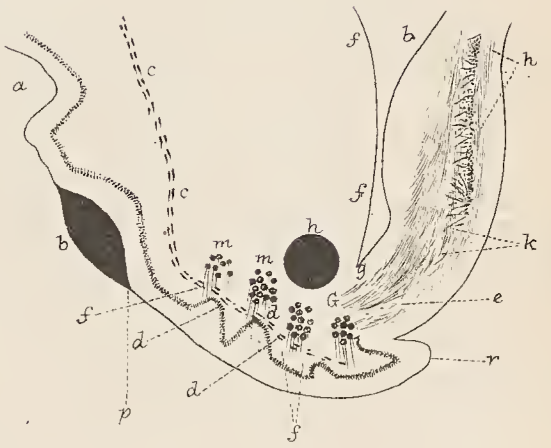


Fig. 9.

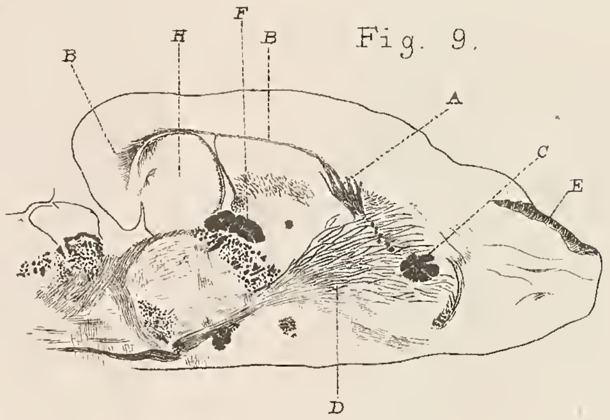


Fig. 11.

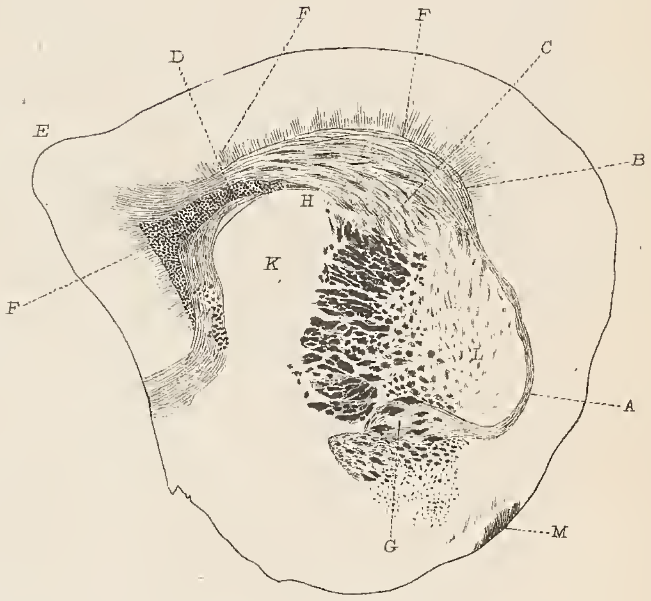


Fig. 10.

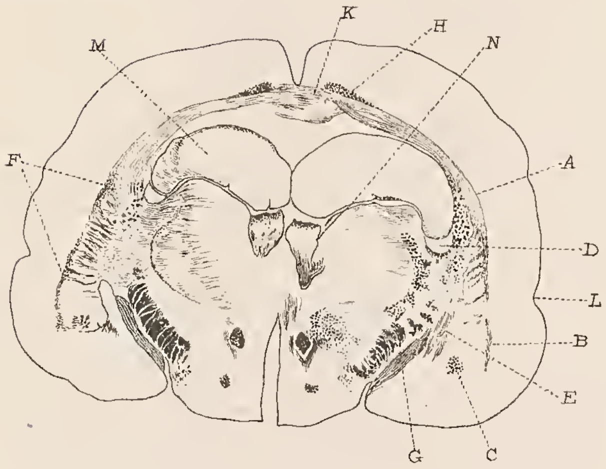


Fig. 12.

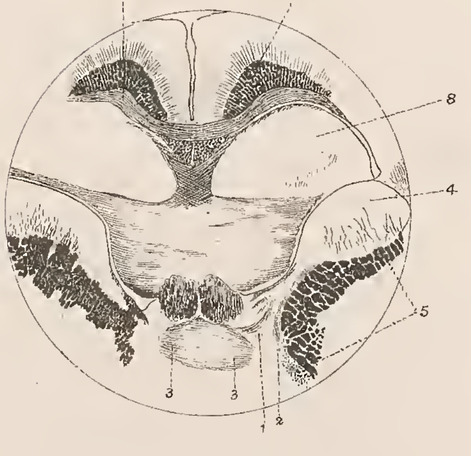


Fig. 13.

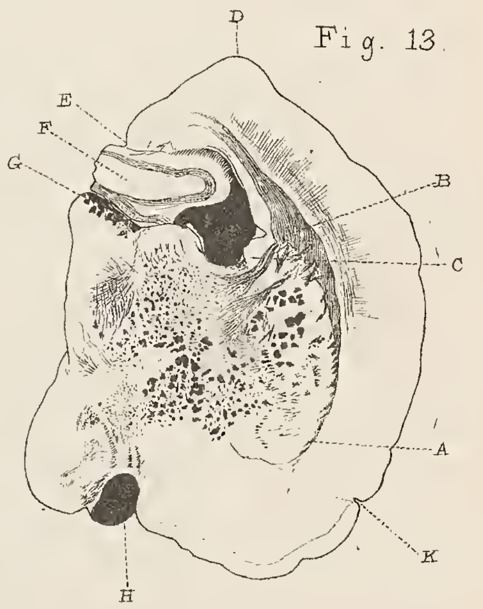


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Fig. 4.

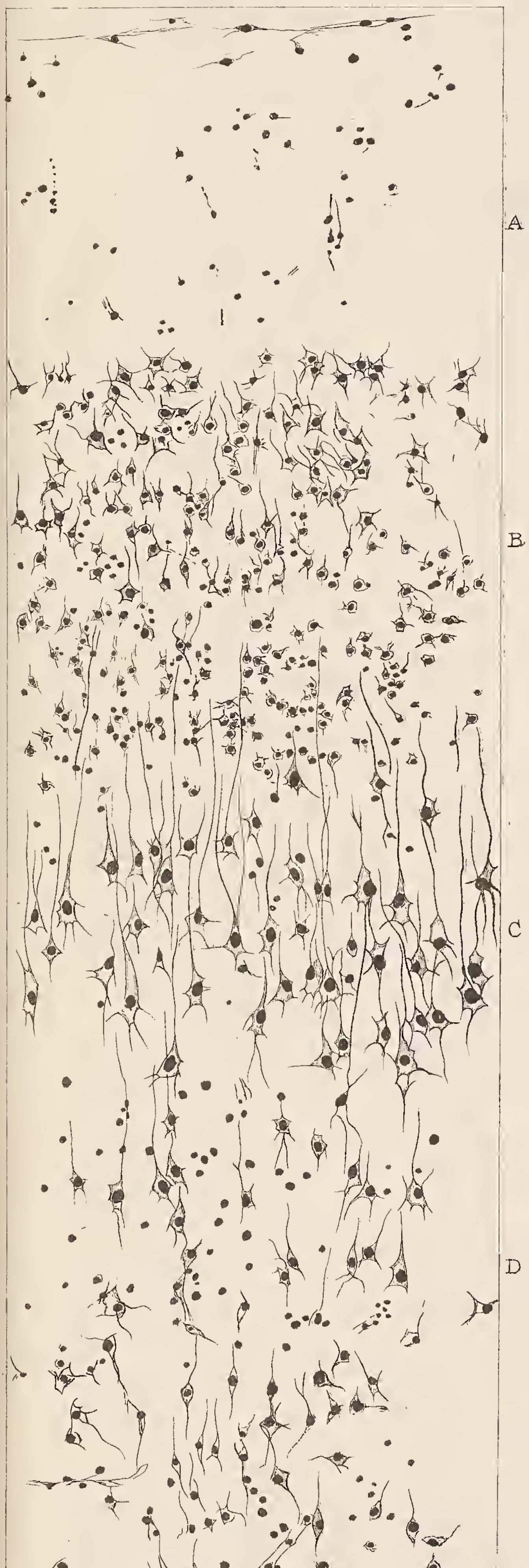


Fig. 5.

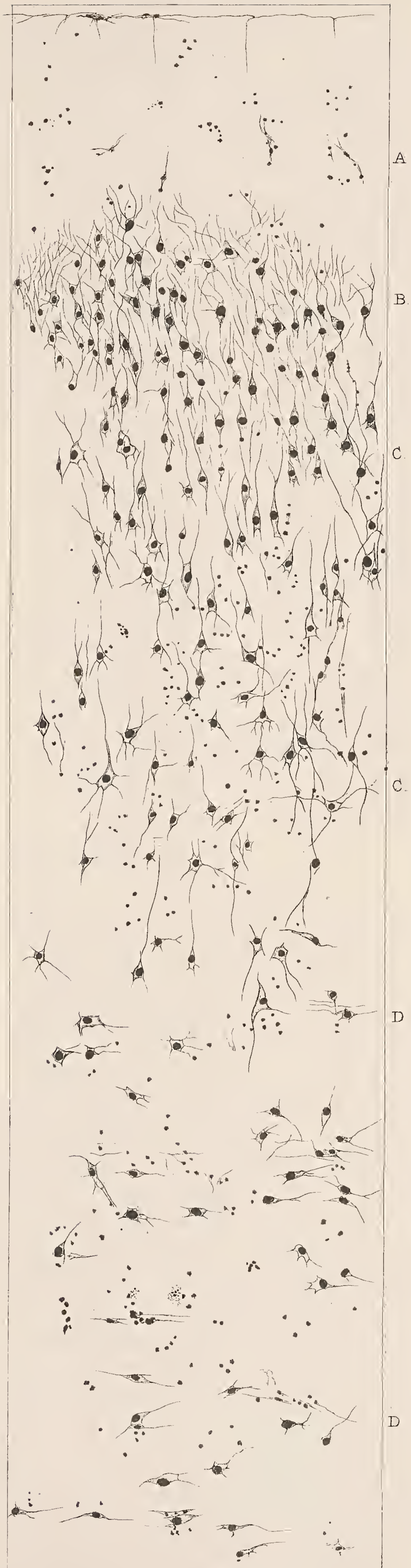
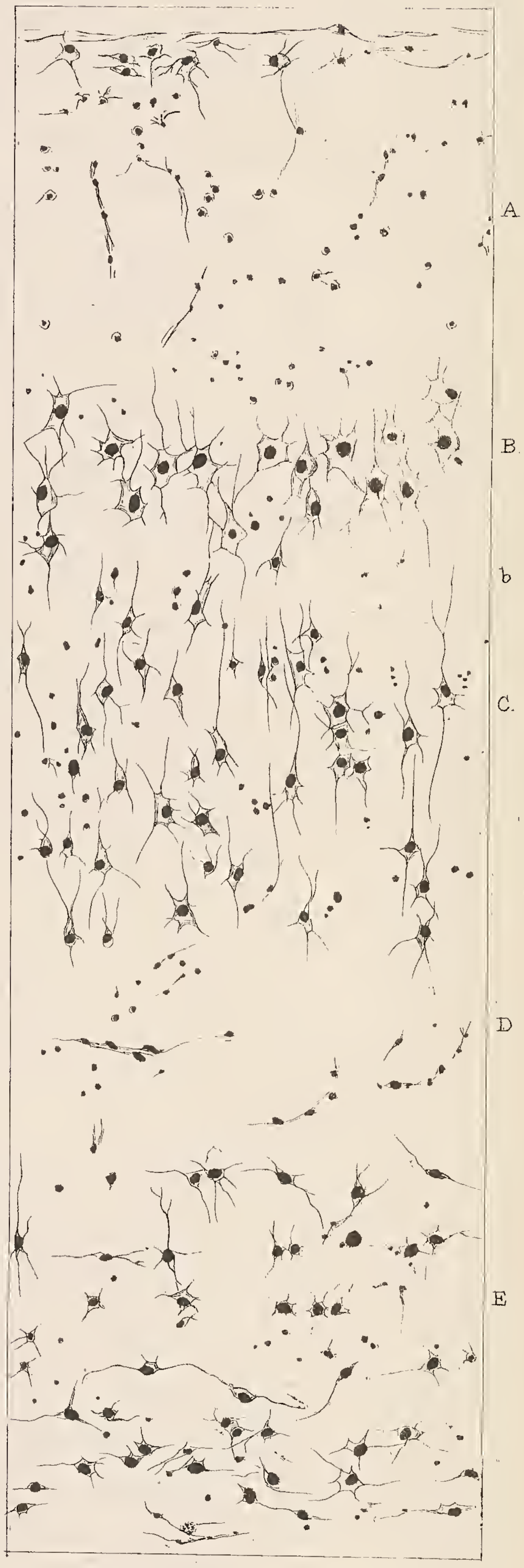


Fig. 6.



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